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# OPTIMAL MANIPULATION RULES IN A MIXED DUOPOLY

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# Optimal Manipulation Rules in a Mixed Duopoly

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**Abstract** We study the optimal manipulation rules of a public firm's objective function in a mixed duopoly with imperfect product substitutability. We compare the solutions under quantity and price competition, and the way in which they are affected by the degree of product substitutability. This allows us to show that partial privatization, strategic delegation and other specific government's commitments on the objective function of the public management can be looked at as special cases of these optimal rules, and to evaluate the viability of these policies under the two modes of competition. In this framework, we also discuss the equivalence between manipulation of the objective function and Stackelberg leadership. *JEL Classification:* D43, L13, L32.

**Keywords** Mixed oligopoly, strategic manipulation, partial privatization.

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# 1 Introduction

The recent literature on mixed oligopolies has highlighted that when a public firm competes with private firms, the welfare enhancing goals pursued by the government can be better achieved by undertaking a partial privatization of the publicly-owned firm (e.g., Matsumura, 1998; Matsumura and Kanda, 2005; Fujiwara, 2007). In a game-theoretic framework, the degree of privatization is seen as a government strategic decision, aimed at exploiting at best the properties of the strategic environment faced by the public firm.

Indeed, the decision to partially privatize a public firm amounts to allowing for a change of its objective function, from pure welfare – where the consumers’ surplus and the profits of all competing firms enter with the same weight – to an unbalanced mix of the welfare components, with a greater weight to its own profits. Therefore, partial privatization can be interpreted as a credible device through which the government, in a welfare maximizing perspective, strategically manipulates the decision rules followed by the public firms. However, partial privatization is a very specific kind of manipulation. As put forth by White (2002), it is in the nature of public firms that they can in principle be assigned by the policy-maker a much wider set of objective functions. For example, the manipulation may consist in assigning to the public firm the maximization of a generalized welfare function, where the consumers’ surplus, the rivals’ profits and its own profits enter separately, with possibly different weights.

In this paper we study this general optimal manipulation problem in the two cases of quantity and price competition. Our reference model is a mixed duopoly with differentiated product, which also allows us to capture the interplay between the strategy space and the degree of substitutability in shaping the optimal manipulation rules. In particular, we show that, since the latter allow for two degrees of freedom, they encompass in principle more specific rules (such as the optimal degree of privatization, the optimal strategic delegation, the optimal pro-consumer composition of the governing board), through an appropriate definition of specific additional constraints on the relationship among the relevant weights in the generalized welfare function. Moreover, in order to clarify the different systemic properties of the solutions under quantity and price competition, we explore the case where the set of optimal manipulations is constrained by a criterion of distance minimization from pure welfare objectives. This approach allows to point out that under quantity competition the optimal manipulation rule implies that the welfare

increase must be obtained through a redistribution from the consumer's surplus to profits, while under price competition the opposite occurs, with a reduction of the weight of profits in favour of consumers – which accounts for different policies being supported as implementation of the optimal rule in the two different strategic environments. Finally, we revisit in our mixed duopoly framework the equivalence between the unilateral adoption of a credible manipulation of an agent's objective function, and a time commitment making that agent a Stackelberg leader (Basu, 1995).

The paper is organized as follows. In Section 2 we derive the optimal manipulation rules under quantity and price competition. In Section 3 we show how they nest various implementation policies, and discuss their structure *vis à vis* the degree of product substitutability. Section 4 briefly comments on the equivalence between manipulation and Stackelberg leadership, while some final remarks are offered in Section 5.

## 2 Optimal manipulation

We consider a duopolistic market for a differentiated product, where the two varieties are produced by a public firm and a private firm (respectively indexed by 1 and 2). These two firms share a constant returns technology, and produce at a constant average and marginal cost  $c < 1$ . On the demand side, there is a continuum of identical consumers (normalized to 1), whose preferences are given by the following semi-linear utility function:

$$U(q_1, q_2; m) = V(q_1, q_2) + m \tag{1a}$$

$$V(q_1, q_2) = (q_1 + q_2) - \frac{1}{2}(q_1^2 + q_2^2 + 2\gamma q_1 q_2) \tag{1b}$$

where  $q_1$  and  $q_2$  are the quantities consumed of the two varieties of the differentiated good,  $m$  is a *numéraire* good exchanged in a perfectly competitive market, and  $\gamma \in (0, 1)$  is the parameter measuring product substitutability. As is well known, semi-linearity implies that the demand functions for the two varieties can be derived directly by maximization of  $V$  and exhibit no income effect.

On the supply side, we assume that while the government has an ultimate objective in terms of social welfare, defined as the unweighted sum of consumers' surplus and overall profits,

$$W = CS + \pi_1 + \pi_2 \tag{2}$$

it has the option of assigning to the public firm an objective function of the generalized welfare type:

$$GW = \alpha CS + \beta \pi_1 + \delta \pi_2$$

i.e., a linear combination of the welfare components. The positive coefficients  $\alpha$ ,  $\beta$ , and  $\delta$  are set by the government in a strict welfare-maximizing perspective, prior to the market interaction of the public firm with the profit-maximizing private firm. In this sense, any configuration different from  $\alpha = \beta = \delta = 1$  can be seen as an optimal manipulation of the public firm's objective function, which can actually be realized through an appropriate composition of its governing board, or through a system of incentives to public managers. The behaviour of this market is therefore described by a two-stage game: at the first stage the government strategically defines  $GW$ , then the public and private firm compete, with respect to either quantities or prices, in the product market. We start by investigating the quantity competition case.

## 2.1 The two-stage game with quantity decisions

Given preferences defined in (1), the inverse demand functions are:

$$p_1 = 1 - q_1 - \gamma q_2 \tag{3a}$$

$$p_2 = 1 - q_2 - \gamma q_1 \tag{3b}$$

so that the consumers' surplus can be written as:

$$\begin{aligned} CS(q_1, q_2) &= V(q_1, q_2) - p_1 q_1 - p_2 q_2 \\ &= \frac{1}{2} \left( (1 - \gamma) (q_1^2 + q_2^2) + \gamma (q_1 + q_2)^2 \right) \end{aligned}$$

Therefore, at the second stage of the game the objective function of the public firm is the following generalized welfare function:

$$\begin{aligned} GW(q_1, q_2) &= \frac{\alpha}{2} \left( (1 - \gamma) (q_1^2 + q_2^2) + \gamma (q_1 + q_2)^2 \right) + \\ &\quad + \beta (1 - q_1 - \gamma q_2 - c) q_1 + \delta (1 - q_2 - \gamma q_1 - c) q_2 \end{aligned}$$

Since the public firm maximizes  $GW$  and the private firm maximizes its profits, the reaction functions are:

$$\begin{aligned} q_1 &= \frac{1}{2\beta - \alpha} (\beta (1 - c) + (\alpha - \beta - \delta) \gamma q_2) \\ q_2 &= \frac{1 - c - \gamma q_1}{2} \end{aligned}$$

Solving for  $q_1$  and  $q_2$  yields the equilibrium quantities in terms of the *GW* function coefficients:

$$q_1(\alpha, \beta, \delta) = \frac{(1-c)(\gamma(\beta - \alpha + \delta) - 2\beta)}{2(\alpha - 2\beta) + \gamma^2(\beta - \alpha + \delta)} \quad (4a)$$

$$q_2(\alpha, \beta, \delta) = \frac{(1-c)(\alpha - \beta(2 - \gamma))}{2(\alpha - 2\beta) + \gamma^2(\beta - \alpha + \delta)} \quad (4b)$$

At the first stage of the game the government chooses the optimal manipulation of the public firm's motives. By substituting (4a,b) into the welfare function (2),  $W(q_1, q_2) = CS(q_1, q_2) + \pi_1(q_1, q_2) + \pi_2(q_1, q_2)$ , and maximizing it with respect to  $\alpha$ ,  $\beta$ , and  $\delta$ , we obtain a linear homogeneous system of rank 1, the solution of which is the optimal manipulation rule:

$$\begin{aligned} \alpha &= B^q(\gamma)\beta + \Delta^q(\gamma)\delta \\ B^q(\gamma) &= \frac{4 + \gamma^2 - 4\gamma}{4 - 2\gamma^2 - \gamma}, \quad \Delta^q(\gamma) = \frac{2\gamma - 2\gamma^2}{4 - 2\gamma^2 - \gamma} \end{aligned} \quad (5)$$

In (5),  $B^q(\gamma)$  is a U-shaped function of  $\gamma$ , tending to 1 as  $\gamma$  tends to zero or 1, with a minimum for  $\gamma = 2/3$ ;  $\Delta^q(\gamma)$  is hump-shaped, tending to zero as  $\gamma$  tends to zero or 1, with a maximum for  $\gamma = 2/3$ . Moreover, for all  $\gamma \in (0, 1)$   $B^q(\gamma) > \Delta^q(\gamma)$  and  $B^q(\gamma) + \Delta^q(\gamma) < 1$ .

It is easily seen that all values of  $\alpha$ ,  $\beta$ , and  $\delta$  satisfying (5) generate the following level of production for the two firms:

$$\begin{aligned} q_1 &= \frac{(4 - 3\gamma)(1 - c)}{(4 - 3\gamma^2)} \\ q_2 &= 2 \frac{(1 - \gamma)(1 - c)}{(4 - 3\gamma^2)} \end{aligned} \quad (6)$$

and ensure the following level of welfare:

$$W_q^* = \frac{1}{2} \frac{(7 - 6\gamma)(1 - c)^2}{(4 - 3\gamma^2)} \quad (7)$$

## 2.2 The two-stage game with price decisions

Assume now that the two firms compete with respect to their prices. The inverse demand system (3a,b) can be solved to obtain the direct demand

functions:

$$q_1 = \frac{(1 - \gamma) - p_1 + \gamma p_2}{1 - \gamma^2} \quad (8a)$$

$$q_2 = \frac{(1 - \gamma) - p_2 + \gamma p_1}{1 - \gamma^2} \quad (8b)$$

By substituting the above expressions into the  $GW$  function, the latter can be expressed in terms of prices as:

$$GW(p_1, p_2) = \alpha \left( \frac{p_1^2 + p_2^2 - 2\gamma p_1 p_2 + 2(1 - \gamma)(1 - (p_1 + p_2))}{2(1 - \gamma^2)} \right) + \beta \frac{(1 - \gamma) - p_1 + \gamma p_2}{1 - \gamma^2} (p_1 - c) + \delta \frac{(1 - \gamma) - p_2 + \gamma p_1}{1 - \gamma^2} (p_2 - c)$$

The price game between the profit-maximizing private firm and the  $GW$ -maximizing public firm has the following general solution:

$$p_1(\alpha, \beta, \delta) = \frac{(2 - \gamma^2 - \gamma(1 - c))\alpha + (\gamma^2 + \gamma(1 - c) - 2(1 + c))\beta + (\gamma^2 - \gamma(1 - c))\delta}{(2 - \gamma^2)\alpha - \beta(4 - \gamma^2) + \gamma^2\delta} \quad (9a)$$

$$p_2(\alpha, \beta, \delta) = \frac{\alpha(1 + c - \gamma^2) + \beta(\gamma^2 + \gamma(1 - c) - 2(1 + c)) + c\gamma^2\delta}{(2 - \gamma^2)\alpha - (4 - \gamma^2)\beta + \gamma^2\delta} \quad (9b)$$

Following the same procedure described above for the solution of the first stage of the game, we obtain the optimal manipulation rule under price competition:

$$\alpha = B^p(\gamma)\beta + \Delta^p(\gamma)\delta \quad (10)$$

$$B^p(\gamma) = \frac{4 - \gamma^3 - 3\gamma^2}{4 - \gamma^3 - 3\gamma^2 + \gamma}, \quad \Delta^p(\gamma) = \frac{2\gamma - \gamma^3}{4 - \gamma^3 - 3\gamma^2 + \gamma}$$

In (10)  $B^p(\gamma)$  is a decreasing function of  $\gamma$  ranging from 1 to 0 as  $\gamma$  goes from 0 to 1, while  $\Delta^p(\gamma)$  is increasing in  $\gamma$  from 0 to 1 along the same the interval, the two functions crossing at  $\gamma = \hat{\gamma} \approx 0.87$ . Moreover  $B^p(\gamma) + \Delta^p(\gamma) > 1$  for all  $\gamma \in (0, 1)$ .

Clearly, the equilibrium prices are the same for any triplet  $(\alpha, \beta, \delta)$  satisfying (10):

$$p_1 = \frac{4c - \gamma^2(1 + 2c) + \gamma(1 - c)}{4 - 3\gamma^2} \quad (11a)$$

$$p_2 = \frac{(1 - c)(\gamma^3 - 2\gamma) - (1 + 2c)\gamma^2 + 2(1 + c)}{4 - 3\gamma^2} \quad (11b)$$

yielding welfare

$$W_p^* = \frac{1}{2} \frac{(7 - \gamma^3 - 5\gamma^2 + \gamma)(1 - c)^2}{(\gamma + 1)(4 - 3\gamma^2)} \quad (12)$$

### 3 Policy implementations of the optimal rules

The optimal rules derived in the previous section clearly allow a high degree of freedom to the policy maker in assigning different weights to the welfare components. This property has been originally discussed by White (2002) in a quantity setting framework with homogeneous product, where the focus is on the possibility that the government exploits these degrees of freedom to disguise its true objectives through the rules it sets for the public management. Our interest is instead in showing how the discussion on the effectiveness of particular welfare-enhancing unilateral policies (e.g., partial privatization, strategic delegation, etc.) can easily be nested within the framework of the optimal manipulation rules – a unified framework, which accordingly encompasses different specific implementation strategies simply by imposing additional constraints on the relevant parameters. This analysis will also allow us to capture the structural features distinguishing the optimal manipulation rules in the price- as opposed to the quantity-setting.

#### 3.1 Partial privatization

Strategic partial privatization is the welfare-enhancing policy of transforming the public firm into a mixed (public–private) firm, where the government chooses optimally the share to be sold to the private sector. The idea is that this mixed ownership structure results into an objective function at the market stage which is a convex linear combination of welfare and the firm’s profits, i.e., a function of the type  $M = (1 - \psi)(CS + \pi_1 + \pi_2) + \psi\pi_1 = (1 - \psi)(CS + \pi_2) + \pi_1$ , where  $\psi$  coincides with (or is univocally related to) the shares held by the private shareholders. It is straightforward to see that, in terms of the optimal manipulation rule, this amounts to looking for a solution of the rule under the restrictions  $\alpha = \delta$  and  $\beta = 1$  which satisfies  $\alpha = \delta \in (0, 1)$ .

In the quantity competition case, by solving (5) under the partial privatization restrictions we get the positive value  $\alpha = \delta (= 1 - \psi) = (\gamma^2 + 4 - 4\gamma) / (4 - 3\gamma) < 1$ , which implies an optimal private share  $(1 - \alpha) = (1 - \gamma)\gamma /$



$(4 - 3\gamma)$ . By contrast, in a price-setting framework, if we solve (10) under the same restrictions, we get  $\alpha = \delta = (\gamma^2 + 4\gamma + 4) / (3\gamma + 4) > 1$ , which explains why under price competition partial privatization is not consistent with the optimal manipulation rule (Ohnishi, 2010; Ghosh and Mitra, 2008).

### 3.2 Strategic delegation

Strategic delegation for a public firm is the welfare-enhancing strategy of delegating market decisions to a manager, whose incentive scheme is a convex linear combination of welfare and another variable of interest for the manager.<sup>1</sup> For simplicity, we consider the so-called "relative performance" case, in which the manager's variable of interest is the profit differential, so that at the market stage the objective function becomes  $M = \phi(CS + \pi_1 + \pi_2) + (1 - \phi)(\pi_1 - \pi_2) = \phi CS + \pi_1 + (2\phi - 1)\pi_2$ .<sup>2</sup> In terms of optimal manipulation, this boils down to looking for a solution of the rule under the restrictions  $\delta = 2\alpha - 1$  and  $\beta = 1$ , which satisfies  $\alpha \in (0, 1)$ .

In the quantity-setting case, if we solve (5) under these strategic delegation restrictions we obtain  $\alpha (= \phi) = (3\gamma^2 - 6\gamma + 4) / (2\gamma^2 - 5\gamma + 4) < 1$  and  $\delta = (4\gamma^2 - 7\gamma + 4) / (2\gamma^2 - 5\gamma + 4)$ , which proves that the optimal manipulation rule can be implemented *via* this kind of strategic delegation. In the price case, the solution of (10) under the same restrictions delivers  $\alpha = (4 - 3\gamma^2 - 2\gamma) / (\gamma^3 - 3\gamma^2 - 3\gamma + 4)$  and  $\delta = (4 - \gamma^3 - 3\gamma^2 - \gamma) / (\gamma^3 - 3\gamma^2 - 3\gamma + 4)$ , such that  $\alpha \in (0, 1)$  with  $\delta > 0$  is observed only for  $\gamma > \hat{\gamma}$ .

### 3.3 Consumer-oriented management

A third economically meaningful possibility is that the government implements a welfare-enhancing policy *via* a consumer-oriented composition of

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<sup>1</sup>Strategic delegation in a private market has been originally discussed by Vickers (1985) and Fershtman and Judd (1987), who consider the cases in which the variables of interest for the manager are respectively sales and revenues. The issue of strategic delegation by a public firm has been firstly discussed by Barros (1995) by assuming an incentive scheme based on the public firm's profits and revenues, while the delegation mechanism based only on profits analyzed by Heywood and Ye (2009) actually collapses to a partial privatization scheme.

<sup>2</sup>In a fully private market, a relative performance based strategic delegation has been originally studied by Fumas (1992). See also Miller and Pazgal (2001).

the governing board, which will instruct the managers to behave according to an objective function which is a linear convex combination of social welfare and consumers' surplus:  $M = \varrho(CS + \pi_1 + \pi_2) + (1 - \rho)CS = CS + \varrho(CS + \pi_1 + \pi_2)$ . In our manipulation rules this amounts to imposing the constraints  $\alpha = 1$  and  $\beta = \delta$ , looking for a solution such that  $\beta = \delta \in (0, 1)$ . Solving (5) under these restrictions yields  $\beta = \delta (= \rho) = (\gamma + 2\gamma^2 - 4) / (2\gamma + \gamma^2 - 4) > 1$ , implying that under quantity competition the choice of a consumer-oriented management is not a viable implementation of the optimal manipulation rule. The opposite applies to the price-setting case: solving (10) under  $\alpha = 1$  and  $\beta = \delta$  gives  $\beta = \delta = (3\gamma^2 - \gamma + \gamma^3 - 4) / (2\gamma^3 - 2\gamma + 3\gamma^2 - 4) < 1$ , which makes a consumer-oriented commitment consistent with the optimal manipulation rule when firms compete with respect to prices.

### 3.4 Minimizing the distance from pure welfare maximization

The implementation of the optimal manipulation rules discussed above results into imposing *a priori* specific constraints on the weights of the generalized welfare function. The key properties of these rules, however, can be more easily captured by imposing a systemic constraint, namely by choosing the triplets  $(\alpha, \beta, \delta)$  satisfying (5) or (10), which minimize the distance from pure welfare maximization. The rationale for such a choice can be envisaged in the existence of symmetric manipulation costs for each parameter – which can be justified in terms of the political bargaining required for any detachment from a simple welfare maximization rule. By using a quadratic definition of distance, this application of the optimal rule amounts to identifying the triplets solving the following minimization problems for the quantity-setting and price-setting cases, respectively:

$$\min_{\alpha, \beta, \delta, \lambda} \left( (\alpha - 1)^2 + (\beta - 1)^2 + (\delta - 1)^2 + \lambda \left( \alpha - \frac{(4\gamma - \gamma^2 - 4)}{\gamma + 2\gamma^2 - 4} \beta - \frac{2\gamma(\gamma - 1)}{\gamma + 2\gamma^2 - 4} \delta \right) \right)$$

$$\min_{\alpha, \beta, \delta, \lambda} \left( (\alpha - 1)^2 + (\beta - 1)^2 + (\delta - 1)^2 + \lambda \left( \alpha - \frac{3\gamma^2 + \gamma^3 - 4}{3\gamma^2 - \gamma + \gamma^3 - 4} \beta - \frac{\gamma^3 - 2\gamma}{3\gamma^2 - \gamma + \gamma^3 - 4} \delta \right) \right)$$

where  $\lambda$  is the Lagrange multiplier associated to each optimal manipulation rule.

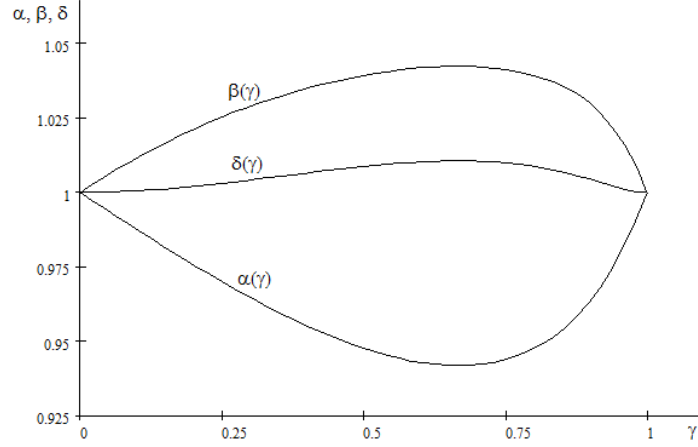


Figure 1: Distance-minimizing optimal weights under quantity competition

For the quantity case we obtain:

$$\begin{aligned}\alpha &= \frac{1}{k} (7\gamma^4 - 11\gamma^3 + 18\gamma^2 - 44\gamma + 32) \\ \beta &= \frac{1}{k} (8\gamma^4 - 7\gamma^3 + 5\gamma^2 - 36\gamma + 32) \\ \delta &= \frac{1}{k} (11\gamma^4 - 16\gamma^3 + 15\gamma^2 - 40\gamma + 32)\end{aligned}$$

with  $k = (9\gamma^4 - 12\gamma^3 + 13\gamma^2 - 40\gamma + 32)$ . As shown in Figure 1, the optimal weights are such that those assigned to the firms' profits are both higher than 1, the highest being the public firm's. On the contrary, the consumers' surplus is assigned a weight lower than 1. These features are preserved for all values of  $\gamma \in (0, 1)$ .

For the price-setting case, distance minimization yields

$$\begin{aligned}\alpha &= \frac{1}{h} (4\gamma^6 + 15\gamma^5 + 10\gamma^4 - 29\gamma^3 - 42\gamma^2 + 12\gamma + 32) \\ \beta &= \frac{1}{h} (2\gamma^6 + 9\gamma^5 + 13\gamma^4 - 15\gamma^3 - 43\gamma^2 + 4\gamma + 32) \\ \delta &= \frac{1}{h} (2\gamma^6 + 12\gamma^5 + 15\gamma^4 - 22\gamma^3 - 45\gamma^2 + 8\gamma + 32)\end{aligned}$$

with  $h = (3\gamma^6 + 12\gamma^5 + 12\gamma^4 - 22\gamma^3 - 43\gamma^2 + 8\gamma + 32)$ . As shown in Figure 2, according to this implementation of the optimal rule, for all values of  $\gamma$

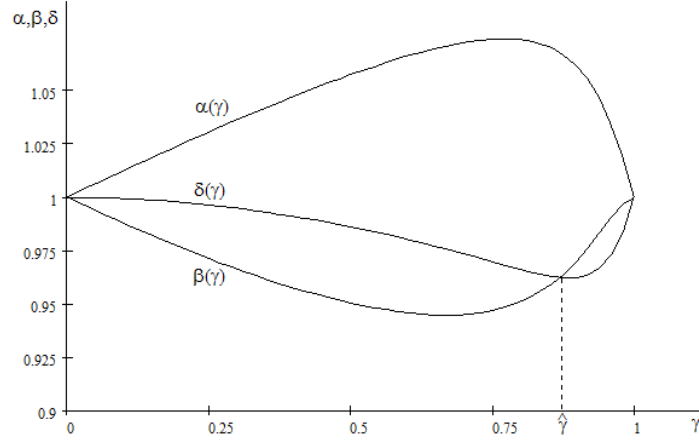


Figure 2: Distance-minimizing optimal weights under price competition

the consumers' surplus is given a weight greater than 1, while those of the profits of both firms are lower than 1, with  $\delta > \beta$  for  $\gamma < \hat{\gamma}$ , which is reversed beyond this threshold value.

### 3.5 Comparing manipulation rules under quantity and price setting

The analysis of the distance minimizing implementation of the manipulation rules highlights the optimal structure of weights associated to the different modes of competition – which in turn mirrors the properties of equilibrium in the non-manipulation case. Under quantity competition, distance minimization implies that the weight assigned to both profits is higher than 1 and the weight assigned to the  $CS$  is lower than 1. In the standard model with no manipulation, the reaction function of the public firm is such that for, any quantity of the rival, it sets a quantity such that its price equals marginal cost. This is due to the fact that any marginal decrease in  $q_1$  does not affect, for given  $q_2$ , the marginal contribution to welfare of its private competitor, while it increases, provided  $p_1 > c$ , its own contribution. This aggressiveness impacts positively on welfare in terms of overall production; however, in a strategic substitutability environment, it turns out to induce a marked difference between the quantities produced by the two firms, which

is beneficial in terms of consumers' surplus, but is indeed welfare detrimental. At equilibrium there is a potential for increasing overall welfare via an increase in profits and a reduction of the consumers' surplus, obtained by changing the balance between the contribution to welfare of the size of market production, and that of the distribution of this production across firms. By assigning a higher weight to its own profits, the public firm reduces its produced quantity, thus expanding the demand faced by its rival: the quantity differential shrinks, with an increase in profits for both firms which more than compensates the reduction of the consumers' surplus associated to the contraction in overall production and its more even distribution. This explains why in the optimal rule  $B^q(\gamma) > \Delta^q(\gamma)$  and  $B^q(\gamma) + \Delta^q(\gamma) < 1$ , and why the implementation of the optimal rule may take the features of a partial privatization, as well as those of a strategic delegation on a relative performance base; it also explains why it does not support a consumer-oriented management. Clearly, the extent of manipulation is larger for intermediate values of  $\gamma$ : when  $\gamma$  is low the public firm can only marginally affect the rival's decisions, while for high  $\gamma$  near homogeneity of product makes pure welfare maximization almost optimal.

Under price competition, if a distance minimization criterion is adopted, the weight of the  $CS$  is always greater than 1, while  $\pi_1$  and  $\pi_2$  are both given a weight lower than 1. With no manipulation, the public firm sets a price higher than marginal cost, which allows for a higher price set by the private firm: given  $p_2$ , any marginal decrease in  $p_1$  reduces the contribution of the private firm to welfare, and this limits the aggressiveness of the public firm. The interaction between the two firms is such that at equilibrium there is room for an increase in welfare through a redistribution of weights which favours consumption. The public firm's behavior may be moved towards a generalized efficient pricing, by exploiting strategic complementarity: by overvaluing the consumers' surplus and undervaluing its own profits and the profits of the rival, it lowers its price and pushes the rival in the same direction. Notice that for  $\gamma \leq \hat{\gamma}$  the marginal effect on welfare of the public firm pricing *via* its own profits dominates its effect through the rival's profits, while the opposite occurs beyond that level. All these observations imply that the manipulation rule cannot be consistent with partial privatization, but rather with a consumer-oriented management. Moreover, it explains why it is consistent with a relative performance oriented strategic delegation – which favours the public firm's own profits over the rivals' – only for  $\gamma > \hat{\gamma}$ , which is also the interval where  $\Delta^p(\gamma) > B^p(\gamma)$ .

## 4 Optimal manipulation and time commitment

The optimal rules (5) and (10) confirm in our mixed duopoly setups the equivalence between any unilateral credible manipulation of an agent's objective function, and the time commitment characterizing sequential market games (e.g. Basu, 1995; White, 2002). The main implication of this equivalence is stated in the following remark:

**Remark 1** *The optimal manipulation rule for a public firm acting at the market stage as a Stackelberg leader is equivalent to pure welfare maximization, and it generates the same solution as the optimal manipulation rule for a public firm playing simultaneously on the market. Therefore, once the government is allowed to optimally manipulate the public firm's objectives, the solution of the market game is independent of the timing (simultaneous or sequential with public leadership) of market decisions.*

Indeed, by solving the same optimal manipulation problem for a public firm acting as a Stackelberg leader in the product market, we obtain the following optimal rules for the quantity case and the price case, respectively:

$$\alpha = \frac{3\gamma^3 - 2\gamma^2 + 8 - 8\gamma}{(2 - \gamma)(4 - 3\gamma^2)}\beta + \frac{4\gamma - 4\gamma^2}{(2 - \gamma)(4 - 3\gamma^2)}\delta$$

$$\alpha = \frac{6\gamma^2 + \gamma^3 - 8}{6\gamma^2 - 4\gamma + 3\gamma^3 - 8}\beta + \frac{2\gamma^3 - 4\gamma}{6\gamma^2 - 4\gamma + 3\gamma^3 - 8}\delta$$

These general rules are consistent with  $\alpha = \beta = \delta = 1$  - i.e., with pure welfare maximization; moreover, by embodying them in the leader's and follower's optimal quantities or prices, and evaluating the correspondent welfare functions, we obtain again eqts (7) and (12).

## 5 Conclusions

Starting from the idea that, in a mixed oligopoly, the public firm's unilateral optimal strategic commitment can take the form of assigning to the public management an objective in terms of a generalized welfare function, in this paper we have analyzed the structure of this function in the duopoly case with quantity setting and price setting, under the assumption that products

are imperfect substitutes. By assuming that the private and public firms share the same technology, our results rely exclusively on the properties of the strategic interaction between firms, thus ruling out any cost disadvantage on the public side.

We show that in the quantity-setting case the optimal generalized welfare function assigned to the public management overvalues the public firm's and the rival's profits, while it undervalues the consumers' surplus: the welfare gain obtained through manipulation of the objective function is obtained *via* a redistribution from consumers to profits – which explains why this optimal structure can be obtained through partial privatization or strategic delegation policies based on relative performance. In the price-setting case, we obtain an opposite result: the welfare enhancing effects of the optimal manipulation of the objective function are obtained *via* a redistribution from profits to consumers, which suggests implementation policies based on a consumer-oriented management – a framework where partial privatization is accordingly inconsistent with the optimal manipulation rule.

Finally, in this general optimal manipulation framework we have reformulated the equivalence between adopting any unilateral strategic commitment, and playing the role of a Stackelberg leader. This equivalence suggests that when the endogenization of the timing of decisions supports public leadership, there are no efficiency arguments in favour of partial privatization, or of any other strategic manipulations of the public firm's objective function, consistent with the optimal rule;<sup>3</sup> by contrast, strategic manipulation is a powerful welfare-enhancing instrument when the incentives of the public and private firms are to act simultaneously on the market.

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<sup>3</sup>The endogenization of the timing of decisions within a mixed oligopoly has been discussed by Pal (1998) and Matsumura (2003) in the case of quantity competition with homogeneous product, and by Barcena-Ruiz (2007) in the case of price competition with differentiated product.

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