

# Errors in rational expectations matter

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Received 23 July 1992

Accepted 1 September 1992

## Abstract

Muth (*Econometrica*, 1961, 29, 299–306) and Sargent and Wallace (*Journal of Monetary Economics*, 1976, 2, 169–183) claimed that the main results of rational expectations theory allow for possibly large independent errors. This paper shows that in such a situation the pseudo rational expectation is no longer optimal.

## 1. Introduction

Uncertainty plays an important role in educational choice. This choice determines the future occupational possibilities, and since many courses take several years, students have to form expectations about the complex and non-transparent labour market. The uncertainty students face has been studied both by cobweb models [Freeman (1971)] and by rational expectations models [Siow (1984) and Zarkin (1985)]. The problem with the cobweb model is that it imposes an expectation behaviour which is a priori non-optimal. On the other hand, rational expectations theory assumes that all prediction errors are due to uncertainty about the future state of the world, and therefore implicitly assumes that students do not make errors in the forecasts themselves. They are assumed to have perfect insight in the functioning of the labour market. Muth (1961), and Sargent and Wallace (1976) claim, however, that the main results of the rational expectations theory are not influenced by the introduction of an independent random error term. In this paper this proposition is challenged by showing that in the case of such random errors this pseudo rational expectation is no longer optimal.

## 2. Information versus interpretation errors

Suppose students have to forecast the wage they will earn after some kind of schooling, to make their enrolment decision. In general their prediction,  $w^{\text{pre}}$ , will not equal the realisation,  $w^{\text{rea}}$ . There are two ways to explain the difference between the realisation and the expectation (i.e. the prediction error). Firstly, following Arrow (1951), it is possible to assume that students' expectations are optimal given the information available at the moment of choice, but that there will be some future events, unpredictable at that moment, which influence the realised wage. In this state of the world view on prediction errors the quality of the forecast is determined by the amount of information available at the moment of choice. It is the world that might deviate from

the prediction:

$$w^{\text{rea}} = w^{\text{pre}} + \epsilon^{\text{re}}. \quad (1)$$

This *information error*,  $\epsilon^{\text{re}}$ , is independent of the prediction. This type of uncertainty is typical of rational expectations models.

A second way to explain prediction errors is to assume that students are not fully capable of interpreting the available information. The labour market is very complex, and it is not evident in which way available information should be used to deduce optimal forecasts about future wages. A student should decide which past data are relevant for the situation he faces, and should decide on the extent of the similarity of these past events and future developments. Furthermore, there is no reason why theoretical thought might not provide 'data' relevant for the expectation formation. The expectation therefore depends critically upon some random personal interpretations about the relevant evidence:

$$w^{\text{pre}} = w^{\text{rea}} + \epsilon^{\text{exp}}. \quad (2)$$

This *interpretation error* is independent of the realisation. The realisation is constant, but due to errors in interpretation the prediction differs from the realisation.

Of course, both models are extreme variants. It might be the case that both types of error occur in a forecast. In that case there exists an unobserved optimal forecast,  $w^{\text{re}}$ , such that

$$\begin{aligned} w^{\text{rea}} &= w^{\text{re}} + \epsilon^{\text{re}}, \\ w^{\text{pre}} &= w^{\text{re}} + \epsilon^{\text{exp}}. \end{aligned} \quad (3)$$

$\epsilon^{\text{re}}$  is independent of  $w^{\text{pre}}$  and  $w^{\text{re}}$ , while  $\epsilon^{\text{exp}}$  is independent of  $w^{\text{rea}}$  and  $w^{\text{re}}$ .

The possibility of errors in forecasts is also observed by proponents of rational expectations, in which errors are always due to unforecastable exogenous shocks. Muth (1961, p. 321) and Sargent and Wallace (1976, p. 180), however, state that it is possible that there are deviations from rationality, but since these will be independent errors, they do not matter for the analysis. In the next section it is shown that this proposition is false. The presence of errors in forecasts influences the optimality property of the rational expectation and causes students to deviate from this pseudo rational expectation.

### 3. The optimal prediction

There are many different approaches to making a forecast and there is no reason to choose one of these methods in advance. One approach is to base the forecast on similar past events. The student has to select cases that appear to him identical to some extent, that can be used to construct a probability distribution about the future wage. The problem is to decide which past events are still seen as similar and which are not. The larger the class of similar cases the less informative the probability distribution will become, i.e. the information error rises.

On the other hand it is possible to restrict the class of similar cases. Such a restriction will be based on theoretical arguments about what is typical for the event that has to be predicted. In the case where the future event is thought to be very typical, it might be the case that no similar events can be found in the empirical data. The only comparable events are constructions of the

mind. In that case the forecast will be purely theoretical. The more restrictive the selection of comparable events is, the more informative the selection becomes, but also the larger the probability that a wrong selection is made, i.e. the interpretation error rises.

The student has to choose between the information errors of a very large set of comparable events, and the interpretation errors in the case of a small set of comparable events. It is therefore not sensible to distinguish only one (unobservable) perfect prediction, as in Eq. (3), but there will be many 'perfect' predictions, rankable from a low degree of informativity to a high degree of informativity. In this paper the analysis is restricted to two different unobservable perfect predictions, but this can easily be extended to more. The ranking provides a trade off between information errors and interpretation errors.

Let  $w_1^{re}$  be a perfect expectation with a high degree of abstraction, and therefore a low degree of informativity, and let  $w_2^{re}$  be a more specific perfect expectation. Equation (3) can be extended to

$$w^{rea} = w_2^{re} + \epsilon_2^{re}, \quad (4)$$

$$w_2^{re} = w_1^{re} + \epsilon_1^{re}, \quad (5)$$

$$w_2^{exp} = w_2^{re} + \epsilon_1^{exp} + \epsilon_2^{exp}, \quad (6)$$

$$w_1^{exp} = w_1^{re} + \epsilon_1^{exp}, \quad (7)$$

in which the error-terms are always independent of the other right-hand-side variables and have a zero expectation.  $w_1^{re}$  is an abstract perfect prediction of the realised wage, but can also be seen as a perfect prediction of the more specific perfect prediction,  $w_2^{re}$ . Both perfect predictions are only observed with an additional measurement error. The specific prediction,  $w_2^{exp}$ , contains at least the error of the abstract prediction,  $w_1^{exp}$ .

In a rational expectations model the measurement errors,  $\epsilon_i^{exp}$ , equal 0. In that case the specific prediction dominates the abstract prediction, so it is optimal to use the most specific prediction. The propositions of Muth, and Sargent and Wallace, however, is that this is still the case if measurement errors are introduced into the model. On the basis of an MSE loss function it is, however, easy to show that it is possible to gain from a combination,  $w^{pre} = \lambda w_1^{exp} + (1 - \lambda) w_2^{exp}$ , of both expectations. The prediction error of this combined prediction equals

$$\Delta w = w^{pre} - w^{rea} = \lambda(-\epsilon_1^{re}) + (1 - \lambda)\epsilon_2^{exp} + \epsilon_1^{exp} - \epsilon_2^{re}. \quad (8)$$

The combination is optimal for a value of  $\lambda$  for which the variance is minimal. Denoting the variances of the errors by respectively  $\sigma_{re1}^2$ ,  $\sigma_{re2}^2$ ,  $\sigma_{exp1}^2$ , and  $\sigma_{exp2}^2$ , the variance of the prediction error equals

$$\sigma_w^2 = \lambda^2 \sigma_{re1}^2 + (1 - \lambda)^2 \sigma_{exp2}^2 + \sigma_{exp1}^2 + \sigma_{re2}^2. \quad (9)$$

Minimising Eq. (9) with respect to  $\lambda$  gives

$$\lambda = \frac{\sigma_{exp2}^2}{\sigma_{re1}^2 + \sigma_{exp2}^2}. \quad (10)$$

This result shows that only the additional expectation errors are relevant for the choice.  $\sigma_{exp2}^2$  is the error of the specific expectation compared with the abstract expectation. By introducing the

specific expectation an additional interpretation error appears.  $\sigma_{re1}^2$  is the additional information error of the abstract expectation, compared with the specific expectation. The specific expectation is optimal only if  $\sigma_{exp2}^2 = 0$  or  $\sigma_{re1}^2 = \infty$ . The proposition of Muth, and Sargent and Wallace, is therefore not true in this construction. The only exception is the case in which only one expectation exists, or  $\sigma_{re1}^2 = \infty$ , i.e. the abstract expectation is completely uninformative.

The result (10) can be given a Bayesian interpretation.  $w_1^{exp}$ , with  $\sigma_{re1}^2$ , can be regarded as a prior distribution for the realised wage. The optimal prediction is the posterior distribution after inference with  $w_2^{exp}$ . This Bayesian interpretation of expectation formation differs strongly, however, from the usual models of Bayesian learning [e.g. Cyert and DeGroot (1987)]. In these models the prior distribution is based on some a priori theoretical insight into the possible distribution of the parameters, while empirical events provide data for inference. In the model of this paper the prior distribution is formed by the frequency of past events, while theoretical insights are viewed as data for inference. While in the usual model forecasts improve from new data, the main improvements in this model come from additional theoretical insights.

#### 4. Implications

The model in this paper shows that if students deviate from the rational expectation by independent random errors, it becomes optimal for them to deviate from this pseudo rational expectation and to make some systematic errors in their forecasts. This result has two important implications. First, such systematic deviations from the rational expectation make it possible to use policies which manipulate people's expectations. In Borghans (1992a) it is shown, however, that the possibilities of such expectations manipulation policies are restricted. Secondly, the recognition of errors in forecasts by the students makes it possible to model an economic policy which aims at the improvement of students' expectations by the provision of public labour market information as described in Borghans (1992b). Such a provision of public labour market information can be interpreted as a decrease of errors in the forecasts made by students ( $\sigma_{exp2}^2$ ).

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