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Non-self-centered inequity aversion matters.
A model

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Non-self-centred inequity aversion matters. A model.

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Abstract. The model by Fehr and Schmidt introduces envy and altruism in the utility function of a representative agent. The aim of this paper is to provide two extensions – non linearity and non self-centredness – to this model. This extension turns out to be more consistent with experimental evidence than the original model.

1. Introduction

The inequity-aversion model by Fehr and Schmidt (1999) is a major theoretical contribution to fairness studies. It states that the utility function of a subject not only is influenced by her own revenue, but also envy an altruism matter. In particular, an unfair distribution of resources among subjects results in disutility for inequity-averse people. The theoretical results of this model are consistent with a lot of experimental results (i.e. in power to take games, public good games and so on) to be considered - correctly - a suitable proxy of human decisional process when fairness matters.

However, some of its features – linearity and self-centredness – make it inconsistent with some experimental results and with some common sense intuitions. The hypothesys of linearity leads to the fact that absolute value of differences among revenues matter. However, experimental evidence suggests that also difference in percentage matters and that different monetary stakes are relevant in subjects' decision processes (i.e. Slonim and Roth, 1998; Munier and Zaharia, 2002).[1] Moreover, linearity allows to provide a good explanation for people's extreme choices (I give nothing, I give half of my income) but not to the significant percentage of interior choices (i.e. ultimatum game, Güth et al., 1982; and dictator game, Forsythe et al., 1994). Finally, self-centredness assumes that fair-minded people's utility is influenced only by the comparison between their own payoff and the other subjects' payoffs, but not by the differences among the other players' payoffs. The opposite seems to emerge in experiments (Engelmann and Strobel, 2002[2]; Ottone, 2007), people consider also differences among others in their utility function.

The aim of this paper is to provide two extensions – non linearity and non self-centreness – to the model of inequity aversion by Fehr and Schmidt. These two aspects are not new in economic literature. Non linearity was

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[1] Bolton and Ockenfels (2000) take account of the relevance of different monetary stakes when proposing a utility function such that players are concerned with their share of the total payoff: a given payoff difference is less important if absolute payoffs are higher. However, experimental results suggest that the Fehr – Schmidt model better explains people's behavior (see for instance Engelmann and Strobel, 2000). This is why we focus on the Fehr – Schmidt model.

[2] Participants declared their aversion to inequality among others in a questionnaire after the experiment.

analysed by Frohlich et al. (2004), starting from the FS model, in a particular scenario where they include "just desert". Non self centred inequity aversion – even if treated from a more general point of view – can be found in Hochman and Rodgers (1969). However, the former is focused on the peculiar context of production. On the other hand, the latter is too generic.

The way we approach these extensions allows to better analyse people's behavior in ultimatum game, dictator game and third party punishment game without loosing the predictive power of the FS model in other games.

Nevertheless, our aim is not simply to find an explanation for people's behaviour in the lab. The reason why we focus on results obtained in some experimental games (Ultimatum Game, Dictator Game, Third Party Punishment Game) is that they are in line with subjects' attitude in real life situations. We may provide some examples. For instance, there are subjects who intervene when someone is hurt along the street. The middle class gives money to poor people, but the rich do not make donations in favour of the middle class, and so on and so forth. Explaining players choices in such games means understanding subjects' decisional process in everyday life.

2. The extension to the Fehr – Schmidt model

The inequity-aversion model by FS states that:

$$U_i\left(x_i, x_j\right) = x_i - \frac{\alpha_i}{n-1}\sum_{j=2}^{n}\max\left\{x_j - x_i, 0\right\} - \frac{\beta_i}{n-1}\sum_{j=2}^{n}\max\left\{x_i - x_j, 0\right\}, \qquad i \neq j$$

(1)

where: n is the number of players, $x_{i,j}$ is the payoff of player i (the subject) or j (the others), α_i is a parameter of envy, β_i is a parameter of altruism. $0 \leq \beta_i < 1$ and $\alpha_i \geq \beta_i$ since the disutility arising from a position of disadvantage is higher than the disutility arising from a position of advantage. $\partial U_i / \partial x_j \geq 0$ iff $x_i \geq x_j$ since an increase in others' income is utility – increasing if and only if they have a lower level of income with respect to subject i.

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A first questionable implication is that intermediate choices are not well explained. Consider, for example, the *Dictator Game*. The Fehr – Schmidt model yields corner solutions depending on the value of β. In particular, when $\beta_i \in [0, 0.5)$, player i always maximizes her own payoff choosing not to transfer any sum to player j. On the other hand, when $\beta_i \in (0.5, 1)$, player i always maximizes her own payoff choosing to share equally the total amount of money with player j. Finally, when β_i is equal to 0.5, player i is indifferent between any distribution of the total payoff S in the range $x_i \in [S/2, S]$. In other words, this model is unable to clearly explain players' interior choices, that are the most common result (for a survey, Camerer and Fehr, 2003).

The assumption of a linear utility function makes the model simple to handle but is the reason why interior solutions are not well defined. Kohler (2003, p. 7) argues that 'an increasing degree of difference aversion resolves the shortcoming that only two "focal" equilibria exist'. However, his model holds only when the initial endowment S is normalized to 1. Consequently, this does not allow any analysis concerning different levels of endowment, a feature that may well be relevant. Moreover, if we consider the actual value of the initial endowment, the quadratic difference[3] (as Kohler suggests) becomes extremely high with high numbers and even an almost selfish Dictator will decide to transfer half of the sum.

Another point is that linearity in the FS model does not allow to consider the relevance of different monetary stakes in subjects' decision processes. In an *Ultimatum Game*, given the value of α_i, the minimum amount (as a percentage of the total sum to be divided) accepted by the Responder is always the same. However, Slonim and Roth (1998) and Munier and Zaharia (2002) show that the higher the sum to be shared, the lower the minimum percentage accepted by the Responder.

Finally, the assumption of self-centredness cannot explain some results we found out while implementing a series of *Third Party Punishment Game*

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[3] He simply suggests to square the differences $(x_i - x_j)$ and $(x_j - x_i)$.

experiments. In particular, this model does not predict punishment when the Dictator's payoff is lower than the Observer's.

We suggest to modify the initial inequity-aversion utility function (1) of player i by taking into account not only the difference between his and player j's payoffs, but also their absolute value[4], as well as the difference among the other players' payoffs. This yields a non-linear and non-self-centred utility function:

$$V_i\left(x_i,x_j\right)= f\left(x_i\right)-\frac{\alpha_i}{n-1}\sum_{j=2}^{n} f\left(x_j - x_i, x_i\right)-\frac{\beta_i}{n-1}\sum_{j=2}^{n} f\left(x_i - x_j, x_j\right)+$$
$$-\frac{\theta_i}{(n-1)(n-2)}\sum_{\substack{j=2\\ j\neq k}}^{n} f\left(\left|x_j - x_k\right|,\min\left\{x_j,x_k\right\}\right)$$

(2)

where: α_i is a parameter of envy, β_i is a parameter of altruism and θ_i a parameter of non-self-centred fairness. Parameter k represents other people different from j.

This is an additive function (as in Fehr and Schmidt) with four terms. In particular, the fourth term is devoted to measure the non self centerness, while the first, the second and the third ones are the same as in FS.

We assume that the second term of the previous function is increasing with respect to the difference in brackets and decreasing with respect to the value of x_i. At the same time, the third term is increasing with respect to the difference in brackets and decreasing with respect to the value of x_j. Finally, the fourth term is increasing with respect to the difference in brackets and decreasing with respect to the minimum value between x_j and x_k.

An simple testable form could be:

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[4] It stands to reason that the discomfort due to inequality decreases as the income of the worse-off player increases. Consider, for example, two subjects in two different scenarios. In the first case, player A has 10 euro and player B 0 euro. In the second case player A has 1000 euro and player B 990 euro. Player A will suffer more in the first situation.

$$U_i = x_i - \alpha_i \frac{1}{n-1} \sum_{j=2}^{n} \max\left\{ \frac{x_j - x_i}{\gamma_i x_i + 1}, 0 \right\} - \beta_i \frac{1}{n-1} \sum_{j=2}^{n} \max\left\{ \frac{x_i - x_j}{\sigma_i x_j + 1}, 0 \right\} +$$

$$-\theta_i \frac{2}{(n-1)(n-2)} \sum_{\substack{j=2 \\ j \neq k}}^{n} \left\{ \frac{|x_j - x_k|}{\pi_i \min\{x_j, x_k\} + 1} \right\}$$

$$(3)$$

where: γ_i is a weight of envy, σ_i is a weight of altruism and π_i a weight of non-self-centred fairness and are used to represent non-linearity. Moreover, $\pi_i \geq \gamma_i$, $\sigma_i \geq \gamma_i$ and $\pi_i, \sigma_i, \gamma_i \in [0, 1]$.

3. Some applications

3.1 Dictator Game

Consider a **Generalized Dictator Game** where the Dictator receives an endowment (k) and he has to decide the sum (s) to transfer to the Receiver. The Receiver obtains ms, with $m \geq 1$. Now, $s \in [0, \frac{k}{m+1}]^5$ and:

$$U_i = k - s - \beta_i \left(\frac{k - s - ms}{\sigma_i ms + 1} \right) \qquad (4)$$

In equilibrium:

$$s* = \begin{cases} k/(m+1) & \text{if } \beta_i \geq h \\ \dfrac{-1 + \sqrt{\beta_i m + \beta_i \sigma_i km + \beta_i}}{\sigma_i m} & \text{if } l < \beta_i < h \\ 0 & \text{if } \beta_i \leq l \end{cases}$$

$$(5)$$

where:

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[5] We consider only the part of the utility function's domain where the Dictator has a payoff that is equal or higher than the payoff of the Receiver.

$$l = \frac{1}{m+1+\sigma_i km} \qquad \text{and} \qquad h = \frac{m+1+\sigma_i km}{(m+1)^2}$$

From (5), it emerges that an increase in β (with $\beta \in (l,h)$) leads to an increase in the optimal value of s chosen by the Dictator.

3.2 Ultimatum Game

Consider an **Ultimatum Game**, where: player i is the Proposer; player j is the Responder; s is the offer of the Proposer; k is the sum to be divided.

Since the Proposer will never offer to the Responder more than half of the total sum k, the utility function of player j is:

$$U_j(s) = s - \alpha_j \left\{ \frac{k-2s}{\gamma_i s+1} \right\}, \qquad\qquad i \neq j. \qquad (6)$$

The Responder will accept any sum that yields a positive value of equation (6), since rejection of the Proposer's offer yields a level of utility equal to 0. The Responder will reject any offer:

$$s < s'(\alpha_j, k) \equiv \frac{1}{2} \frac{-1 - 2\alpha_i + \sqrt{1 + 4\alpha_i + 4\alpha_i^2 + 4\alpha_i \gamma_i k}}{\gamma_i} \qquad (7)$$

This implies that, the higher k, the lower the portion of k the Receiver is willing to accept, as equation (7) states. In fact, the first derivative $(\frac{\partial s}{\partial k})$ is positive, while the second one $(\frac{\partial^2 s}{\partial k^2})$ is negative. This is consistent with the experimental results in Slonim and Roth (1998) and Munier and Zaharia (2002).

3.3 Third-Party Punishment Game

Consider three subjects – the Dictator (player j), the Receiver (player k) and the Observer (player i). W hen $x_i > x_j > x_k$, the Observer's utility is:

$$U_i = x_i - wc - \frac{\beta_i}{2}\left(\frac{x_i - wc - x_j + zc}{\sigma_i(x_j - zc) + 1} + \frac{x_i - wc - x_k}{\sigma_i x_k + 1}\right) +$$

$$- \theta_i\left(\frac{x_j - zc - x_k}{\pi_i x_k + 1}\right)$$

$$(8)$$

where w is the cost of punishment for the Observer; z is the cost of punishment for the Dictator and c is the quantity of punishment. Moreover, $z > w \geq 0$ and $x_i \geq wc$.

$$\frac{\partial U_i}{\partial c} = -w - \frac{\beta_i}{2}\left(\frac{z - w}{\sigma_i(x_j - zc) + 1} + \frac{(x_i - wc - x_j + zc)\sigma_i z}{(\sigma_i(x_j - zc) + 1)^2}\right) +$$

$$+ \frac{\beta_i}{2}\left(\frac{w}{\sigma_i x_k + 1}\right) + \frac{\theta_i z}{\pi_i x_k + 1}$$

$$(9)$$

As equation (9) may be positive, $c^* > 0$ is possible (as confirmed by the experimental evidence in Ottone, 2007). More precisely, $c^* = \left(0, \dfrac{x_j - x_k}{z}\right]$.[6] It is worth noting that the corresponding result obtained with the Fehr – Schmidt model is $c^* = 0$.

4. Conclusions

Fehr and Schmidt assume that fair-minded people exist and describe them by a self-centred linear model. In this paper we provide a non-linear, non-self-centred utility function for fair-minded people that better explains the experimental evidence and, as a consequence, real life situations. In particular, we assume that the disutility due to unfair distribution of outcomes is influenced

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[6] The Observer will never punish the Dictator so as to make her poorer than the Receiver.

not only by the difference between the payoffs but also by the absolute value of the payoff of each player and by the differences among the other players' payoffs. This hypothesis looks plausible and allows to explain the behavior of players involved in some experimental games (Ultimatum Games, Dictator Games and Third-Party Punishment Games) as well as in everyday life situations. In addition, the model is consistent with the empirical evidence also in other games – Public Good Game (Ledyard for a survey, 1995), Gift Exchange Game (Fehr et al., 1993), Trust Game (Berg et al., 1995; Camerer and Fehr, 2003), as the model by FS is.

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