Gravity or Dummies?
The Limits of Identification in Gravity Estimations

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Abstract

Trade economists often estimate gravity equations of international trade with fixed effects. Anderson and van Wincoop (2003, American Economic Review 93, 170–192) have shown the importance of controlling for multilateral trade resistances when estimating a gravity equation. This can be done by including exporter-time and importer-time fixed effects in a panel or exporter and importer fixed effects in a cross section estimation. I argue that this approach limits the identifiability of policy parameters that capture the effect of certain "club memberships" (EU, NAFTA, euro area, WTO, etc.) on trade flows. I show that, in the baseline case, only one effect can be identified, which precludes, for example, the estimation of separate effects on the exporter and the importer side. The magnitude, and even the sign, of the estimated club effect are very sensitive to the precise identification assumptions, which are often left unspecified in empirical studies. The underlying problem is that club membership provides some, but very little bilateral variation. When heterogeneous club effects are to be identified, the membership dummies can become perfectly collinear with the fixed effects. Empirical researchers may not be aware of the lack of identification, because standard estimation techniques often permit them to run perfectly collinear regressions. I illustrate the findings with estimating the effect of EU enlargement in 2004 on the trade flows of new and old members. Finally, I discuss potential solutions.

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1 Introduction

Measuring the effects of economic clubs (trade agreements, customs unions, currency unions) on bilateral trade has always been a central issue in the empirical trade literature. Many studies estimate gravity equations to quantify the effects of such club memberships, which are typically captured by dummy variables. Sometimes more than one club dummies are included to measure heterogenous effects.[1] The seminal paper of Anderson and van Wincoop (2003) has shown that gravity estimations should control for the multilateral trade resistances of the exporter and the importer countries to obtain unbiased estimates. This is often done by including fixed effects: exporter and importer effects in cross section estimations and country pair, exporter-time and importer-time effects in panels.

In this paper I consider the above fixed effects gravity specification and argue that its ability to identify club membership effects is very limited. In the baseline case, which includes cross section estimations and panels without sequential changes in club memberships, only one parameter can be identified, which implies that heterogeneous club effects cannot be estimated. For instance, it is not possible to separately identify the effect of joint versus one-sided membership. Similarly, one cannot estimate differential effects on the two directions of trade between countries. Moreover, the magnitude and also the sign of the estimate for the one identifiable parameter is very sensitive to the identification assumption. Depending on whether only joint membership or both joint and one-sided membership is assumed to affect trade, the estimate can turn into its negative. This casts doubt on the economic sense of the estimates.

The explanation for these problems is the following. The fixed effects net out all variation from the data except for the bilateral variation in cross sections and the time-varying bilateral variation in panels. At the same time, the club membership dummy has some, but very little bilateral (time-varying bilateral) variation. This limited variation allows the identification of only one parameter. Estimates under different identifying assumptions are simple linear transformations of this one parameter. When two or more club dummies are included, identification becomes infeasible due to perfect collinearity of the club dummies with the fixed effects.

I derive the main findings analytically for the simplest cross section case. Then I show, on the example of EU enlargement in 2004, that the identification problems extend to multi-

[1] Both the WTO and the euro is found to have affected the trade of two members and the trade of a member and a non-member differently. See Rose (2004), Baldwin, Skudelny and Taglioni (2005) and Flam and Nordström (2006).

period panel estimations. Attempting to measure the trade effect of EU entry on a database of bilateral trade flows of EU entrants and insiders I find that, depending on the identification assumption, the parameter estimates vary between -0.007 and 0.007, not different from zero statistically. Heterogeneous effects (on trade of an entrant with an insider versus on trade of two entrants) cannot be estimated because of perfect collinearity.

The presence of perfect collinearity is not always apparent from the estimation results. This carries the risk that the researcher overlooks the problem. I demonstrate it by using two popular methods in STATA: Fixed Effects Least Squares Dummy Variables (FE-LSDV) estimation and the method of OLS on the demeaned variables.[2] Under unidentifiability both methods fail to drop the perfectly collinear club dummy and report false estimates. The former drops one of the country-time dummies instead of the club dummy. The latter drops the club dummy only if the database is balanced, also including self-trade, which is rarely the case with trade databases.

The above identification problems do not always persist. They are not present for dummies with richer bilateral variation. Dummies like common language, common border, or FTA in general, typically incorporate more than one category (of language, border, FTA) and provide more bilateral variation even in cross section data. In panel databases, sequential entries to (or exits from) a club create extra variation and helps avoid the identification problem.

A further potential remedy, which I demonstrate on the example of EU enlargement, is to extend the database of EU entrants and insiders with trade flows of outsider (i.e. non-EU) countries. On such an extended database the fixed effects gravity can identify heterogeneous club effects up to four (linearly independent) club dummies. Nevertheless, I will show, this approach also has its limitations. Trade protection between entrants and outsiders fell significantly with EU enlargement. If I include an additional dummy variable to control for this change, the original identification problem returns.

Finally, one can look for alternative ways to account for the multilateral trade resistances in the gravity equation, a recent summary of which is in Anderson (2011). In this paper I provide alternative estimates for the EU's trade effect by using the trade cost index proposed by Novy (2008) and Head and Ries (2001). These estimates show that trade expanded by

[2]The former is fixed effects estimation with country pair fixed effects, while exporter-time and importer-time effects are included among the regressors as dummies. The latter method first nets out all the fixed effects from the RHS and LHS variables with the corresponding within transformation and then estimates by OLS. In principle, the two methods lead to identical estimates.

as much as 40% in the entering countries, and trade of two entrants was affected twice as strongly as trade between entrants and insiders.

The paper is structured as follows. Section 2 presents the cross section gravity equation and examines identifiability of club membership effects by solving for the estimates analytically under four different identification assumptions. Section 3 discusses the panel data case and presents the empirical application. The panel is extended with outsider countries in Section 4. Section 5 presents the alternative estimates based on the trade cost index. Summary and discussion is in Section 6.

2 Cross section gravity

The traditional way of estimating gravity equations has been criticized since the equation gained firm theoretical grounds. Anderson and van Wincoop (2003) show that the traditional gravity, which does not account for the multilateral trade resistances of the exporter and the importer countries, yields biased estimates. In the theory-based gravity, bilateral trade depends on the ratio of bilateral to multilateral trade barriers, in the form,

$$X_{ij} = \frac{Y_i Y_j}{Y^w} \left(\frac{\tau_{ij}}{\Pi_i P_j} \right)^{1-\sigma}, \tag{1}$$

where X_{ij} is the trade flow from country i to j, Y_i and Y_j are nominal income levels of the exporter and the importer, Y^w is world income, τ_{ij} denotes bilateral trade barriers and Π_i and P_j are the multilateral trade resistances. σ is the elasticity of substitution between all goods. The multilateral trade resistance of the exporter (Π_i) is an average of the bilateral trade barriers that the exporter faces in all the destination markets in the world. The importer's multilateral trade resistance (P_j) is an average of the bilateral trade barriers that the importer imposes on goods from all the countries in the world. Since these two depend on all the bilateral trade barriers of the exporter and the importer, omitting them from the empirical gravity equation introduces a bias in the estimation of the effect of any bilateral trade barrier.

Log-linearizing (1) yields

$$x_{ij} = y_i + y_j - y^w + (1 - \sigma) \ln \tau_{ij} - (1 - \sigma) \pi_i - (1 - \sigma) p_j, \tag{2}$$

where lower-case x, y, π and p denote natural logarithms of trade, income and multilateral trade resistance terms.

Getting an estimable equation from (2) is not straightforward. First, π_i and p_j are not observable variables. Although more recently several approaches have been developed to account for these two terms (e.g. Novy, 2008, Baier and Bergstrand, 2009), the most easily implementable, and hence the most popular, remains estimation with fixed effects. Second, $\ln \tau_{ij}$ is usually assumed to be some linear function of different observable bilateral trade barriers. In this paper I assume that it is a function of club membership, D_{ij}, with some parameter ρ and an additive measurement error, such that $\ln \tau_{ij} = \rho D_{ij} + \epsilon_{ij}$.[3] The ϵ_{ij} error is assumed to be uncorrelated with the club dummy.

Then, the estimable fixed effect gravity becomes

$$x_{ij} = \beta D_{ij} + \alpha_i + \eta_j + \varepsilon_{ij}. \tag{3}$$

The coefficient that gives the trade effect of club membership is $\beta = (1-\sigma)\rho$. The exporter and importer fixed effects, α_i and η_j, respectively, soak up both the income variables and the multilateral trade resistances. The error term is $\varepsilon_{ij} = (1-\sigma)\epsilon_{ij}$.

2.1 Identification assumptions

So far I have not specified the club membership dummy in (3). In order to do so, notice that there are two types of countries with respect to club membership, member (A) and non-member (B), which generate four types of trade flows. On Figure 1, x_{AA} denotes observations of trade between two members, x_{AB} trade from a member to a non-member, x_{BA} trade from a non-member to a member and x_{BB} trade between two non-members.

Figure 1: Trade flows of members and non-members

$i \backslash j$	member	non-member
member	x_{AA}	x_{AB}
non-member	x_{BA}	x_{BB}

For which observations D_{ij} is 1 and for which it is 0 depends on the precise identification assumption on the effects of club membership. It is customary to assume that trade between non-members (x_{BB}) is unaffected and to use these observations as benchmark (control group, reference group) in the estimation. What to assume about the three other types of trade flows is ultimately an empirical question. In what follows I consider four different identification assumptions:

1. x_{AA} is affected, the other three are the benchmark;

[3] I abstract from other bilateral trade barriers like geographical distance in order to make the following derivations as simple as possible. This can be done without loss of generality of the main results.

2. x_{AA}, x_{AB} and x_{BA} are all affected to the same extent, x_{BB} is the benchmark;

3. x_{AA}, x_{AB} and x_{BA} are all affected, but x_{AA} is to a different extent than the other two, x_{BB} is the benchmark;

4. x_{AB} and x_{BA} are affected to the same extent, x_{AA} and x_{BB} are the benchmark.

The identification assumption determines the exact formulation of the club membership dummy. Under the first assumption, D_{ij} is 1 for trade between two members and 0 otherwise. Under the second, D_{ij} is 1 for trade of pairs with *at least one* member and 0 for trade of two non-members. Under the third assumption, there are two club dummies: the first is 1 only for trade of two members and the second is 1 for trade between a member and a non-member. Under the fourth assumption, the club dummy is 1 for trade between a member and a non-member and 0 otherwise.

The difference between the first and the second identification assumptions is whether one-sided membership is also believed to affect trade. The third identification assumption allows the effect of joint membership to be different (usually stronger) than the effect of one-sided membership. Rose (2004, 2005) follows a similar approach by examining separate trade effects for joint and one-sided WTO membership. An example of the fourth identification assumption is the US-Canada border effect literature, initiated by McCallum (1995), which looks at how much less US and Canadian states trade across the border (international trade) than within the border (intranational trade). In this context, "member" denotes a US state and "non-member" a Canadian state, or vice versa.

2.2 Identifiability

I examine the identifiability of the club membership effect under each of the four identification assumptions by analytically deriving the β estimates. For that I assume that the cross section database includes all trade flows between n_A member and n_B non-member countries. The total number of observations is then N^2, where $N = n_A + n_B$. Notice that this sample is balanced and also includes self-trade, i.e. trade of a country within its own borders. In the empirical part I show that the main findings on identifiability extend to samples without self-trade, although the exact β estimates are then different.

A simple way to solve for the club effect estimate is to net out the exporter and importer fixed effects in (3) from the left-hand side variable and from the club membership dummy and run OLS regression on the demeaned variables. The ij-th element of the vector of the demeaned left-hand side variable, $\ddot{x}$, is

$$\ddot{x}_{ij} = x_{ij} - \frac{1}{N}\sum_{i=1}^{N} x_{ij} - \frac{1}{N}\sum_{j=1}^{N} x_{ij} + \frac{1}{N^2}\sum_{j=1}^{N}\sum_{i=1}^{N} x_{ij}, \tag{4}$$

and similarly for the club dummy, $\ddot{D}$.[4] Then, the estimate for β can be obtained via the OLS formula $\hat{\beta} = \left(\ddot{D}'\ddot{D}\right)^{-1}\ddot{D}'\ddot{x}$.

Let us first derive the demeaned club dummy under the first identification assumption (denoted as $\ddot{D}^{(1)}$). The club dummy can be expressed in the vector form,

$$D^{(1)} = \begin{bmatrix} \mathbf{1}_{n_A^2} \\ \mathbf{0}_{n_A n_B} \\ \mathbf{0}_{n_A n_B} \\ \mathbf{0}_{n_B^2} \end{bmatrix}, \tag{5}$$

where $\mathbf{1}_{n_A^2}$ is the column vector of ones of dimension n_A^2, $\mathbf{0}_{n_A n_B}$ is the column vector of zeros of dimension $n_A n_B$ and $\mathbf{0}_{n_B^2}$ is the column vector of zeros of dimension n_B^2. Applying the within transformation formula (4), the observations in the first row of (5) become

$$\begin{aligned} &\ddot{D}^{(1)}_{ij\in AA} = \\ &= 1 - \frac{1}{N}(n_A\cdot 1 + n_B\cdot 0) - \frac{1}{N}(n_A\cdot 1 + n_B\cdot 0) + \frac{1}{N^2}(n_A^2\cdot 1 + n_A n_B\cdot 0 + n_A n_B\cdot 0 + n_B^2\cdot 0) = \frac{n_B^2}{N^2}. \end{aligned}$$

The demeaned observations in the second row of (5) are

$$\begin{aligned} &\ddot{D}^{(1)}_{ij\in AB} = \\ &= 0 - \frac{1}{N}(n_A\cdot 0 + n_B\cdot 0) - \frac{1}{N}(n_A\cdot 1 + n_B\cdot 0) + \frac{1}{N^2}(n_A^2\cdot 1 + n_A n_B\cdot 0 + n_A n_B\cdot 0 + n_B^2\cdot 0) = \frac{-n_A n_B}{N^2}. \end{aligned}$$

Similarly demeaning the observations in the third and fourth rows yields the demeaned club dummy vector,

$$\ddot{D}^{(1)} = \frac{1}{N^2}\begin{bmatrix} n_B^2\cdot\mathbf{1}_{n_A^2} \\ -n_A n_B\cdot\mathbf{1}_{n_A n_B} \\ -n_A n_B\cdot\mathbf{1}_{n_A n_B} \\ n_A^2\cdot\mathbf{1}_{n_B^2} \end{bmatrix} = \frac{1}{N^2}z.$$

The demeaned club dummies under the other three identification assumptions can be similarly obtained.

[4] This formula, also called within transformation formula, is present in several Econometrics textbook like e.g. Baltagi (2001). The demeaning formula for the panel equation (8) is more complicated. I provide a derivation of it in Appendix B.

The demeaning of the left-hand side variable (x_{ij}) can be simplified by observing that only variation across the country pair groups AA, AB, BA and BB matters for identification. Then, individual observations for x_{ij} within each group can be replaced by their group means, and the left-hand side variable can be expressed in the following vector form

$$x = \begin{bmatrix} \bar{x}_{AA} \cdot \mathbf{1}_{n_A^2} \\ \bar{x}_{AB} \cdot \mathbf{1}_{n_A n_B} \\ \bar{x}_{BA} \cdot \mathbf{1}_{n_A n_B} \\ \bar{x}_{BB} \cdot \mathbf{1}_{n_B^2} \end{bmatrix},$$

where $\bar{x}_{AA}$ is the mean of trade flows between two members, $\bar{x}_{AB}$ is the mean of trade flows from a member to a non-member and so on. Applying the within transformation formula (4) as before yields the demeaned vector of left-hand side variable,

$$\ddot{x} = \frac{\Delta}{N^2} \begin{bmatrix} n_B^2 \cdot \mathbf{1}_{n_A^2} \\ -n_A n_B \cdot \mathbf{1}_{n_A n_B} \\ -n_A n_B \cdot \mathbf{1}_{n_A n_B} \\ n_A^2 \cdot \mathbf{1}_{n_B^2} \end{bmatrix} = \frac{\Delta}{N^2} z,$$

where

$$\Delta = \bar{x}_{AA} - \bar{x}_{AB} - \bar{x}_{BA} + \bar{x}_{BB}. \tag{6}$$

Using $\ddot{x}$ and the $\ddot{D}$s, it is straightforward to calculate the OLS estimates for the club membership effects.

I present the demeaned club dummies together with the β estimates in Table 1. The rows represent the different identification assumptions. It is immediately apparent that the demeaned club dummies are all perfectly collinear, since all of them are multiples of the same vector by a factor of 1, -1 or -2. This explains that more than one club dummies (3. identification assumption) cannot be separately identified. The fixed effects gravity equation in a cross section database cannot tell whether joint membership has stronger trade effect than one-sided membership. (An alternative way to show unidentifiability under the third identification assumption is presented in Appendix A.)

When the club effect is identified (1., 2. and 4. identification assumptions), the estimate is Δ or some simple transformation of it. Since Δ is determined as in (6), the estimates are based on very little data variation, namely the variation across the four groups of flows in Figure 1. As a result, the estimates are very sensitive to the change in the identification assumption. When only joint membership is assumed to have an effect on trade, the estimated effect is Δ. When both joint and one-sided membership is believed to affect trade, the estimate is $-\Delta$. Under the last identification assumption, the estimate is $-\Delta/2$.

Table 1: The demeaned club dummies and the βs

IA	Demeaned club dummy ($\ddot{D}$)	$\hat{\beta}$
1	$N^{-2}z$	Δ
2	$-N^{-2}z$	$-\Delta$
3.1	$N^{-2}z$	not identified
3.2	$-2N^{-2}z$	separately
4	$-2N^{-2}z$	$-\Delta/2$

3 The panel analogue

Most of the recent gravity estimations work with panel data. These applications assume that the gravity equation holds in all the time units (t) of the panel. The panel version of (2) is

$$x_{ijt} = y_{it} + y_{jt} - y_t^w + (1-\sigma)\ln\tau_{ijt} - (1-\sigma)\,\pi_{it} - (1-\sigma)\,p_{jt}. \tag{7}$$

Notice that the multilateral trade resistances also vary with time. Controlling for them with fixed effects therefore requires exporter-time and importer-time effects.

Let the bilateral trade barrier function be additively separable in its time-varying and time-constant components, where the time-varying component is the club membership dummy and an additive error term, such that $\ln\tau_{ijt} = \kappa_{ij} + \rho D_{ijt} + \epsilon_{ijt}$. All time-constant bilateral barriers are captured in κ_{ij}, and the ϵ_{ijt} error term is assumed to be uncorrelated with the club dummy. Then, the panel fixed effects gravity equation is

$$x_{ijt} = \beta D_{ijt} + \zeta_{ij} + \delta_{it} + \theta_{jt} + \varepsilon_{ijt}, \tag{8}$$

where δ_{it} and θ_{jt} exporter-time and importer-time fixed effects, respectively, control for the income variables and the multilateral resistances. This panel fixed effects gravity specification was recently suggested by several papers (Baltagi, Egger and Pfaffermayr, 2003; Baldwin and Taglioni, 2006; Baier and Bergstrand, 2007) as the theory-consistent fixed effects gravity.[5]

Earlier panel gravity estimations, which do not control for the time-varying multilateral resistances, use only country or country-pair fixed effects. Separate exporter and importer effects were proposed by the early paper of Mátyás (1997). Egger and Pfaffermayr (2003) and Cheng and Wall (2005) argue for country pair effects (such as ζ_{ij} in (8)). Many elements of bilateral trade costs, like those related to culture or institutions, cannot be observed and hardly change with time. Unless the regressor of interest is also time-invariant, it has become

[5]Eicher and Henn (2009) also uses this specification.

customary to include country pair fixed effects to avoid omitted variable biases stemming from these unobserved trade barriers.

An important difference from the cross section case is that the panel fixed effects gravity equation (8) identifies β only from the changes of club membership in time, because the country pair fixed effects net out all the time-constant variation. This means that one can estimate the effect of club membership only if some countries in the sample enter the club during the sample period.[6] Identification is therefore not based on the difference between members' and non-members' trade as in the cross section case, but on the difference between the evolution of trade of entering countries relative to countries that do not change their membership status.

It can be shown that the identification problems in Section 2 also extend to the estimation of (8) on panel data. The necessary condition for that is that club entry occurs at the same time for all entering countries, i.e. there are no sequential entries. In this case, the time period of the panel can clearly be divided into a pre-entry and a post-entry period. For such a panel, (8) can be transformed into a cross section equation like (3) by time-averaging the observations in both the pre-entry and the post-entry periods and time-differencing across these two periods to get

$$dx_{ij} = \beta dD_{ij} + \alpha_i + \eta_j + d\varepsilon_{ij}, \tag{9}$$

where $d.$ denotes change from the pre-entry to the post-entry period. All the derivations in Section 2.2 apply for (9), with x_{ij} and D_{ij} in (3) replaced by dx_{ij} and dD_{ij}.

Again, the exact specification of dD depends on the identification assumption. I assume that the panel includes two types of countries with respect to club membership: entrants, who enter the club, and insiders, who are members of the club during the sample period.[7] Figure 2 shows the four different flows of trade with entrants and insiders.

Figure 2: Trade flows of entrants and insiders

$i \backslash j$	entrant	insider
entrant	dx_{AA}	dx_{AB}
insider	dx_{BA}	dx_{BB}

The four identification assumptions are the same as in Section 2.1, with entrants and insiders replacing members and non-members. The change in trade between two insiders (dx_{BB}) is always part of the benchmark. It is based on the assumption that club membership

[6] I do not consider the symmetric case of exits.

[7] I consider countries who are not members in Section 4.

affects trade *growth* only shortly after entry, so the growth of insiders' mutual trade is not affected by their membership status any more.

3.1 EU enlargement example

I demonstrate the analytical findings on identifiability and their extension to panel data on the example of the enlargement of the European Union in 2004. The EU is a customs union, which means tariff-free intra-EU trade and a common external trade protection.[8] I use a database of all annual bilateral trade flows of eight entering countries (Czech Republic, Estonia, Hungary, Latvia, Lithuania, Poland, Slovakia, Slovenia) and twelve insiders (Austria, Denmark, Finland, France, Germany, Greece, Ireland, Italy, Portugal, Spain, Sweden, United Kingdom) in years between 2001 and 2006.[9] Since the entry occurred at the same time for all entrants, the time span can be divided into a pre-entry and a post-entry period (2001-2003 and 2004-2006).[10] I use annual data and not a two-period panel to demonstrate that the findings equally apply to multi-period panels.

I estimate (8), using two popular estimation methods. First, I estimate with the Fixed Effects Least Squares Dummy Variables (FE-LSDV) method, which nets out the ζ_{ij} by using the corresponding within transformation and controls for δ_{it} and θ_{jt} by directly including exporter-year and importer-year dummies among the regressors. Second, I estimate by OLS on the demeaned variables, which means demeaning the left-hand side variable and D_{ijt} in (8) from all the fixed effects and then estimating β with OLS on the demeaned variables. (Appendix B derives the within transformation formula for (8).) Both types of estimations are performed in STATA.

The two methods should, in principle, give identical results. In practice, OLS on the demeaned gives unbiased estimates only in balanced panels with self-trade (domestic trade) observations, because the within transformation formula is incorrect otherwise.[11] For demonstration purposes, I do the estimations both with and without self-trade.[12]

[8] For evidence on EU enlargement-induced trade-creation see Hornok (2010, 2011).

[9] Trade data is from Eurostat.

[10] I put 2004 in the post-entry period, although the enlargement was in May 2004.

[11] See Mátyás and Balázsi (2011) for a derivation of the bias. The same problem applies for the within transformation formula in (4), but not for the within transformation that nets out ζ_{ij} in the FE-LSDV method.

[12] I construct self-trade for each country as gross output of all non-services sectors minus total exports of goods. Self-trade is similarly constructed, among others, in Wei (1996), Novy (2008), Jacks, Meissner and Novy (2011) and Hornok (2011).

The EU membership dummies under the first, second and fourth identification assumptions are as follows. Superscripts denote the number of the identification assumption. The first EU dummy is

$$D_{ijt}^{(1)} = \begin{cases} 1 & \text{if AA and t} \geq 2004 \\ 0 & \text{otherwise} \end{cases},$$

which is based on the assumption that EU enlargement affected only the trade of two entrants.[13] The second EU dummy is

$$D_{ijt}^{(2)} = \begin{cases} 1 & \text{if (AA or AB or BA) and t} \geq 2004 \\ 0 & \text{otherwise} \end{cases},$$

based on the assumption that both the trade of two entrants and the trade between entrants and insiders was affected in 2004. The fourth EU dummy is

$$D_{ijt}^{(4)} = \begin{cases} 1 & \text{if (AB or BA) and t} \geq 2004 \\ 0 & \text{otherwise} \end{cases},$$

which looks at the effect on trade between entrants and insiders relative to trade between two entrants or two insiders. Finally, under the third identification assumption there are two EU dummies, where $D_{ijt}^{(3.1)}$ is identical to $D_{ijt}^{(1)}$ and $D_{ijt}^{(3.2)}$ is identical to $D_{ijt}^{(4)}$.[14]

3.2 Estimation results

Table 2 presents the estimation results by identification assumption and by estimation method for both with and without self-trade observations. The β estimates reinforce the analytical findings. Based on the estimates with self-trade, the value of Δ is -0.007. It can also be calculated from the pre- to post-entry average changes in the logarithm of trade according to $\Delta = \bar{dx}_{AA}$ - $\bar{dx}_{AB}$ - $\bar{dx}_{BA}$ + $\bar{dx}_{BB}$ = 0.604 - 0.488 - 0.235 + 0.112. The estimates under the different identification assumptions relate to each other as expected. The estimate under the second identification assumption is the negative of the estimate under the first, and the estimate under the fourth is half of the estimate under the second. None of them are statistically different from zero.

[13] Notice that it is identical to saying that one-sided membership in the EU has the same effect as joint membership, since the trade of entrants with insider was not any more affected in 2004.

[14] It is ultimately the time variation in D, which matters for identification, since the panel fixed effects gravity identifies only from time changes. Notice that different definitions of D can have identical time variation.

Table 2: Estimates for EU with entrants and insiders

	FE-LSDV				OLS on demeaned
IA	$\hat{\beta}$	Cluster s.e.	Within R^2	Identified?[1]	$\hat{\beta}$
with self-trade					
1	-0.007	0.059	0.626	Yes	-0.007
2	0.007	0.059	0.626	Yes	0.007
3.1	0.459^a	0.157	0.626	No (1)	dropped
3.2	0.233^a	0.077			0.003
4	0.003	0.029	0.626	Yes	0.003
without self-trade					
1	0.071	0.056	0.658	Yes	0.073
2	-0.071	0.056	0.658	Yes	-0.067
3.1	1.208^a	0.257	0.658	No (1)	0.566
3.2	0.569^a	0.127			0.247
4	-0.036	0.028	0.658	Yes	-0.035

Notes: (8) is estimated with FE-LSDV and OLS on demeaned. N of obs is 2400 with, 2280 without self-trade. The sample includes country pairs of 12 old EU countries and 8 of the countries that joined the EU in 2004. Dependent variable is log bilateral exports. Time dimension is years between 2001 and 2006. Pair fixed effects, exporter-year and importer-year dummies included. [1] Number of extra country-year dummies dropped in bracket. [a] significant at 1%, [b] at 5%. Significance is not reported for OLS on demeaned (last column).

The two EU effects under the third identification assumption cannot be separately identified because of perfect collinearity with the country-time dummies. Yet, quite misleadingly, the FE-LSDV estimation method reports sizeable and strongly significant estimates. Only if one checks how many country-year dummies are dropped (out of the total $2{\cdot}20{\cdot}6 = 240$), it turns out that one is dropped instead of the perfectly collinear EU dummy. When self-trade is in the database, the method of OLS on the demeaned variables (estimates reported in the last column of Table 2) drops the perfectly collinear EU dummy. However, when self-trade is not included, it also reports false estimates.[15]

4 Panel with outsiders

So far I abstracted from countries that never become members of the club (outsiders). Below I show that extending the sample with n_C such countries may help solve the unidentifiability of the club effect. This approach however also has its limitations. When additional dummy variables are included to control for outsider countries' trade changes, the identification problems can return.

[15]Note that, when self-trade is missing, the estimates obtained with the OLS on the demeaned method are not unbiased, which shows that the within transformation formula is incorrect for unbalanced data.

Let us consider a panel with three country types with respect to club membership: entrants, insiders and outsiders. Outsiders are countries that never become members during the sample period. These three country types generate nine different trade flows, as shown on Figure 3.

Figure 3: Trade flows with entrants, insiders and outsiders

$i\backslash j$	entrant	insider	outsider
entrant	x_{AA}	x_{AB}	x_{AC}
insider	x_{BA}	x_{BB}	x_{BC}
outsider	x_{CA}	x_{CB}	x_{CC}

We want to estimate the club effects under the four identification assumptions as before. Notice however that the benchmark (control group) of the estimation now also includes trade flows with outsiders ($x_{C\cdot}$ and $x_{\cdot C}$). This relies on the implicit assumption that what happens to outsiders' trade is uncorrelated with the club entry, i.e. these observations are valid benchmark.

The inclusion of outsider countries allows for the identification of heterogeneous club effects up to four linearly independent club dummies (shown in Appendix A) and leads to a less restrictive range of estimated effects. One can solve for the β estimates by following the same steps as in Section 2.2. The estimated coefficients can be expressed as linear combinations of the elements of the vector of the left-hand side variable,

$$dx' = \begin{bmatrix} \bar{d}x_{AA} & \bar{d}x_{AB} & \bar{d}x_{AC} & \bar{d}x_{BA} & \bar{d}x_{BB} & \bar{d}x_{BC} & \bar{d}x_{CA} & \bar{d}x_{CB} & \bar{d}x_{CC} \end{bmatrix},$$

with some parameter vector, where the elements of the parameter vector are functions of n_A, n_B and n_C. (Details of the analytical solution are in Appendix C.) I present the parameter vectors in Table 3 under the simplifying assumption that the number of countries by type is equal, i.e. $n_A = n_B = n_C$. Linear combinations of the elements of dx with these give the β estimates. For instance, the estimated effect under the first identification assumption can be expressed as $\hat{\beta}^{(1)} = \bar{d}x_{AA} - 0.5 \cdot \left(\bar{d}x_{AB} + \bar{d}x_{AC} + \bar{d}x_{BA} + \bar{d}x_{CA}\right) + 0.25 \cdot \left(\bar{d}x_{BB} + \bar{d}x_{BC} + \bar{d}x_{CB} + \bar{d}x_{CC}\right)$.

4.1 EU estimates with outsiders

I extend the EU database with trade flows of eight non-EU countries (Switzerland, Israel, Iceland, Japan, South Korea, Mexico, Norway, United States). At the same time, I reduce the number of insiders to eight, so that $n_A = n_C = n_C$ and comparison with the analytical

Table 3: β estimates in panels with outsiders ($n_A = n_B = n_C$)

IA	Elements of vector of LHS variable								
	$\bar{dx}_{AA}$	$\bar{dx}_{AB}$	$\bar{dx}_{AC}$	$\bar{dx}_{BA}$	$\bar{dx}_{BB}$	$\bar{dx}_{BC}$	$\bar{dx}_{CA}$	$\bar{dx}_{CB}$	$\bar{dx}_{CC}$
1	1	-0.5	-0.5	-0.5	0.25	0.25	-0.5	0.25	0.25
2	0	0.5	-0.5	0.5	-0.5	0	-0.5	0	0.5
3.1	1	0	-1	0	-0.25	0.25	-1	0.25	0.75
3.2	0	0.5	-0.5	0.5	-0.5	0	-0.5	0	0.5
4	-0.4	0.5	-0.1	0.5	-0.4	-0.1	-0.1	-0.1	0.2

Notes: β estimates are linear combinations of the elements of dx with the parameter values in the rows.

solutions in Table 3 is possible.[16] The estimation results (with self-trade) are reported in Table 4. All the effects, including the heterogeneous effect under the third identification assumption, are identified.

Table 4: Estimates for EU with entrants, insiders and outsiders

	FE-LSDV				OLS on demeaned
IA	$\hat{\beta}$	Cluster s.e.	Within R^2	Identified?	$\hat{\beta}$
1	-0.266^a	0.074	0.478	Yes	-0.266
2	-0.230^a	0.060	0.479	Yes	-0.230
3.1	-0.496^a	0.115	0.486	Yes	-0.496
3.2	-0.230^a	0.059			-0.230
4	-0.032	0.034	0.472	Yes	-0.032

Notes: (8) is estimated with FE-LSDV and OLS on demeaned. N of obs: 3456, also including self-trade. The sample includes country pairs of 8 old EU members, 8 of the countries that joined the EU in 2004 and 8 non-EU countries. Dependent variable is log bilateral exports. Time dimension is years 2001-2006. Pair fixed effects, exporter-year and importer-year dummies included. ^{a} significant at 1%, ^{b} at 5%, ^{c} at 10%. Last column shows coefficient estimates from OLS on demeaned, where significance is not reported.

One can check whether the estimates are in line with the analytical solutions for the βs in Table 3. The vector of the left-hand side variable, which includes the averages of the dx_{ij}s for the nine different types of trade relations, is

$$dx' = \begin{bmatrix} 0.604 & 0.481 & 0.602 & 0.265 & 0.075 & 0.094 & 0.590 & 0.166 & 0.060 \end{bmatrix}.$$

The β estimate under the first identification assumption can be calculated as linear combination of the elements of this vector with the corresponding parameter values in the first row of Table 3, i.e. $\hat{\beta}^{(1)}$ = 0.604 - 0.5·(0.481 + 0.602 + 0.265 + 0.590) + 0.25·(0.075 + 0.094 + 0.166 + 0.060) = -0.266, which is equal to the estimate in the first row of Table 4.

[16] Denmark, Greece, Ireland and Sweden are dropped from the original 12 insiders. The choice is arbitrary.

The estimates for the EU effect with outsiders are strikingly different from the baseline estimates in Table 2. They are all negative, mostly large in absolute value and statistically significant. The difference is due to the change in the benchmark observations, which now include all trade with outsiders. Trade between entrants and outsiders (3rd and 7th elements of dx') increased especially strongly around EU enlargement, which drives the β estimates down.

4.2 Controlling for outsider effect

The strong growth in trade between entrants and outsiders is most probably related to EU entry. The EU is a customs union, which means that entering countries have to adjust their levels of trade protection with outsiders to the common external trade protection level of the customs union. Historical data on tariffs suggests that protection levels between entrants and outsiders had to decrease considerably with EU entry. If this is not controlled for in the estimation and trade between entrants and outsiders is part of the benchmark, the estimated coefficients will be downward biased.

Controlling for changes in trade barriers with hard data is often problematic. Available data on bilateral tariffs is deficient and often not good quality, let alone data on non-tariff trade barriers. An alternative solution is to include a dummy variable, which controls for the outsider effect,

$$D^o_{ijt} = \begin{cases} 1 & \text{if (AC or CA) and t} \geq 2004 \\ 0 & \text{otherwise} \end{cases} .$$

The augmented panel fixed effects gravity equation is then

$$x_{ijt} = \beta D_{ijt} + \gamma D^o_{ijt} + \zeta_{ij} + \delta_{it} + \theta_{jt} + \varepsilon_{ijt}, \tag{10}$$

where γ is the coefficient of the outsider dummy.

Since in panels with outsiders the fixed effects gravity can identify at most four club dummies, the inclusion of one extra dummy can, in principle, allow the identification of the βs. Estimation results in Table 5 however show that it is not the case in the current example. The estimation of (10) is subject to the same identification problems as estimation of (8) on data without outsiders. The β estimates, which become again small and not significant, relate to each other as shown in Section 2.2 and heterogeneous effects cannot be identified separately. Unidentifiability under the third identification assumption is again due to perfect collinearity among the fixed effects and the dummies (shown in Appendix A).

Table 5: Estimates for EU with entrant-outsider effect

		FE-LSDV				OLS on demeaned
IA	Coefficient	Estimate	Cluster s.e.	Within R^2	Identified?[1]	Estimate
1	β	-0.036	0.068	0.486	Yes	-0.036
	γ	0.230^a	0.059			0.230
2	β	0.036	0.068	0.486	Yes	0.036
	γ	0.266^a	0.074			0.266
3	β_1	0.551^b	0.269	0.486	No (1)	dropped
	β_2	0.294^b	0.132			0.018
	γ	0.524^a	0.136			0.248
4	β	0.018	0.034	0.486	Yes	0.018
	γ	0.248^a	0.057			0.248

Notes: (10) is estimated with FE-LSDV and OLS on demeaned. N of obs is 3456. The sample includes country pairs of 8 of the EU-15 countries, 8 of the countries that joined the EU in 2004 and 8 non-EU countries. Dependent variable is log bilateral exports. Time dimension is years in 2001-2006. Pair fixed effects, exporter-year and importer-year dummies included.
[1] Number of extra country-year dummies dropped in bracket. [a] significant at 1%, [b] at 5%. Significance is not reported for OLS on demeaned.

The advantages of adding outsider observations to the sample are lost, when I control for entrant-outsider trade effects with an additional dummy variable. Depending on the empirical application, additional outsider dummies may take different forms. Sometimes separate direction-specific entrant-outsider effects or insider-outsider effects should be controlled for. Table 6 shows identifiability of the club effect for these cases. There is no identification problem with only insider-outsider dummies. When entrant-outsider dummies are included (common or separate for the two directions), the club dummies under the third identification assumption cannot be identified. Finally, none of the club effects can be identified, when separate dummies are included for both insider-outsider and entrant-outsider groups.

Table 6: Identifiability with outsider effect dummies

	Additional dummies for outsiders					
IA	AC, CA		BC, CB		AC, CA, BC, CB	
	common	separate	common	separate	common	separate
1	yes	yes	yes	yes	yes	no
2	yes	yes	yes	yes	yes	no
3	no	no	yes	yes	no	no
4	yes	yes	yes	yes	yes	no

Notes: "yes" and "no" refer to identifiability of the club effect under identification assumptions 1-4, when additional dummies for outsiders' trade are included. "Common" stands for a common dummy, "separate" for two direction-specific dummies.

5 Identifying from the trade cost index

The gravity equation, which controls for multilateral trade resistances with fixed effects, cannot produce reliable estimates for the trade effects of EU enlargement. Below I present estimates from an alternative method, which is proposed by Novy (2008) and Head and Ries (2001), and which is readily implementable with the available data.

An index of relative bilateral trade barriers can be derived from the theoretical gravity equation (1),

$$\Theta_{ij} = \left(\frac{\tau_{ij}\tau_{ji}}{\tau_{ii}\tau_{jj}} \right)^{\frac{1}{2}} = \left(\frac{X_{ii}X_{jj}}{X_{ij}X_{ji}} \right)^{\frac{1}{2(\sigma-1)}}, \tag{11}$$

which is already net of the multilateral trade resistances. The index captures relative (international to domestic) trade barriers between two countries and, as shown in Novy (2008), it can be expressed as the ratio of domestic trade flows in the two countries (self-trade) to their international trade flows. Notice that the index is an average of the two directions of trade and hence, it is not able to identify direction-specific effects.

I calculate Θ for each country pair and year in the sample with the 8 insiders, 8 entrants and 8 outsiders. In line with the literature, which estimates the elasticity of substitution to be in the range of 5 to 10, I assume $\sigma = 7.5$.[17] The effect of EU enlargement on bilateral trade costs can then be estimated with the equation

$$\ln \Theta_{ijt} = \rho D_{ijt} + \omega D^{o}_{ijt} + \kappa_{ij} + \mu_t + \epsilon_{ijt}, \tag{12}$$

where κ_{ij} are country pair fixed effects and μ_t captures a common time trend in trade barriers. The estimates for ρ and ω measure how much bilateral trade barriers fell with EU enlargement, the latter capturing the fall between entrants and outsiders. The expected sign of the estimates is negative. Estimates are relative to the benchmark, which includes trade of two insiders, trade of two outsiders, and trade between insiders and outsiders.

The estimates can be transformed into changes in relative trade flows according to

$$d \ln \left(\frac{X_{ij}X_{ji}}{X_{ii}X_{jj}} \right)^{\frac{1}{2}} = (1-\sigma)\,\hat{\rho}. \tag{13}$$

The estimation results are presented in Table 7, with the implied changes in trade flows in the last column. Under every identification assumption I estimate a significant decrease in bilateral trade barriers and, hence, increase in bilateral trade flows. The trade increase for entrant-entrant and entrant-insider pairs (second identification assumption) is estimated to

[17] See the assessment of Anderson and van Wincoop (2004) on the empirical estimates of the elasticity of substitution.

be around 40 per cent, which is induced by a 5.6 percentage points (ad valorem) decrease in bilateral trade costs.[18] Nearly half as strong a trade increase is estimated for trade between entrants and insiders than trade of two entrants.

Table 7: Estimates with trade cost index

IA	coefficient	estimate	cluster s.e.	within R2	Trade effect
1	ρ	-0.074^a	0.007	0.484	0.483
	ω	-0.061^a	0.010		0.399
2	ρ	-0.056^a	0.005	0.495	0.364
	ω	-0.076^a	0.010		0.494
3	ρ_1	-0.089^a	0.007	0.512	0.578
	ρ_2	-0.042^a	0.005		0.271
	ω	-0.076^a	0.010		0.494
4	ρ	-0.025^a	0.005	0.4427	0.162
	ω	-0.059^a	0.010		0.384

Notes: (12) is estimated with pair fixed effects and common time dummies. N = 1656. The sample includes country pairs of 8 of the EU-15 countries, 8 of the countries that joined the EU in 2004 and 8 non-EU countries. Dependent variable is log of the trade cost index in (11). Time dimension is years in 2001-2006. ^{a} significant at 1%.

6 Summary and Discussion

This paper argues that the theory-consistent fixed effects gravity equation, which controls for the multilateral trade resistances with fixed effects, has serious limitation in identifying the effects of certain "club membership" dummies. The fixed effects leave only the bilateral (time-varying bilateral) variation in the data, while the club membership dummy has very little variation in this dimension. As a result, in several settings, only one parameter can be identified. The estimated effects under different identification assumptions differ in a non-intuitive way and heterogeneous club effects (e.g. joint versus one-sided membership) cannot be identified separately. Standard estimation methods do not necessarily report these problems.

These findings, though quite general, do not extend to all kinds of club dummies and databases. The identification limitations are less severe, if the bilateral (time-varying bilateral) variation in the club dummy is richer than in the presented baseline case. Possible ways to overcome the identification problems are as follows.

[18] These estimated effects reflect decreases both in tariff and non-tariff type trade barriers. The latter includes various types of barriers such as administrative costs, time costs, institutional and informational barriers, or political risk.

In cross section applications, the identification problems persist as long as the club dummy represents only one club. This means that one cannot identify the effect of WTO, NAFTA, euro area, or being English-speaker. One can however identify the effect of free trade agreements, common currency or common language, if these dummy variables incorporate several categories of trade agreement, currency and language. The more categories they include, the more bilateral variation they offer for identification.

In panel applications, one way to increase the time-varying bilateral variation of the club dummy is to look at sequential entries to the club, i.e. when different countries enter the club in different dates. Again, the more entries one has at different dates, the more variation there is for identification. Naturally, this solution is not available if one wants to analyze the effect of a single event, such as the EU enlargement in 2004 or the euro area creation in 1999.

Another possibility with panel data, discussed in Section 4, is to include three types of countries in the panel with respect to club membership: entrants, insiders and outsiders. I showed that in such a database heterogeneous club effects can be identified up to 4 different (linearly independent) club dummies. This approach however also has its potential pitfalls. Introducing additional dummies to capture club-related changes in outsider countries' trade can bring the identification problems back.

Finally, it is worth considering to apply alternative methods of controlling for the multilateral trade resistances. I demonstrate it by applying the method of Novy (2008) on the example of EU enlargement in 2004. Other alternative methods are structural estimation (Anderson and van Wincoop, 2003, Bergstrand, Egger and Larch, 2012) or linear approximation of the multilateral trade resistances (Baier and Bergstrand, 2009). A drawback is that the alternative methods are often more computation- and/or data-demanding than fixed effects estimation.

Although this paper considers the gravity equation of international trade *per se*, it conveys a more general message. It demonstrates a pitfall of relying on fixed effects extensively to control for the unobserved variables in the estimation. If the fixed effects net out most of the useful data variation, identification problems can be present even if standard estimation methods do not report them. These lessons are potentially also useful for empirical researchers in other fields, where multidimensional panel data sets are common and fixed effects are often used.

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A Unidentifiability of heterogeneous club effects

One way to see that more than one club dummies cannot be identified in (3) is to write out the matrix of regressors under the third identification assumption:

$$\begin{bmatrix} \alpha & \eta & D \end{bmatrix} = \begin{bmatrix} \mathbf{1} & \mathbf{0} & \mathbf{1} & \mathbf{1} & \mathbf{0} \\ \mathbf{1} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{1} \\ \mathbf{0} & \mathbf{1} & \mathbf{1} & \mathbf{0} & \mathbf{1} \\ \mathbf{0} & \mathbf{1} & \mathbf{0} & \mathbf{0} & \mathbf{0} \end{bmatrix}. \tag{14}$$

The elements of the matrix are column vectors of ones or zeros of dimensions ${n_A}^2$ in the first, $n_A n_B$ in the second and third and ${n_B}^2$ in the fourth rows of the matrix. The first two columns of the matrix are the exporter fixed effects, the third column includes the importer fixed effects for members (importer fixed effects for non-members omitted) and the last two columns are the two club dummies. Since the number of linearly independent columns should always be equal to the number of linearly independent rows, the five column vectors of this matrix cannot be linearly independent. The exporter and importer fixed effects already take 3 out of the maximum 4 linearly independent column vectors, so there is room left for only one linearly independent club dummy.

Of course, having only one club dummy is a necessary but not sufficient condition for linear independence and, hence, identification. Even if there is only a single club dummy, identification is not possible if the regressor matrix is of deficient rank. This means that the club dummy is constructed so that it is perfectly collinear with one or more of the country fixed effects. This would be the case e.g. if one wanted to estimate the effect on x_{AA} and x_{AB}, relative to x_{BA} and x_{BB}. In this case the club dummy is, by construction, perfectly collinear with the exporter fixed effect for members (first column of the regressor matrix).

In panel estimation of (8), when trade of entrants, insiders and outsiders are all included in the database (discussed in Section 4), the club effect under the third identification assumption is also identified. This can be seen by looking at the corresponding matrix of regressors in the time-differenced equation (9),

$$\begin{bmatrix} \alpha & \eta & dD \end{bmatrix} = \begin{bmatrix} \mathbf{1} & \mathbf{0} & \mathbf{0} & \mathbf{1} & \mathbf{0} & \mathbf{1} & \mathbf{0} \\ \mathbf{1} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{1} & \mathbf{0} & \mathbf{1} \\ \mathbf{1} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{1} & \mathbf{0} & \mathbf{1} & \mathbf{0} & \mathbf{0} & \mathbf{1} \\ \mathbf{0} & \mathbf{1} & \mathbf{0} & \mathbf{0} & \mathbf{1} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{1} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{1} & \mathbf{1} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{1} & \mathbf{0} & \mathbf{1} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{1} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \end{bmatrix}, \tag{15}$$

where the rows are in order AA, AB, AC, BA, BB, BC, CA, CB, CC, and the elements of the matrix are vectors of ones or zeros of the following dimensions: ${n_A}^2$ in the first, $n_A n_B$ in the second and fourth, $n_A n_C$ in the third and seventh, ${n_B}^2$ in the fifth, $n_B n_C$ in the sixth and eighth and ${n_C}^2$ in the ninth rows of the matrix, where n_C is the number of outsider countries. The first three columns of the matrix are the exporter dummies, the fourth and fifth columns are the importer dummies (importer dummy for outsiders omitted) and the last two columns are the two club dummies under the third identification assumption. The extension of the database with outsider countries increases the number of rows of the regressor matrix to nine, which also increases the maximum possible number of linearly independent column vectors to nine. Since five columns are reserved for the country dummies, it is possible to identify *at most* four club effects separately.[19]

When entrant-outsider effects are controlled for by an additional dummy variable as in (10), unidentifiability under the third identification assumption is due to a deficient rank

[19] An alternative way to check identifiability is to write out the regressor matrix ($\mathbf{M}$) and check whether the determinant of $\mathbf{M'M}$ is zero (singular matrix) or approximately zero (near singular matrix). A singular or near singular matrix indicates perfect collinearity.

regressor matrix,

$$\begin{bmatrix} \alpha & \eta & dD & dD^o \end{bmatrix} = \begin{bmatrix} \mathbf{1} & \mathbf{0} & \mathbf{0} & \mathbf{1} & \mathbf{0} & \mathbf{1} & \mathbf{0} & \mathbf{0} \\ \mathbf{1} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{1} & \mathbf{0} & \mathbf{1} & \mathbf{0} \\ \mathbf{1} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{1} \\ \mathbf{0} & \mathbf{1} & \mathbf{0} & \mathbf{1} & \mathbf{0} & \mathbf{0} & \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \mathbf{1} & \mathbf{0} & \mathbf{0} & \mathbf{1} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{1} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{1} & \mathbf{1} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{1} \\ \mathbf{0} & \mathbf{0} & \mathbf{1} & \mathbf{0} & \mathbf{1} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{1} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \end{bmatrix}. \tag{16}$$

Although the number of rows (nine) is greater than the number of columns (eight), there is perfect collinearity among the column vectors. Perfect collinearity arises from the linear relationship among the exporter and importer fixed effects for entrants, the two club dummies and D^o of the form $2 \cdot v_6 + v_7 + v_8 - v_1 - v_4 = \mathbf{0}$, where the vs are the column vectors of (16) in order.

B The demeaning formula for the panel specification

I derive the demeaning (within transformation) formula for the error structure of the fixed effects panel estimation (8). The derivation is based on the general solution in Davis (2002). More recently, the formula was also derived in Mátyás and Balázsi (2011).

The fixed effects panel specification for international trade data can be represented with the error structure

$$u_{ijt} = \zeta_{ij} + \delta_{it} + \theta_{jt} + \varepsilon_{ijt}, \tag{17}$$

where $i = 1, ..., N$ denote exporters, $j = 1, ..., M$ importers and $t = 1, ..., T$ time, ζ_{ij}, δ_{it} and θ_{jt} are the unobservable pair-specific, exporter-year and importer-year effects, respectively. In vector form,

$$u = Z_\zeta \zeta + Z_\delta \delta + Z_\theta \theta + \varepsilon, \tag{18}$$

where ζ, δ and θ are vectors of parameters to estimate of dimension $NMT \times NM$, $NMT \times NT$ and $NMT \times MT$, respectively, and $Z_\zeta = I_{NM} \otimes \iota_T$, $Z_\delta = I_N \otimes \iota_M \otimes I_T$ and $Z_\theta = \iota_N \otimes I_{MT}$. I is the identity matrix and ι is the vector of ones of given dimension and $\otimes$ denotes the Kronecker product.[20]

[20] A useful property of the Kronecker product (mixed-product property) is that $(A \otimes B) \cdot (C \otimes D) = AC \otimes BD$, given that the dimensions of the matrices are such that taking their product is possible.

The projection matrix, which projects onto the range of $Z = (Z_\zeta; Z_\delta; Z_\theta)$, is $P_{[Z]} = Z\left(Z'Z\right)^{-1}Z'$. The orthogonal projection matrix is $Q_{[Z]} = I - P_{[Z]}$. P and Q are symmetric and idempotent. Note that $P_{[Z_\zeta]} = I_{NM} \otimes \bar{J}_T$ averages the data over t, where $\bar{J}_T = \frac{1}{T}J_T$ with J_T being the matrix of ones of dimension T. Similarly, $P_{[Z_\delta]} = I_N \otimes \bar{J}_M \otimes I_T$ averages the data over j and $P_{[Z_\theta]} = \bar{J}_N \otimes I_{MT}$ averages the data over i. For example, in the last case, $\left(\bar{J}_N \otimes I_{MT}\right)u$ has a typical element $\bar{u}_{.jt} = \frac{1}{N}\sum_{i=1}^{N} u_{ijt}$.

The general solution for the within transformation matrix according to Davis (2002) is

$$Q_{[Z]} = Q_{[A]} - P_{[B]} - P_{[C]}, \tag{19}$$

where $A = Z_\theta$, $B = Q_{[A]}Z_\delta = Q_{[Z_\theta]}Z_\delta$ and $C = Q_{[B]}Q_{[A]}Z_\mu = Q_{[Q_{[Z_\theta]}Z_\delta]}Q_{[Z_\theta]}Z_\zeta$.

It is straightforward to show that

$$Q_{[A]} = \left(I_N - \bar{J}_N\right) \otimes I_{MT}.$$

The second term can be expressed as

$$P_{[B]} = \left(I_N - \bar{J}_N\right) \otimes \bar{J}_M \otimes I_T,$$

where I used that $Q_{[A]}Z_\delta = \left(I_N - \bar{J}_N\right) \otimes \iota_M \otimes I_T$. The third them is

$$P_{[C]} \quad = \quad \left(I_N - \bar{J}_N\right) \otimes \left(I_M - \bar{J}_M\right) \otimes \bar{J}_T,$$

where I used that $Q_{[A]}Q_{[B]}Z_\zeta = \left(I_N - \bar{J}_N\right) \otimes \left(I_M - \bar{J}_M\right) \otimes \iota_T$.

Collecting terms,

$$\begin{aligned} Q_{[Z]} &= \left(I_N - \bar{J}_N\right) \otimes \left(I_M - \bar{J}_M\right) \otimes \left(I_T - \bar{J}_T\right) \\ &= I_{NMT} - \bar{J}_N \otimes I_{MT} - I_N \otimes \bar{J}_M \otimes I_T - I_{NM} \otimes \bar{J}_T + \\ &+ I_N \otimes \bar{J}_{MT} + \bar{J}_N \otimes I_M \otimes \bar{J}_T + \bar{J}_{NM} \otimes I_T - \bar{J}_{NMT} \end{aligned}$$

with a typical element

$$\ddot{u} = Q_{[Z]}u = u_{ijt} - \bar{u}_{.jt} - \bar{u}_{i.t} - \bar{u}_{ij.} + \bar{u}_{i..} + \bar{u}_{.j.} + \bar{u}_{..t} - \bar{u}_{...}, \tag{20}$$

where $\bar{u}_{.jt} = N^{-1}\sum_i u_{ijt}$, $\bar{u}_{i.t} = M^{-1}\sum_j u_{ijt}$, $\bar{u}_{ij.} = T^{-1}\sum_t u_{ijt}$, $\bar{u}_{i..} = (MT)^{-1}\sum_t\sum_j u_{ijt}$, $\bar{u}_{.j.} = (NT)^{-1}\sum_t\sum_i u_{ijt}$, $\bar{u}_{..t} = (NM)^{-1}\sum_j\sum_i u_{ijt}$ and $\bar{u}_{...} = (NMT)^{-1}\sum_t\sum_j\sum_i u_{ijt}$.

The estimation method 'OLS on the demeaned' is done by demeaning the variables as in (20) and estimating the regression equation with the demeaned variables. It is important to add that this formula is derived for a "full" trade matrix. This means that the database is balanced and, if some countries are both exporters and importers in the database (which is almost always the case), data on trade of these countries with themselves (self-trade) should also be included.

C Deriving the estimates for panel with outsiders

If I do not assume $n_A = n_B = n_C$, the demeaned left-hand side variable can be expressed as

$$\ddot{dx} = a_1\bar{dx}_{AA} + a_2\bar{dx}_{AB} + a_3\bar{dx}_{AC} + a_4\bar{dx}_{BA} + a_5\bar{dx}_{BB} + a_6\bar{dx}_{BC} + a_7\bar{dx}_{CA} + a_8\bar{dx}_{CB} + a_9\bar{dx}_{CC},$$

where the as are vectors, whose elements are functions of n_A, n_B and n_C. Expressing the as in terms of the number of countries it becomes

$$\begin{aligned}
\begin{bmatrix} \ddot{dx}_{AA} \\ \ddot{dx}_{AB} \\ \ddot{dx}_{AC} \\ \ddot{dx}_{BA} \\ \ddot{dx}_{BB} \\ \ddot{dx}_{BC} \\ \ddot{dx}_{CA} \\ \ddot{dx}_{CB} \\ \ddot{dx}_{CC} \end{bmatrix}
&= \frac{1}{N^2} \begin{bmatrix} (n_B+n_C)^2 \\ -n_A(n_B+n_C) \\ -n_A(n_B+n_C) \\ -n_A(n_B+n_C) \\ n_A^2 \\ n_A^2 \\ -n_A(n_B+n_C) \\ n_A^2 \\ n_A^2 \end{bmatrix} \bar{dx}_{AA} + \frac{1}{N^2} \begin{bmatrix} -n_B(n_B+n_C) \\ (n_A+n_C)(n_B+n_C) \\ -n_B(n_B+n_C) \\ n_A n_B \\ -n_A(n_A+n_C) \\ n_A n_B \\ n_A n_B \\ -n_A(n_A+n_C) \\ n_A n_B \end{bmatrix} \bar{dx}_{AB} + \\
&+ \frac{1}{N^2} \begin{bmatrix} -n_C(n_B+n_C) \\ -n_C(n_B+n_C) \\ (n_A+n_B)(n_B+n_C) \\ n_A n_C \\ n_A n_C \\ -n_A(n_A+n_B) \\ n_A n_C \\ n_A n_C \\ -n_A(n_A+n_B) \end{bmatrix} \bar{dx}_{AC} + \frac{1}{N^2} \begin{bmatrix} -n_B(n_B+n_C) \\ n_A n_B \\ n_A n_B \\ (n_A+n_C)(n_B+n_C) \\ -n_A(n_A+n_C) \\ -n_A(n_A+n_C) \\ -n_B(n_B+n_C) \\ n_A n_B \\ n_A n_B \end{bmatrix} \bar{dx}_{BA} + \\
&+ \frac{1}{N^2} \begin{bmatrix} n_B^2 \\ -n_B(n_A+n_C) \\ n_B^2 \\ -n_B(n_A+n_C) \\ (n_A+n_C)^2 \\ -n_B(n_A+n_C) \\ n_B^2 \\ -n_B(n_A+n_C) \\ n_B^2 \end{bmatrix} \bar{dx}_{BB} + \frac{1}{N^2} \begin{bmatrix} n_B n_C \\ n_B n_C \\ -n_B(n_A+n_B) \\ -n_C(n_A+n_C) \\ -n_C(n_A+n_C) \\ (n_A+n_B)(n_A+n_C) \\ n_B n_C \\ n_B n_C \\ -n_B(n_A+n_B) \end{bmatrix} \bar{dx}_{BC} + \\
&+ \frac{1}{N^2} \begin{bmatrix} -n_C(n_B+n_C) \\ n_A n_C \\ n_A n_C \\ -n_C(n_B+n_C) \\ n_A n_C \\ n_A n_C \\ (n_A+n_B)(n_B+n_C) \\ -n_A(n_A+n_B) \\ -n_A(n_A+n_B) \end{bmatrix} \bar{dx}_{CA} + \frac{1}{N^2} \begin{bmatrix} n_B n_C \\ -n_C(n_A+n_C) \\ n_B n_C \\ n_B n_C \\ -n_C(n_A+n_C) \\ n_B n_C \\ -n_B(n_A+n_B) \\ (n_A+n_B)(n_A+n_C) \\ -n_B(n_A+n_B) \end{bmatrix} \bar{dx}_{CB} + \frac{1}{N^2} \begin{bmatrix} n_C^2 \\ n_C^2 \\ -n_C(n_A+n_B) \\ n_C^2 \\ n_C^2 \\ -n_C(n_A+n_B) \\ -n_C(n_A+n_B) \\ -n_C(n_A+n_B) \\ (n_A+n_B)^2 \end{bmatrix} \bar{dx}_{CC}.
\end{aligned}$$

The demeaned club dummy is $\ddot{dD} = a_1$ under the first identification assumption, $\ddot{dD} = a_1 + a_2 + a_3$ under the second and $\ddot{dD} = a_2 + a_3$ under the fourth. The matrix of the two demeaned club dummies under the third identification assumption is $\ddot{dD} = \left[\begin{array}{cc} a_1 & a_2 + a_3 \end{array} \right]$, where a_1 and $a_2 + a_3$ are column vectors of the matrix. To express the policy effect estimates as functions of the n's and the $\bar{dx}$s, one needs to solve for the OLS formula $\hat{\beta} = \left(\ddot{dD}'\ddot{dD}\right)^{-1} \ddot{dD}'\ddot{dx}$ under each identification assumption separately.