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Rates in the Aggregate Life-Cycle/Permanent Income Cum Rational Expectations Model

by Kim J. Kowalewski

of this interest is not so much due to deterioration in the ability of economists to predict future output and prices, although that is clearly important.

The main impetus is the challenge of the "New Classical" school. Barro (1974) argued that rational private agents do not view bond-financed increases in government spending or decreases in taxes as increases in wealth, because they know that the new bonds must be retired by additional future taxes. Rational private agents therefore will increase current saving to pay for these future taxes, no matter how far into the future they come due. This additional saving is exactly enough to purchase all of the new debt; interest rates and aggregate wealth remain unchanged. This implies that bond-financed increases in government spending have a multiplier value of 1, and that bond-financed tax cuts have a zero multiplier.

Money-financed increases in government spending also have a zero multiplier, because rational private agents view the faster growth in money as leading to a higher inflation rate in the future. This higher inflation is another "tax" that private agents will save for. That is, money-financed tax cuts have no effect on real variables, because one tax is just substituted for another.

These "New Classical" results are a direct challenge to the Keynesian and Monetarist schools, which assign higher values to these multipliers (at least in the short run), because the effects of fiscal policy actions are distinguished by how they are financed.[1]

Barro's result depends, among other things, on the assumption that private agents have the opportunity to offset these government actions. This, in turn, assumes that capital markets are perfect—that there are no transactions or other costs that drive a wedge between borrowing and lending interest rates, and that there are no informational asymmetries that are controlled with down payments,

1. See Tobin (1980) and Tobin and Buiter (1980) for good discussions of these points.

security interests, rationing the quantity of credit, and other non-price loan provisions. Thus, with perfect capital markets, the length of a consumer's spending horizon (that is, the time span over which a permanent increase in life-cycle wealth/permanent income is consumed) is as long as his remaining lifetime. It may be longer if, as Barro assumes, a consumer's utility function includes the utility of his direct descendants. A consumer can borrow any amount up to the current value of his net nonhuman wealth, plus the present value of all his expected future after-tax labor income, all discounted at the common rate of interest. An increase in life-cycle wealth/permanent income will be consumed over the remainder of the horizon, making the amount consumed in the short run very small.

If capital markets are imperfect, however, then the length of a consumer's planning horizon may be shortened. A consumer may not be able to borrow against all of his life-cycle wealth (or permanent income), or may do so only at a penalty rate of interest. Increases in life-cycle wealth/permanent income will be consumed over this shorter horizon, enlarging the (short-run) impact of bond-financed tax cuts or spending increases. Clearly, shorter horizons make it possible for stabilization policies to affect real variables, at least in the short run.

Thus, the recent interest in consumption behavior centers on learning the length of consumer spending horizons. The approach taken by most recent studies is to test some variant of the life-cycle/permanent income cum rational expectations (RE-LC/PI) model assuming perfect capital markets. Rejection of the RE-LC/PI model, incorporating perfect capital markets, is taken to mean that horizon lengths may not be long enough to diminish the power of stabilization policies.

Hall (1978), Flavin (1981,1985), Hayashi (1982), Muellbauer (1983), Wickens and Molana (1984), Bernanke (1982), Mankiw (1983), DeLong and Summers (1984), Boskin and Kotlikoff (1984), Kotlikoff and Pakes (1984), and Mankiw, Rotemberg, and Summers (1985) test the RE-LC/PI model with aggregate time series data, while Hall and Mishkin (1982), Bernanke (1984), and Hayashi (1985) use cross-section or panel data on individual households.

Of the studies employing micro-data, only Bernanke (1984) can reject the model. Of the studies that employ aggregate time series data, Hall (1978), Hayashi (1982), Mankiw (1983), Bernanke (1984), and Delong and Summers (1984) cannot reject the model during the post-World War II period. Kotlikoff and Pakes (1984) can reject the model, but conclude that the differences from the model are not large enough to matter in practice.[2]

These studies are not the first to be concerned with the length of consumer spending horizons. For example, Tobin (1951) argued that capital market imperfections may have accounted for the different savings behaviors of black and white Americans in the late 1940s. Houthakker (1958), in his review of Friedman's (1957) permanent income hypothesis, argued that the exclusion of capital market imperfections was the main defect of Friedman's work. Friedman (1963) argued that consumer horizon lengths were about three years.

Before rational expectations came into vogue, there were numerous tests of the life-cycle and permanent income models, beginning with Modigliani and Brumberg (1954) and Friedman (1957). The debate about the efficacy of the 1968 temporary tax increase focused on the length of consumer spending horizons see, for example, Okun (1971) and Blinder (1981). There has been considerable theoretical work done on the impact of capital market imperfections (see, for example, Tobin and Dolde [1971], Dolde [1973], Pissarides [1978], Heller and Starr [1979], Foley and Hellwig [1975], and Watkins [1975,1977]).

What is new about these recent studies is their assumption of rational expectations. Unfortunately, richness of detail seems to have been sacrified for this assumption. For exam-

2. Muellbauer (1983) and Wickens and Molana (1984) reject the model using U.K. consumption and income data.

ple, none of the recent models that are estimated with U.S. aggregate time series data allows for uncertain real interest rates. All of the models, except Bernanke (1982) and Mankiw (1983) assume that the real interest rate is constant. Bernanke (1982) and Mankiw (1983) allow real interest rates to vary, but assume that consumers know all future real interest rates.

It is rather curious that stochastic real interest rates have been ignored, because the real interest rate is a key variable in the life-cycle/permanent income model (and in many New Classical models). The interest rate measures the exchange rate between consuming today and saving today to consume more tomorrow. The life-cycle/permanent income model determines the utility-maximizing allocation of life-cycle wealth (permanent income) across time by balancing the marginal rate of transforming consumption today into consumption tomorrow (the interest rate) with the marginal rate of substitution (the discounted marginal utility from consuming tomorrow relative to that from consuming today). Changes in interest rates, expected or unexpected, should lead to a reallocation of consumption spending across time. Thus, an allowance for stochastic real interest rates should provide a more powerful test of the RE-LC/PI model and indirectly of the (maximum) length of the representative consumer's spending horizon.

In this article, we estimate a RE-LC/PI model that allows for uncertain future interest rates. The model is developed by Muellbauer (1983), which he estimated with United Kingdom (U.K.) data. To put Muellbauer's model into perspective, the Hall and Flavin (1981) models are also discussed and estimated. Updating the Hall and Flavin results with the 1980s data also may reveal any structural instabilities and shifts in the distribution of horizon lengths across consumers, which is a possibility ignored by all of the recent RE-LC/PI tests. Section II reviews the RE-LC/PI models, section III briefly outlines the procedures followed in estimating the three models and explains the results, and the third section concludes our study.

I. The Life-Cycle/Permanent Income Model With Rational Expectations

Tests of the RE-LC/PI model begin with Hall (1978). The consumer is assumed to maximize the expected present discounted value of current and future utility. Income is exogenous and is known in the current period, but unknown thereafter; the consumer's choice variable is the level of consumption each period. The horizon begins with the current period and ends at the (known) last period of the consumer's lifetime. There are no bequests and no capital market imperfections. Expectations are rational—functions of all information available in the current period. Real interest rates and rates of time preference are assumed to be constant. The model is:

$$(1)\qquad \max_{C_t} E_t \sum_{i=0}^{T-t} [\delta^i U(C_{t+i})]$$

subject to

$$\sum_{i=0}^{T-t} (R^i C_{t+i}) - \sum_{i=0}^{T-t} (R^i y_{t+i}) = A_t,$$

where

6 is the inverse of 1, plus the pure rate of time preference, assumed constant,
R is the inverse of 1 plus the real, after-tax rate of interest r, also assumed constant, $(\delta \geq R)$,
C is real life cycle consumption (not NIPA personal consumption expenditures),
y is real labor income,
A is current real nonhuman wealth,
$U(\bullet)$ is the instantaneous utility function, and
E_t is the expectations operator, conditioned on the information available at time t (variables dated t-1 and earlier).

The first order conditions for this problem are:

(2a) $E_tU'(C_{t+i}) = (R/\delta)E_tU'(C_{t+i-1})$, for $i=1$ to $T-t$; in particular, for $i=1$

(2b) $E_tU'(C_{t+1}) = (R/\delta)U'(C_t)$.

There are two things to note about (2b). First, C, can be thought of as a sufficient statistic for C_{t+1}; that is, no variable except C_t helps predict future marginal utility of consumption $U'(C_{t+1})$. Second, with the assumption of rational expectations, marginal utility follows the regression relation:

(3) $U'(C_{t+1}) = \gamma U'(C_t) + \epsilon_{t+1}$.

The term ϵ_{t+1} represents the impact on marginal utility of all new information that becomes available in period t + 1 about the consumer's lifetime well-being. Under rational expectations, $E_t\epsilon_{t+1} = 0$ and ϵ_{t+1} is orthogonal to $U'(C_t)$. Moreover, ϵ should be white noise, that is, unpredictable using variables in the information set.

If the utility function is quadratic or "the change in marginal utility from one period to the next is small, both because the interest rate is close to the rate of time preference and because the stochastic change is small." (See Hall [1978, p. 975].) Then equation **(3)** becomes:

(4) $C_t = \gamma C_{t-1} + \epsilon_t$.

That is, life-cycle consumption follows an AR (1) process—no other variables dated t-1 or earlier affect C_t. If y = 1, then consumption follows a random walk. It is important to notice that **(4)** is not a structural model of life cycle consumption behavior. Because it is only the first-order condition for utility maximization, it is only an implication of the life-cycle model under rational expectations. Indeed, it is only a necessary condition for this RE-LC model to be true.

Hall also shows that lifetime resources evolve as a random walk with trend. First, nonhuman wealth follows the relation:

(5) $A_t = R^{-1}(A_{t-1} + y_{t-1} - C_{t-1})$.

Second, human wealth, H_t, is the sum of current labor income and the expected present discount value of future labor income:

(6) $H_t = \sum_{i=0}^{T-t} (R^iE_ty_{t+i})$,

where

$E_ty_t = y_t$,

from which it follows that:

(7a) $H_t = R^{-1}(H_{t-1} - y_{t-1}) + \mu_t$,

where μ_t represents the present value of the changes in expectations of future income that occur between period t-1 and t:

(7b) $\mu_t = \sum_{i=0}^{T-t}[R^i(E_ty_{t+i} - E_{t-1}y_{t+i})]$.

Again, under rational expectations, $E_{t-1}\mu_t = 0$, and μ_t **should** be white noise. Under certainty equivalence, $\delta_t = a_t\mu_t$, where a_t is an annuity factor modified to take account of the fact that the consumer plans to make consumption grow at a proportional rate y over his remaining lifetime. Then the equation for total wealth is:

(8) $A_t + H_t = R^{-1}(1-\alpha_{t-1})(A_{t-1} + H_{t-1}) + \mu_t$.

Flavin (1981) estimates a different version of the permanent income model using the insight from (7) to eliminate the unobserved H_t. She starts by defining current consumption as the sum of permanent and transitory consumption. By equating permanent consumption with permanent income (y_t^p), she has:

(9) $C_t = y_t^p + \epsilon_{2t}$, where ϵ_{2t} is transitory consumption.

Thus, permanent income is defined to be the annuity value of the expected present discounted value of human and nonhuman wealth (A_t + H_t), assuming the real, after-tax rate of interest, r, is constant:

(10) $y_t^p = r(A_t + \sum_{i=0}^{\infty} [R^{i+1}E_ty_{t+i}])$.

Flavin shows that $E_t y^p_{t+1}$ = y fusing the insight implicit in equation (7b). Substituting (10) into (9) and using the nonhuman wealth constraint:

$$(11) \quad A_{t+1} = R^{-1} A_t + y_t - C_t.$$

Unlike equation (5), current period saving does not earn interest in equation (11). Equation (9) can be used to solve for C_{t+1} in terms of C_t:

$$(12) \quad C_{t+1} = C_t + r \sum_{i=0}^{\infty} [R^{i+1}(E_{t+1} - E_t) y_{t+i+1})] - R^{-1}\epsilon_{2t} + \epsilon_{2t+1}.$$

Flavin notes that because the coefficient of ϵ_{2t} is not -1, C_t will not evolve as a random walk unless the transitory consumption term ϵ_{2t} is zero for all t.

Equation (12) contains revisions in expectations of future real labor income. Flavin notes that "[a]s an empirical matter however, unanticipated capital gains and losses on nonhuman wealth probably constitute a significant fraction of the revisions in permanent income this model is trying to capture." (See Flavin [1981, p. 988].) She defines unanticipated capital gains as the present value of the revision in the expected earnings associated with the current nonhuman wealth position. By then assuming "... that changes in the rate of return to capital ... are quantitatively more important than the endogenous changes (in nonhuman wealth) in determining the time-series properties of the observed path of nonlabor income ...", unanticipated capital gains can be approximated as the present value of the revision in expected future nonlabor income. (See Flavin [1981, p. 988].) This permits her to use disposable personal income (YD) in place of labor income (y) in equation (12).[3]

Flavin next derives an expression for the revision in expectations of future YD by assuming that YD follows an ARMA process. She shows that the revision in the expectation of YD_{t+s} ($s>0$) between periods t and t-1 is the product of the moving average error of YD in period t (u_t) and the s[th] coefficient from the corresponding moving average representation for YD (B_s). Then the present discounted value of the set of revisions is:

$$(13) \quad (\sum [R_s B_s]) u_t.$$

Thus, she demonstrates that the revision in income expectations is white noise.

The ARMA model for YD plus the equation formed by substituting (13) into (12) is Flavin's permanent income consumption model. Note that (13) still contains an unobserved variable u_t. This term is included with the other error terms in estimation, making her consumption equation very similar to Hall's. The difference is that Hall's model can be viewed as a reduced form of Flavin's structural model. Flavin argues that the error terms in the two equations are correlated because her model is incomplete. The income equation error will contain additional terms because the information set probably contains variables other than past income. These omitted information set variables will also appear in the consumption equation error through (13), thus producing the correlation between the two equation errors. She dismisses this apparent specification bias by assuming that these omitted information set variables are serially uncorrelated and uncorrelated with the lagged income terms.

Hayashi (1982) also uses equation (7) to eliminate the unobserved H_t. He starts with the permanent income model in level form:

$$(14) \quad C_t = \alpha(A_t + H_t) + \epsilon_t,$$

where ϵ_t is defined as "transitory consumption"—a shock to preferences or measurement error in C_t and A_t. He notes that α, the propensity to consume, is a function of the expected real rates of return from nonhuman wealth and the subjective rate of time preference: but, like Hall and Flavin, assumes that these factors are constant over time and individuals. Using (7a) with an "overall" discount

3. This assumption is not unreasonable, given that her model explains short-run changes in consumption. However. in her later paper, Flavin (1985) uses annual data where it seems less likely that changes in the rate of return to capital dominate endogenous changes in wealth accumulation.

rate $1+d$ in place of R, Hayashi eliminates H, from *(14):*

$$(15) \quad C_t = (1+d)C_{t-1} + \alpha[A_t - (1+d)(A_{t-1} + y_{t-1})] + v_t,$$

where $v_t = u_t - (1+d)u_{t-1} + \alpha\mu_t$. Like Flavin, Hayashi also uses a two-equation model, composed of equation *(15)* and a stochastic version of equation *(5)*. He adds an error term to Hall's nonhuman wealth identity to capture unanticipated movements in asset prices and measurement errors in A_t, A_{t-1}, y_{t-1}, and C_{t-1}. Note that Hayashi's model uses labor income instead of YD and is slightly more general than either Hall's or Flavin's, because it does not assume that $1+d = R^{-1}$.

Hall, Flavin, and Hayashi test their models by adding other variables to the right-hand side of (4), the modified version of (12), and *(14)*. It is clear that by doing so they test the ***joint*** hypothesis that both the life-cycle/permanent income model and the rational expectations assumption are correct. If they were interested in testing only the assumption of rational expectations, conditional upon the LC/PI model, for example, they would have compared their models with suitable transformations based on different hypotheses about expectations formation. If the joint hypothesis is correct, then no other variable in the information set except C_{t-1} will help forecast C_t. Although any set of variables could be used to test these models, income is an obvious choice, because a direct relationship between consumption and current income in these models would be strong evidence against the simple life-cycle/permanent income model assuming perfect capital markets and against Barro's neutrality hypothesis.

Recall that there is no direct structural relationship between consumption and income in these models. Current income may be correlated with current consumption, but the correlation arises only indirectly, because current income represents new information about human wealth/permanent income. Unlike Friedman *(1957)* and Modigliani and Brumberg (1954), who allowed for the possibility that some unexpected changes in income would not alter a consumer's estimate of his permanent income or life-cycle wealth, all unexpected income changes in the Hall, Flavin, and Hayashi models lead to revisions in permanent income or life-cycle wealth and, hence, consumption.

The models are estimated and tested with post-World War II U.S. aggregate time series data. Unfortunately, it is difficult to compare their results because they use different data and sample periods. This is partly due to the lack of reliable data on life-cycle/permanent consumption. Hall uses real, per capita PCE-nondurables and services as the consumption variable, ignoring the service flow from consumer durables because of the lack of reliable data. Flavin uses only real per capita PCE-nondurables as the consumption variable. She notes that the consumption of durable services should exhibit a lagged response to changes in permanent income due to the transactions costs of adjusting durable good stocks. The same is true of housing services, which form a large part of PCE-services. By using only PCE-nondurables, she says that she gives the benefit of the doubt to the random walk hypothesis of one-quarter adjustment.

However, this point is probably irrelevant, because Flavin detrends the consumption and income data before estimation. The strong trend in PCE-services most likely would be eliminated with detrending, allowing her to use PCE-nondurables and services as the dependent variable. Indeed, as shown below, Flavin's model rejects the RE-LC/PI model, using PCE-nondurables and services as the dependent variable. Hayashi uses real, per capita annual data constructed by Christensen and Jorgenson *(1973* and updates) for the consumption variable and a modification of their labor income variable for y. The consumption data contain imputations for the service flow of consumer durables. Flavin uses real per capita YD for the income variable, and all three use this variable (or its lagged value) for testing their models.

Hall's first test consists of adding three additional lagged C terms to the right-hand side of (4) and finds them to be statistically insignificant individually and taken together. He finds the same result when one, four, and 12 lagged YD terms are added. In all cases, the coefficient on C_{t-1} is not significantly different from 1, which leads Hall to conclude that aggregate consumption is a *random walk* process.

However, when Hall adds four lagged stock price variables (Standard and Poor's comprehensive index of stock prices deflated by the implicit deflator for PCE-nondurables and services and divided by population), he finds that they are individually and collectively statistically significant. Hall argues that this evidence does not contradict the joint hypothesis, if it is assumed that "some part of consumption takes time to adjust to a change in permanent income. Then any variable that is correlated with permanent income in period $t-1$ will help in predicting the change in consumption in period t, since part of that change is the lagged response to the previous change in permanent income." (See Hall [1978, p. 985].) He also says that "the discovery that consumption moves in a way similar to stock prices actually supports this modification of the random walk hypothesis, since stock prices are well known to obey a random walk themselves." (See Hall [1981, p. 973].) In all tests, the Durbin-Watson statistic, which is biased downwards in these models when the autocorrelation of the errors is positive, cannot reject the hypothesis of no first-order autocorrelation. Hall thus concludes that the model cannot be rejected.

This is a rather curious inference. Hall finds a variable that contradicts the null hypothesis, and he subjectively rationalizes it! Moreover, it seems highly improbable that two truly random walks will be strongly correlated with each other. Since the two series are correlated, does this mean that the two series are not random walks, that they are random walks around a common trend, that there is a structural relationship between the two series, that the correlation is simply spurious, or that they are an artifact of aggregate time series data? Unfortunately, Hall does not report any tests of these possibilities.

Flavin adds the current and first seven lagged changes in real per capita YD to equation (12) with ΔC_t as the dependent variable. By adding these eight terms, she obtains a just-identified system. The reduced form of her model thus becomes:

$$\text{(12a)}\quad YD_t = \mu_1 + \alpha_1 YD_{t-1} + \alpha_2 YD_{t-2} + \ldots$$
$$+ \alpha_8 YD_{t-8} + \eta_{1t}$$
$$\Delta C_t = \mu_2 + \beta_0 (U_1 + (\alpha_1 - 1) YD_{t-1}$$
$$+ \alpha_2 YD + \ldots + \alpha_8 YD_{t-8})$$
$$+ \beta_1 \Delta YD_{t-1} + \beta_2 \Delta YD_{t-2} + \ldots$$
$$+ \beta_7 \Delta YD_{t-7} + \eta_{2t},$$

where n_2, contains ϵ_2, and (13). The β's are "measures of the 'excess sensitivity' of consumption to current income, that is, sensitivity in excess of the response attributable to the new information contained in current income." (See Flavin [1981, p. 990].) Thus, a test of the joint statistical significance of the β's is a test of the RE-PI model. Over the 1949:IIIQ to 1979:IQ sample, Flavin can reject the model at a 0.5 percent significance level. The coefficient β_0 on the ΔYD_t term allows her to test for a direct effect of current income on C, although her estimate of β_0 is quite large relative to those of the other ΔYD terms, its t-statistic is only 1.3, suggesting that the test "falls short of providing conclusive evidence that the permanent income-rational expectations hypothesis fails in a quantitatively significant way." (See Flavin [1981 p. 10021.)

Hayashi adds YD_t to equation (14) and finds its coefficient to be of the same order of magnitude as the estimate of the discount factor, but statistically insignificant in his two-equation model. He also finds that the discount rate is statistically different from the constant real rate of return, contrary to Hall's and Flavin's assumptions. Although this is

4. It is not clear how Bernanke lets the real interest rate vary over time.

evidence in favor of the permanent income cum rational expectations hypothesis, Hayashi argues that "... the relevant measure of consumption for the liquidity-constrained households is personal consumption expenditures as defined in the National Income and Product Accounts (NIPA), which ***excludes*** service flows from consumer durables and ***includes*** expenditures on consumer durables. The foregoing test of the permanent income hypothesis seems to be in some sense unfair to the alternative hypothesis of liquidity constraints." (See Hayashi [1978, p. 908].) When he uses PCE as the dependent variable and estimates only the consumption equation (because the asset equation includes consumer durables), he finds the coefficient on current YD to be fairly large (0.892) with a t-statistic of about 20. On the basis of this result, he is persuaded to reject the permanent income cum rational expectations model. Here again is a rather curious inference. In effect, Hayashi is saying that only PCE-durables purchases can be liquidity-constrained.

Other authors have tried to relax some of the assumptions made by these writers. Bernanke (1982) and Mankiw (1983) focus on the separability issue by adding consumer durables to the life-cycle cum rational expectations model. They argue, like Flavin, that lagged stock adjustment and accelerator effects may lead to an incorrect rejection of the model. This is even true when durables are excluded from the analysis, if nondurables and durables are not separable in consumer utility functions. Moreover, as Hayashi points out, imperfections in capital markets are likely to show up in the pattern of durables purchases.

Bernanke derives a two-equation system in current period PCE-nondurables and services and next period's stock of consumer durables as the solution to the utility maximization problem. A quadratic utility function containing quadratic costs of adjusting consumer durable stocks is used. Mankiw also obtains a two-equation model, only based on the first-order conditions for utility maximization. Both show that consumption is not a random walk. In Bernanke's model, this is due to the adjustment costs, which supports Hall's assertion that adjustment costs can be consistent with the life-cycle cum rational expectations model. In Mankiw's model, consumption is not a random walk, because the real rate of interest and the relative price of durables are non-constant.

Both economists test their models with post-World War II U.S. aggregate time series data. Under the assumption of constant real interest rates, Bernanke finds that the response of consumers to an income innovation is significantly greater than predicted by the theoretical model and thus rejects the life-cycle cum rational expectations model. He claims, but unfortunately does not prove the evidence, that a similar result obtains if the real interest rate is allowed to vary.

Mankiw adds disposable income growth terms to both equations in his model and finds them statistically insignificant. He thus finds no evidence against the life-cycle cum rational expectations model and argues that his model "...is a useful framework for examining the linkage between interest rates, prices, and consumer demand." (See Mankiw [1983, p. 23].) As in many past studies, he also finds that consumer durables are quite sensitive to the real rate of interest. Depending on the parameter values chosen, the short-run elasticity of the stock of consumer durables with respect to the real interest rate varies between -1.7 and -4.3. Mankiw's results also suggest that the assumption of rational expectations is unimportant because he obtains results similar to those studies that do not assume rational expectations.

Real interest rates are not handled very satisfactorily by Mankiw.[4] Consumers are assumed not to know future income, but are assumed to know future interest rates (and the relative price of durables). Thus, interest rates are allowed to vary over time in a very uninteresting way. Muellbauer (1983) and

Wickens and Molana (1984) allow for random and unknown future real interest rates.

Wickens and Molana show that when the interest rate in the life-cycle cum rational expectations model is random, the first order condition for utility maximization becomes:

$$(16) \quad E_{t-1}U'(C_{t+i-1}) = \delta E_{t-1}[(1/R_{t+i})U'(C_{t+i})] \quad (i \geq 0).$$

This expression is obtained by substituting C_t out of the utility function with the period-to-period budget constraint (11) and maximizing the present discounted value of expected future utility with respect to A_t. Expectations are formed with the information set available at the end of period t-1, which includes variables dated t-1 and earlier. With the necessary assumptions, (16) can be written as:

$$(17) \quad E_{t-1}C_{t+i} = E_{t-1}\gamma_{t+i}(E_{t-1}C_{t+i-1}), \quad (i \geq 0).$$

where 2 is a function of the interest rate and the rate of time preference. Thus, as in Hall's equation (2a), the coefficient on the lagged consumption term varies with the real interest rate.[5] With the appropriate assumptions, Muellbauer obtains an expression in potentially observable variables:

$$(18) \quad \Delta lnC_t = \mu_0 + \delta_3 E_{t-1}r_{t-1} + \delta_1\sigma_{1t} + \delta_2\sigma_{2t} + \epsilon_{t+1},$$

where σ_1 and σ_2 are the innovations in period t real disposable income and the real interest rate based on information available at the end of period t-1, which includes variables dated t-1 and earlier. The Wickens and Molana model differs only slightly from this, using r_{t+1} instead of Y,.,,because of a minor difference in the dating of the interest rate in the cash flow constraint. Both papers use post-World War II U.K. aggregate time series data.

Also note that apart from the logarithms and the dating difference on Y, Flavin's model is nested in (18). However, Muellbauer and Wickens and Molana estimate their models differently than Flavin, because the variables they use to test their consumption equations are all lagged at least one period. Recall that the Flavin model is simultaneous, because she uses AYD, as one of her test variables. When deriving the reduced form of her two-equation system, the equation for YD is used to substitute out the current YD term in AYD_t. The revision to permanent income due to new information provided by current YD (13) cannot be identified and thus is thrown into the error term. Because Muellbauer and Wickens and Molana only use lagged variables to test their models, the income and interest-rate innovations remain identified by the income and interest-rate equations. Thus, unlike Flavin, they can estimate the coefficients on the innovation terms.

Ignoring the interest-rate terms in Muellbauer's and Wickens and Molana's model, it is not clear that their test is more powerful than Flavin's. The presence of AYD_t in the consumption equation gives Flavin a direct test of the impact of current income on current consumption. If the RE-LC/PI model is rejected, there is some knowledge about what the correct alternative may be, or at least in what direction the search for the correct alternative might go, but she cannot test for the impact of the income innovation, an important variable of the null hypothesis. By not adding any current income terms, Muellbauer and Wickens and Molana cannot test for a direct effect of current income on current consumption, but they do have a direct test of the impact of innovations in income.

The estimation procedure used by Muellbauer and Wickens and Molana requires two steps. The first step estimates with ordinary least squares (OLS) the simple reduced forms for disposable income and the real interest rate to generate the income and interest-rate innovations and expected values. Muellbauer's $lnYD$ equation uses the first two lags of $lnYD$ and lnC_{t-1} as the information set. For his real interest-rate equation, Muellbauer argues that apart from seasonal factors, the U.K. real interest rate varies randomly about

5. In general, when real interest-rate expectations are probabilistic the coefficient on C_{t-1} depends on the joint distribution of expected real incomes and real interest rates. In both cases, the optimal forecast of current consumption requires more information than provided by C_{t-1}.

a constant from the 1950s until the pound sterling began to float in 1972:IIQ; it follows a random walk thereafter. Wickens and Molana say that a broader information set than one that includes only lagged values of income and real interest rates, should be used with their more general model. They use the first four lags of *ln* YD, *lnC, r, lnA,* the latter being the log of real consumer liquid assets, as the information set for both real disposable income and the real interest rate.

The second step uses the residuals for the innovation terms and fitted values for the expected value terms in OLS regressions of the consumption equations. Both papers find that their models appear to fit the U.K. data very well. Wickens and Molana do not test the joint life-cycle rational expectations hypothesis; Muellbauer does by adding the information set variables to the right-hand side of (18) and tests for their joint statistical significance. He finds the additional lagged terms to be significantly different from zero. He concludes that allowing for stochastic interest rates does not seem to be a major cause for the failure of the simple Hall model to explain U.K. consumption found earlier by Daly and Hadjimatheou (1981).

II. Updates of the Aggregate Life Cycle Cum Rational Expectations Model

We update the estimates, test the Hall (1978) and Flavin (1981) models, and present estimates of the Muellbauer model using post-World War II U.S. aggregate time series data.[6]

Updating the Hall and Flavin models serves at least four purposes. First, the updates help put the results from Muellbauer's model in perspective. The importance of allowing for stochastic interest rates is immediately clear. Second, by estimating the models through 1984, we can estimate their stability. Third, it is interesting to know how the 1980s data fit these models. Real output and prices varied over wide latitudes during the 1980s and, hence, offer macroeconometricians a rich set of high-influence data, which may help them estimate coefficients more precisely. It is likely that the 1980s data provide even stronger evidence against the RE-LC/PI model than found by Flavin.

Finally, the different models are estimated with different information sets (reduced forms) and different sample periods. It is reasonable to wonder if either the content of the information set or the estimation period has a large influence on the estimates. Our interest in these models does not lie solely in determining whether the RE-LC/PI model is accepted or rejected, although that is a very important consideration. If these models are to be useful for policymaking and forecasting, however, they should be robust to different assumptions about the underlying structure used to derive the reduced forms.

The Hall and Flavin models are updated with their original samples, specifications, and estimation techniques. To make the three models comparable, we had to make at least four decisions. The first concerns the specification of the dependent and independent variables. Hall uses per capita PCE-nondurables and services, Flavin uses the change in per capita PCE-nondurables, and Muellbauer uses the change in the logarithm of per capita (U.K.) PCE-nondurables and services. The consumption definition used in these tests is per capita PCE-nondurables and services. Although Flavin's reasons for ignoring PCE-services may be valid, most of these problems should be eliminated once the data are detrended. The change in the logarithm of consumption and the logarithm of income are used here to facilitate comparison with the Muellbauer specification. This logarithmic specification should also minimize heteroskedasticity problems. The income definition is real disposable income per capita. The log real per capita income and consumption data are detrended by their average growth trends over the 1947:IQ to 1984:IVQ period. When the same dependent

6. It was decided not to update Hayashi's model, because it is not so easily compared with the Hall and Flavin models. The Wickens and Molana model was not updated either, because it is similar to Muellbauer's, apart from some additional terms that complicate the estimation procedure.

7. See Kowalewski (1985) for more detail on this point.

variable is used, Flavin's consumption equation is, for all practical purposes, the same as Muellbauer's with constant interest rates.

The second decision involves seasonal adjustment of the data. Muellbauer uses seasonally unadjusted data, while Hall and Flavin use seasonally adjusted data. We used seasonally adjusted data to maintain comparability with other U.S. consumption results.

A third choice concerns estimation techniques. Hall uses OLS, Flavin uses maximum-likelihood to estimate her consumption equation jointly with her income forecasting equation, and Muellbauer uses a two-step OLS procedure. The original estimation techniques used by Hall and Flavin are used to update their models with the most recent data. Maximum-likelihood is used to estimate Muellbauer's model, because the computer-generated coefficient standard errors produced by the two-step method are incorrect.[7]

A fourth choice is that of the definition of the real interest rate. Instead of using an *ex post* real interest rate, Muellbauer uses something like an *ex ante* rate—a nominal interest rate minus an *expected* inflation rate. He computes this real rate by subtracting from the nominal rate a fitted value from an inflation equation. This choice of real rate is rather odd, for it means that instead of using an expected real interest rate as his theory requires, he is using an expected *expected* real interest rate in his consumption equation. It also means that he is using a three-step estimation process, with the estimation of the inflation equation as the first step. Moreover, the inflation equation uses an information set different from that used for the income and interest-rate equations. A logical extension and correction of his model would be to specify separate forecasting equations for the nominal rate and the inflation rate, to use the same information set for all of the equations, and to use the fitted values and residuals from both equations to compute the expected real rate and its innovation. An equivalent strategy employed here is to use an *ex post* rate, as Wickens and Molana do. This requires only one forecasting equation. The *ex post* real three-month U.S. Treasury bill rate, (nominal rate, minus current-quarter compounded annual actual growth rate in the PCE-nondurables and services deflator) is used as the real interest rate in the estimations of Muellbauer's model shown below.

Because there is no reason to think that U.S. real interest rates have behaved as random walks during the post-World War II period, the real interest-rate equation for Muellbauer's model will have information set variables as regressors, and these will be the same as those used for the income equation—the first two lags of income, the first two lags of the real interest rate, and the first lag of consumption. This is a simple extension of Muellbauer's original information set, which consisted of the first two lags of income and the first lag of consumption.

The estimation results are shown in tables 1 to 5. The data used for the computations contain revisions through the second revised estimates for 1984:IVQ dated March 31,1985. The models in tables 1 to 3 were estimated over their original samples and over 1949:IIIQ to 1984:IVQ. For the re-estimates of Hall's model, the data were not detrended. For the re-estimates of Flavin's model, the consumption and income data were detrended using their average growth rates over the 1947:IQ to 1979:IQ period. When the two models are updated with the data through 1984:IVQ, the consumption and income data are detrended using their average growth rates over the 1947:IQ to 1984:IVQ period, and a dummy variable is added to control for the credit controls of 1980:IIQ. Detrending biases the test in favor of the random walk hypothesis, because it removes the main source of correlation from these variables. Detrending may also remove structural correlation between C and YD, again favoring the random walk hypothesis. It unfortunately leaves the trend unexplained. The dummy variable is part of the maintained

8. Serially correlated errors may not signal a breakdown of the model, if as Hall argues when rationalizing the statistically significant stock price index terms, consumers take more than one quarter to assimilate new information and act upon a changed expectation of life-cycle wealth.

hypothesis and is not included among the variables included in the test of the RE-LC/PI model.

The first table contains OLS estimates of Hall's model. The first equation shows the re-estimates of Hall's model with only one lagged income term. The coefficients, though different from Hall's published numbers, yield the same apparent inference: the RE-LC/PI model cannot be rejected. The next equation shows the original Hall model updated through 1984:IVQ. Note that the addition of the 1980s data did not change the conclusion of the hypothesis test—the coefficient on lagged personal income is small, has the wrong sign, and is statistically insignificant. However, the Durbin h-statistic rejects the hypothesis of positive serially uncorrelated errors at better than a 5 percent significance level using a one-tailed test. Because the theory predicts that the error should be white noise, the addition of the 1980s data may be signaling a breakdown of the model.[8]

The third equation contains the change in the detrended log of per capita PCE-nondurables and services as the dependent variable and the detrended logarithm of real per capita disposable personal income as the income variable. The estimation period is 1948:IQ to 1977:IQ. Neither coefficient is large, the t-statistics are very low, and the adjusted R^2 is negative. The results change very little when the estimation period is extended through 1984:IVQ; all of the explanatory power of the right-hand side variables comes from the

Table 1 Hall Estimates

$$C_t = \alpha_0 + \alpha_1 C_{t-1} + \alpha_2 Y_{t-1} + \alpha_3\ DUM802 + \epsilon_t$$

	#1	#2	#3	#4
C	*NDS/POP*	*NDS/POP*	Chg in detrended log of *NDS/POP*	Chg in detrended log of *NDS/POP*
Y	*YD72/POP*	*YD72/POP*	Detrended log of *YD72/POP*	Detrended log of *YD72/POP*
Sample	48:1Q-77:IQ	48:1Q-84:4Q	48:1Q-77:1Q	49:3Q-84:4Q
α_0	-0.0376 (-2.2620)	-0.0059 (-0.5835)	0.0007 (1.1492)	0.0005 (0.8562)
α_1	1.0811 (24.8721)	1.0081 (31.3779)		
α_2	-0.0480 (-1.6283)	-0.0008 (-0.0341)	-0.0063 (-0.4627)	-0.0060 (-0.4902)
α_3		-0.0441 (3.0626)		-0.0131 (-2.3346)
adj R^2	0.9989	0.9994	-0.0068	0.0263
Durbin h	1.358	1.7327	1.7752[a]	1.7460[a]
SER	0.0136	0.0290	0.0058	0.0056

Variables:

NDS = PCE-nondurables plus services, 1972 dollars.
YD72 = disposable personal income, 1972 dollars.
POP = non-institutionalized, civilian population.

a. Durbin-Watson statistic.

NOTE:
1. The variables in equations #3 and #4 are detrended over the 1947:IQ to 1984:IVQ period.
2. The t-statistics are shown below the coefficient estimates.

9. Flavin (1981), proves the equivalence of these two procedures in appendix II.

10. When the consumption and income variables are detrended with their average growth rates between 1947:IQ and 1984:IVQ, the LRS for the joint test of the Δ ln YD terms becomes 13.5, which implies the rejection of the null hypothesis at about a 10 percent significance level.

dummy variable. Thus, Hall's model can find no evidence to reject the RE-LC/PI model.

The results for Flavin's model (12a) are shown in tables 2 and 3. Only the coefficients of the A YD (Δln YD) terms (the β coefficients in equation (12a) are shown because only they are relevant for the test of the RE-LC/PI model. Recall that these terms must be jointly statistically different from zero in order to reject the model. The first equation in table 2 shows the re-estimates of her original specification. Like the updates of Hall's model, these coefficients are not quantitatively the same as the original estimates; qualitatively, however, they are very similar. The coefficient on ΔYD_t, β_0, though fairly large, has a very low *t*-statistic; of the β's, only β_1 is significant at better than 5 percent using a one-tailed test. The likelihood ratio statistic (LRS) tests the joint significance of the ΔYD terms. Surprisingly, the RE-LC/PI model *cannot* be rejected at the original significance level.

Table 2 Flavin Re-estimates

Coef	Var	#1	#2
β_0	ΔYD_t	0.3194	0.2712
		(1.1164)	(1.4596)
β_1	ΔYD_{t-1}	0.0605	0.0650
		(1.8388)	(2.3574)
β_2	ΔYD_{t-2}	0.0079	-0.0099
		(0.2493)	(-0.3659)
β_3	ΔYD_{t-3}	-0.0662	-0.0535
		(-1.2940)	(-1.4499)
β_4	ΔYD_{t-4}	0.0415	0.0136
		(0.8088)	(0.3915)
β_5	ΔYD_{t-5}	-0.0081	-0.0082
		(-0.1410)	(-0.1908)
β_6	ΔYD_{t-6}	0.0068	0.0050
		(0.2163)	(0.1834)
β_7	ΔYD_{t-7}	0.0074	0.0169
		(0.2381)	(0.6146)
C SER		0.0103	0.0104
C D-W		2.0101	2.0521
Y SER		0.0329	0.0325
Y D-W		2.0008	2.0009
LR Statistic		11.754	17.148
Sample		**49:3Q-79:1Q**	**49:3Q-84:4Q**

NOTE: Detrending occurs from 1947:1Q to 1979:1Q.

Flavin's original likelihood ratio statistic, which is asymptotically distributed as $X^2(8)$, is 27.0, significant at better than 0.5 percent. The LRS for the test of equation (1) is only 11.8, significant at slightly better than 25.0 percent. Identical test results are obtained by estimating only the consumption-reduced-form equation with OLS and by testing for the joint significance of the lagged income terms.[9] Apparently, the results are sensitive to revisions in the data and to the use of different trend values for PCE-nondurables and YD.[10]

Equation (2) in table 2 updates Flavin's original model through 1984:IVQ. The 1947:IQ-1979:IQ trend values are used to detrend the post-1979:IQ data. Interestingly, the model can now be rejected at better than a 5.0 percent significance level; the LRS is 17.2, while the $X^2(8)$ cut-off value is 15.5 at 5.0 percent. The coefficient β_0 is now smaller, but its *t*-statistic is larger; the coefficient and *t*-statistic on $\Delta ln YD_{t-1}$ are also larger. Moreover, the fit of the equation is improved over the longer period; the standard errors of the two equations are smaller in the longer sample. Thus, as was expected, the 1980s data appear to tighten up coefficient standard errors and help reject the RE-LC/PI model.

Equations (3) and (4) in table 3 use the change in the logarithm of per capita real PCE-nondurables and services as the dependent variable, and the log per capita consumption and income data are detrended over the 1947:IQ to 1984:IVQ period. They compare to the Hall equations (3) and (4) in table 1. The third equation shows the unconstrained results over the 1949:IIIQ to 1979:IQ sample period. Notice that they are qualitatively similar to those of equation (1); β_0 is about 0.3 and is statistically insignificant; β_1 is large and is statistically significant. Testing the joint significance of the Aln YD terms yields a LRS of 27.1, which is significant at better than 0.5 percent, Flavin's original significance level. Note that this result is much stronger than Flavin's original result, because the consumption variable includes PCE-services, which Flavin argued would bias the results against the RE-LC/PI model.

The fourth equation shows the estimation results over the 1949:IIIQ to 1984:IVQ sample. Qualitatively, these results are similar to those of equation (3). The LRS of the test of the lagged Δln YD terms is now 29.4, greater than the LRS over the 1949:IIIQ to 1979:IQ sample; the standard errors of the equation also are smaller in the longer sample. Again, it appears that the 1980s data provide additional stronger evidence against the RE-LC/PI model.

Tables 4 and 5 contain the estimates of Muellbauer's models. Only the coefficients on the information set, innovation, and expected interest-rate terms are shown. The dependent variable is the change in the logarithm of real per capita PCE-nondurables and services; detrending of the log real per capita consumption and income data occurs over the 1947:IQ to 1984:IVQ period. Table 4 shows the estimates of equation (18) without the interest-rate terms $E_{t-1}r_{t-1}$ and σ_{2t}. The coefficient δ_1 on the income innovation should be positive, because positive innovations in current income should lead to upward revisions in life-cycle wealth/permanent income and, hence, in consumption. The first equation shows the results using the 1949:IIIQ to 1979:IQ sample. This equation compares to Flavin's equation (3) in table 3. The coefficient is δ_1 positive and statistically significant. Surprisingly, the RE-LC/PI model cannot be rejected by this form of Muellbauer's model, even though Flavin's model could. The LRS is only 3.8, significant at slightly less than 30 percent. Again, the results appear to be sensitive to the specification of the test.

The second equation in table 4 updates Muellbauer's model without the interest-rate terms over the 1949:IIIQ to 1984:IVQ sample. As was true of Flavin's model, Muellbauer's model without the interest-rate terms fits better with the 1980s data. Moreover, the LRS is now 14.2, significant at better than 1 percent. Again, the 1980s data lead to a convincing rejection of the RE-LC/PI model. Note that the coefficients on the information set variables are the same order both of magnitude and statistical significance in equations (1) and (2); the difference is that the model fits better with the 1980s data.

Table 5 contains the estimates of Muellbauer's model including the interest-rate terms. Recall from equation (18) that δ_3, the coefficient on the expected interest-rate term, is a positive function of the ratio of one, plus the interest rate, to one, plus the rate of time preference; hence, it should be positive. Presumably, the coefficient δ_2 on the interest-rate innovation is negative, since a higher-than-expected interest rate should cause consum-

Table 3 Flavin Estimates Using Logs

Coef	Var	#3	#4
β_0	ΔYD_t	0.2794	0.2652
		(0.8282)	(0.9903)
β_1	ΔYD_{t-1}	0.1208	0.1280
		(2.9091)	(3.3398)
β_2	ΔYD_{t-2}	0.0709	0.0597
		(1.7267)	(1.6026)
β_3	ΔYD_{t-3}	-0.0977	-0.0762
		(-1.7462)	(-1.5329)
β_4	ΔYD_{t-4}	0.0577	0.0457
		(0.6548)	(0.6887)
β_5	ΔYD_{t-5}	-0.1296	-0.1095
		(-2.7135)	(-2.5909)
β_6	ΔYD_{t-6}	0.0444	0.0459
		(1.1380)	(1.2202)
β_7	ΔYD_{t-7}	0.0162	0.0423
		(0.4045)	(1.1439)
C SER		0.0051	0.0050
C D-W		1.8636	1.9003
Y SER		0.0102	0.0098
Y D-W		1.9942	2.0022
LR Statistic		27.068	29.360
Sample		**49:3Q-79:1Q**	**49:3Q-84:4Q**

NOTE: Detrending occurs over the 1947:1Q-1984:4Q.

ers to save more in the current period. Equation **(3)** shows the results over the 1949:IIIQ to 1979:IQ sample. The two interest-rate coefficients appear to be small in magnitude, but this is simply a scaling difference because, interest rates are measured in percentage points. The interest-rate innovation coefficient δ_2 is statistically insignificant, while δ_3 is significant at slightly better than 10 percent. The LRS for the test of the RE-LC/PI model is 27.8, which is asymptotically distributed as $X^2(5)$, and is significant at better than 1 percent. Compared with equation (1) in table 4, the allowance for stochastic interest rates now leads to the rejection of the RE-LC/PI model. Again, the specification of the test has an important effect on the results.

Table 4 Muellbauer Estimates Without the Interest Rate

Coef	Var	#1	#2
61	*YRESID*	0.2185	0.2191
		(4.7533)	(5.3016)
β_1	$lnYD_{t-1}$	0.1207	0.1557
		(2.2775)	(3.4233)
β_2	$lnYD_{t-2}$	-0.1683	-0.1585
		(-3.4897)	(-3.5539)
β_3	lnC_{t-1}	0.0418	-0.0112
		(1.0391)	(-0.4075)
C SER		0.0053	0.0050
Y SER		0.0104	0.0104
LR Statistic		3.800	14.200
Sample		**49:3Q-79:1Q**	**49:3Q-84:4Q**

NOTE. YRESID is the current income innovation term

Table 5 Muellbauer Estimates with the Real Interest Rate

Coef	Var	#3	#4
δ_1	*YRESID*	0.2318	0.2431
		(5.2208)	(5.8538)
δ_2	*RRESID*	0.0001	-0.0001
		(0.2042)	(-0.3660)
δ_3	Er_{t-1}	0.0042	0.0026
		(1.8764)	(2.1257)
β_1	$lnYD_{t-1}$	0.0871	0.1433
		(1.6383)	(2.9831)
β_2	$lnYD_{t-2}$	-0.0000	-0.0419
		(-0.0005)	(-0.6306)
β_3	r_{t-1}	0.0006	0.0003
		(2.2494)	(1.0161)
β_4	r_{t-2}	-0.0024	-0.0017
		(-2.1887)	(-2.6248)
β_5	lnC_{t-1}	-0.0738	-0.1109
		(-1.0183)	(-1.7685)
C SER		0.0049	0.0050
Y SER		0.0111	0.0108
r SER		1.9737	2.0102
LR Statistic		27.800	26.200
Sample		**49:3Q-79:1Q**	**49:3Q-84:4Q**

NOTE: YRESID and *RRESID* are the current income and interest-rate innovations. Er_{t-1} is the expectation of last period's real interest rate based on Information available last period.

Equation (4) in table 5 shows the estimates of Muellbauer's model with the interest-rate terms over the 1949:IIIQ to 1984:IVQ period. All of the coefficients are estimated more precisely, but unlike the previous results, the equation fits the longer period less well. The coefficients **62** and δ_3 now have the correct signs and about the same statistical significance as the earlier estimates. The LRS statistic for the test of the RE-LC/PI model is 26.2, rejecting the model at better than a 1 percent significance level, but it is a bit smaller than the LRS from the shorter sample period. Nevertheless, the results are qualitatively the same for both estimation periods, unlike the results of the Flavin tests.

The worse fit using the 1980s data occurs because the interest-rate equation fits less well in the longer period. This is not surprising, given that interest rates behaved so differently in the 1980s than in the earlier period.[11] Does this mean that the test is invalid because the equation generating the interest-rate expectations is wrong? This does not seem likely. Although the t-statistics on δ_2 and **63** do not provide support for the model, the LRS of the joint significance of the two interest-rate terms in equation (4) is 46.1. Thus, the interest-rate terms are undoubtedly important, even if they are poorly computed. Moreover, it is not clear how quickly interest-

11. The standard error of the consumption equation also increased, but this is probably due to the poorer fit of the interest-rate equation through the cross-equation constraints.

rate forecasting models were adjusted in the 1980s. Given the lag in the learning process, the number of quarters for which the interest-rate equation may be wrong is probably smaller than 20. Even if the interest-rate equation is wrong, it is not necessarily irrational. Finally, the fit of the model did not worsen so much that this is likely to be the sole reason that the RE-LC/PI model is rejected.

III. What Has Been Learned?

The estimation results provide ample evidence to reject this form of the RE-LC/PI model during the postwar period, especially when the 1980s data are included. Even though Hall's specification cannot reject the model, minor generalizations of Flavin and Muellbauer can, and Muellbauer's specification including uncertain interest rates can reject the model with or without the 1980s data. It would appear that an important assumption for Barro's neutrality hypothesis does not hold.

Unfortunately, this rejection of the RE-LC/PI model does not offer an explicit alternative as a replacement. As mentioned earlier, these tests cannot distinguish the assumption of rational expectations from that of the life-cycle/permanent income model. All that can be inferred from these tests is that the joint hypothesis can be rejected. Flavin (1985) attempts to determine whether the rejection of the RE-LC/PI model is due to the assumption of perfect capital markets or to that of the permanent income model. She uses her original model augmented with an equation for the unemployment rate, which is a proxy for the number of liquidity-constrained consumers. However, there are many problems using such a crude variable for such a complex hypothesis; her tests undoubtedly have little power.

Nor do these tests provide many clues about the exact length of consumer spending horizons, or how the distribution of horizon lengths changes as interest rates, the distribution of income, or the supply of consumer credit changes.

That the distribution of consumer horizon lengths may vary over time is suggested by the increased significance of the likelihood ratio tests when the 1980s data are included. The early 1980s were apparently a time when the distribution of horizons lengths was skewed toward the shorter end, increasing the correlation of aggregate consumption to current disposable income. Additional evidence about changes in the distribution of consumer spending horizons is provided by Kowalewski (1982), who studies the time series behavior of aggregate personal bankruptcy filings in the United States. Personal bankruptcy filings are countercyclical, increasing in recessions and falling in recoveries. For a variety of reasons discussed in the article, it is likely that just before they file for bankruptcy, personal bankrupts have about the shortest spending horizons of all consumers.

Thus, increases in the number of personal bankruptcy filings might indicate a shift in the distribution of consumer spending horizons towards shorter lengths. In a regression explaining per capita personal bankruptcy filings, transitory income had a much larger impact than permanent income, suggesting that liquidity is very important for these financially distressed consumers. The composition of consumer portfolios was also significantly related to the behavior of personal bankruptcy filings. Unfortunately, this evidence is only about one tail of the distribution. It is clear that much work remains to be done before the time series behavior of aggregate consumption is understood.

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