A SIMPLE CONTINUOUS MEASURE OF CREDIT RISK

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ABSTRACT. This paper introduces a simple continuous measure of credit risk that associates to each firm a risk parameter related to the firm's risk-neutral default intensity. These parameters can be computed from quoted bond prices and allow assignment of credit ratings much finer than those provided by various rating agencies. We estimate the risk measures on a daily basis for a sample of US firms and compare them with the corresponding ratings provided by Moody's and the distance to default measures calculated using the Merton (1974) model. The three measures group the sample of firms into various risk classes in a similar but far from identical way, possibly reflecting the models' different forecasting horizons. Among the three measures, the highest rank correlation is found between our continuous measure and Moody's ratings. The techniques in this paper can be used to extract the entire distribution of inter-temporal risk-neutral default intensities which is useful for time-to-default estimations as well as for pricing credit derivatives.

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1. Introduction

Credit risk, or the risk of counterparty default, is an important factor in the valuation and the risk management of financial assets. The losses arising from the string of recent corporate collapses, including Enron, WorldCom and Kmart, provide ample evidence of its importance. However, the risk of default, by its very nature, is difficult to quantify, with the traditional credit ratings supplied by agencies such as Moody's and Standard & Poor's being far from ideal. For example, the discrete nature of these ratings result in groups of firms being assigned the same rating despite the differences in their credit worthiness, and the changes in the credit worthiness of firms are not immediately reflected in their ratings.

Moreover, the measures of credit risk implied from models, be it traditional scoring models or structural models like the Merton (1974) model, typically rely on balance sheet data and historical stock volatilities. Consequently these measures tend to rely as much on historical events as on the current market situation, and are subject to the availability of accurate and up to date balance sheet data.

This paper introduces a new measure of credit risk that complements the traditional credit ratings in several ways. Firstly, being continuous in nature, the measure allows a finer separation of firms according to their credit worthiness. Secondly, being computed from current bond data, the measure is inherently forward looking and adjusts immediately to changes in the credit worthiness of firms. Finally, unlike the symbolic ratings that are without economic meaning, the measure has a convenient interpretation as the risk neutral probability of immediate default.

It should be noted that the measure introduced in this paper, being the risk-neutral intensity of immediate default, focuses on the risk of default in the short term. In contrast, Moody's ratings instead take a more long term view, typically focusing on at least one entire business cycle while Merton's model typically produces one to two year default probabilities. Nevertheless, a comparison of the these measures reveal a high degree of correlation, and, in any case, for practical purposes they should probably be seen as complements rather than substitutes for one another.

The data requirements to compute the measure introduced in this paper are minimal. The only requirements are the bonds issued by the firms under consideration and proxies for the risk-free bonds, which may be the treasury bond or the bonds issued by a firm of highest credit rating. However, to ensure sensible default intensities, we found it necessary to construct the risk-free yield curve with the condition that the risk-free yields lie below the yields of all corporate bond yields. The model is also quite simple, with the instantaneous forward rates and zero coupon bond prices playing the key roles in determining the risk measures and default intensities.

Accurate estimates of the expected *time* to default of firms are useful and are addressed in this paper by extending the credit measure beyond its use as a simple rating. In particular, it is shown that a firm's entire inter-temporal distribution of default intensities can be inferred from instantaneous forward rates and zero coupon bond prices. This distribution can then be used to extract the expected time to default.

The structure of the remainder of the paper is as follows. Section 2 gives a brief review of the existing measures of credit risk, Section 3 defines the new risk measure and outlines a method for its computation, Section 4 presents the data used in the empirical study of this paper, and Section 5 provides the results. The paper finally concludes with Section 6.

2. TRADITIONAL MEASURES OF CREDIT RISK

All investors, regardless of whether they invest in equities or fixed income instruments, need to take the risk of counterparty default into consideration when making their investment decisions, and the ability to quantify this so called credit risk is important for both pricing and risk management purposes.

Now, there are several ways in which the risk of default can be gauged. One way is to rely on rating agencies, such as Moody's and Standard & Poor's, that rate individual firm's capability to service and repay their debt. An alternative way is to rely on models that attempt to quantify the level of credit risk using accounting information and stock price data.

Unfortunately, the precise processes by which the ratings agencies arrive at their credit ratings remain the proprietary information of the respective agencies. However, they most likely involve analysts' views formed from public and private information. A particularly simple and popular approach for inferring credit risk information using accounting and stock market information is the contingent claims based model of Merton (1974). This model views a firm's liabilities (equity and debt) as contingent claims issued against the firm's underlying assets. By backing out asset prices and volatilities from quoted equity prices and balance sheet information, it provides estimates of the firm's default probabilities. Since we compare this risk measure to ours in the empirical study in Section 5 we now describe it in some detail.

2.1. Review of the Merton Model.

Recall that equity holders have the residual claim on a firm's assets while being subject to limited liability. Merton (1974) recognised that an equity in a firm is equivalent to a long position in a call option on the firm's assets, and used this correspondence to derive the market value and volatility of the firm's underlying assets. More precisely, Merton used the Black and Scholes (1973) framework to solve for the asset value and volatility implied by the option price and the option price volatility[1]. The asset value and the asset volatility can then be combined into a risk measure called *distance to default* that is directly related to the credit worthiness of the equity issuing firm.

At the heart of the Merton (1974) model lies a modified version of the Black-Scholes formula, viz.

$$V_E = V_A \mathcal{N}(d_1) - e^{-r(T-t)} D \mathcal{N}(d_2), \tag{2.1}$$

[1]Note that in this case the option is equity and the underlying asset is the firm's assets.

linking the market value of equity and the market value of assets, where

$$\begin{aligned}
V_E &= \text{market value of the firm's equity},\\
V_A &= \text{market value of the firm's assets},\\
D &= \text{total amount of the firm's debt},\\
T-t &= \text{time to maturity of the firm's debt},\\
r &= \text{risk free interest rate},\\
d_1 &= \frac{\ln(V_A/D) + (r + \frac{1}{2}\sigma_A^2)(T-t)}{\sigma_A\sqrt{T-t}},\\
d_2 &= d_1 - \sigma_A\sqrt{T-t},\\
\mathcal{N}(\cdot) &= \text{cumulative normal distribution}.
\end{aligned}$$

Moreover, it is easily shown that the equity and asset volatility are related by the expression

$$\sigma_E = \frac{V_A}{V_E}\mathcal{N}(d_1)\sigma_A, \tag{2.2}$$

where

$$\begin{aligned}
\sigma_E &= \text{volatility of the firm's equity returns},\\
\sigma_A &= \text{volatility of the firm's asset returns}.
\end{aligned}$$

Solving the nonlinear system of equations (2.1)–(2.2) gives V_A and σ_A, and the distance to default is defined by the expression

$$\gamma = \frac{\ln(V_A/D) + (r - \frac{1}{2}\sigma_A^2)(T-t)}{\sigma_A\sqrt{T-t}}. \tag{2.3}$$

This is simply the number of standard deviations that the firm value is from the threshold point, and the smaller the value of γ the larger the (risk neutral) probability that the firm will default on its debt.

In this paper we compare the rankings implied by this risk measure to the rankings obtained from our continuous risk measure and those provided by Moody's. Note that while the risk measure implied by Merton's model is based on observations from the stock market the measure introduced in this paper will be based on observations from the bond market. Consequently, the comparison of the two results could shed some light on whether the stock and the bond markets carry similar information about the credit worthiness of firms.

2.2. **Motivations for a New Risk measure.** Although useful, traditional credit ratings and scoring models relying on accounting information have several drawbacks as outlined in Section 1. The most obvious limitations with the rankings provided by the ratings agencies include the low frequency with which they are updated, often only once or twice a year, and the discrete nature of the rankings. Accounting based scoring models, on the other hand, rely on infrequently updated data that are not only released with a time lag, but also with possible accounting manipulations. Moreover, accounting information is inherently backward looking, based on historic information rather than the market's assessment of the future. Both approaches also suffer from producing measures without meaningful economic interpretations. Neither a credit rating nor a credit score is easily transformed into an actual probability of default and neither gives much information about the actual loss given such a default.

An alternative to these credit ratings and scores involves the extraction of information about credit risk from up to date market data. If credit risk is incorporated into market prices then there must be ways of filtering the information contained in these prices to extract the component that can be attributed to credit risk. Various ways of extracting this information from either the stock or the bond data have been suggested.

Public information is likely to be instantaneously and simultaneously embedded into individual security prices in the stock and the bond markets. This makes stock and bond prices contemporaneously correlated. In contrast, private information conveyed by informed investors that systematically trade in either the stock or the bond market is transmitted more through one of the markets than through the other.[2] Stock and bond markets consequently carry somewhat different information regarding credit risk.

The most well known stock market based approach of providing estimates of the firm's default probability, the Merton (1974) model, was described above. Now, data from the bond market can also be used as the basis for extracting credit risk information, and this paper provides one such method that assigns a continuous credit measure using bond prices. As with most credit risk measures based on debt market information the underlying idea is to compare bonds *with* credit risk (corporate bonds) to those essentially *without* credit risk (treasury bonds or high quality corporate bonds). The term structure of survival probabilities can then be constructed, as for instance, in Jarrow, Lando and Turnbull (1997). Clearly, there is a correspondence between the shape of these survival probability curves and the risk of default of the corresponding firms, and this paper introduces the slope at the short end of these curve as a simple proxy for the credit *rating* of the firms. Since this slope reduces to the difference between the instantaneous short rates implied by the corporate and the risk-free bonds, the problem of determining the risk measure reduces to that of constructing reliable forward rate curves.

3. A New Continuous Credit Risk Measure

This section provides a detailed description of the continuous credit risk measure introduced in this paper. The measure relies solely on bond market data and neither accounting data nor external agency ratings are used. Compared to ordinary ratings our rating has the advantage that it is continuous rather than discrete and that it is based on daily market quotes instead of sporadic and subjective decisions by rating agencies. As mentioned previously, being continuous, this measure avoids the problems of traditional ratings regarding widely different *(ex post)* default rates within the same rating class and the large overlaps in default rates between different rating classes. It also assigns a single number to each firm rather than the multiple ratings of ordinary rating agencies, due mainly to their focus on debt issues instead of the actual firm. Finally, recall that this measure has the useful interpretation as the risk-neutral intensity of immediate default.

[2] The reasons for informed traders to systematically prefer one market to the other include difference in risk aversion, transaction costs, insider-trading laws or different institutional constraints

As with all other market based measures of default risk our measure relies on markets being efficient.[3] We expect the validity of this assumption, and the reliability of our measure, to improve with time as bond markets around the world develop in a manner similar to the way in which the global equity markets have developed over the past few decades. Compulsory periodic emission of large quantities of subordinated debt by banks, in order to impose discipline on these entities as suggested by Calomiris (1999), could be one factor speeding up the growth of bond markets around the world.

Another crucial assumption is the independence of the risk-free interest rate process and the default processes of the firms. This assumption is common in the literature, and is made for example in Das and Tufano (1996), Jarrow, Lando and Turnbull (1997) and Leland and Toft (1996), but whether it is reasonable or not is an outstanding empirical issue. It should be noted that several studies indicate violations of this assumption, most notably Duffee (1998) and Longstaff and Schwartz (1995) who find that spreads on corporate bonds fall when risk-free interest rates rise.

A related issue is the specification of a proxy for the risk-free interest rates and corresponding risk-free bonds from which risk-free interest rates must be inferred. Although treasury bonds and the corporate bonds of the highest quality are both commonly used proxies for the risk-free bonds in the literature, the latter is often the better choice, as suggested for example in Kwan (1996), since a large part of the spread between the corporate and treasury bond yields arises from differences in factors such as taxation and liquidity (Huang and Huang (2002)), and using high quality corporate bonds allows the impact of some of these non-credit related causes of the credit spread to be minimised.

In this paper, we construct the risk-free forward rate curves by requiring that they lie below all the risky, or corporate, forward rate curves. These *risk-free* forward rates are then used to derive the *risk-free* zero-coupon prices and the default intensities. This procedure ensures that all risky bonds are priced below their risk-free counterparts. It also ensures that the differences, or the spreads, between the risky and the risk-free bond prices are due primarily to credit risk.

Finally, it is assumed that the recovery rate, or the amount that is recovered by the bondholder in case of default (as a share of the principal plus any accumulated interest rate payments) is given exogenously. For the purposes of this paper, we use an averaged historical value quoted in Jarrow, Lando and Turnbull (1997). Although it is possible to relax this assumption by estimating the recovery rate from the bond price data along with the credit measure, we nevertheless treat it as an exogenous constant for simplicity since the recovery rate is of less importance for the purposes of determining the *relative* credit rankings of firms in this paper.

3.1. Derivation of the Risk Measure and the Role of Forward Rate Curves.

The first step in determining the risk measure involves inferring the risk-neutral survival probabilities, or the probabilities that a firm will not default within given time horizons, from the bond market data.

[3] Even if the bond markets are inefficient the measure could be interpreted as the bond market implied credit worthiness of a particular firm. It would not be possible to interpret the measure as an actual credit rating, however.

Let $p(t,T)$ denote the time t price of a T-maturity risk-free zero coupon bond and let $v^\pi(t,T)$ be the corresponding price for bonds issued by firms with a certain external *credit rating* π. Then the assumption of independence of the risk free interest rate process and the default processes implies

$$v^\pi(t,T) = p(t,T)\left[\delta^\pi + (1-\delta^\pi)\,F^\pi(T|\,t)\right], \tag{3.1}$$

where δ^π is the recovery rate for firms with external credit rating π, and $F^\pi(T|t)$ is the corresponding risk-neutral conditional survival probability to time T given that they have survived to time $t \le T$. Rearranging this equation gives the expression,

$$F^\pi(T|\,t) = \frac{v^\pi(t,T)/p(t,T) - \delta^\pi}{1-\delta^\pi}, \tag{3.2}$$

for the risk-neutral conditional survival probabilities, which is a measure of the credit worthiness of firms. If it is assumed that the recovery rates are given exogenously, then the only task that remains in order to calculate survival probabilities, $F^\pi(T|\,t)$, is to extract zero coupon bond prices from traded coupon bond prices, and a detailed description of the method used for this task in this paper is given in §3.2.

Note that if π_1 represents a higher credit rating than π_2, then it should be the case that

$$v^{\pi_1}(T|t) \ge v^{\pi_2}(T|t), \tag{3.3}$$

and it should then follow from (3.2) that

$$F^{\pi_1}(T|t) \ge F^{\pi_2}(T|t), \tag{3.4}$$

for all $t \le T$. This observation, of course, relies on the assumption of a constant recovery rate over the rating classes.

Now, provided that the bonds are priced correctly and consistently in the market, the increase in the risk of default, represented by decreasing π, should be reflected in the steepening of the corresponding curve for $F^\pi(T|t)$.[4] Consequently, in an efficient market, the slope of the curve $F^\pi(T|\,t)$ at $T=t$ can be interpreted as a measure of a firm's short term default risk, or as a short term *credit rating*. Since the slope of $F^\pi(T|t)$ at $T=t$ is negative, and the negative of the slope permits the convenient economic interpretation as the intensity of immediate default, we have chosen to use the negative of the slope as the measure of risk. An important advantage of this measure compared to external ratings is that it is continuous and hence enables a much finer separation of firms according to their risk of default.

It follows from (3.2) that the slope of $F^\pi(T|\,t)$ at $T=t$ is given by the equation

$$\left.\frac{\partial F^\pi(T|\,t)}{\partial T}\right|_{T=t} = \frac{1}{1-\delta^\pi}\frac{\partial}{\partial T}\left[\frac{v^\pi(t,T)}{p(t,T)}\right]\Bigg|_{T=t}. \tag{3.5}$$

Now, if we denote the risk-free forward rate curve by $r(t,T)$ and the forward rate curve for the firm π by $r^\pi(t,T)$, then by definition

$$v^\pi(t,T) = e^{-\int_t^T r^\pi(t,u)\,du} \quad \text{and} \quad p(t,T) = e^{-\int_t^T r(t,u)\,du},$$

[4]It is not necessarily the case that the slope monotonically increases with deteriorating credit rating. Due to the different views of rating agencies and the bond market, there will never be a strict one-to-one correspondence between the slope and the rating class.

and so we have

$$\begin{aligned}\frac{\partial}{\partial T}\left[\frac{v^\pi(t,T)}{p(t,T)}\right]\bigg|_{T=t} &= \frac{\partial}{\partial T}\left[e^{-\int_t^T (r^\pi(t,u)-r(t,u))\,du}\right]\bigg|_{T=t} \\ &= -\left(r^\pi(t,T)-r(t,T)\right)e^{-\int_t^T (r^\pi(t,u)-r(t,u))\,du}\Big|_{T=t} \\ &= -\left(r^\pi(t,T)-r(t,T)\right)\left[\frac{v^\pi(t,T)}{p(t,T)}\right]\bigg|_{T=t},\end{aligned} \tag{3.6}$$

and (3.5) becomes

$$\frac{\partial F^\pi(T|\,t)}{\partial T}\bigg|_{T=t} = -\frac{(r^\pi(t,t)-r(t,t))}{1-\delta^\pi}\left[\frac{v^\pi(t,t)}{p(t,t)}\right] = -\frac{(r^\pi(t,t)-r(t,t))}{1-\delta^\pi}$$

since $v^\pi(t,t) = p(t,t) = 1$. Consequently, the credit measure proposed in this paper is

$$m(\pi) = -\frac{\partial F^\pi(T|\,t)}{\partial T}\bigg|_{T=t} = \frac{(r^\pi(t,t)-r(t,t))}{1-\delta^\pi}. \tag{3.7}$$

Note that since the conditional probability of default, $\tilde{F}^\pi(T|\,t)$, satisfies the equation $\tilde{F}^\pi(T|\,t) = 1 - F^\pi(T|t)$, it follows that

$$m(\pi) = \frac{\partial \tilde{F}(t,T)}{\partial T}\bigg|_{T=t} = \lim_{T\downarrow t}\tilde{f}(T|t), \tag{3.8}$$

where $\tilde{f}(T|t)$ represents the intensity of default. This shows that the measure, $m(\pi)$, can be interpreted as the intensity of *immediate* default.

3.2. **Construction of Forward Rate Curves.** Since the instantaneous forward rates, and in particular the instantaneous short rates, are key determinants of the risk measure introduced in this paper, it is important to ensure that the zero coupon bond prices are extracted in such a way that the corresponding forward rates are as *accurate*[5] as possible. Moreover, since the shapes of forward rate curves play a key role in determining the risk neutral default intensities, it is also important to ensure that the curve of these forward rates are as *smooth* as possible. In practice, however, these requirements cannot be satisfied simultaneously and, depending on the situation, the accuracy or smoothness must be sacrificed.

For the purposes of this paper, accurate short rates and smooth forward rate curves are required. Now, the only sure way to satisfy the latter requirement is to assume a parametric form for the forward rate curves, and the specification that provides a good compromise between flexibility and ease of implementation is the parametric form introduced in Nelson and Siegel (1987). The $(T-t)$-maturity instantaneous forward rate, $r(t,T)$, under their specification is assumed to be determined by the equation

$$r(t,T) = a_0 + a_1\,e^{-\kappa(T-t)} + a_2(T-t)\,e^{-\kappa(T-t)}, \tag{3.9}$$

where a_0, a_1, a_2 and κ are parameters subject to constraints

$$a_0 > 0, \quad a_0 + a_1 > 0 \quad \text{and} \quad \kappa > 0. \tag{3.10}$$

[5]We consider the extracted forward rates to be *accurate* if the prices they imply for the market traded bonds coincide with the actual market quoted prices.

Note that $a_0 = \lim_{(T-t)\uparrow\infty} r(t,T)$ corresponds to the long rate and $a_0 + a_1 = r(0)$ corresponds to the short rate.

Now, for each firm and date of interest, the aim is to determine the four parameters in (3.9) from the corresponding bond data. In this paper, we have determined the parameters by minimising the total weighted squared error between the model implied bond prices and their actual market quoted counterparts. That is, we have determined the parameters a_0, a_1, a_2 and κ by minimising the quantity

$$\epsilon^2 = \sum_i \omega_i^2 \left(P_i(a_0, a_1, a_2, \kappa) - \hat{P}_i \right)^2, \tag{3.11}$$

where P_i is the model implied price of the i-th bond, $\hat{P}_i$ is the market observed price, and

$$\omega_i = \frac{1/D_i}{\sum_i 1/D_i},$$

with D_i being the duration of the i-th bond. Note that the introduction of the weighting factors, ω_i, has the effect of reducing the natural tendency for the model to over fit the long dated bonds at the expense of the short dated bonds.

Numerical implementations of the above procedure require the input of initial starting values of the parameters in one form or another, and as observed for example in Bolder and Stréliski (1999), the parameters can be very sensitive to these initial values. The approach suggested in Bolder and Stréliski (1999) first approximates the parameters a_0 and a_1 by setting the short and the long rates equal to the yield on the shortest and the longest maturity bonds respectively. The remaining parameters are then determined by dividing the (a_2, κ) space into several rectangular regions, computing the local minimum within each region, and then setting the parameters equal to the minimum among these local minima.

We have essentially adopted the above approach with one important modification. Since we require accurate short rates, we first constructed the forward rate curves using the technique introduced in Kwon (2002). This technique computes forward rate curves that maximise a given smoothness measure, and results in forward rate curves that are splines with certain functional forms. The technique also adjusts the market bond prices, within their bid-ask spreads, to improve the smoothness of the resulting forward rate curves. The smoothness measure used for the initial forward rate curves in this paper was

$$\int_0^{\bar{T}} r'(t,u)^2 \, du, \tag{3.12}$$

where $\bar{T}$ is the maturity of the longest dated bond, and corresponds to quadratic splines. Unfortunately, the resulting forward rate curves were not sufficiently smooth for the purposes of inferring the entire curve of default intensities, thereby necessitating the use of Nelson-Siegel curves.

Once constructed, the short and the long forward rates from these more accurate curves were used to approximate the Nelson-Siegel parameters a_0 and a_1, and the remaining parameters were then determined according to the method outlined above. It should be noted that our approach provides a more accurate approximation for a_0 and a_1 than that suggested by Bolder and Stréliski (1999), since our approximation uses actual forward rates rather than bond yields and our short rate is the actual short rate rather than being the yield of the shortest dated bond whose maturity may be more than one year.

As discussed earlier, common proxies for the risk free bonds in the literature are the treasury bonds or the bonds from a firm of the highest credit rating. However, with the data used in this paper, neither of these approaches were satisfactory since in many cases the corresponding 'risk-free' forward rates were higher than their risky counterparts, resulting in negative default intensities. To overcome this serious problem, we constructed the risk-free rate forward rate curves with the main aim of ensuring that it remained below all the risky forward rate curves. For this, we divided the maturity interval into one year subintervals, identified the lowest forward rate in each subinterval among all the risky forward rates curves for the date under consideration, and then fit a Nelson-Siegel curve through all these minimal forward rates. As a final step, the 'risk-free' curve was shifted down, if required, to ensure that the risk-free forward rate curve was below all the risky forward rate curves.

Figure 3.1 gives examples of typical forward rate curves that result from the above construction procedure. In general, it appears that the forward rate curves for the riskier firms are inverted while the curves for the safer firms tend to have the normal upward sloping shape.[6]

3.3. **Construction of Inter-Temporal Default Density Distributions.** The technique described above for computing the intensity of immediate default can be extended to give the entire distribution of default intensities in a simple way. In determining the risk measure $m(\pi)$, we have focused on the slope of the curve $F^\pi(T|t)$ only at the short end. However, applying the same ideas to other points along this curve gives default intensities for other maturities. More specifically, for any $t \leq t^* \leq T$, setting $T = t^*$ in (3.6) gives the intensity of default, $\tilde{f}(t^*|t)$, at time t^* as

$$\tilde{f}(t^*|t) = \frac{(r^\pi(t,t^*) - r(t,t^*))}{1-\delta^\pi} \left[\frac{v^\pi(t,t^*)}{p(t,t^*)}\right]. \tag{3.13}$$

Moreover, since we have approximated the forward rate curves using the Nelson-Siegel family, it is possible to go further and derive explicit expressions for the curve of default intensities. For this, denote by a_0, a_1, a_2, κ and $a_{\pi,0}, a_{\pi,1}, a_{\pi,2}, \kappa_\pi$ the Nelson-Siegel parameters for the risk-free forward rate curve and the forward rate curve corresponding to firm π respectively. Then since

$$\begin{aligned} r^\pi(t,T) &= a_{\pi,0} + a_{\pi,1}\, e^{-\kappa_\pi(T-t)} + a_{\pi,2}(T-t)\, e^{-\kappa_\pi(T-t)} \\ r(t,T) &= a_0 + a_1\, e^{-\kappa(T-t)} + a_2(T-t)\, e^{-\kappa(T-t)}, \end{aligned}$$

[6] These firms are chosen among the 23 firms in the empirical study in Section 5. They are chosen from the top and bottom end of firms ranked according to creditworthiness

the bond prices $v^\pi(t,T)$ and $p(t,T)$ are given by

$$v^\pi(t,T) = \exp\Big[-a_{\pi,0}(T-t) - \frac{a_{\pi,1}}{\kappa_\pi}\left(e^{-\kappa_\pi t} - e^{-\kappa_\pi T}\right) - \frac{a_{\pi,2}}{\kappa_\pi^2}\left(e^{-\kappa_\pi t} - e^{-\kappa_\pi T}\right) - \frac{a_{\pi,2}}{\kappa_\pi}\left(te^{-\kappa_\pi t} - Te^{-\kappa_\pi T}\right)\Big]$$

$$p(t,T) = \exp\Big[-a_0(T-t) - \frac{a_1}{\kappa}\left(e^{-\kappa t} - e^{-\kappa T}\right) - \frac{a_2}{\kappa^2}\left(e^{-\kappa t} - e^{-\kappa T}\right) - \frac{a_2}{\kappa}\left(te^{-\kappa t} - Te^{-\kappa T}\right)\Big],$$

and substituting these expressions into (3.13) gives the desired expressions for the default intensities in terms of the Nelson-Siegel parameters.

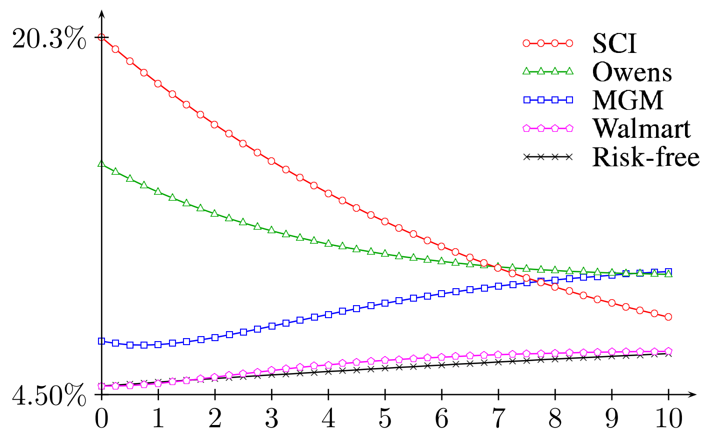

FIGURE 3.1. Nelson-Siegel forward rate curves for March 7, 2001

4. THE DATA

The data used in this paper consists of daily closing mid prices of 23 firms in various industries that are traded at the NYSE from March 7, 2001 to December 31, 2001. A list of these firms is given in Table 2. Most of the firms are large multinationals and for the time period considered they had at least three outstanding bonds with maturities less than ten years. We have chosen to calculate our risk measure at dates where all bonds have at least 6 trading days to the next coupon or principal payment, viz. the 7^{th}, 8^{th}, 9^{th}, 20^{th}, 21^{st} and 22^{nd} of each month.[7] All the corporate bonds in our sample are bonds without any contractual provisions such as convertible, callable, or putable features. The bond price data was obtained from DataStream. Finally, following Jarrow, Lando and Turnbull (1997) we have set $\delta^\pi = 0.3265$ as the exogenous recovery rate for all firms regardless of rating class π. This assumption can easily be relaxed by setting a different recovery rate for each rating class π.

For comparative reasons we also obtained daily credit ratings from Moody's (more specifically senior unsecured debt ratings), and the daily stock prices and debt levels for each firm from DataStream. Moody's ratings were then converted to numerical values

[7]This was to avoid having to address the rather ad hoc manner in which ex-coupon dates were determined.

between 1 and 21 with the larger numerical value corresponding to poorer credit quality.[8] The debt levels required for Merton's model were approximated by interpolating the total debt figures contained in the year-end balance sheet reports and the maturities of these debts were assumed to be one year. Market capitalisations were calculated from quoted stock prices and the number of outstanding shares, equity volatilities were estimated using the 3-month historical sample volatilities, and the risk-free rates were proxied by the 3-month US treasury bill rates. These choices are all in line with other studies in the literature.

5. Empirical Results

This section gives the results obtained from implementing the technique described in Section 3 on the bond data described in Section 4. Since the ultimate task of our measure is to rank different firms according to their credit worthiness, an evaluation of the measure's performance is warranted. However, due to the well known difficulties associated with evaluating credit risk models (default is a very rare event), we have decided on a somewhat indirect approach of comparing our measure with other well known, and thoroughly tested, credit measures. Considering the fact that our risk measure is primarily a rating tool giving information on the relative health of a group of firms, we chose to concentrate on the measure's performance in producing relative rankings.

The first issue we consider is the degree of correlation among the three risk measures.[9] Although it was mentioned previously that the target horizon of the three measures are quite different, it would not be unreasonable to expect a certain level of correlation between the measures. Since the data representing the risk measures consists of ranks and the assumption of normality without outliers does not hold for this data set, we present both the ordinary Pearson correlation coefficients and Spearman's rank correlation coefficients in Table 1.

Since a large distance to default is associated with low risk, Merton's distance to default measure is of course negatively correlated with the other two risk measures. In fact, the level of correlation between the distance to default risk measure and the other two measures is approximately -0.5. This holds both for the ordinary Pearson correlation coefficient and for the Spearman rank correlation coefficient. Meanwhile, the correlation between our continuous risk measure and Moody's ratings is significantly higher; Pearson's correlation coefficient is 0.71 and Spearman's rank correlation coefficient is 0.64. This higher correlation is interesting since our measure is constructed to be much more myopic than Moody's, and supports the popular view that the rating agencies' interests are more closely aligned with the interests of bond holders than with those of equity holders.

The rankings of the individual firms using the three different measures in our study are shown in Table 2. These rankings are the averages of the 41 daily rankings in the sample. The strongest agreement between the three measures occur at the extremes. For example, all three measures suggest that Wal-Mart and Bellsouth are very stable while they all suggest the opposite for Owens-Illinois, SCI and MGM.

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[8] This cardinalisation of course imposes an implicit assumption that Moody's ratings are linearly related to the actual risk of the firm.

[9] In order to calculate correlations Moody's ratings are first cardinalized as described above.

We next consider in greater detail the relationship between our continuous credit measure and Moody's ratings. In Figure 5.1 and in Table 3, the relationship between the two measures of credit worthiness are presented on an aggregated level. The two curves in Figure 5.1 represent the mean and the median risk measures among all the firms over the entire year within the different Moody's rating classes. It is evident that our measure closely tracks Moody's ratings, with a certain degree of overlap between the two. This monotonic relationship is broken only once, viz. the Baa1 rated firms are on average considered safer by our measure than both the A2 rated and the A3 rated firms. Overall, however, our measure very much confirms Moody's views on the credit worthiness of firms, at least at the aggregate level.

TABLE 1. Correlation between the risk measures.

Spearman Rank Correlation

	Moody's	**Merton**	**Continuous**
Moody's	1	−0.479	0.636
Merton	−0.479	1	−0.426
Continuous	0.636	−0.426	1

Correlation (Pearson)

	Moody's	**Merton**	**Continuous**
Moody's	1	−0.551	0.707
Merton	−0.551	1	−0.501
Continuous	0.707	−0.501	1

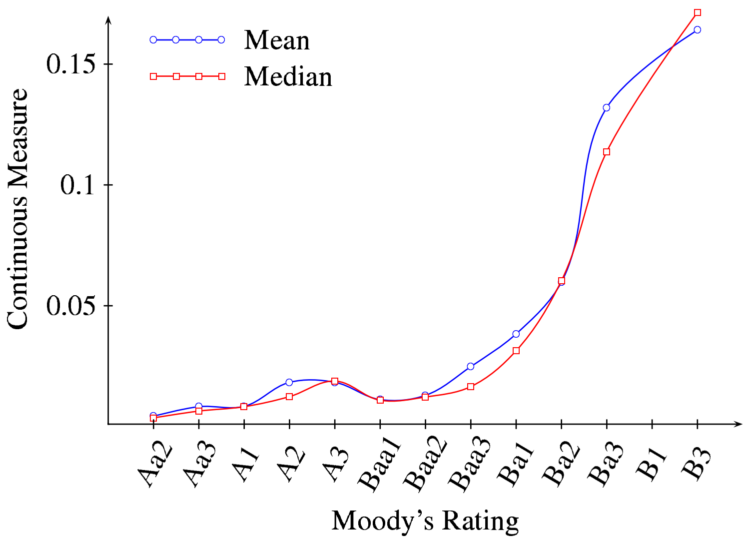

FIGURE 5.1. The continuous measure as a function of Moody's ratings.

TABLE 2. Average ranking of the firms over the year.

Moody's	Merton	Continuous
Wal-Mart (Aa2)	Coca Cola	Wal-Mart
Bellsouth (Aa2)	Wal-Mart	GE
GE (Aa3)	Bellsouth	Seagram (Pernod)
Coca Cola (Aa3)	Allstate	Bellsouth
Pacific Bell (Aa3)	Pacific Bell	Allstate
Banc One (Aa3)	Carolina P&L	Bank One
Allstate (A1)	Philip Morris	Coca Cola
Target (A2)	Occidental	Time Warner
Philip Morris (A2)	Coastal (El Paso)	Philip Morris
Sears, Roebuck & Co. (A3)	Marriott	Pacific Bell
Carolina P&L (A3)	Lockheed Martin	Union Pacific
Time Warner (Baa1)	Time Warner	Occidental
Pohang Iron & Steel (Baa2)	Union Pacific	Pohang Iron & Steel
Illinois Power (Baa2)	Seagram (Pernod)	Illinois Power
Seagram (Pernod) (Baa2)	Target	Lockheed Martin
Coastal (El Paso) (Baa2)	Bank One	Coastal (El Paso)
Lockheed Martin (Baa3)	GE	Sears, Roebuck & Co.
Occidental (Baa3)	Pohang Iron & Steel	Carolina P&L
Union Pacific (Baa3)	Sears, Roebuck & Co.	Target
MGM (Baa3)	Illinois Power	Marriott
Marriott (Ba2)	MGM	MGM
SCI (B1)	SCI	SCI
Owens-Illinois (B1)	Owens-Illinois	Owens-Illinois

TABLE 3. The continuous measure as a function of Moody's ratings.

	Mean	Median	σ	Max	Min	Observations
Aa2	0.004291017	0.003608018	0.00251	0.00796	0	82
Aa3	0.008240535	0.006429102	0.00487	0.0185	0	164
A1	0.008374165	0.008225687	0.00219	0.0103	0.00193	41
A2	0.018262806	0.012397921	0.00693	0.0298	0.00505	82
A3	0.018262806	0.018856719	0.00576	0.0270	0	123
Baa1	0.011165553	0.01091314	0.00181	0.0122	0.00497	41
Baa2	0.01294729	0.012249443	0.00569	0.0359	0	143
Baa3	0.024795843	0.016629547	0.0128	0.0519	0.00384	144
Ba2	0.03830735	0.031328879	0.0122	0.0482	0.0123	38
Ba3	0.059836674	0.060430586	0.00120	0.0412	0.0389	3
B1	0.13199703	0.113734224	0.0310	0.159	0.0568	54
B3	0.16481069	0.170749814	0.0288	0.163	0.0561	28
All	0.02657758	0.01355605	0.0274	0.163	0	943

In addition to producing a continuous risk measure based on market information, it was shown how the methodology can be extended to give the entire inter-temporal default density, or default intensity, distribution. This density distribution not only provides information on the likely *timing* of the firms' default, but also plays a key role in the pricing and risk management of various credit derivatives. Moreover, it is a simple task to compute the expected time to default from the inter-temporal default intensity function. Figure 6.1

shows the default intensity distribution for Wal-Mart, Target, SCI, and Owens-Illinois, and Figure 6.2 shows the corresponding cumulative default distribution functions. An interesting observation is the clear dependence of the shape of the inter-temporal default distributions on the riskiness of the firm. The riskier firms have a skewed distribution with most of the mass at short maturities. This is consistent with the view that although the riskier firms are more likely to default in the short term, if they somehow manage to survive the current problems, then the likelihood of default over the subsequent periods is likely to decrease (the cumulative probability is of course steadily increasing). Safer firms, on the other hand, exhibit a more symmetrical, slightly bell-shaped, distribution. The safer firms are unlikely to default in the short term, with the likely time of default some distance away. Figure 6.1 shows that the probability of immediate default for SCI, the riskiest firm on March 7, is approximately 15%, while the corresponding value for Wal-Mart, the safest firm, is close to 0%. It can be seen from Figure 6.2 that even the riskiest firm has a 50% chance of survival beyond ten years.

Finally, the time series of inter-temporal default intensity curves for SCI is shown in Figure 6.3. Such time series plots provide an opportunity to trace the changes in the inter-temporal risk profile of a firm over time, and should be useful in predictions of default probabilities over various horizons.

6. Conclusions

This paper introduced a simple continuous measure of credit risk that is easily computed from market bond data. The measure can be identified as the instantaneous risk-neutral default intensity and is therefore a relevant measure of a firm's risk of distress in the short run.

A comparison of the relative rankings produced by our measure with those produced by Moody's and the distance to default type measure based on the Merton (1974) model for a set of US firms shows that the three measures produce similar results, particularly at the extremes. Rank correlation estimates reveal a closer relationship between our measure and Moody's ratings than that between our measure and the distant to default measure. Due to the lack of actual default events in our sample, it was not possible to investigate the relative merits of the various measure as predictors of future default.

It was also shown how the approach of this paper can be used to extract the entire inter-temporal (risk-neutral) default intensity distribution of firms, which is useful for inferring the expected time of default and for the pricing and risk management of credit derivatives.

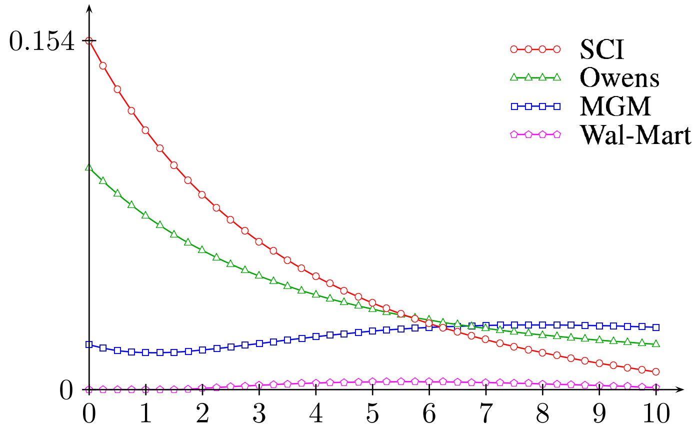

FIGURE 6.1. Risk-neutral default intensities for March 7, 2001

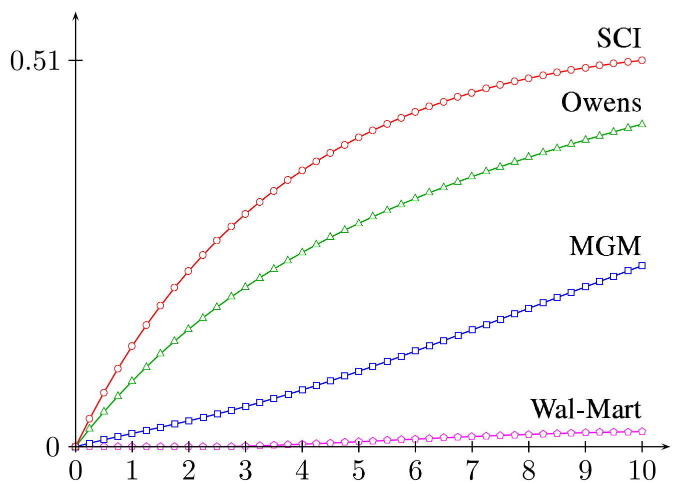

FIGURE 6.2. Cumulative default intensities for March 7, 2001

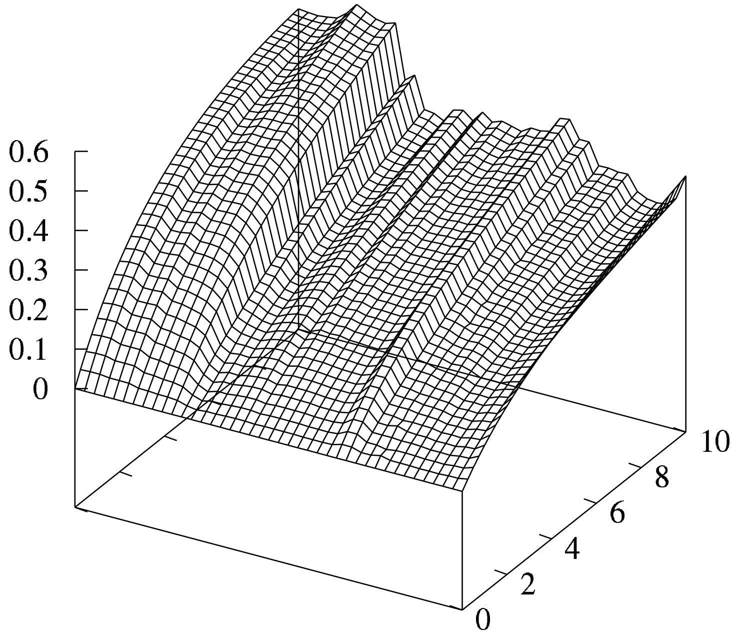

FIGURE 6.3. Time series plot of the inter-temporal default intensity distribution for Service Corporation International (SCI) between March 7, 2001 and December 31, 2001.

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