PRICE LEVEL DETERMINATION IN A HETEROGENEOUS MONETARY UNION

PAUL R. BERGIN

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Department of Economics
University of California
Davis, California 95616-8578

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Paul R. Bergin*

University of California, Davis

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Abstract:

A monetary union requires that a common central bank be shared among multiple nations, where governments and households may well be heterogeneous across national borders. A dynamic stochastic general equilibrium model of a two-country monetary union provides a natural setting in which to examine the implications of agent heterogeneity for price-level determination. The model suggests, first, that significant heterogeneity in government fiscal policies can be accommodated within a monetary union. Second, household heterogeneity gives monetary policy a reallocative dimension which affects price-level determination. For example, dissimilar preferences for holding money tend to enhance the potency of a monetary contraction to lower inflation. Fiscal federalism may reverse this effect.

1. Introduction

Recent research in the theory of price level determination has emphasized the interdependence between monetary and fiscal policies in achieving price stability. This interdependence takes on new dimensions in a monetary union, since a common central bank is shared by multiple national governments, which are likely to be heterogeneous in the fiscal policies they pursue. A monetary union is a relatively novel arrangement, and its implications for price-level determination have not been much explored.[1] A further dimension is added by the fact a monetary union involves households which may well be heterogeneous across national borders. Heterogeneity in households' demands for money gives monetary policy a reallocative dimension, which can alter its effects on the price level.

I analyze price level determination in the context of a dynamic stochastic general equilibrium model of a two-country monetary union. My analysis has its roots in the classic work in price-level determination by Sargent and Wallace (1981), and builds on the recent research in this area by Leeper (1991 and 1993), Sims (1994) and Woodford (1995 and 1996).[2] The analysis is distinct from recent research on public finance aspects of monetary unions, such as Sibert (1992 and 1994) and Canzoneri and Diba (1991).[3] Given the centrality of price stability in policy discussions of monetary union in Europe, it is perhaps useful to focus on this issue separately and apply a modeling strategy developed for this purpose. This analysis describes distinct classes of equilibria under different classes of policy rules. Monetary policy is characterized by a reaction function, describing the degree to which the common central bank is willing to raise the nominal interest rate to fight inflation. The fiscal policies of both countries are similarly characterized by a reaction function of lump-sum taxes to the size of government debt. The analysis identifies threshold values in the policy parameters that divide different classes of equilibria.

One class of equilibria exists in the case where monetary policy is willing to fight inflation vigorously and the national fiscal authorities are prevented from letting their debt grow too quickly. In this equilibrium the price level is insulated from jumps in the debts of the fiscal authorities, and the central bank is able to safeguard price stability. This may be viewed as a rationalization of the proposed monetary and fiscal rules for the European Monetary Union in the Maastricht

[1] See Leeper 1993 and Wopdford 1996 for some exploration of this issue.

[2] See also Aiyagari and Gertler (1985). The modeling strategy employed in this paper is adapted from Leeper (1991).

[3] See also Buiter and Kletzer (1990), Casella and Feinstein (1988), Canzoneri and Roubini (1993), Eichengreen and Bayoumi (1994), and Kletzer (1995).

Treaty, although the analysis suggests some modifications to the treaty's rules. Within the limits of the policy rules, the budgetary policies pursued by the national fiscal governments can be quite heterogeneous and yet be consistent with the single seignorage policy of the central bank.

The likelihood that households are heterogeneous across borders in their preferences for holding money gives monetary policy a new redistributive dimension. This redistribution of wealth between households in turn can affect the implications of monetary policy for price-level determination. Such heterogeneity is a dimension not present in existing price-level determination literature, which has to date focused on closed economies with a single representative agent. A monetary contraction here tends to lower the wealth of the household whose preference for money holding is lower. This wealth redistribution has real effects on consumption and money demand. In particular, aggregate money demand here tends always to move endogenously in a direction to enhance the potency of a monetary contraction in lowering inflation, although the magnitude of this effect in the present model appears to be fairly small.

A second class of equilibria exists if monetary policy is not sufficiently aggressive in fighting inflation and one fiscal authority is not fiscally responsible. In this equilibrium a contractionary monetary policy perversely produces inflation. Further, a rise in debt of the fiscally irresponsible government affects the price level for all members of the union. This result may be understood as a two-country version of a finding familiar from the Unpleasant Monetarist Arithmetic of Sargent and Wallace (1981), or it may be understood in terms of the fiscal theory of price level determination more recently developed in Sims (1994) and Woodford (1995).

A third case explores an equilibrium which combines both aggressive monetary policy and irresponsible fiscal policy. This hybrid equilibrium becomes possible in the context of a particular type of fiscal federalism. In this equilibrium a monetary contraction is associated with inflation, despite the aggressive stance of the central bank. While heterogeneity in households' money demands gives monetary policy the same reallocative dimension as above, fiscal federal transfers can potentially reverse the implications of this wealth reallocation for aggregate money demand.

The next section of this paper develops a model of a two-country monetary union and discusses the role of fiscal solvency in price level determination. Sections three through five, explore the three classes of equilibria. Section six concludes.

2. The Model

This section constructs a simple framework for analyzing a hypothetical monetary union, in which two distinct countries share a common central bank. One new feature of modeling a monetary union is that a specification of monetary policy involves decisions about how to allocate between the two countries the seignorage revenue as well as the open market purchases of national bonds. **A** second feature is that money demands of households may differ across national borders. Finally, it is the joint solvency condition of the two governments that matters for price-level determination, not each separately. The framework is simplified sufficiently to allow analytical solutions in most cases and thereby make results more transparent.

2.1. The Consumer

Consider two countries. Country one, for example, is populated by an infinitely-lived household, receiving a constant endowment of $\overline{y}_1$ units of the common consumption good each period. The household consumes c_{1t} of this endowment. There is one fiat currency issued by the common central bank. It earns no interest, and real balances provide consumers with utility separably from consumption. The household in country one holds real balances, m_{1t}, which is the ratio of nominal balances, M_{1t}, and the common price level, p_t. The household may also save one-period nominal government debt issued by the government of country one, B_{11t}, or that issued by the government of country two, B_{21t}. These have real values b_{11t} and b_{21t}, and yield nominal interest of R_{1t} and R_{2t}. Since cross-border asset trade is limited to non-contingent government debt, asset markets are incomplete and households are unable to insure against shocks affecting countries asymmetrically. The household pays τ_{1t} units of the consumption good in lump-sum taxes each period. Utility is discounted at the rate β, in the interval (0,1).

The household solves the following problem:

$$\begin{array}{ll} \max & E_0 \sum_{t=0}^{\infty} \beta^t U_1 \left(c_{1t}, \frac{M_{1t}}{p_t} \right) \\ s.t. & c_{1t} + \frac{M_{1t}}{p_t} + \frac{B_{11t}}{p_t} + \frac{B_{21t}}{p_t} + \tau_{1t} = \overline{y}_1 + \frac{M_{1t-1}}{p_t} + R_{1t-1} \frac{B_{11t-1}}{p_t} \\ & \quad + R_{2t-1} \frac{B_{21t-1}}{p_t} \\ \text{where} & M_t \geq 0, \; B_{11t} \geq 0, \; B_{21t} \geq 0 \end{array} \tag{2.1}$$

The utility function takes the form:

$$U_1\left(c_{1t}, \frac{M_{1t}}{p_t}\right) = \log\left(c_{1t}\right) + a_1 \log\left(\frac{M_{1t}}{p_t}\right) \tag{2.2}$$

Optimal behavior by households requires they plan to fully utilize their total wealth. This transversality condition requires that the wealth of the consumer not explode. This combined with the consumer's budget constraint implies an intertemporal budget constraint:

$$\frac{W_{1t}}{p_t} = \sum_{s=t}^{\infty}\left(\prod_{j=t}^{s-1} r_j^{-1}\right)\left[c_{1s} + \tau_{1s} - \bar{y}_1 + \frac{R_s}{R_s - 1} m_{1s}\right] \tag{2.3}$$

$$where\ \frac{W_{1t}}{p_t} = \frac{R_{t-1}\left(B_{11t-1} + B_{21t-1}\right) + M_{1t-1}}{p_t} \tag{2.4}$$

Here W_{1t} is the nominal value of predetermined, beginning of period wealth. The real rate of return on bonds is represented by $r_t = R_t\left(p_t/p_{t+1}\right)$.

The first-order necessary conditions require that $R_{1t} = R_{2t}$, indicating agents regard the bonds issued by different governments as perfect substitutes. Hereafter, all first-order equations will refer to the common interest rate, R_t. The remaining first-order conditions reduce to the following:

$$m_{1t} = a_1\left(\frac{R_t}{R_t - 1}\right) c_{1t} \tag{2.5}$$

$$\frac{1}{R_t} = \beta E_t\left[\frac{1}{\pi_{t+1}} \frac{c_{1t}}{c_{1t+1}}\right] \tag{2.6}$$

where inflation is defined as $\pi_t = \frac{p_t}{p_{t-1}}$. Equation *(5)* is a liquidity preference relation, where real money demand increases with consumption and decreases with the interest cost of holding money. The condition resembles a standard LM equation, except it depends only on the private part of output. Equation (6) is essentially an intertemporal optimizat'on-based version of an IS equation.

The household of country two faces an analogous problem, with analogous Euler equations.[4]

'Since home and foreign bonds are perfect substitutes, there is an indeterminacy in the allocations of portfolios. The following allocation rule is specified, saying agents split their portfolios between the two types of bonds in the same proportion as they comprise the total body of issues outstanding:
$\frac{B_{12t}}{B_{11t}+B_{12t}+B_{1mt}} = \frac{B_{22t}}{B_{21t}+B_{22t}+B_{2mt}}$. Such a portfolio would be the average result of random purchases in the bond market.

2.2. The National Governments

Now consider each of the two national governments. They have no control over the creation of money. The government in country one uses direct lump-sum taxes, τ_{1t}, transfers from the common central bank, v_{1t}, and new issues of debt to finance the constant level of purchases each period, $\overline{g}_1$, all subject to the following budget constraint:

$$\overline{g_1} + R_{t-1}\left(\frac{B_{1pt-1} + B_{1mt-1}}{p_t}\right) = \tau_{1t} + v_{1t} + \left(\frac{B_{1pt} + B_{1mt}}{p_t}\right) \tag{2.7}$$

where B_{1pt} is issues of country-one government debt held by private agents of either nationality, $B_{11t} + B_{12t}$.

Fiscal policy is characterized here by the following tax reaction function.

$$\tau_{1t} = \gamma_{01} + \gamma_{11}\left(\frac{B_{1pt-1}}{p_{t-1}}\right) + \gamma_{21}\left(\frac{B_{1mt-1}}{p_{t-1}}\right) + \psi_{1t} \tag{2.8}$$

$$\psi_{1t} \sim N(0, \sigma_1)$$

The parameters γ_{11} and γ_{21}describe how tax revenues collected by the national government respond to changes in debt held by private households and that held by the central bank, respectively. (The counterparts for the foreign government are γ_{12} and γ_{22}.) The size of these response parameters in this reaction function define the degree of fiscal responsibility. The term ψ_t represents a mean zero i.i.d. innovation to tax policy. An analogous rule is specified for the government in country two. The price-level determination literature typically is criticized for imposing such exogenous policy rules as these, since it is usually assumed rules cannot be imposed in practice and they do not consider strategic behavior. However, it could be argued that the present case is especially well suited to this approach. Rules governing both monetary and fiscal policy have in reality been proposed for the European Monetary Jnion, and it is these rules that are at the heart of current policy debates.

2.3. The Common Central Bank

The common central bank is distinct from each of the national governments. It controls the issue of new money, M_t, through open market purchases of bonds issued by the national governments. The fact the central bank must hold and trade bonds issued by the two national fiscal authorities generates a certain link between fiscal and monetary policies. The central bank, itself, does not issue debt, nor does it levy taxes. Instead it returns this interest revenue back to the national governments, in the form of rebates v_{1t} and v_{2t}. The central bank's budget constraint is as follows:

$$\frac{B_{1mt}+B_{2mt}}{p_t}+v_{1t}+v_{2t}=R_{t-1}\left(\frac{B_{1mt-1}+B_{2mt-1}}{p_t}\right)+\frac{M_t-M_{t-1}}{p_t} \tag{2.9}$$

Monetary policy is characterized by a reaction function, specifying that interest rates be targeted in response to inflation:

$$R_t=\alpha_o+\alpha_1\pi_t+\theta_t. \tag{2.10}$$

$$\theta_t \tilde{} N(0,\sigma_3)$$

The policy parameter α_1 specifies the degree to which the common central bank is willing to raise the nominal interest rate to fight inflation. This reaction function may be thought of as coming from an optimizing central bank that wishes to smooth price level and the interest rate. The parameter α_1 represents the relative weight of these two variables in the objective function. The term θ_t represents a mean zero i.i.d. innovation to monetary policy.

An additional dimension to monetary policy unique to a monetary union is the composition of the central bank's purchases in open market operations. For the present time, the central bank is specified to buy the two types in equal shares:

$$B_{1mt}=B_{2mt}. \tag{2.11}$$

Note that by determining the purchases of bonds, this rule determines the allocation of seignorage.

For the present time, it is assumed that the central bank rebates all interest revenues directly back to the government that paid them. This means rebates and expenditure are set as follows:

$$v_{1t}=(R_{t-1}-1)\left(\frac{B_{1mt-1}}{p_t}\right) \tag{2.12}$$

$$v_{2t}=(R_{t-1}-1)\left(\frac{B_{2mt-1}}{p_t}\right) \tag{2.13}$$

Finally is the money market-clearing condition:

$$M_{1t}+M_{2t}=M_t. \tag{2.14}$$

2.4. Fiscal Solvency, the Distribution of Household Wealth, and the Price Level

Government solvency conditions apply here in a form somewhat different from a closed-economy setting. Any government solvency conditions necessary for equilibrium in this model arise from the requirement that optimizing households be willing to hold government debt. In other words, they arise from the transversality condition combined with the intertemporal budget constraint of the household optimization problems. In the present model, these are represented in the wealth condition of equations **2.3** and **2.4** and the counterparts for the household of the other country:

$$\frac{W_{2t}}{p_t} = \sum_{s=t}^{\infty} \left(\prod_{j=t}^{s-1} r_j^{-1} \right) \left[c_{2s} + \tau_{2s} - \bar{y}_2 + \frac{R_s}{R_s - 1} m_{2s} \right] \tag{2.15}$$

$$\text{where } \frac{W_{2t}}{p_t} = \frac{R_{t-1}\left(B_{12t-1} + B_{22t-1}\right) + M_{2t-1}}{p_t} \tag{2.16}$$

These conditions define two distinct wealth conditions, one for each of the two representative households. In the more typical one-country case, with one government and one household, the household wealth condition is identical to the government solvency constraint. In that case it is irrelevant whether the one condition is regarded as describing government solvency or household wealth. Here, in contrast, there are two distinct household wealth conditions. While the sum of these are the same as the sum of the two government solvency conditions, the household conditions are distinct in the way they group types of debt. As a result the household wealth conditions have somewhat different implications than the government solvency conditions.

One implication is to impose a requirement on the cross-border distribution of bonds between households. Even if the level of debt issued by both governments is stable, an equilibrium can be undermined if the bond holdings of one country explodes upward while that of the other country explodes downward. This cross-border distribution of wealth between households is an additional dimension of equilibrium in a monetary union.

Secondly, the condition would permit an equilibrium in which one government's debt grows explosively and the wealth of the other government grows at the same explosive rate, so that the latter purchases the debt of the former. This point is emphasized in Woodford (1996). However, while the model does not explicitly preclude such an equilibrium, it is difficult to conceive that a government would willingly purchase continuously the debt of another without ever being repaid. Even the simplest specification of an optimizing government would introduce a government transversality condition that would preclude such a case. Such an equilibrium will not be explored in this paper, which rules it out by requiring $B_{1pt} > 0$ and $B_{2pt} > 0$.

It has been argued in Woodford (1995 and 1996) that a household wealth condition can usefully be regarded as the equilibrium condition determining the equilibrium price level. The condition specifies that given the predetermined nominal value of net government liabilities W_{1t} or W_{2t} on the left-hand side, and given expectations at date t regarding current and future values of relative quantities and relative prices on the right hand side, there is a specific price level that makes the equality hold. If the real quantities on the right side are inconsistent with the nominal value of current government labilities at the current price level, then the price level must adjust. In intuitive terms, if a national government is fiscally irresponsible, in that it cuts taxes and increases bond financing without plans to compensate with future tax increases, this makes domestic households think they are wealthier. Since their hudget set seems to have expanded, households will wish to raise consumption. But this will only generate excess aggregate demand, requiring a rise in the price level to clear the goods market. This price level will lower the real value of wealth until aggregate demand is consistent with aggregate supply.

2.5. The Linearized System

A more practical analysis of fiscal rules in a monetary union will be facilitated by looking at a linearized version of the model. Linearization will provide convenient demarcation between contrasting classes of equilibria.

A complete dynamic system is specified by equations (2.1-2.14) and counterparts for the other country. This system is linearized around an initial steady state where output is symmetric across the two countries. This linear system can be transformed into a more tractable six-variable system that can be dealt with analytically. The variables of this transformed system are consumption (c_1), inflation (π), country-one debt held by the central bank (b_{1m}), debt issued by the government in country one held in private hands $(b_{1p} = b_{11} + b_{12})$, privately-held debt issued by the government in country two $(b_{2p} = b_{21} + b_{22})$, and finally, the net foreign assets of households in country one (b, $= b_{21} - b_{12}$). Note that all variables of the linearized system are marked by tildes, indicating deviations from steady state. Steady state values of the variables are marked by overbars. The terms e_{1t} and e_{2t} are expectational errors.

$$\begin{bmatrix} \widetilde{c}_{1t} \\ \widetilde{\pi}_t \\ \widetilde{b}_{1mt} \\ \widetilde{b}_{1pt} \\ \widetilde{b}_{2pt} \\ \widetilde{b}_{xt} \end{bmatrix} = A_1 \begin{bmatrix} \widetilde{c}_{1t-1} \\ \widetilde{\pi}_{t-1} \\ \widetilde{b}_{1mt-1} \\ \widetilde{b}_{1pt-1} \\ \widetilde{b}_{2pt-1} \\ \widetilde{b}_{xt-1} \end{bmatrix} + B_1 \begin{bmatrix} \theta_t \\ \psi_{1t} \\ \psi_{2t} \end{bmatrix} + B_2 \begin{bmatrix} \theta_{t-1} \\ \psi_{1t-1} \\ \psi_{2t-1} \end{bmatrix} + H \begin{bmatrix} e_{1t} \\ e_{2t} \end{bmatrix} \quad (2.17)$$

where:

$$A_1 = \begin{bmatrix} \boxed{1} & 0 & 0 & 0 & 0 & 0 \\ 0 & \boxed{\alpha_1\beta} & 0 & 0 & 0 & 0 \\ 0 & \xi_1 & \boxed{1} & 0 & 0 & 0 \\ 0 & \xi_2 & 0 & \boxed{\frac{1}{\beta}\text{-}\gamma_{11}} & 0 & 0 \\ 0 & \xi_2 & 0 & 0 & \boxed{\frac{1}{\beta}\text{-}\gamma_{12}} & 0 \\ -1 & \xi_3 & 0 & 1 & 0 & \boxed{\frac{1}{\beta}} \end{bmatrix}$$

See Appendix A for B_1, B_2, and H.

Typically, the existence of a unique equilibrium will coincide with the number of unstable roots being equal to the number of endogenous expectational error terms, which here is two (Blanchard and Kahn, 1980). The compacted system here was derived so that the six roots of the system may be read off the diagonal of the coefficient matrix A_1: 1, $\alpha_1\beta$, 1, $\frac{1}{\beta} - \gamma_{11}$, $\frac{1}{\beta} - \gamma_{12}$, and $\frac{1}{\beta}$. Further, this compacted system captures all the roots of interest from the full 22-variable system, since the remaining 16 roots are all zeros in the full model. The sixth root, $\frac{1}{\beta}$, is always greater than unity, since 4 is always less than unity; the first and third roots are necessarily exactly unity. Thus for the number of unstable roots to equal two, it is necessary to choos policy function parameter values α_1, γ_{11} and γ_{12} so that exactly one of the remaining roots exceed unity. It may be recalled that α_1 specifies the responsiveness of the monetary authority to inflation, while γ_{11}, and γ_{12} specify the responsiveness of the national fiscal authorities to their respective debt stocks.[7]

[7] **The two unit roots reflect two nonstationarities in the steady state, which indicates transitory shocks have permanent effects. The first simply reflects the specification of monetary policy as permanent shifts in money supply. The second reflects the incompleteness of asset markets, which**

While roots counts are useful in identifying classes of equilibria, complete proofs of existence and uniqueness of these equilibria are more involved. In particular, the root-counting method of Blanchard and Kahn (1980) would require that expectational terms enter without linear dependencies, which is not true here. See appendix B for a discussion of a method for proving existence and uniqueness in the present situation and the proofs for each of the classes of equilibria presented in the paper.

3. Case 1: Maastricht Equilibrium

The linearized model above suggests there is a knife-edge threshold value for each of the policy parameters. The nature of the equilibrium generally depends on whether the fiscal authorities' tax response to debt, γ_{11} and γ_{12}, exceed the value $\frac{1}{\beta}-1$, which is the long-run real interest rate. Large responses indicate that governments are fiscally responsible, in the sense that they take on the responsibility for making sure debt does not grow too quickly. The nature of the equilibrium also depends on whether the monetary policy response to inflation, α_1, exceeds $\frac{1}{\beta}$. A large response indicates the central bank values low inflation relative to smoothing interest rates, and thus is willing to fight inflation aggressively by raising the interest rate.

Consider first a case in which national governments act fiscally responsibly and the common central bank values price stability highly . This case generally reflects the intentions for the European Monetary Union laid out in the Maastricht Treaty, and so it is here referred to as the Maastricht Equilibrium. In particular, suppose regarding fiscal policy, taxes must respond to debt by more than the steady state real interest rate: $\gamma_{11} > \frac{1}{\beta}-1$ and $\gamma_{12} > \frac{1}{\beta}-1$ And regarding monetary policy, suppose that the nominal interest rate is raised in response to inflation in a proportion greater than the steady-state interest rate plus one: $\alpha_1 > \frac{1}{\beta}$. In terms of the model framework above, there will be exactly two large roots: the second root, $\alpha_1\beta$, and the sixth, while but the fourth and fifth roots,

are restricted to non-contingent government bonds. If asset markets were assumed complete, shocks would affect wealths of both countries symmetrically. This result is clearly counterfactual, and it would preclude analysis of the cross-country heterogeneity of households. It has been demonstrated in Baxter and Crucini (1995) that the usual solution methods of linearizing around an initial steady state can still can be employed in such a case, so long as–the state space is expanded to track the distribution of wealth between the two agents (here b_x). The fact that the final steady state is affected by the model dynamics suggests that the linear approximation will be less good in the long run, but for analysis of short-run dynamics this is not likely to be a significant problem.

$\frac{1}{\beta} - \gamma_{11}$ and $\frac{1}{\beta} - \gamma_{11}$, are less than unity. Appendix B contains the proof that this arrangement does imply a unique stable equilibrium.

Proposition 1: Price stability. In the Maastricht Equilibrium defined above, prices are stable in the following sense: inflation is insulated from jumps in government debt and the central bank has the ability to control inflation through its monetary policy.

Proof: Much can be learned from the stability condition associated with the unstable second root, which becomes a condition for equilibrium:

$$\widetilde{\pi}_t = -\frac{1}{\alpha_1}\theta_t \tag{3.1}$$

This condition states that current inflation is a function only of changes in monetary policy (θ_t), and not of changes in fiscal policy (ψ_{1t} or ψ_{2t}). In particular, it stipulates that a monetary contraction (positive θ_t) translate directly into a fall in inflation. ■

Figure 1 illustrates a contractionary monetary shock (θ_t=0.01), where impulse responses are plotted as percent deviations from steady state.[8] Money supply falls and lowers inflation. Note that in Figure 1 both governments raise taxes. This is because the open market operations implied by this monetary contraction raise the level of government debt in private hands. The fiscal rules then require a rise in taxes to keep debt from growing explosively. This fiscal response is an integral part of establishing the equilibrium. In fact, from the perspective of a general equilibrium model, it cannot truly be said whether it is the fall in money supply or the rise in taxes that is the channel through which the shock θ_t makes inflation fall. The quantity theory of money offers a story of the former channel. The fiscal theory of price level determination of Woodford (1995), discussed in the previous section, offers a story for the latter channel. Recall that in this story a rise in taxes lowers household wealth and hence lowers demand for goods. Equilibrium in the goods market then requires a fall in the pr'ce level to raise real household wealth. But regardless of the story to explain the equilibrium, the model suggests policy responses within certain boundaries are integral components of that equilibrium.

If the features of this equilibrium are desirable to policy makers in Europe, the model suggests some justification for policy rules that mandate the policy responses that support the equilibrium. However, the policy specifications used in

[8]Parameters are set at: $\alpha_1 = 1.2, \gamma_{11} = \gamma_{21} = 0.2, a_1 = 0.02, a_2 = 0.01, \overline{y}_1 = \overline{y}_2 = 1, \overline{g}_1 = \overline{g}_2 = 0.3, \overline{\tau}_1 = \overline{\tau}_2 = 0.306$ (to reproduce realistic steady-state debt levels), $\beta = 0.99$. The shock is $\theta_t = 0.01$ for t=1 and 0 for t>1.

the model above suggest forms for rules somewhat different from those proposed in the Maastricht Treaty. For instance, it is not necessary to impose an absolute ceiling on debt, as is specified in the treaty. It is sufficient to impose a limit on the rate debt grows or to impose a rule that has this effect $\left(\gamma_{11} > \frac{1}{\beta} - 1 \text{ and } \gamma_{12} > \frac{1}{\beta} - 1\right)$. Further, the monetary authority need not hold price-stability as its sole objective. It may value interest-rate smoothing to some degree, so long as the relative weighting of price stability, α_1, is sufficiently large $\left(\alpha_1 > \frac{1}{\beta}\right)$.

One concern with a monetary union is how a single central bank can set seignorage to be consistent with the fiscal behavior of multiple fiscal authorities that may be heterogeneous in their fiscal behavior. But such fiscal heterogeneity is not a problem in the current class of equilibria..

Proposition 2: Government heterogeneity. Most types of heterogeneity in the behavior of fiscal authorities (defined in terms of policy parameters in the fiscal rules) are irrelevant for the equilibrium price level in this class of equilibria.. First, the absolute sizes of their debt need not be the same in steady state. Second, the responsiveness of governments to the portion of debt held in private hands need not be equal, so long as they both are sufficiently large. Further, no restriction at all needs be placed on the responsiveness to the remaining portion of debt, that part held by the central bank.

Proof: In the proof of the existence of the Maastricht Equilibrium, the only restriction on fiscal rules were that $\gamma_{11} > \frac{1}{\beta} - 1$ and $\gamma_{12} > \frac{1}{\beta} - 1$. So it is permissible that $\gamma_{01} \neq \gamma_{02}$, meaning that steady state levels of debt could be any value. Secondly, it is possible that $\gamma_{11} \neq \gamma_{12}$, implying different responses to privately-held debt. Further no assumption at all was made for γ_{21} and γ_{22}, controlling the responsiveness to debt held by the central bank.■

The last portion of the proposition depends on the particular open market operations rule specified in the model. To this point it has been assumed the central bank was careful to allocate its open market operations evenly between the bonds of the two countries, which basically means the seignorage benefits were evenly distributed. Consider what happens if instead the common central bank treats both bonds as perfect substitutes. In that case, the share of country-one bonds in the purchases of the central bank would be expected to be on average equal the share of these bonds in the market in general. Thus the open market operations allocation rule, equation (11) needs to be replaced by the following rule:

$$\frac{B_{1mt}}{B_{11t}+B_{12t}+B_{1mt}} = \frac{B_{2mt}}{B_{21t}+B_{22t}+B_{2mt}} \tag{3.2}$$

The model cannot be easily solved analytically under this rule, but the case can be explored numerically in the calibrated version of the model used in Figure 1. It is found that a rule that ignores debt held by the central bank does not eliminate the third unstable root. But if γ_{12} and γ_{22} are set to exceed the value $\frac{1}{\beta}-1$, as are γ_{11} and γ_{21}, then the Maastricht Equilibrium is restored. As a general principle, then, the proper specification of a fiscal rule depends in part on the specification of the open market operations rule.

In addition the heterogeneity among fiscal authorities, a second concern in a monetary union is potential heterogeneity between households, which may differ in their preferences for holding money. Such household heterogeneity is usually not an issue in a closed economy model, but it is a central issue in a multi-country setting.

Proposition **3:** Household heterogeneity. If households have different demands for money, innovations in monetary policy may redistribute wealth between households of the two countries. This redistribution of wealth between households with contrasting money demands may in turn affect aggregate money demand and thereby affect the implications of monetary policy for the equilibrium price level. In particular, aggregate money demand in the present model changes endogenously in a direction that always makes monetary policy more potent in affecting inflation. **A** related implication is that monetary policy has asymmetric real effects on consumption in the two countries.

Proof: The second stability condition, associated with the sixth unstable root, states that consumption must respond to changes in the net foreign asset position of the household. When combined with the first stability condition, this condition reduces to the simple statement:

$$\tilde{c}_{1t} = \left(\frac{1}{\beta} - 1\right)\tilde{b}_{xt} \tag{3.3}$$

which states that consumption in country one must change in direct proportion to the current real net foreign asset position of households in country one in interest bearing assets. This proportion, $\frac{1}{\beta}-1$, is the steady-state real interest rate, so as to prevent interest bearing debt to the other country from growing explosively. Maintaining dynamic stability of the cross-border distribution of household wealth is essential to the existence of an equilibrium, as was specified in the intertemporal wealth conditions discussed in the previous section.

In turn, the relative wealth position may be solved in terms of lagged values as:

$$\widetilde{b}_{xt} = \widetilde{b}_{xt-1} + \xi_8 \theta_t \tag{3.4}$$

which suggests that relative wealth of households is affected by monetary policy shocks (θ_t). ($\xi_8$is defined in the appendix.) Or solving forward:

$$\widetilde{b}_{xt} = \sum_{i=0}^{t-1} [\xi_8 \theta_{t-i}] + \widetilde{b}_{x0} \tag{3.5}$$

It is clear that $\xi_8 > 0$ if $a_1 > a_2$ and given that the model is linearized around a symmetric steady state (in which $\bar{c}_1 = \bar{c}_2$). So a contractionary monetary policy (positive θ_t) raises the relative wealth position of the household that has a higher demand for money, and thus raises the consumption of that household.

Using the liquidity preference relation (2.5), it can be shown that real money demand moves with consumption and hence with b_{xt}. So in a contractionary monetary shock, the money-loving agent controls a larger fraction of aggregate wealth, total real money demand will rise.

$$\widetilde{m}_t = \widetilde{m}_{t-1} + \frac{1}{\beta}(a_1 - a_2)\xi_8 \theta_t \tag{3.6}$$

Solving for the path of nominal money supply.

$$\widetilde{M}_t = \widetilde{M}_{t-1} - \frac{1}{\alpha}\theta_t + \frac{1}{\beta}(a_1 - a_2)\xi_8 \theta_t \tag{3.7}$$

The term $\frac{1}{\beta}(a_1 - a_2)\xi_8$ is always non-negative, regardless of which country is the relative money lover. And it becomes larger as the absolute value of the difference between a_1 and a_2 rises. So in the case of a contractionary monetary policy ($\theta_t > 0$), the new wealth effect will moderate the degree to which nominal money supply must decrease. And in the case of a monetary expansion ($\theta_t < 0$) it will moderate the degree to which nominal money supply must increase. In either direction, the relative wealth effect makes changes in monetary supply more potent in affecting inflation. ■

See again Figure 1 for an illustration of this effect for a contractionary monetary policy shock ($\theta_t = .01$). The households are heterogeneous in their demand for money, where country one has twice the money' demand of country two ($a_1 = 0.02$, $a_2 = 0.01$). The parameter values were chosen so that the initial steady state levels of money in the model roughly replicate M1 as a ratio to

GDP in Germany and Italy, respectively. Relative wealth of the money-loving household does indeed rise a small amount, since the negative seignorage tax associated with the fall in price level affects more heavily the household that holds more money. This rise in relative wealth is reflected in the rise of consumption of the money-loving household and a rise in its money demand. Consumption falls an equal amount in the other household and its money demand falls. But since money demand of the money-lover is larger in proportion to its consumption, aggregate money demand summed over both households rises. The fact that the fall in nominal money supply induces real money demand to fall, suggests the equilibrium price level that clears the money market must fall all the more.

Calibrated with the parameter values above, which produce realistic steady state real money levels, this wealth effect can be seen in the figure to be fairly small.[9] The effect would be larger for economies in which steady state money demand is larger relative to consumption, or economies in which the elasticity of money demand to changes in wealth is larger relative to that of consumption. There is probably no reason why the effect here should be peculiar to a monetary union. Heterogeneity between different types of agents within a single national economy could also give rise to the redistributive wealth effects which are seen here to alter subtly the implications of monetary policy for price-level determination.

4. Case 2: Bailout Equilibrium

In the Maastricht class of equilibria above, it was seen that significant heterogeneity in government fiscal policies can be accommodated within a monetary union, without threatening certain desirable properties of the price-level determination process. However, the model does suggest the existence of other equilibria, in which some types of fiscal behavior by one member government can produce inflationary problems throughout the monetary union. Consider a case in which one fiscal government is fiscally irresponsible, in the sense that it does not raise taxes sufficiently in the face of rising debt to prevent fiscal insolvency. Suppose also that the common central bank values interest-rate smoothing as well as price-smoothing. It either pursues a simple interest-rate pegging rule, or it

[9]The present case in which money preferences in country one are double those of country two may be compared to a benchmark case with the same total money demand, but with symmetric preferences between the two countries. Compared to this benchmark, the approximate one-percent drop in infiation ($\theta_t' = .01$) requires a contraction in nominal money supply that is 0.1 percent smaller in the case of heterogeneous preferences than in the benchmark case. However, there are conditions under which this small effect can be greatly multiplied, as will be shown in the final class of equilibria.

only cautiously raises the interest rate when inflation is observed. In terms of the policy rules in the model, this scenario is described by $\gamma_{11} < \frac{1}{\beta} - 1$, $\gamma_{12} > \frac{1}{\beta} - 1$, and $\alpha_1 < \frac{1}{\beta}$. So the fourth root of the model, $\frac{1}{\beta} - \gamma_{11}$, is greater than unity, but the second root, $\alpha_1\beta$, now has been brought inside the unit circle. There are still a total of two large roots. Appendix B contains the proof that this arrangement does imply a unique stable equilibrium. However, this equilibrium has some unusual properties, which suggest a label for this case of Bailout Equilibrium.

Proposition 4: Price instability. In this Bailout equilibrium, inflation is affected by jumps in government debt of the fiscally irresponsible government. Further, a contractionary monetary policy innovation produces positive inflation throughout the monetary union rather than lowering it.

Proof: Again the stability conditions associated with the unstable roots are informative. In solving for the equilibrium, the following stability condition, associated with the fourth root, replaces the condition which had been associated with the second root and had been central to proving Proposition 1:

$$\widetilde{\pi}_t = -\frac{1}{\alpha_1}\theta_t - \left(\frac{1 - \gamma_{11} - \alpha_1\beta}{\xi_2}\right)\widetilde{b}_{1pt} \tag{4.1}$$

This resembles the earlier condition, except that now inflation is a function not only of monetary policy innovations, but also of the current size of debt. A complete solution for the equilibrium appears intractable. However, since a monetary contraction involves open market operations of country-one debt, it is clear that $\widetilde{b}_{1pt}$, will rise. Since the government of country one does not respond to the rise in debt with new taxes, the debt would grow exponentially. The nominal interest rate would rise, and the endogerious response of monetary policy would be to backtrack on part of the monetary contraction. Ultimately, the central bank will bail out the government of country one for the rise in debt plus interest in the meantime, so in the end money supply will rise above the initial level. Since money demand is a function of expected future inflation, money demand will drop immediately greater than the fall in money supply, producing inflation. ■

Figure 2 illustrates this story by simulating a monetary contraction similar to that in Figure 1 (in this case, the shock takes the form $\theta_t = .0001$). However, here shock that produces positive inflation. [10] This result may be understood in terms of the unpleasant monetarist arithmetic of Sargent and Wallace (1981), that any

"Parameters are set at: $\alpha_1 = 0.5, \gamma_{11} = 0, \gamma_{21} = 0.2, a_1 = 0.02, a_2 = 0.01, \overline{y}_1 = \overline{y}_2 = 1, \overline{g}_1 = \overline{g}_2 = 0.3, \overline{\tau}_1 = \overline{\tau}_2 = 0.306$ (to reproduce realistic steady-state debt levels), $\beta = 0.99$. The shock is $\theta_t = 0.0001$ for t=1 and 0 for t>1.

increase in one country's debt unbacked by future taxes may cause household expectations of a monetization sometime in the future. This expected future inflation affects money demand and inflation immediately. Another interpretation can be found in the fiscal theory of price-level determination of Woodford (1995) and Sims (1995). A rise in debt held by private households unbacked by future taxes obligations makes households feel their nominal intertemporal wealth has increased. Equilibrium requires, the resulting excess demand for goods be eliminated by a rise in price level to lower real wealth.

5. Case 3: Fiscal Federalism Equilibrium

Proposition 3 has suggested that redistributions of wealth between home and foreign households can affect the equilibrium price level, although the effects found in the Maastricht class of equilibria were fairly small. Is there a case where these effects are large? This becomes possible in the present model if it is augmented with a particular form of fiscal federalism. Fiscal federalism, which has been proposed widely and in various forms in policy circles for the new European Monetary Union, transfer:; wealth between countries. By introducing a new means of wealth redistribution, fiscal federalism opens up the possibility of a new class of equilibria in the present model.

The previous two classes of equilibria were identified by manipulating the three roots that included policy parameters so that one of these three exceed unity. It was taken as given that the sixth root would always be the second of the two unstable roots. It is tempting to ask if the sixth root could be manipulated to be stable, thereby allowing both the second and fourth roots to be unstable at the same time ($\gamma_{11} < \frac{1}{\beta} - 1$, $\gamma_{12} > \frac{1}{\beta} - 1$, and $\alpha_1 > \frac{1}{\beta}$)?[11] The simplest way to manipulate the sixth root, which characterizes the rate of growth in relative wealth between national households, is to introduce a tax rule that transfers wealth between them. Consider adding τ_{21} on the left side of the household budget constraint (2.1), and τ_{22} for the budget constraing of the other household, where:

$$\tau_{21} = \phi_0 + \phi_1(\widetilde{b}_{xt}) \tag{5.1}$$

$$\tau_{22} = -\tau_{21} \tag{5.2}$$

[11] In the terminology of Leeper (1991), the question is if there can be an equilibrium in which both monetary and fiscal policy are active simultaneously?

This fiscal federalism tax rule states that for any asymmetric gain relative to the initial relative wealth position, a portion must be transfered to the country that has become relatively poorer.

Various types of fiscal federalism along this line have been proposed for the European monetary union, either to promote equity or to substitute for the loss of an independent monetary policy as a way of dealing with asymmetric shocks.[12] Fiscal federalism has the potential to improve welfare because the asset markets here between countries are incomplete. Asset trade is limited to non-contingent government debt. Perhaps surprisingly, even in this limited asset market, consumption can be smoothed remarkably well in the face of asymmetric shocks to income. If output in one country were to fall, households could cushion the impact on current consumption by borrowing abroad. This would take the form of selling government bonds, and would be reflected in a drop in that country's net foreign asset position $(\tilde{b}_{xt})$. As a result of this asset trade, the shock to consumption would be pooled almost equally among home and foreign households; it would be exactly pooled except for the fact that the poorer household must generate enough saving to pay interest payments each period on its net foreign debt. But fairly small fiscal federal transfers can erase this vestige of market incompleteness. Since a country's consumption smoothing would be reflected in a drop in its net foreign assets $(\tilde{b}_{xt})$, transfers gauged to this measure of relative wealth can be made to repatriate the interest payments due on this borrowing. Under such a rule, the complete-markets outcome could be mimicked, in which asymmetric shocks to consumption would be perfectly pooled. For the full interest payment to be repatriated, the policy parameter in the fiscal federal tax rule ϕ_1 would need to be set exactly to $\frac{1}{\beta} - 1$, which is the steady-state real interest rate.

For the sixth root of the linear system to be stable, it must be that $\phi_1 > \frac{1}{\beta} - 1$. This is the result if the fiscal federal transfer rule happens to be set by policy makers at a level even minutely higher than the optimal level. With the sixth

[12]Most schemes of fiscal federalism proposed in practice would work to tax countries as a whole, based on the aggregate wealth in the country. They do not tax agents differently depending on individual wealth. It is appropriate to model these transfers as not affecting the decisions of individual households, who do not see themselves as affecting the aggregate national wealth.

While fiscal federalism is typically modelled as insuring against asymmetric shocks to income, which are not present in this model, the fiscal federal arrangements that currently exist in practice in Europe could be viewed as functions of relative wealth. Examples are the European structural funds, which generally transfer wealth from Northern to Southern countries. Another example would be transfers within Germany from western to the new eastern regions following German monetary union.

root inside the unit circle, it allows the fourth root to exceed unity without violating conditions for an equilibrium. This new class of equilibria, which allows a fiscal authority to follow a fiscally irresponsible policy rule at the same time the central bank fights inflation aggressively, will be called here the Fiscal Federalism Equilibrium.

Proposition 5: In the Fiscal Federalism Equilibrium defined above ($\phi_1 > \frac{1}{\beta}-1, \gamma_{11} < \frac{1}{\beta}-1, \gamma_{12} > \frac{1}{\beta}-1$, and $\alpha_1 > \frac{1}{\beta}$), a fall in the money supply generates inflation, despite the fact that the central bank fights inflation aggressively.

Proof: Since condition (3.2) holds:

$$\widetilde{\pi}_t = -\frac{1}{\alpha_1}\theta_t \tag{5.3}$$

it is clear that a positive shock to θ_t will lower inflation just as in the Maastricht Equilibrium. But when this stability condition is combined with that associated with the fourth root (equation 4.2), it reduces to the simple statement:

$$\widetilde{b}_{1pt} = 0 \tag{5.4}$$

Real debt of the government in country one must never deviate from its initial steady-state level, or else given the lax fiscal rule in country one, it would grow explosively. This condition has the surprising implication that when a monetary contraction raises the level of nominal debt of the government in country one, price level must rise rather than fall, so as to keep the real level of debt unchanged. To verify this conclusion, note that the path of real money may be expressed:

$$\widetilde{m}_t = \widetilde{m}_{t-1} + \frac{2}{\alpha_1}\left(\bar{b}_{1m} + \frac{\bar{b}_{1p}}{\beta}\right)\theta_t + \psi_t \tag{5.5}$$

This states that a positive θ_t, which lowers inflation, will imply a rise in real money supply. The path of the nominal money supply is:

$$\widetilde{M}_t = \widetilde{M}_{t-1} + \frac{2}{\alpha_1}\left(\bar{b}_{1m} + \frac{\bar{b}_{1p}}{\beta} - \frac{1}{2}\right)\theta_t + \psi_t \tag{5.6}$$

which similarly suggests a positive θ_t will imply a rise in nominal money supply, so long as debt levels are not very small. ■

Figure 3 illustrates this equilibrium for a monetary contraction (which here is a shock opposite to that in figures one, $\theta_t = -.01$).[13] This result is distinct from that in the Bailout Equilibrium. There a monetary contraction produces inflation because of the rational expectation that the fall in money supply will ultimately be reversed. Here it is the monetary contraction itself, not an implicit reversal, that produces the inflation. This odd property results from the wealth redistribution discussed in proposition **3** of the Maastricht Equilibrium, although here fiscal federalism reverses and multiplies the implications of this wealth redistribution for consumption and money demand. The monetary contraction lowers the wealth of the money-hating household as in the Maastricht Equilibrium, but here fiscal transfers subsidize the consumption of this household. Since fiscal federalism unlinks consumption choices from wealth reflected in current asset holdings, consumption and money demand of the money-hating household rises despite the fall in the current-period wealth position. The opposite happens for the money-loving household, and since money demand moves proportionately more with the money-lover, aggregate money demand falls. In addition to reversing the effect on money demand, fiscal federalism also amplifies the effect. Since consumption of the money-hating household rises despite the fall in current wealth, the wealth of this household falls further. This additional shift in relative wealth initiates another round of fiscal transfers. Fiscal federalism thus acts as a sort of multiplier, raising consumption of the money-hating household and equally lowering that of the money-loving household. Since money demand of the money lovers moves more strongly with consumption, aggregate money demand falls. In fact real money demand drops more than the fall in money supply that initially generated it, thus requiring a rise in the price level to clear the money market.

This case illustrates one scenario in which wealth redistribution between heterogeneous agents has especially strong implications for price-level determination. For this equilibrium to be generated in practice would require fairly large fiscal transfers, with a value for ϕ_1 just above $\frac{1}{\beta} - 1$. But since it might be argued that an optimal fiscal federal scheme would exactly pool consumption by setting ϕ_1 exactly equal to $\frac{1}{\beta} - 1$, transfers of this size may not be unreasonable. Further, such fiscal federal transfers are not unreasonably large compared to fiscal transfer systems already functioning within Europe between northern and southern countries or transfers between western and eastern regions within Germany after their monetary union.[14]

[13] Parameters are set at: $\alpha_1 = 1.2$, $\gamma_{11} = 0$, $\gamma_{21} = 0.2$, $\phi = 0.2$, $a_1 = 0.02$, a;? = 0.01, $\bar{y}_1 = \bar{y}_2 = 1$, $\bar{g}_1 = \bar{g}_2 = 0.3$, $\bar{\tau}_1 = \bar{\tau}_2 = 0.306$ (to reproduce realistic steady-state debt levels), $\beta = 0.99$. The shock is $\theta_t = -0.01$ for t=1 and 0 for t>1.

[14] Fiscal federal transfers from western Germany to eastern German regions have remained at

6. Conclusion

By modeling the price-level determination of a two-country monetary union, several classes of equilibria were identified, distinguished by their specifications for monetary and fiscal policy rules. The first class of equilibria offers some justification for the general type of fiscal and monetary rules proposed for in the Maastricht Treaty, in which governments must limit the growth of their debts and the central bank fights inflation vigorously. This equilibrium suggested significant government heterogeneity can be accommodated within a monetary union without threatening the goal of price stability. This equilibrium also suggests that household heterogeneity gives monetary policy a reallocative dimension which can affect the equilibrium price level. In the model of the paper, this household heterogeneity enhances the potency of a monetary contraction to lower inflation.

A second class of equilibria suggests some types of fiscal behavior by one member government can produce inflationary results throughout the monetary union. A third class of equilibria suggests that household heterogeneity and wealth redistribution from fiscal federalism can also generate surprising results for price-level determination. In the present model, a particular type of fiscal federalism redistributes wealth among households that are heterogeneous in their demands for money. This redistribution can generate large changes in aggregate money demand and hence the equilibrium price level.

The interdependence between monetary and fiscal policies has long been recognized as central to price-level determination. The equilibria above suggest that the relatively novel arrangement of a monetary union, which may be on the horizon in Europe, adds new dimensions to this interdependence. First, the common central bank must interact with multiple and potentially heterogeneous national governments. Further, it must interact with households potentially heterogeneous between nations in their demand for money. The analysis highlights the importance of considering the distribution of wealth between heterogeneous agents in analyzing the determination of the equilibrium price level.

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about 5 percent of west German GDP since the political and monetary union between the two groups commenced.

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7. Appendix A: Linearized Model

$$\begin{bmatrix} \widetilde{c}_{1t} \\ \widetilde{\pi}_t \\ \widetilde{b}_{1mt} \\ \widetilde{b}_{1pt} \\ \widetilde{b}_{2pt} \\ \widetilde{b}_{xt} \end{bmatrix} = A_1 \begin{bmatrix} \widetilde{c}_{1t-1} \\ \widetilde{\pi}_{t-1} \\ \widetilde{b}_{1mt-1} \\ \widetilde{b}_{1pt-1} \\ \widetilde{b}_{2pt-1} \\ \widetilde{b}_{xt-1} \end{bmatrix} + B_1 \begin{bmatrix} \theta_t \\ \psi_{1t} \\ \psi_{2t} \end{bmatrix} + B_2 \begin{bmatrix} \theta_{t-1} \\ \psi_{1t-1} \\ \psi_{2t-1} \end{bmatrix} + H \begin{bmatrix} e_{1t} \\ e_{2t} \end{bmatrix} \qquad (7.1)$$

where:

$$A_1 = \begin{bmatrix} \boxed{1} & 0 & 0 & 0 & 0 & 0 \\ 0 & \boxed{\alpha_1\beta} & 0 & 0 & 0 & 0 \\ 0 & \xi_1 & \boxed{1} & 0 & 0 & 0 \\ 0 & \xi_2 & 0 & \boxed{\frac{1}{\beta}\text{-}\gamma_{11}} & 0 & 0 \\ 0 & \xi_2 & 0 & 0 & \boxed{\frac{1}{\beta}\text{-}\gamma_{12}} & 0 \\ -1 & \xi_3 & 0 & 1 & 0 & \boxed{\frac{1}{\beta}} \end{bmatrix}$$

$$B_1 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ -\frac{\beta^2(a_1\bar{c}_1+a_2\bar{c}_2)}{2(1-\beta)^2} & 0 & 0 \\ \frac{\beta^2(a_1\bar{c}_1+a_2\bar{c}_2)}{2(1-\beta)^2} & -1 & 0 \\ \frac{\beta^2(a_1\bar{c}_1+a_2\bar{c}_2)}{2(1-\beta)^2} & 0 & -1 \\ -\frac{\beta^2(a_1\bar{c}_1-a_2\bar{c}_2)}{2(1-\beta)^2} & 0 & 0 \end{bmatrix} \qquad B_2 = \begin{bmatrix} 0 & 0 & 0 \\ \beta & 0 & 0 \\ \frac{\xi_1}{\alpha_1} & 0 & 0 \\ \frac{\xi_2}{\alpha_1} & 0 & 0 \\ \frac{\xi_2}{\alpha_1} & 0 & 0 \\ \frac{\xi_3}{\alpha_1} & 0 & 0 \end{bmatrix}$$

$$H = \begin{bmatrix} 1 & 0 \\ -\frac{1}{\bar{c}_1} & 1 \\ -\frac{\xi_4}{\bar{c}_1} + \frac{a_1-a_2}{2(1-\beta)} & \xi_4 \\ -\frac{\xi_5}{\bar{c}_1} - \frac{a_1-a_2}{2(1-\beta)} & \xi_5 \\ -\frac{\xi_6}{\bar{c}_1} - \frac{a_1-a_2}{2(1-\beta)} & \xi_6 \\ -\frac{\xi_7}{\bar{c}_1} + \frac{a_1-a_2}{2(1-\beta)} - 1 & \xi_7 - \frac{a_1\bar{c}_1}{1-\beta} \end{bmatrix}$$

Constants:

$$
\begin{aligned}
\xi_1 &= \alpha_1\beta\left(1-\alpha_1\beta^2\right)\tfrac{a_1\bar{c}_1+a_2\bar{c}_2}{2(1-\beta)^2} - \alpha_1\beta\bar{b}_{1m} \\
\xi_2 &= -\xi_1 - \alpha_1\beta\bar{b}_{1m} \\
\xi_3 &= -\alpha_1\beta\left(1-\alpha_1\beta^2\right)\tfrac{a_1\bar{c}_1-a_2\bar{c}_2}{2(1-\beta)^2} \\
\xi_4 &= -\bar{b}_{1m} + \tfrac{a_1\bar{c}_1+a_2\bar{c}_2}{2(1-\beta)} - \tfrac{\alpha_1\beta^2(a_1\bar{c}_1+a_2\bar{c}_2)}{2(1-\beta)^2} \\
\xi_5 &= -\bar{b}_{1p}\beta^{-1} - \tfrac{a_1\bar{c}_1+a_2\bar{c}_2}{2(1-\beta)} + \tfrac{\alpha_1\beta^2(a_1\bar{c}_1+a_2\bar{c}_2)}{2(1-\beta)^2} \\
\xi_6 &= -\bar{b}_{2p}\beta^{-1} - \tfrac{a_1\bar{c}_1+a_2\bar{c}_2}{2(1-\beta)} + \tfrac{\alpha_1\beta^2(a_1\bar{c}_1+a_2\bar{c}_2)}{2(1-\beta)^2} \\
\xi_7 &= -\bar{b}_x\beta^{-1} + \tfrac{a_1\bar{c}_1+a_2\bar{c}_2}{2(1-\beta)} - \tfrac{\alpha_1\beta^2(a_1\bar{c}_1+a_2\bar{c}_2)}{2(1-\beta)^2} + \tfrac{\alpha_1\beta^2 a_1\bar{c}_1}{(1-\beta)^2} \\
\xi_8 &= \tfrac{1}{\alpha_1}\left(\tfrac{1}{\beta} + \tfrac{a_1+a_2}{2\beta}\right)^{-1}\left(\tfrac{a_1\bar{c}_1-a_2\bar{c}_2}{2(1-\beta)} + \tfrac{\bar{b}_x}{\beta}\right)
\end{aligned}
$$

8. Appendix B: Proofs of Existence and Uniqueness

While counting roots is helpful in identifying classes of equilibria, complete proofs of existence and uniqueness will involve more than this. The root-counting method of Blanchard and Kahn (1980) requires that expectational terms enter without linear dependencies, which is not true here (that is, H is not [I 0]'). The system may be rewritten:

$$\begin{bmatrix} x_t \\ Z_t \end{bmatrix} = \begin{bmatrix} A_1 & B_2 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_{t-1} \\ Z_{t-1} \end{bmatrix} + \begin{bmatrix} B_1 \\ I \end{bmatrix} Z_t + \begin{bmatrix} H \\ 0 \end{bmatrix} e_t \tag{8.1}$$

or

$$x_t^* = A^* x_{t-1}^* + F Z_t + H^* e_t \tag{8.2}$$

Let the Jordan decomposition of A* be represented by $C^{-1}JC$ with m roots greater than unity in the lower right corner of **J.** Partition C and **F** into the top rows relating to the stable roots, $C(1,:)$ and $F(1,:)$, and the lower rows relating to these m unstable roots, $C(2,:)$ and $F(2,:)$. A sufficient condition for existence and uniqueness of a solution to the system is that $C(2,:)H^*$ is square and full rank. A necessary condition for existence is that the column space of $C(2,:)H^*$ spans that of $C(2,:)F$. These criteria are discussed in Sims (1996), available at ftp://ftp.econ.yale.edu/pubsims/gensys.

8.1. Case 1

Consider a monetary union that requires both fiscal authorities be fiscally responsible ($\gamma_{11} > \frac{1}{\beta} - 1$ and $\gamma_{12} > \frac{1}{\beta} - 1$) and that requires the common central bank fights inflation aggressively ($\alpha_1 > \frac{1}{\beta}$). Then the model implies:

$$C(2,:)H^* = \begin{bmatrix} 1 & -\frac{1}{\bar{c}_1} \\ \xi_8 & -\frac{1}{\bar{c}_1}\xi_8 + \frac{1}{\beta}\left(1 + \frac{a_1+a_2}{2}\right) \end{bmatrix} \tag{8.3}$$

$$\text{where } \xi_8 = -\left(\frac{1}{\beta} - 1\right)\left(\xi_7 - \frac{a_1 \bar{c}_1}{1-\beta}\right)$$

and where ξ_4 is defined in the appendix A. This matrix is indeed square and full rank since $\frac{1}{\beta}\left(1 + \frac{a_1+a_2}{2}\right) \neq 0$, satisfying the sufficient condition for existence and uniqueness of an equilibrium.

8.2. Case *2*

If one fiscal authority is fiscally irresponsible $(\gamma_{11} < \frac{1}{\beta} - 1,\ \gamma_{12} > \frac{1}{\beta} - 1)$ and the common central bank does not fight inflation aggressively enough $(\alpha_1 < \frac{1}{\beta})$, this arrangement would have a unique equilibrium.

$$C(2,:)H^* = \begin{bmatrix} \xi_{10} & -\frac{1}{\bar{c}_1}\xi_{10} - \frac{a_1 - a_2}{2(1-\beta)} \\ \xi_{11} & -\frac{1}{\bar{c}_1}\xi_{11} + \frac{2 + a_1 + a_2}{2\beta} \end{bmatrix} \tag{8.4}$$

$$\text{where} \quad \begin{aligned} \xi_{10} &= (1 - \alpha_1\beta)^{-1}\xi_2 + \xi_5 \\ \xi_{11} &= -\left(\frac{1-\beta}{1-\alpha_1\beta^2}\right)\xi_3 - \left(\frac{1}{\beta} - 1\right)\left(\xi_7 - \frac{a_1\bar{c}_1}{1-\beta}\right) \end{aligned}$$

which is indeed square and full rank.

8.3. Case 3

Consider the case of fiscal federalism presented in case **3** in the text, in which the monetary authority fights inflation aggressively while one of the fiscal authorities is fiscally irresponsible. $(\phi_1 > \frac{1}{\beta} - 1,\ \gamma_{11} < \frac{1}{\beta} - 1,\ \gamma_{12} > \frac{1}{\beta} - 1, \alpha_1 < \frac{1}{\beta})$. In this case

$$C(2,:)H^* = \begin{bmatrix} 1 & -\frac{1}{\bar{c}_1} \\ \xi_5 & -\frac{1}{\bar{c}_1}\xi_5 - \frac{a_1 - a_2}{2(1-\beta)} \end{bmatrix} \tag{8.5}$$

which is indeed square and full rank, as long as the two representative households do not have exactly identical preferences for money holding.

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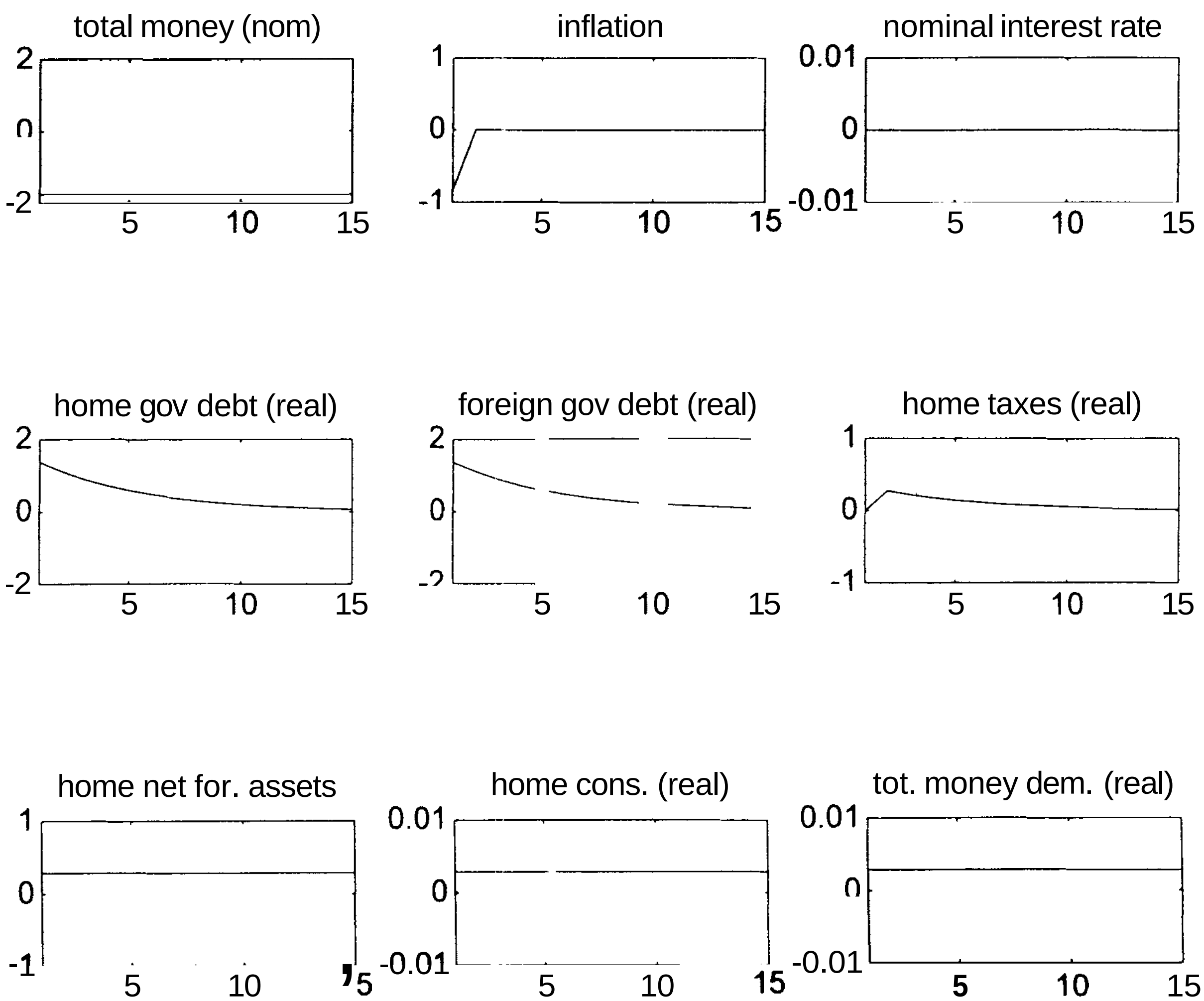
Figure 1
Maastricht Equilibrium
Monetary Policy Shock



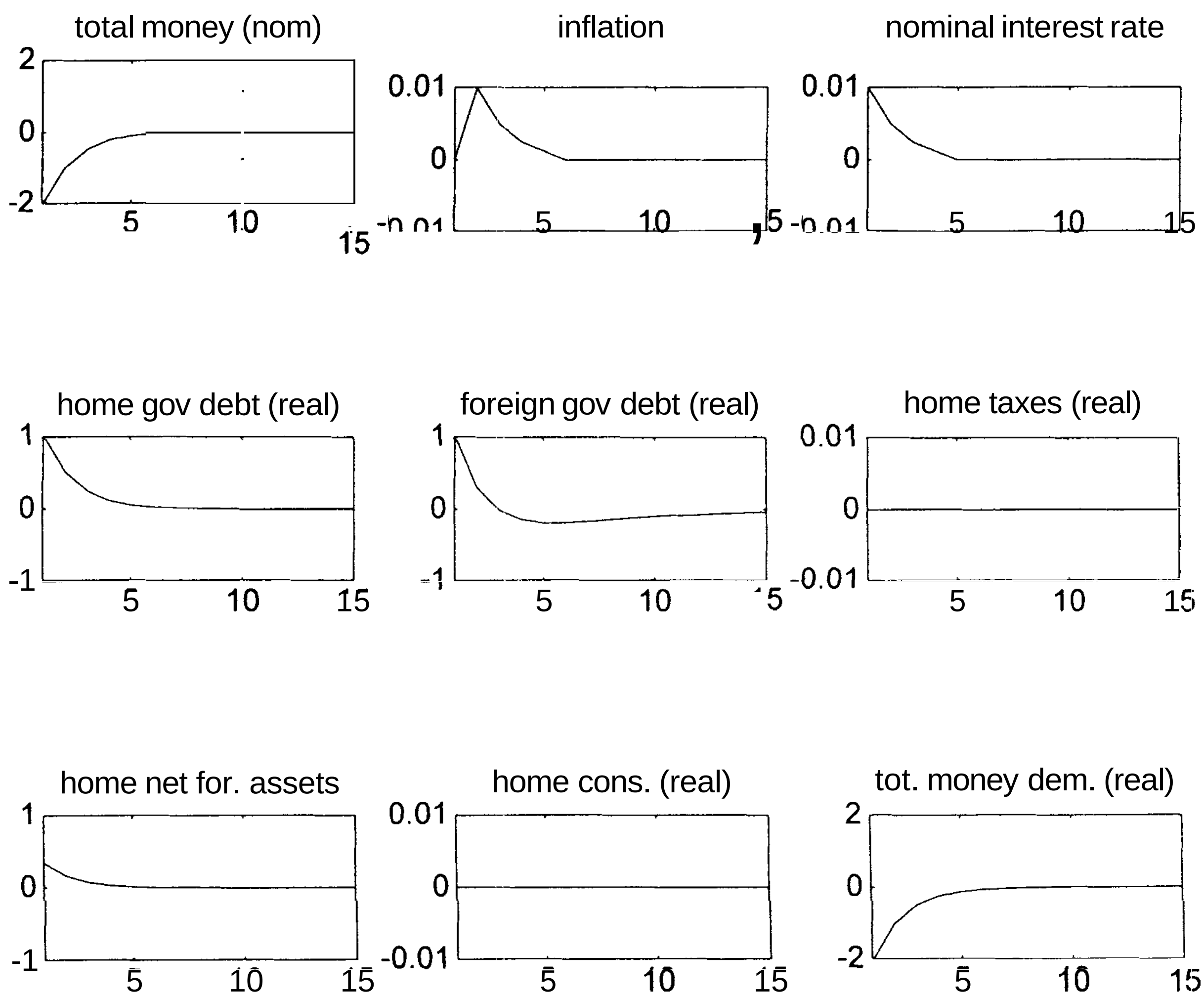
units: percent deviations from steady state
parameter setting: $\alpha 1 = 1.2$, $\gamma_{11} = \gamma_{12} = 0.2$,
$\beta = 0.99$, $a_1 = 0.02$, $a_2 = 0.01$,
$y_1 = y_2 = 1.0$, $g_1 = g_2 = 0.3$, $\tau_1 = \tau_2 = 0.306$

shock: $\theta_t = 0.01$ for $t = 1$, 0 for $t > 1$

Figure 2

Bailout Equilibrium

Monetary Policy Shock



units: percent deviations from steady state

parameter setting: $\alpha 1 = 0.5$, $\gamma_{11} = 0$, $\gamma_{12} = 0.2$,

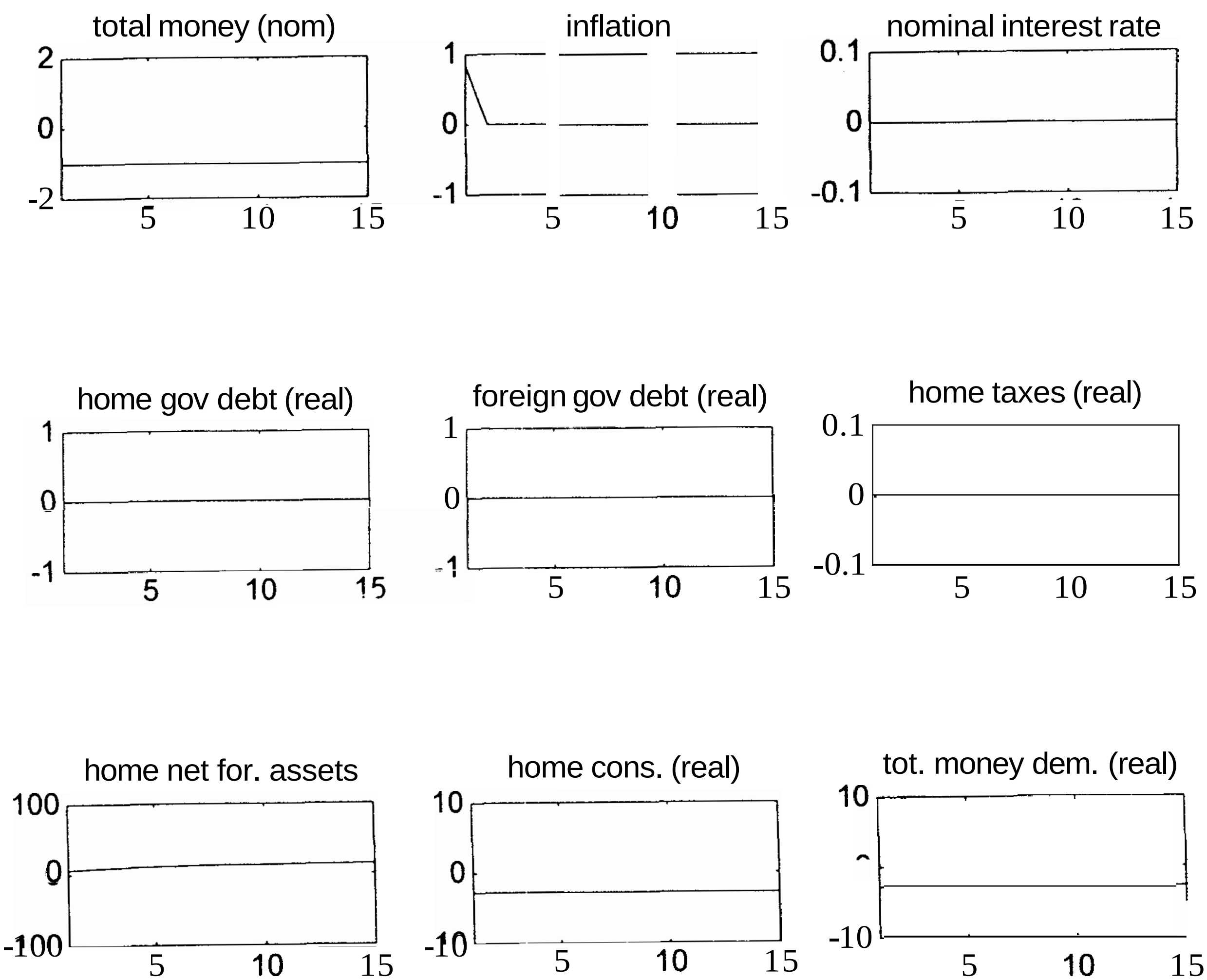
$\beta = 0.99$, $a_1 = 0.02$, $a_2 = 0.01$,

$y_1 = y_2 = 1.0$, $g_1 = g_2 = 0.3$, $\tau_1 = \tau_2 = 0.306$

shock: $\theta_t = 0.01$ for $t = 1$, 0 for $t > 1$

Figure 3

Fiscal Federalism Equilibrium
Monetary Policy Shock



units: percent deviations from steady state

parameter setting: $\alpha 1 = 1.2$, $\gamma_{11} = 0$, $\gamma_{12} = 0.2$, $\phi = 0.2$

$\beta = 0.99$, $a_1 = 0.02$, $a_2 = 0.01$,

$y_1 = y_2 = 1.0$, $g_1 = g_2 = 0.3$, $\tau_1 = \tau_2 = 0.306$

shock: $\theta_t = -0.01$ for $t = 1$, 0 for $t > 1$