Learning, Rationality, the Stability of Equilibrium and

by John B. Carlson

John B. Carlson is an economist at the Federal Reserve Bank of Cleveland. The author would like to acknowledge helpful criticisms from Richard Kopcke, Mark Sniderman, E.J. Stevens, Alan Stockman, Owen Humpage, Steve Strongin, William T. Gavin, Randall W. Eberts, and Charles Carlstrom, on earlier drafts.

Introduction

It is sometimes argued that the strength in models that assume rational expectations is the weakness of their competitors. For example, McCallum (1980) says: "Each alternative expectational hypothesis, that is, explicitly or implicity posits the existence of some particular pattern of systematic expectational error. This implication is unattractive, however, because expectational errors are costly. Thus, purposeful agents have incentives to weed out all systematic components."

This alluring intuition, however, glosses over a very difficult problem that remains unsolved in general: How do agents acquire the information and understanding sufficient to enable them to "weed out" systematic error? The acquisition of information is costly and no one actually believes anyone knows the true underlying model of the economy. Discovering systematic error is one thing; knowing what to do about it is another. The central issue is one of learning.

The problem of learning in models that assume rational expectations has received increasing attention lately.[1] The approach taken in many papers treats stability of equilibrium as a problem in learning. That is, the issue of convergence to rational expectations equilibrium (REE) is presumed tantamount to the question of how agents acquire sufficient information to weed out systematic expectational error. While several modeling approaches have found such "stability" under different and reasonably plausible assumptions, there are no general theorems. More importantly, however, even the limited results found in these models presume continuous market clearing. Thus, the meaning of stability is quite restricted. The fundamental issue—how individual behavior will lead to the necessary price adjustment—is never explicitly modeled. Neglect of this issue is not new; it has long hindered progress in general equilibrium theory.

The purpose of this paper is to examine carefully the assumptions about individual behavior required for stability in models where agents learn to form rational expectations. Section one provides a restatement of the importance of stability analysis for deriving meaningful results from equilibrium models, and introduces the idea of developing learning models to describe the transition process to systemic equilibrium.

To illustrate the correspondence between learning processes and stability of REE, two examples are presented. The first, presented in section two, presumes rational agents know the structure (that is, the functional form) of the true economic model, but not the parameters. The example presented in section three presumes agents don't even know the model structure while they are learning. The precise meaning of stability in both models is discussed in section four. A distinction is made between expectational equilibrium and equilibrium of the aggregative economy. In section five, we discuss the difficulties facing the researcher who seeks to model

1 For a concise review of these models see Blume, Bray, and Easley (1982).

learning in an aggregative economy. The issues are developed in a general model employing a notion of equilibrium proposed by Frank Hahn (1973). Section six offers concluding remarks.

I. Importance of Stability

Analysis of positions and characteristics of equilibrium is by far the most widely accepted mode of economic analysis. Typically, such equilibria are derived from (or presumed to be) the solution of individual optimization problems. A key hypothesis that begets coordination of individual plans (aggregative consistency) is that certain variables—usually prices—take on values that make all individual plans mutually consistent. Under these circumstances, no individual has any incentive for further change. Economists rarely specify a behavioral process that could account for how variables, like prices, adjust to recoordinate individual plans when conditions change. Rather "changes" in equilibrium outcomes are generally developed in comparative static analysis, which compares equilibria corresponding to different values of underlying parameters.

The use of comparative statics in economics was first explained in rigorous detail by Samuelson (1947). He recognized, however, that to obtain definite *operationally meaningful* theorems in comparative statics, one has to specify a hypothesis about the dynamical properties that will lead to equilibrium values. The 'duality' between the problem of stability and the problem of deriving fruitful theorems in comparative statics is what Samuelson called the Correspondence Principle.

The importance of dynamical foundations has recently been restated by Fisher (1983). He argues that if general equilibrium models are to be of any use then we must have some confidence that the system is *stable,* that is, that it must converge to an equilibrium, *and* that such convergence to equilibrium must take place relatively quickly:

> If the predictions of comparative statics are to be interesting in a world in which conditions change, convergence to equilibrium must be sufficiently rapid that the system, reacting to a given parameter shift, gets close to the predicted new equilibrium before parameters shift once more. If this is not the case, and *a fortiori,* if the system is unstable so that convergence never takes place, then what will matter will be the 'transient' behavior of the system as it reacts to disequilibrium. Of course, it will then be a misnomer to call such behavior 'transient' for it will never disappear. (p. 3)

Fisher goes on to emphasize his point in the context of models assuming rational expectations:

> In such models, analysis generally proceeds by finding positions of rational expectations equilibrium if they exist. At all other points, agents in the model will have arbitrage opportunities; one or another group will be able systematically to improve its position; ....The fact that arbitrage will drive the system away from points that are *not* rational expectations equilibria does not mean that arbitrage will force the system to converge to points that are rational expectations equilibria. The latter proposition is one of stability and it requires a separate proof. Without such a proof—and, indeed without a proof that such convergence is rapid—there is no foundation for the practice of analyzing only equilibrium points of a system which may spend most or all of its time far from such points and which has little or no tendency to approach them. (pp. 3-4)

Fisher argues that analysis of this problem requires a full-dress model of disequilibrium — one that is based on explicit behavior of optimizing agents.[2] A general model would accommodate trading, consumption and production while the model is out of equilibrium. That is, such an approach would provide a theoretically based alternative to the Walrasian auctioneer. Arbitrage would follow from *individual* rationality. Unfortunately, practitioners of this approach have not advanced the subject enough to address the stability of model-consistent (that is, of rational) expectations.

The stability of REE has been addressed, extensively, however, on a less fundamental level. This approach presumes that markets clear and that REE is the true underlying *long-run* equilibrium. It examines different processes by which agents might acquire (learn) the information necessary for an expectations equilibrium consistent with REE. An important paper by Cyert and DeGroot (1974) defends the use of models of the learning process:

> The attempt to develop process models immediately opens us to the criticism of developing *ad hoc* models. We acknowledge that there may be a large number of models that could potentially describe the process to equilibrium. Our position is

2 Fisher (1983) does make a contribution in this direction but only under the assumption of perfect foresight His monograph illustrates the burden that lies ahead of any serious theoretician in this matter.

> that, while the models have a certain amount of face validity, our major contribution is the introduction of an explicit learning process described in Bayesian terms. The notion of developing models to describe the transition process toward equilibrium of a system disturbed by some random shocks may be questioned by some economists. The development of comparative statics and the neglect of dynamic analysis is in part a reflection of such attitudes in the profession. Yet without well-developed process models, the concept of rational expectations is essentially a black box. (p. 522)

Thus, models of the learning process are essentially provisional tools that enable us to interpret REE in a more realistic way. We may think of the development of such models as an attempt to justify the use of the rational expectations hypothesis.

These models, at the very least, allow us to ask if it is conceivable that agents could "learn their way" to equilibrium in the model at hand. This problem is not simple. Because agents are presumed to base their decisions on their own estimates of a model's parameters, their actions cannot be considered exogenous to parameter estimation. If estimates of parameters change, agents adjust their behavior accordingly. Moreover, agent actions generate the data on which the estimates of parameters are made, making learning an endogenous process. To correctly specify the model, agents would need to take the endogeneity into account. Conventional econometric techniques are typically not well-suited for this task.

The question of convergence to REE has been examined in two frameworks. The first assumes that agents know the functional form of the model or, at least, the appropriate specification of the likelihood function underlying the generation of the data. In this framework, agents are presumed to learn about the value of parameters either through classical statistical methods, repeated use of Baye's Theorem, or some other statistical method. The second framework does not require that agents know the model, although some of this work assumes that agents base their expectations on the basis of one model chosen from a set that includes the true model.

II. Learning When Agents Know the Model

To illustrate a process of learning and its connection to the stability of REE, we first examine one approach taken by Cyert and DeGroot (1974). They proposed to design models that describe the process by which rational expectations may develop within a market. They build on a version of the cobweb model used by Muth (1961) to propose the concept of rational expectations. Muth posited a partial equilibrium model for a homogeneous good with a production lag. Using the notation of Bray and Savin (1986), the market equations have the following form in any period t:

(1) $d_t = m_1 - m_2 p_t$ (demand)

(2) $s_t = m_0 + m_5 p_t^e + v_{2t}$ (supply)

(3) $d_t = s_t$ (equilibrium),

where m_0, m_1, m_2, and m_5 are fixed parameter values; p_t is the market-clearing price of the good; p_t^e is the market-anticipated price before trade takes place; and v_{2t} is an exogenous shock to supply. It is assumed that all units demanded are consumed in period t and that firms make production decisions before trade takes place. Thus, the deterministic component of supply is fixed in period t.

The assumption of market clearing yields:

(4) $p_t = M - a p_t^e + u_t$,

where $M = (m_1 - m_0)m_2^{-1}$, $a = m_5 m_2^{-1}$ and $u = -m_2^{-1} v_{2t}$.

Under the usual assumption of rational expectations, the market-anticipated price equals the objective mathematical expectation for price given the model and as conditioned on the data available when the expectation was formed.[3] That is, $p_t^e = E_{t-1}(p_t)$. Cyert and Degroot propose a similar basis for determining p_t^e. They assume that expectations are consistent, meaning that the firms' expectations are based on the mechanism implied by the model. The essence of this distinction is that while agents are presumed to know the correct likelihood functions, they are not required to know the parameter values. Cyert and DeGroot derive an explicit expression for market-anticipated price by taking expections of both sides of (4), substituting p_t^e for $E_{t-1}(p_t)$ and solving for p_t^e:

(5) $$p_t^e = \frac{E_{t-1}M - E_{t-1}[u_t]}{1 + E_{t-1}(a)}$$

Note that since the parameter values are unknown, the market-anticipated price is expressed in terms of *expected* values of the parameters, not true values. Agents (firms) learn to form rational expectations if, with additional data, the expected values of the parameters converge to their true values. Note also that market-anticipated price will differ from actual market

3 It is perhaps more accurate to call such expectations model-consistent instead of "rational." (See Simon 1978).

price both because of expectional error and the supply shock.

The economic process evolves as follows: In each period, the firms form consistent expectations of the price in the next period h-om (5) based on expected parameter values (priors). The actual price is then generated according to the model incorporating the consistent expectations, that is, price is given by (4). The observed values of actual price contains new information that leads firms to change their expectations of the values of the parameters and, hence, to change their expectations of the price in the following period. The actual price in the next period is again generated by the model and the process continues in this manner.

Cyert and DeGroot verify that such a process can, in fact, converge to REE when slope coefficients m_2 and m_5 are known, even if intercepts m_0 and m, are not. In this example, the authors assume that the random (supply intercept) error has a normal distribution with mean 0 and known precision (inverse of variance). Moreover, they posit a posterior distribution for M at the end of period t-1 that is normal with finite mean and precision. Finally, they show that a Bayesian updating of parameter values does converge to the true value of M.

The convergence result was encouraging. It showed that one need not assume all knowledge is innate, but that, h-om a Bayesian point of view, the relationship between expectations and other variables in the model arises naturally when economic agents form expectations in a manner internally consistent with the mechanism generating the data. In simple terms, this means that agents can learn parameter values even though their expectations affect outcomes of the model. An essential assumption is that all agents can correctly specify likelihood functions of unknown parameters, that is, that they "know" the structure of the model.

An implicit assumption underlying this and all other models obtaining convergence when agents know the model is that the solution concept being employed is Nash equilibria. This means that each agent has no reason to alter his specification of the likelihood function, given his own specification and those of all other agents. Thus, the approach assumes not only that agents know the model, but also that agents know that other agents know the model. The implications of this are discussed by Blume, Bray, and Easley (1982):

> The concept of a Nash equilibrium in learning strategies has much to commend it. Any other learning process is to some degree ad hoc; if some or all of the agents are learning by using mis-specified models, at some stage they should realize this and change the specification. Nash equilibria in learning strategies are rational expectations equilibria in which agents take into account their uncertainty about features of the world which they are assumed to know in standard models of rational expectations equilibria. However, Nash equilibria in learning strategies are liable to be considerably more informationally demanding than conventional rational expectations equilibria, as agents require extensive knowledge about the structure and dynamics of the model that prevails while they learn. There may also be problems with the existence of equilibrium. Thus, while this approach yields convergence to a conventional rational expectations equilibrium, its extreme informational demands make it an unsatisfactory answer to the initial question of how agents learn how to form rational expectations. (p.315)

In sum, employing the Nash solution concept begs the question as to how agents learn the structural form of the underlying model. Moreover, it provides no economic justification for why any agent should believe that all other agents will know what forecast methods other agents use. What incentives are there for such behavior?

III. Learning When Agents Don't Know the Model

When agents know the structural form of the economy, it is a relatively straightforward task to identify informational requirements sufficient to obtain convergence to REE. As we have seen, however, these requirements are quite demanding. They presume that agents have extensive knowledge about what other agents believe as they all learn about the parameters. It is somewhat interesting, however, that in situations where agents don't know the model, convergence can occur under somewhat weaker assumptions about the learning process. These results, however, are model specific. Other, equally reasonable, approaches lead to instability of REE. Achieving convergence depends not only on the nature of learning but on the structural and stochastic parameters of the underlying model.

When agents don't know the model, the problem of learning has been addressed in two distinct ways. The first approach provides an explicit model that allows agents to modify their forecasting rules in light of observable outcomes (see Blume and Easley [1982]). Typically, they choose among a set of models that includes the true one. Convergence occurs when all agents eventually adopt the true model. In this approach, we find that the results are mixed. In some models, rational expectations equilibria are locally stable but not unique.

The second approach examines the possibility of convergence when agents never switch models, despite the fact that they may have misspecified the model while they are learning. Essentially, this approach considers whether "irrational" learning can lead to rational expectations equilibrium.

An interesting model by Bray and Savin (1986) examines the second kind of learning. An appealing feature of this model is that agents learn using conventional techniques—such as by estimating the parameters of a standard linear-regression model. While this is the correct econometric specification for their postulated model in *equilibrium,* the econometric model is misspecified while people are learning. Moreover, Bray and Savin use simulations to examine the rate at which convergence takes place and to assess the possibility that agents discover that their estimated model is misspecified.

Following Townsend (1978), they extend the cobweb model to include stochastic demand, to allow for exogenous shocks to aggregate supply, and to accommodate diversity of firm expectations and decisions. All firms are assumed to face the same technology as defined by a quadratic cost function

$$c_{it} = q_{it}^2/2m_5,$$

where $m_5 > 0$ and q_{it} is the output of firm i at date t. Under the profit-maximizing postulate, firm i chooses an output level equal to $m_5 p_{it}^e$ where p_{it}^e is the mean of its prior on market-anticipated price.4

The aggregate of these expectations over all firms is denoted as $\bar{p}_t^e$. Their model is thus given by:

$$(6)\quad d_t = m_1 - m_2 p_t + v_1, \qquad \text{(demand)}$$
$$(7)\quad s_t = m_5 \bar{p}_t^e + x_t' m_4\ v_{2t}, \qquad \text{(supply)}$$
$$(8)\quad d_t = s_t \qquad \text{(equilibrium)},$$

where $x_t' m_4 + v_{2t}$ is an exogenous supply shock and x_t' is observable. Market clearing implies that:

$$(9)\quad p_t = x_t' m + a\bar{p}_t^e + u_t,$$

where x,' is redefined to include 1 as the first component and $m = [m_1 : m_4]\, m_2^{-1}$ and as in (4) $a = m_5 m_2^{-1}$, but $u_t = (v_{1t} - v_{2t}) m_2^{-1}$.

If agents knew both the model structure and the values of the parameters, the REE price forecast would be:

$$(10)\quad p_{it}^e = x_t' m (1-a)^{-1}$$

for all i, assuming $a \neq 1$. Together (9) and (10) imply that the REE price, for each t, is:

$$(11)\quad p_t = x_t' m (1-a)^{-1} + u_t.$$

The linear relationship between actual price and exogenous-supply influences applies only in equilibrium when agents all share the same expectations. This simple relationship does not hold when agents are learning the values of the parameters. To illustrate this, Bray and Savin assume agents maintain the hypothesis that:

$$(12)\quad p_t = x_t' b + u_t$$

satisfies the assumptions of the standard linear model, and estimate b accordingly. They consider the consequences that agents may be classical or Bayesian statisticians. If all agents (firms) are Bayesian statisticians who assume u_t is *i.i.d.* as $N(0,\varphi^2)$, and if firm i's initial prior on b is b_{io} and prior on precision is S_0/σ^2, firm i may obtain revised priors on b after observing $(x_1, p_1), \ldots, (x_{t-1}, p_{t-1})$, which will have mean $b_{i,t-1}$, and precision S_{t-1}/σ^2 where,

$$(13)\quad b_{i,t-1} = \left(S_0 + \sum_{j=1}^{t-1} x_j x_j'\right)^{-1} \left(S_0 b_{i,0} + \sum_{j=1}^{t-1} x_j p_j\right)$$

and

$$S_{t-1} = S_0 + \sum_{j=1}^{t-1} x_j x_j'.$$

Note that the classical statistician is essentially a Bayesian Statistician whose initial prior on b is diffuse ($S_o = 0$).

With this revised prior, agent i's forecast of p_t is $p_{it}^e = x_t' b_{it}$. The aggregate of market-anticipated price is $p_t^e = x_t' b_t$, where b_t is an aggregate of b_{it} over all firms. Substituting this in (9) gives:

$$(14)\quad p_t = x_t'(m + ab_{t-1}) + u_t.$$

Equation (14) generates the actual observed price given both the market mechanism and the way agents form expectations. Note that the coefficient of x_{t-1}, $(m + ab_{t-1})$, varies with time. Thus, agents are incorrectly assuming that price is generated by a standard linear model with a constant coefficient. The model is incorrect because it fails to take account of the effects of learning on the parameter values. If agents

4 Bray and Savin consider a continuum of firms producing a homogenous good. The set of firms is the unit interval [0,1] indexed by *i*. Thus, market-anticipated price is a Lebesque intergral. It is in that sense an average expected price.

knew what we know, they would not use linear regressions to form expectations.

Despite the fact that agents may misspecify the model, Bray and Savin are able to show that: (1) the difference between the individual estimates b_{it} and the average estimate b_t tends to zero with probability one as t tends to infinity; and (2) the average estimate b_t cannot converge to any value other than the REE value $m(1-a)^{-1}$. The intuition they offer is that if b_t tends to b for large t, the actual price is $p_t = x_t'(m + ba) + u_t$. Since the data generation process closely approximates the standard linear model with coefficient $m + ba$, the estimate b_t tends to $m + ba$, which is impossible unless $b = m(1-a)^{-1}$.

These results enable Bray and Savin to obtain the restrictions on parameters a and b that are necessary and sufficient for existence, uniqueness, and 'stability' of the REE. The conditions are precisely the same conditions for the existence, uniqueness, and tantonnement stability of a market in which supply and demand are simultaneous, that is, a Walrasian model in which supply at time t is based on actual price at t as opposed to market-anticipated price.

The intuition behind the convergence process of the Bray-Savin model is straightforward. Suppose suppliers' beliefs are such that, in the aggregate, they underestimate price corresponding to a given set of exogenous influences. This would lead them to supply less than they otherwise would have done. Consequently, the auction would assure that the market-clearing price would be above the market-anticipated price. Taking account of the newly observed price, suppliers would, on average, raise their estimate of price corresponding to the same set of exogenous influences. Provided they don't overreact, learning would bring them closer to REE in each successive period.

An important feature of the Bray-Savin approach is that the specified learning process is reasonably simple and plausible despite the fact that the underlying mechanism is much more complicated. A potential problem, however, is that agents might discover that they have incorrectly specified the model. Since the estimated model is not the true one while they are learning, the data may confirm the misspecification. On the other hand, if convergence is sufficiently fast, their test may fail to spot the misspecification.

To examine this possibility, Bray and Savin use computer simulations. The simulations suggest that the rate of convergence can be slow if the ratio of the slopes of demand and supply are near the boundary of the stability region, especially if the initial prior mean is incorrect for REE and the prior precision is high. Thus, the fact that equilibrium may be stable may not mean much. Equilibrium behavior may not provide a reasonable enough approximation of the actual behavior to be meaningful.

Bray and Savin also use the simulations to examine the likelihood that agents will discover that their estimated model is misspecified. Agents are presumed to examine the Durbin-Watson statistic as a diagnostic check for model misspecification. The results suggest that if REE is stable, and if the estimates converge rapidly, agents are unlikely to identify the misspecification. Thus, it is reasonable to expect that agents could persist using simple linear (misspecified!) methods and eventually learn all they need to know to form expectations in a manner consistent with REE.

IV. The Meaning of Stability

The major contribution of the learning models discussed above is that they provide an explicit framework for describing a transition process toward equilibrium of a system disturbed by some random shocks.[5] While they successfully demonstrate how rational expectations may develop in a perfectly competitive market, learning models do not provide the kind of underpinnings sought by general equilibrium theorists in stability analysis. They focus only on the development of *expectational* equilibrium. No attempt is made to specify the dynamics of price formation. Rather, the framework implicitly assumes an auction process not substantively different from that required to achieve standard competitive (Walrasian) equilibrium.

Thus, these models beg the central question that continues to plague general equilibrium theorists: how to derive behavioral foundations for price adjustment. This is not a criticism specific to the models at hand, but is a fundamental problem with all equilibrium models, including fixed-price models. To appreciate the problem, it is useful to review briefly the theoretical foundations of the stability of competitive equilibrium.

Stability analysis of competitive equilibrium builds on the earliest notions about price adjustment, which were imbedded in the "law of supply and demand." It essentially holds that in competitive markets, prices will rise when there is excess demand and fall when there is

5 It is the view of Cyert and DeGroot that such a process has to be developed if the rational hypothesis is to be a scientific truth rather than a religious belief.

excess supply. This argument has the familiar dynamic formulation first proposed by Samuelson in 1941 (see 1947):

$$(15)\quad \frac{dp}{dt} = h(D-S),\ h(0) = 0, \text{and } h' > 0 \text{ and}$$

$$(16)\quad D = D(p,a) \quad S = S(p),$$

where D and S are quantities demanded and supplied for a homogeneous good; p is the market price of that good, and a is an exogenous shift parameter. The properties of the *static* demand and supply functions are derived under the standard hypothesis that households and firms maximize familiar objective functions. Formal proofs for the stability of competitive markets essentially derive sufficient conditions for the dynamic relations expressed by (15) to yield time paths of prices that approach their equilibrium values from arbitrary points.[6] Unfortunately, global stability is obtained only under very severe restrictions on excess demand functions, the most notable being the assumption that all goods be gross substitutes.

While the assumption implicit in (15) seems plausible, it is beset by some important conceptual difficulties. The first problem is that (15) has never been deduced as the maximizing response of economic agents to changing data. Sonnenschein (1973) has shown that the standard assumptions about individual behavior do not imply any restrictions on excess demand functions beyond homogeneity of degree zero and Walras' Law—conditions not sufficient for stability. Thus, adjustment to Walrasian equilibrium lacks the rigorous basis that is accorded to the properties of *static* supply and demand functions. Moreover, it is not clear who changes prices when the system is not in equilibrium. In competitive equilibrium, sellers and buyers are typically treated as price takers. Therefore, it is presumed that there is some implicit market manager who sets price.

The idea of a market manager whose behavioral rule for price adjustment is given by (15) was, of course, the ingenious answer given by Walras. This approach is tantamount to an assumption that all consumers and suppliers gather in one place. The market manager quotes a set of prices for each commodity. Then each trader writes on a piece of paper (a ticket) the amounts of each of the commodities he wishes to buy or sell at the given set of prices. If there is excess demand for the commodity i, the manager raises the price of i, if there is an excess supply for commodity j, he lowers the price of j. Each time a new set of prices is quoted, each trader submits a revised ticket. The process continues until excess demand is zero, that is, equilibrium price is determined. *Until then no trade or production takes place.*[7] Essentially, this is a description of a *timeless* process by which market clearing can be achieved and thus fails to help in understanding the *dynamics* of price.

The only difference between this Walrasian situation and the one implied by the Bray-Savin model is that, under the latter, suppliers commit to production levels prior to trade. Suppliers therefore must base their decisions for output levels on the anticipated price for their good. While these anticipated prices may initially differ when suppliers use Bayesian learning models, the observed market-clearing price at any point in time must be the same for all suppliers. Because the model used by suppliers to determine anticipated price specifies the *single* market-clearing price as the dependent variable, a tantonnement process is necessary to generate data that is essential for the process to be operational. Clearly, the auction process plays an essential role in consolidating information that is necessary for convergence.

A key distinction between the Bray-Savin process and a pure Walrasian process involves a restriction on what suppliers can learn about the *aggregate* supply function. In a standard Walrasian auction, suppliers are free to adjust the quantities they would produce for all the prices quoted. In this way, the auction process also synthesizes for all agents all the relevant properties about both aggregate supply and demand. In the Bray-Savin model, on the other hand, suppliers offer the same quantity for all prices quoted. The auction essentially determines the point on the demand curve that corresponds to the predetermined level of output. That is, the auction synthesizes only responses of consumers to the array of price quotes. Suppliers learn from the (temporaty) equilibrium price about whether they under or overestimated prices, but they do not know how well other suppliers estimated prices and, consequently, how aggregate supply might adjust to different prices. This information is revealed only through a succession of auction outcomes.

Notwithstanding information lags, the situation in the Bray-Savin model may not be very plausible for markets where prices are not

6 See Arrow and Humicz, (1958) and Arrow, Humicz, and Block (1959).

7 The requirement that no trade take place before equilibrium is determined is essential if such a process is to converge to a unique equilibrium. Fisher (1983) shows how trading at "false" prices affects endowments of agents and, hence, the ultimate outcome of the process. Thus equilibrium would depend not only on initial endowments, but also on the process that achieves equilibrium. Such a property is sometimes called hystersis.

determined by auction processes, even though the markets may appear competitive. Arrow (1959) noted that there is an inconsistency between the assumptions required of individuals in a state of equilibrium and those necessary to explain behavior in disequilibrium. He argued that, in situations of excess demand, firms do not behave as price takers but, in fact, use price-setting tactics similar to the profit-maximizing tactics of a monopolist.

The problem is somewhat more complex in that a firm's competitors will also be raising prices. Moreover, on an individual basis, no seller would have the incentive to agree to an auctioneer, since the market-clearing price would be less than what he could obtain in disequilibrium. In situations of excess supply, Arrow shows that firms are still monopolists, but buyers are monopsonists; thus, it is a joint decision that establishes price. The lesson is that disequilibrium price adjustment may need to recognize elements of imperfect competition.

Theories of imperfect competition require elements of strategic behavior, that is, situations in which two or more agents choose strategies that interdependently affect each other. Such problems involve game theory. Arrow (1986) recently concluded that analysis of games with structures that are extended over time leads to very weak implications—in the sense that there are a continua of equilibria. The fact is that we know very little about how economic man interacts with other economic men in situations of excess demand or supply. Unfortunately, the learning models considered above provide no shortcuts around this problem.

V. **Learning** in the Macroeconomy

While Bray-Savin learning shows that agents using "plausible" models can "learn their way" to REE in *auction* markets, it is doubtful that such a result could obtain for a highly decentralized market economy. This section identifies some difficulties, apart from the problems of modeling strategic behavior, that confront a modeler seeking to extend the Bray-Savin result to the macroeconomy. The issues are sketched using a notion of equilibrium proposed by Frank Hahn (1973).

It is the essence of a decentralized economy that individuals have different information.[8] Furthermore, each individual is specialized in certain activities and has, in general, specialized knowledge about those activities. There is no reason to believe that individuals base their expectations on the rather general kind of information that econometricians use. Instead, different individuals base their decisions on different sets of information. In short, a "plausible" model of learning in macroeconomics would need to incorporate the existence of heterogenous information.

The problem of learning when agents have incomplete and different information has recently been studied by Marcet and Sargent (1986b).[9] In their approach, agents use least-squares estimation to formulate expectations that they think are relevant to understanding the underlying law of motion as it affects them. Marcet and Sargent assume that agents do not respecify their regressions over time, but maintain the same "theory" about the world they observe. As with Bray-Savin, their model accommodates feedback from agent expectations to the actual law of motion of the system. Marcet and Sargent show that the existence of informational asymmetries does not preclude convergence to REE when the law of motion is a linear stochastic process.

While the class of learning models studied by Marcet and Sargent imposes some restrictions on the economic environment, the mechanism can accommodate a wide class of economic theories. Nothing inherent in the least-squares learning schemes precludes convergence to a non-Walrasian equilibrium.

The idea that an economic system might converge to a non-Walrasian equilibrium is, no doubt, difficult to accept for some economists. For example, won't arbitrage opportunities arise? Although there would be such opportunities vis-a-vis a Walrasian ideal, it is not evident that agents can perceive the ideal to identify the opportunities. Because agents don't observe continuous market-clearing equilibrium outcomes in a non-Walrasian environment, there is no reason that their expectations will ever become consistent with Walrasian equilibrium in the long run.

The point here is that agents' expectations *could* become consistent with the conventions (including price-setting mechanisms) that determine the laws of motion of the system. While equilibrium expectations would not be systematically inconsistent with observed outcomes of the model, agent choices would not necessarily be Pareto-optimal. Nevertheless, to the extent that market forces operate, it is conceivable that price-setting conventions could develop

8 This point and the following were made by Arrow (1978) as a criticism of the use of Muthian expectations to the aggregate economy.

9 See Marcent and Sargent (1986a).

that would lead to an equilibrium that is "approximately competitive."[10]

To understand what "approximately competitive:' might mean, it is useful to introduce a notion of equilibrium proposed by Hahn (1973). In Hahnian equilibrium, each agent holds his own theory about the way the economy will develop *and* about the consequences of his own actions.[11] The agent abandons his theory when it produces systematic and persistent errors. To the extent the agent maintains a theory, his actions are conditioned on his perceptions about the laws of motion of such a system. The agent is said to be in equilibrium when he maintains his theory. The economy is said to be in equilibrium if it doesn't produce outcomes systematically and persistently inconsistent with agents' perceptions.

In the context of Marcet-Sargent learning, the theories agents hold are embodied in the regressors they choose. Under the assumption that the true law of motion is linear, agents will *ultimately* not be able to falsify their theories.[12] Thus, they would have no reason to abandon the theory. In the context of Hahn's notion, each agent would be considered in equilibrium. Moreover, since the actual outcomes would not be inconsistent with predictions of agents' theories, the economy would be in equilibrium.

Although Hahn was not completely precise about his notion of equilibrium, he clearly intended it to be more general than the equilibrium obtained in Marcet-Sargent learning. For Hahn, the *structure* of true "laws of motion" need not be independent of the theories agents choose. The theories could determine the structure of the laws of motion—a structure that could have nonlinearities that agents could never comprehend. In the model of Sargent and Marcet, the underlying structure is constrained to obey a linear (stochastic) law of motion.

Another important difference is that Hahnian equilibrium would accommodate agent behavior that could be inconsistent at any point in time, but not persistently so. In the Marcet-Sargent limit point, agents ultimately learn enough so that their expectational error is white noise, that is, agent actions lead to a steady-state equilibrium. This means that agent expectations would ultimately become mutually consistent in every period, given what they can know. Because Hahn only imposes that actions (expectations) of agents not be systematically *and persistently* inconsistent, his equilibrium would not be unique. Hence, at any point in time, equilibrium would be distinct from a steady state. Local stability would mean that, for short enough periods and for small enough disturbances, the set of equilibria is large but that it shrinks.

It is useful to stress here that the agents in the Hahnian concept of equilibrium are rational in the spirit of McCallum's intuition. That is, agents do not maintain their "theory" when systematic errors are sufficiently persistent for falsification of the theory. However, the meaning of rationality is much less restrictive (hence more plausible) than is presumed in conventional formulations of rational expectations. Agents in Hahnian equilibrium are rational only in a subjective sense. Nothing inherent in the Hahnian approach would assure that aggregate economic outcomes would converge to a stationary stochastic process with a unique *objective* probability distribution. Without such convergence, agents' subjective expectations could not coincide with an objective expectation of aggregate outcomes. Imposing the restriction that agents' *subjective* expectations be mutually consistent with each other and with a particular *objective* probability distribution underlying a given model seems too restrictive to be very useful in practice. This point has been developed in an alternative model proposed by Swamy, Barth, and Tinsley (1982).[13]

An attractive feature of Hahnian equilibrium concept is that it can accommodate more plausible market structures such as the "approximately competitive" economy suggested above. Agents may adopt stable reaction rules that allow them to cope in a competitive environment without requiring unreasonable computational abilities necessary for analyzing the aggregative

10 The meaning of "approximately competitive" equilibrium devel oped below is different from the sense that allocations in the core are said to be approximately competitive. The latter refers to outcomes of a bargaining process, while the former refers to outcomes derived from habitual behavior that allows agents to "survive" in a competitive economy.

11 Clearly, this notion abstracts from many difficult problems posed by strategic behavior For a more complete description of Hahn's notion of equilibrium and a comparison to the Austrian view, see Littlechild (1982).

12 It is not evident that agents would maintain their theories in the early stages of learning For any given mode one might want to provide sensitivity analysis a-la Bray-Savin.

13 Swamy et. al., show how confounding 'objective' and 'subjective' notions of probability may violate the axiomatic basis of statistical theory. They propose an alternative model for aggregation of subjective expectations. The problem with conventional formulations of the rational expectations hypothesis in macroeconomic models lies not with the concept of individual rationality but with the context in which it is developed—namely in the representative agent model. Once one allows agents to differ both in the information they have and in the theories they hold, a model can accommodate arbitrage opportunities that are deemed essential for a process leading to a rational expectations equilibrium. How agents learn to recognize arbitrage opportunities, however, remains an open, but difficult, issue.

impacts of strategic behavior. Moreover, the equilibrium of such a model would accommodate a wide variety of nonstationarities in the variables. Nevertheless, Hahnian equilibrium too has some severe limitations.

A key difficulty for a researcher modeling approximately competitive environments is that an infinite set of plausible conventions could be developed that would lead to "model consistent" (rational, in the sense of Hahn?) expectations. This may not be relevant for the individual agent in Hahnian equilibrium. The agent could be satisfied with his own conventions for dealing in his specialized corner of the world. A macromodeler, on the other hand, may not have access to all relevant information. His estimates of underlying relationships would be inconsistent because of omission of relevant explanatory variables bias. Thus, it may be impossible for a modeler of *aggregate* economic activity to discover adequately the law of motion for the economy as a whole, even when the economy is in Hahnian equilibrium. This, of course, is the essence of the Austrian criticism of macroeconomics, both Keynesian and New Classical.[14]

The most difficult problem for modeling learning in an approximately competitive model, however, is the situation in which agents change theories.[15] In the context of Hahnian equilibrium, this is the problem of global stability. That is, when a shock to equilibrium is so big, it causes agents to change their theories. Hahn argued that it is impossible to make any claims about global stability. He concluded that this limitation was imposed by the current state of economic knowledge. Economists know very little about how agents adapt to a changing economic environment.

When confronted with the limits of equilibrium analysis, economists are often more willing to invoke a convenient fiction than to modify their fundamental tools. The urge to close the model typically prevails over a venture into a methodological frontier. As is often noted, some people searching for a lost wallet at night prefer to look under a street lamp even though it may be more likely that they lost the wallet in the dark alley. Hahn's proposed reformulation of equilibrium was useful in illuminating the problems of learning in a large, decentralized economy. In this sense, it demonstrates the potential value of building new streetlamps.

VI. **Concluding** Remarks

This paper opened with the idea that rational, purposeful individuals have incentives to weed out systematic errors in their own expectations. Thus, it is argued that economic models should not allow expectational errors to persist. Conventional formulations of rational expectations, which assume Walrasian market-clearing, do not violate this restriction. The implicit auction process works to assure that all decisions are mutually consistent both with what agents can know about the model and with the underlying model.

This paper presented the Bray-Savin result that shows that agents may use "plausible" learning mechanisms to "learn their way" to rational expectational equilibrium in auction markets. Thus, learning models extend the results of tatonnement stability analysis to situations where agents form model-consistent expectations about the environment they are in. The restriction that economic models not permit systematic expectational errors to persist, however, does not require that agents behave in a mutually consistent manner in each period of time as in Walrasian equilibrium. The restriction is weaker than that and hence allows for a broader scope in the meaning of rationality than is generally considered in conventional formulations of the rational expectations hypothesis. That is, the restriction allows a broader class of economic models than the Walrasian economy.

The model of "approximately competitive" equilibrium sketched in this paper illustrates one potential subclass of such models. The sketch provides a plausible example of how rational, self-seeking agents might "learn their way" to non-Walrasian equilibria. Without an auctioneer in each and every market, a modeler cannot rule out such equilibria *a priori* simply by assuming agents have incentives to weed out systematic expectational errors.

14 Another way of looking at the same problem is that the specification of "approximately competitive" behavior in this paper is too general to have empirical content. Nevertheless, the researcher is free to specify his own set of conventions — provided, of course,. that they are logically consistent. Because of the difficulties in falsifying economic theories, one might choose among alternative specifications on the basis of out-of-sample forecasts. The foundations of such a method are found in Swamy, Conway, and von zur Muehlen (1985).

15 This is what Hahn calls learning. It is also the sense of learning examined by Blume and Easley.

REFERENCES

Arrow, Kenneth, J., L Hurwicz, and H.D. Block, "On the Stability of Competitive Equilibrium II," *Econometrica,* vol. 27, (January, 1959) pp. 82-109.

Arrow, Kenneth, J., and L Hurwicz. "On the Stability of the Competitive Equilibrium I," *Econometrica,* vol. 26, (October, 1958) pp. 522-552.

Arrow, Kenneth J. "Rationality of Self and Others in An Economic System." The Journal of *Business,* vol. 59, no. 4, part 2 (October 1986), pp. 385-399.

Arrow, Kenneth J. "Toward a Theory of Price Adjustment," The *Allocation* of Economic Resources, Stanford University Press, (1959), pp. 41-51.

Arrow, Kenneth J. "The Future and the Present in Economic Life," Economic *Inquiry,* vol. XVI, No. 2, (April 1978), pp. 157-169.

Blume, Lawrence E. and D. Easley. "Learning to be Rational,"Journal of Economic *Theory,* vol. 26 no. 2, (April 1982), pp. 340-351.

Blume, LE., M.M. Bray, and D. Easley. "Introduction to the Stability of Rational Expectation Equilibrium,"Journal of Economic *Theory,* vol. 26., no. 2 (April 1982), pp. 317-317.

Bray, M.M., and N.E. Savin. "Rational Expectations Equilibria, Learning and Model Specification," *Econometrica,* vol. 54, no. 5, September 1986. pp. 1129-1160.

Bray, M.M. "Convergence to Rational Expectations Equilibrium." Individual *Forecasters & Aggregate* Outcomes, Cambridge University Press. 1983, (pp. 123-137).

Cyert, Richard M. and Morris H. DeGroot. "Rational Expectations and Bayesian Analysis." Journal of Political *Economics,* vol. 82 no. 3, (May/June 1974), pp. 521-536.

Fisher, Franklin M. "Disequilibrium Foundations of Equilibrium Theory," Econometric Society Monographs in Pure *Theory #6,* New York: Cambridge University Press, 1983.

Hahn, F.H. "On the Notion of Equilibrium in Economics," Cambridge: Cambridge University Press, (1973).

Littlechild, S. C. "Equilibrium and the Market Process," Method Process, and Austnan Economics: Essays in Honor of *Ludwig* Von *Mises,* ed. Israel M. Kirzner, Lexington Books, 1982.

Marcet, Albert, and Thomas J. Sargent. "Convergences of Least Squares Learning Mechanisms in Self Referential Linear Stochastic Models," Manuscript, October 1986.

Marcet, Albert, and Thomas J. Sargent. "Convergence of Least Squares Learning In Environments with Hidden State Variables and Private Information," Manuscript, October 1986.

Muth, John F. "Rational Expectations and the Theory of Price Movements," *Econometrica,* vol. 29, no. 3 (July 1961), pp. 315-55.

McCallum, Bennett T. "Rational Expectations and Macroeconomic Stabilization Policy: An Overview,"Journal of Monty, Credit, and Banking - vol. XII, no. 4, part 2, (November 1980), pp. 697-746.

Samuelson, Paul A. Foundations of Economic Analysis, Harvard University Press, Cambridge, Mass., and London, England (1947).

Simon, H A *Models* of Man, Wiley Press, New York, 1957.

Simon, H A "Rationality as Process and as Product of Thought," American Economic Review, vol. 68, no. 2, (May 1978), pp. 1-16.

Sonnenschien, Hugo. "Do Walras' Identity and Continuity Characterize the Class of Community Excess Demand Functions?" *Journal* of Economic Theoy, vol. 6, no. 4, (August 1973), pp. 345-354.

Swamy, P.A.V.B., R. K. Conway, and P. von zur Muehlen. "The Foundations of Econometrics - Are There Any?" *Econometric* Reviews, vol. 4, no. 1 (1985), pp. 1-61.

Swamy, P.A.V.B., J. R. Barth, and P A Tinsley. "The Rational Expectations Approach to Economic Modelling,"Journal of Economic Dynamics and *Control,* Vol. 4 no. 2, (May 1982), pp. 125-147.

Taylor, John B., "Monetary Policy During a Transistion to Rational Expectations," *Journal* of Policital Economy, vol. 83, no. 5, (October 1975), pp. 1009-1021.

Townsend, Robert M. "Market Anticipations, Rational Expectations, and Bayesian Analysis," International Economic Review, vol. 19, no. 2, (June 1978), pp. 481-494.