

QUANTITATIVE FINANCE
RESEARCH CENTRE



UNIVERSITY OF
TECHNOLOGY SYDNEY



QUANTITATIVE FINANCE
RESEARCH CENTRE

UTS

THINK.CHANGE.DO

QUANTITATIVE FINANCE RESEARCH CENTRE

Research Paper 217

March 2008

The toll of subrational trading in an agent based economy

Paolo Pellizzari

ISSN 1441-8010

www.qfrc.uts.edu.au

THE TOLL OF SUBRATIONAL TRADING IN AN AGENT-BASED ECONOMY

PAOLO PELLIZZARI

ABSTRACT. In an agent-based exchange economy, we measure the loss of wealth for rational agents due to the presence of varying proportions of subrational (boundedly rational) traders that do not know all the needed parameters. We consider two departures from rationality: M-traders use private, stochastic and unbiased signals to build an estimate of the value of the risky asset; chartists only use the last observed price. The exchange takes place using a realistic continuous double auction.

We show by numerical simulations that M-traders' subrational behavior does not reduce the wealth of the rational agents. On the contrary, a sizable fraction of chartists can lead to mispricing of the risky asset and to a reduction of the wealth share of the rational traders. Moreover, as chartists perceive a higher wealth than the others, due to wrong estimates of the fundamental value, their fraction in the market may not dissolve in the long run.

1. INTRODUCTION

The classic approach in financial economics is based on the assumption of fully rational agents that are able to acquire and process all the relevant information to compute the equilibrium price of risky assets traded in the market. The knowledge of the relevant parameters and of the preferences of the agents suffices to attain an efficient allocation of risk. There are huge conceptual and analytical advantages in taking such a rational viewpoint where agent select their optimal endowments maximizing their expected utility. There are also a few heroic assumptions and lack of fit with empirical data.

We avoid in this paper the slippery assumption that all agents correctly know the distribution of the risky asset and the preferences (i.e., risk tolerance coefficients) of the other traders. We define a sensible way to build beliefs about the fundamental values on the part of the non-rational agents as well as embed exchange in a realistic

Date: This version: March 13, 2008.

Key words and phrases. Risk sharing; boundedly rationality; cost of subrational trading, agent-based markets.

Draft for the LSE (12-13 June 2008). I'm indebted to Ron Bird, that introduced me to the problem of evaluating the effects of non-information based trading, and Marco LiCalzi, who sharpened my ideas in innumerable brain-storming sessions. They are not guilty of my errors.

trading process (other than the preternatural Walrasian auction) that can shepherd the agents toward an efficient allocation. The computational agent-based models developed in the last decade appear to be useful to overcome analytical difficulties arising when detailed multi-agent and interactive environments are studied. Good surveys are [LeBaron, 2006, Hommes, 2006].

We are interested in quantifying the extent of the social loss that rational agents suffer because of the presence of less than rational traders. A strong motivation to study this issue is related to the impressive explosion of non-informational based trading in the markets in the past decades. In a broad sense, any trading strategy which is not based on fundamental research aiming at the risk-adjusted maximization of profits, can be dubbed *subrational*. Think, say, to momentum or index funds that programmatically ignore fundamental analysis to keep positions based on past returns or passive replication of the market portfolio. Scholars and practitioners are well aware that there may be good reasons to cease to be rational: based on evidence, passive investing is cheap, most fundamental mutual funds are unable to attain similar results and momentum strategies appear to reap excess returns (albeit for reasons that may still look puzzling or fuzzy). [Grinblatt et al., 1995] shows that 77% of mutual funds actually employs some form of momentum stock picking to assemble their portfolios. On the theoretical side, in [De Long et al., 1990] it is shown that irrational agents can gain higher returns than rational traders (but lower risk-adjusted returns); [Hong and Stein, 1999] argue that momentum traders can profit of news underreaction (even in risk-adjusted terms, see [Chan et al., 1996]). These reasons explain why we prefer the term “subrational” to “irrational” in this work.

It is intuitive, however, that the widespread neglect of fundamental research and information processing could lead to poor pricing with related deficient risk allocation, bad credit allotment and waste of financial resources. Many other agent-based papers have investigated the effects of the coexistence of different strategies and various behavioral “biases”, since the seminal work of [Day and Huang, 1990]. Most of the work, however, focus on asset pricing anomalies or statistical properties of the resulting time-series that are generated by agents’ heterogeneity and robust interaction. [Brock and Hommes, 1998] is a semi-analytical model that shows how boundedly rational agents with heterogeneous beliefs can produce chaotic prices, see also [Chiarella and He, 2001]. The Santa Fe Artificial Stock Market, [Arthur et al., 1997], is another influential model of a market with heterogeneous and learning agents that generate simulated time-series with “psychologically rich” price behavior and persistent deviations from the homogeneous rational expectation equilibrium. To the best of our knowledge, much

smaller consideration has been given to wealth effects and social losses that may arise in the presence of boundedly rational subpopulations.

In this paper, we consider two different markets where rational agents coexist with two breeds of subrational agents, named *M-traders* and *chartists*. All the agents share the same demand function but the M-traders do not know the fundamental value of the risky asset and receive/process a stream of unbiased stochastic signals to build an estimate of the unknown quantity. The chartists just use the price to guess the fundamental value. We investigate whether and how varying proportions of subrational traders affect the certainty equivalent of the rational agents. A natural benchmark to use is the *fair share* (FS) of wealth, that is the amount of certainty equivalent that rational agents would get in a Walrasian world entirely void of any subrationality. In order to get a broader perspective we present our results in 4 parts, inquiring into the convergence of price, the extent of the mispricing possibly occurring, the fraction of FS that is ultimately secured by rational traders and, finally, the reasons that could foster ongoing subrational behavior.

Our findings can be abridged in two statements:

- (1) M-traders are not harmful to rational agents;
- (2) Markets populated by chartists often converge to the wrong equilibrium, producing a visible mispricing when the fraction of subrational traders is large; the fair share of the rational agents is rather volatile in these markets and this could result in severe losses with positive probability; even though chartists earn less fair share than the rational agents, they *perceive* a much higher share of wealth due to their wrong (but self-reinforcing) beliefs and consequently there are reasons to trust in their persistence in the market.

The paper is organized as follows. Section 2 presents the model and describes the microstructural environment used by the agents. In the third section, we show and discuss the results, with several subsections devoted to the aforementioned 4 issues. The final Section summarizes and offers some reflections and policy implications.

2. THE MODEL

We consider a standard exchange economy where a risky stock paying a random amount $Y \sim N(\mu, \sigma^2)$ in the far future T can be traded for cash¹. In the rest of the paper, we will define μ as *fundamental value* of the stock.

The economy is populated by N CARA traders endowed with some initial amount of cash c_{i0} and units of stock s_{i0} , $i = 1, \dots, N$. The i -th

¹Equivalently, we could consider a risky stock and a risk-free bond paying an interest rate r . In our setup we have $r = 0$.

trader has the following excess demand function for the risky asset at price p

$$q_{it}(p) = k_i \tau (\mu(i, t) - p) - s_{it}, \quad (1)$$

where $\tau = 1/\sigma^2$ is the precision of the payoff, k_i is the risk tolerance coefficient, s_{it} is the number of risky units at time t and $\mu(i, t)$ denotes the expected average realization value of the risky asset that may vary across agents and depends for each agent on a sequence of possibly stochastic signals $\vec{w}_{it} = \{w_{ij}\}, i = 1, \dots, N, j = 1, \dots, t$:

$$\mu(i, t) = \mu(i, \vec{w}_{it}).$$

We name *rational* the agents that know and use the true parameters in their demand functions. In the presence of rational agents, well-known work, see [Wilson, 1968], shows that there is a unique efficient allocation. In other words, if $\mu(i, t) \equiv \mu$ the equilibrium price is given by $p^* = \mu - S/(\tau K)$, where $K = \sum_i k_i$ and $S = \sum_i s_{i0}$ are the sum of agents' risk tolerance coefficients and the total quantity of risky asset.

Despite the existence of this efficient allocation, it is still interesting to discuss whether rational agents can attain it in practical situations, given that the computation of p^* and of the optimal risky endowments requires S and K . In particular, the knowledge of K is tricky as no agent can conceivably know all the risk tolerances of his peers in the market. Moreover, no agent knows at the inception of trading if he should be a net buyer or seller, as his final endowment depends ultimately on the endowments and risk tolerances of the other traders. In [LiCalzi and Pellizzari, 2007], however, it is shown that agents equipped with demand functions (1) are able to reach the efficient allocation in finite time, randomly attempting a purchase or a sell at each time t , if $\mu(i, t) \equiv \mu$ for all agents. This result is robust to variations in the trading protocol.

More realistically, we also consider here traders that have imperfect knowledge of the true underlying economic fundamental μ . Hereafter, we call these agents *subrational*. They adapt to the environment in the sense that time is needed to build some knowledge about the initially unknown parameter μ . Let w_{ij} be a doubly indexed sequence of independent and identically distributed signals with correct mean μ and variance σ_w^2 . We take into consideration two kinds of subrational agents with different mechanisms to define $\mu(i, t)$:

- (1) M-traders (M stands for μ and Much More): agents recursively update their estimate of μ with an adjustment term dependent on the last observed signal

$$\mu(i, t+1) = \mu(i, t) + \lambda(w_{i,t+1} - \mu(i, t)). \quad (2)$$

If $0 \leq \lambda < 2$, the AR(1) process for $\mu(i, t)$ is stationary, its unconditional mean is equal to the (true) μ and the variance is

given by

$$\text{Var}[\mu(i, t)] = \frac{\lambda}{2 - \lambda} \sigma_w^2. \quad (3)$$

Observe that standard invertibility results ensure that it is possible to write the process as a function of past innovations contained in $\vec{w}_{i,t+1}$.

In the next section we will consider three representative values for λ :

- (a) $\lambda = 0.5$: the estimated fundamental value $\mu(i, t)$ is a weighted average of past signals, with more weight given to more recent elements of \vec{w}_{it} ;
- (b) $\lambda = 1$: M-traders myopically set $\mu(i, t)$ to the last observed signal, forgetting all the past;
- (c) $\lambda = 1.5$: M-traders exhibit a form of overreaction to the last signal, putting negative weight on the previous value of $\mu(i, t)$.

We believe these instances are interesting because they shed some light on three frequently studied flavours of irrational behavior.

- (2) Chartists: a huge number of research papers suggest that some form of chartist behavior is used by many traders. This means that prices are used as signals, say because the agents have not the capability to get the right μ or process the proper piece of information w_{it} or want to free-ride other agents, bypassing the cost of information acquisition. We suppose that the last (observed) closing price \bar{p}_t is used to build a rough estimate of the right fundamental² value μ .

As the price, say \bar{p}_t , is valid for a unit transaction (see below for the details on the market functioning), the agent pragmatically inverts his demand function

$$\pm 1 = k_i \tau (\mu - \bar{p}) - s_i$$

to get μ (we drop the time index for clarity). Assuming that \bar{p} is the local equilibrium price, given his endowment s_i , he can compute two values for $\mu_{1,2}$:

$$\mu_{1,2} = \frac{s_i \pm 1}{k_i \tau} + \bar{p}.$$

²There are other appealing ways to model the use of a pseudo-signal like $\Delta p_t = p_t - p_{t-1}$ or $R_t = p_t/p_{t-1}$ in order to estimate the unknown equilibrium price p^* . Agents could, for example, conjecture that the same return will persist for d periods and could build at time t the estimate

$$p^* = p_t R_t^d.$$

This could in turn be used to guess whether the risky position should be increased or decreased.

Hence, the fundamental value is guessed to be

$$\hat{\mu} = \frac{\mu_1 + \mu_2}{2} = \bar{p} + \frac{s_i}{k_i \tau}. \quad (4)$$

Observe that this estimate would be accurate had the transaction occurred at a price \bar{p} close to p^* and provided that the stock to risk tolerance ratio of the market is close to that of the agent. However, random departures from the equilibrium price and/or misalignments of the risk borne by the agent and the market as a whole yield grossly fallacious estimate of $\hat{\mu}$.

In the following we assume that a fraction $0 \leq \alpha \leq 1/2$ of traders is either made of M-traders or chartists, while the remaining portion is made of rational traders with $\mu(i, t) \equiv \mu$ in (1). We refer to markets where there are rational agents and M-traders (chartists) as *M-trader markets* (*chartist markets*).

We are interested in measuring the global efficiency of the market and, in particular, to quantify the amount of utility that is lost because of the the presence of the subrational agents. This is meant to assess the extent of the cost that the rational agents bear due to the (sub)behavior of the others.

The comparison of the certainty equivalents³ of the rational and subrational agents allows to gauge the relative performances of the two groups.

In the case of M-trader markets, we have that

$$E[\mu(i, t)] = \mu,$$

for all agents. Subrational agents are able to correctly infer the fundamental value on average but there is scope for endogenous fluctuations

³In a CARA-normal setup, the certainty equivalent of the i -th agent at time t is given by

$$m(i, t) = c_{it} + \left(\mu - \frac{s_{it}}{2\tau k_i} \right) s_{it}. \quad (5)$$

In a Walrasian world, where each trader ends up with cash $c_i^* = c_{i0} - p^*(s_i^* - s_{i0})$ and $s_i^* = (S/K)k_i$ units of stock, after exchanging at the equilibrium price p^* in a unique giant step, the certainty equivalent would be

$$m_i^* = c_i^* + \left(\mu - \frac{s_i^*}{2\tau k_i} \right) s_i^*. \quad (6)$$

We refer to m_i^* as the *fair share*, with the name recalling that this would be the final outcome in a fictitious and idealized Walrasian setup. If any other market protocol is used to exchange the risky asset, the final certainty equivalent $m(i, T)$ of an individual trader can be different from his/her fair share m_i^* , either because the agent is holding the wrong amount $s_{iT} \neq s_i^*$ of risky stock or because the final cash $c_{iT} \neq c_i^*$.

The *perceived certainty equivalent* $\pi(i, t)$ is obtained when the perceived $\mu(i, t)$ replaces μ in (5):

$$\pi_{it} = c_{it} + \left(\mu(i, t) - \frac{s_{it}}{2\tau k_i} \right) s_{it}. \quad (7)$$

of individual estimates $\mu(i, t)$ that is going to decrease the certainty equivalents (or, interchangeably, the utilities) of some traders.

In the case of chartist estimation of $\hat{\mu}$ using the observed price, there is no theoretical guarantee that all the $\mu(i, t)$ converge to the correct fundamental value. Moreover, we can verify that the following self-reinforcing mechanism could generate persistent deviations from the correct valuation of the risky asset. Assume that $\alpha \gg 0$ (sufficiently big) and that for purely random reasons a large price \bar{p} is observed in the previous trading session. The chartist traders implicitly will herd on a relatively large estimate for $\hat{\mu}$. The belief of a large fundamental value triggers the submission of large bids, that are likely to find a counterpart in the set of rational traders, and large asks that have a smaller probability to be executed. This excess demand might sustain or increase further the price level so that prolonged departures from the equilibrium price are observed. The positive feed-back cycle is eventually broken by the budget constraints that avoid a perpetual buy frenzy.

2.1. The market microstructure. In each trading session, agents are randomly selected (without resampling) to submit a limit order for a unit quantity. Hence, the left hand side of (1) is either +1 (attempt to buy one more unit) or -1 (attempt to sell one piece of stock). The side of the order is independently chosen with equal probability. This is consistent with the idea that agents do not know if they should be net buyers or sellers. Of course, risk tolerant (averse) agents with few (many) stocks would randomly produce very aggressive (unattractive) bids and rather unattractive (aggressive) asks, in such a way that their probability to increase (decrease) their stock endowments is large. We assume that agents are budget constrained: they cannot bid more than the available cash and they cannot try to sell units they do not own. A similar environment is discussed at length in [LiCalzi and Pellizzari, 2007].

Without loss of generality, assume that an agent must submit a order to buy. The reservation price p' is computed, given his $\mu(i, t)$ and s_{it} from the demand equation (1)

$$1 = k_i \tau (\mu(i, t) - p') - s_{it} \quad \text{yielding} \quad p' = \mu(i, t) - \frac{s_{it} + 1}{k_i \tau}.$$

Submitting the above reservation price is a dominated strategy as, were the order executed at that price, there would be no increment in the certainty equivalent. Hence, the agents bid by shading their true reservation value using a limit price uniformly sampled in $[p'', p']$ where

$$p'' = \mu - \frac{s_{it} + 2}{k_i \tau}.$$

In words, the agent tries to get a discount by posting a random order with a more favorable limit price. He bids an amount that is in the right neighborhood of the price p'' valid for the purchase of two units (but still the order is valid for one unit). This mechanism is a less extreme instance of the device used in [Gode and Sunder, 1993], where agents with reservation price v_i bid randomly in $[0, v_i]$. The case of a seller is dealt with obvious modifications.

The market platform is a standard continuous double auction where limit orders, valid for one unit, can be posted on the the bid or ask books. If an order is not executed immediately it is stored in the proper book for future use. At any time, the orders to buy (sell) are kept ordered in the bid- (ask-)book in the standard decreasing (increasing) price-time priority, so that b_1 (a_1) denote the best bid (ask). A new buy (sell) order with limit price b (a) is executed if $b \geq a_1$ ($a \leq b_1$), the two matching orders are cancelled and the cash/stock endowments of the seller/buyer are updated. Then another agent is sampled and the process continues until all the agents have submitted their limit orders.

At the end of each trading session, indexed by $t = 1, \dots, T$, the closing price \bar{p}_t (to be used by chartists) is recorded and the book is cleared.

3. RESULTS

This section presents the results obtained by simulation of the previously described markets. We investigate 4 main themes: the eventual convergence of the price to some stable level, the extent of the mispricing with respect to the equilibrium price, the fraction of the fair share that is retained by the rational traders and a comparison of (the wealth of) the two groups of agents. Hence the following 4 subsections seek to answer the questions

- (1) Does the price ultimately converge?
- (2) What's the magnitude of the mispricing with respect to the theoretical equilibrium price?
- (3) How is the wealth of the rational traders affected by a varying proportion of subrational ones?
- (4) Is there any reason justifying the prolonged existence of M-trader or chartist subrationality?

The discussion is based on 100 simulations for each of the two markets. In each run, lasting $T = 500$ days (trading sessions), a random value for α is sampled in $[0, 0.5]$ or, equivalently, $0 \leq 100\alpha \leq 50$ percent of agents behave subrationally. Different batches of simulations are disjointly performed for the two types of behavior we have considered in (2) and (4). All the parameters used in the simulations, with a brief description, are listed in Table 3.

Param.	Value	Description
μ	1000	Constant fund. value
σ^2	120	Payoff variance
σ_w^2	240	Variance of the signal process
T	500	Trading sessions per simulation
N	1000	Number of agents
c_{0i}	50000	Initial cash endowment
s_{0i}	$\{20, \dots, 80\}$	Random initial stock endowment
κ_i	$\{10, \dots, 40\}$	Random risk tolerance

TABLE 1. Values of the parameters used in the simulation, with a brief description.

3.1. **Convergence.** M-traders are different from chartists in two fundamental ways. The former group is using a stochastic signal to build a tentative valuation of the asset and, hence, to post limit orders. Moreover, each M-trader uses private and distinct (unbiased) information in (2). All the members of latter group, conversely, use a public signal (the price \bar{p}) to build different estimates of the fundamental value that is needed to provide limit orders. Observe that no random variable is involved in this process as \bar{p} is taken as given.

We expect a noisy price time series and ever-lasting trades at least in the M-trader populated market. Table 2 depicts the percent standard deviation of the price in the last 100 days, for various levels of α . In this and all the subsequent tables we computed the mean $\hat{e}(\alpha)$ of the quantity of interest using a non parametric smoother. Then we provide the values $\hat{e}(0.1), \dots, \hat{e}(0.5)$. Pre-subscripting the index $r = 1, \dots, 100$ to the quantities obtained in the r -th simulation, we report the entries $100\hat{e}(\alpha)$ for the standard deviation, that is

$$\hat{e}(\alpha) = \frac{1}{100} \sum_{r=1}^{100} \omega(|_r\alpha - \alpha|) \text{sd}({}_r p_t, t = 401, \dots, 500) / {}_r p^*,$$

in which the specific form of the weights $\omega(|_r\alpha - \alpha|)$ depends on the selected smoother and $\text{sd}(\cdot)$ is the standard deviation of the sequence in the argument.

The dispersion of the price is bounded away from zero in all the M-traders' markets and is also monotonically increasing in λ . This is coherent with the dependence of the variance (3) on λ . The left panel of Figure 1 shows indeed a "thermal" noise around the equilibrium level due to the stochastic adjustment of $\mu(i, t)$ performed by the M-traders that keep exchanging marginal units.

The last row of Table 2 shows that, conversely, very little price dispersion is observed in markets populated with chartists. The right part of Figure 1 is depicting two cases (where $\alpha = 0.19$ and 0.33). The price

		α					
		λ	0.1	0.2	0.3	0.4	0.5
Mtr	0.50		0.22	0.23	0.23	0.24	0.25
	1.00		0.27	0.30	0.32	0.34	0.37
	1.50		0.30	0.34	0.38	0.42	0.46
Cha			4.5e-05	0.0033	0.0048	0.00061	0.00063

TABLE 2. Standard deviation of prices in the last 100 trading sessions (normalized by p^* , percent values.)

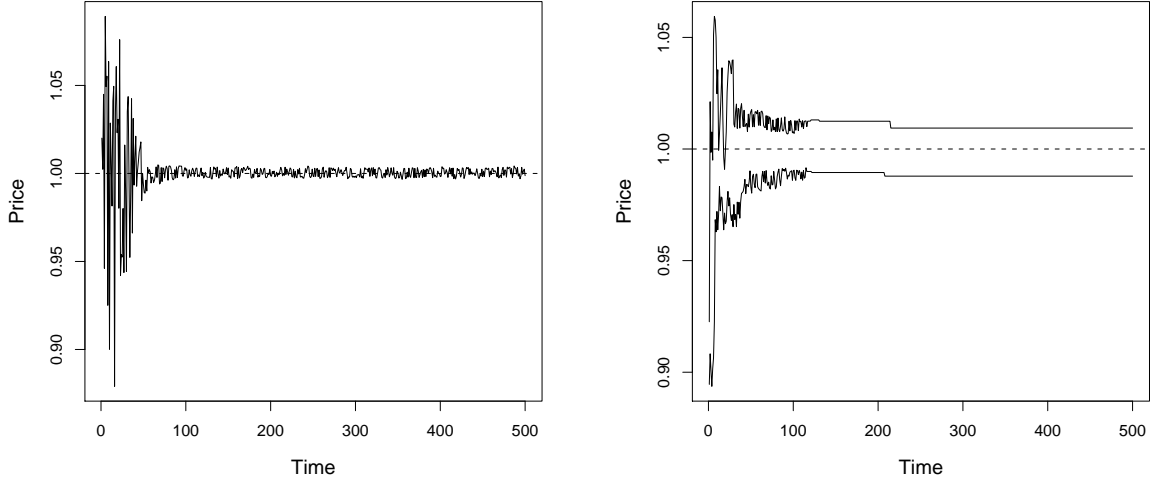


FIGURE 1. Left: price trajectory in a market populated by 9% of M-traders using $\lambda = 0.5$. Right: price trajectories in markets where 19% (upper) and 33% (lower) of chartist traders are present, respectively. The time-series are normalized by the equilibrium price that is shown with a horizontal dashed line.

is converging to a constant value that can be different from the equilibrium price. This conclusion always holds in the simulated chartist markets.

Loosely speaking, in the M-trader market there is only “convergence in mean” while strict price convergence to a possibly “wrong” equilibrium level is the norm in a chartist setup.

3.2. Mispricing. As seen in Figure 1, some mispricing is present in both our markets. This is an obvious source of allocative inefficiency as it prevents agents to achieve an efficient risk allocation. We can measure the mispricing as $|p_T - p^*|/p^*$ for each simulation and provide

the average values⁴ in Table 3. The absolute percent deviation⁵ from p^* is below 0.5% in all markets with M-traders. There is also some evidence that the mispricing is increasing with λ . Observing that the standard deviations reported in Table 2 are quite close to mispricing figures, one can conclude that the deviation from the equilibrium price is essentially due to the variance of the stochastic fluctuations around p^* , as depicted in the left panel of Figure 1.

		α				
		λ	0.1	0.2	0.3	0.4
Mtr	0.50	0.15	0.18	0.19	0.19	0.15
	1.00	0.23	0.27	0.27	0.30	0.34
	1.50	0.25	0.31	0.34	0.38	0.51
Cha		0.15	0.27	0.50	0.75	1.66

TABLE 3. Average relative mispricing $|p_T - p^*|/p^*$ (percent values).

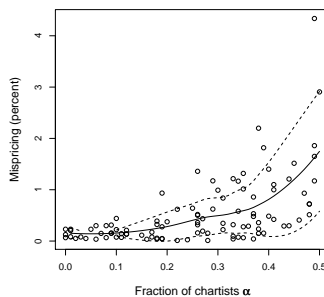
While the mispricing is slowly increasing in the fraction α of the M-traders, the presence of chartist agents is noticeable beyond a certain threshold. The average mispricing is 0.5% when $\alpha = 0.3$ and more chartists cause the figures to increase up to 1.66%. The dynamics of the averages, however, is not telling the whole story. Figure 2 shows a non parametric estimate of the average mispricing together with a one standard deviation “confidence” band⁶.

⁴Hence, we show in Table 3 the values of $100\hat{e}(\alpha)$ where

$$\hat{e}(\alpha) = \frac{1}{100} \sum_{r=1}^{100} \omega(|r\alpha - \alpha|) \frac{|r p_T - r p^*|}{r p^*}.$$

⁵Taking the mean of $(p_T - p^*)/p^*$ produces results that are not significantly different from zero, as negative and positive deviations compensate and indeed there is no reason in our framework to have unbalanced outcomes.

⁶Our results are obtained using Friedman’s “super smoother”, see [Friedman and Stuetzle, 1981], but the findings are robust and virtually the same with other smoothers. For example, using the loess polynomial fitter described in Cleveland et al. (1992) we get the following very similar picture.



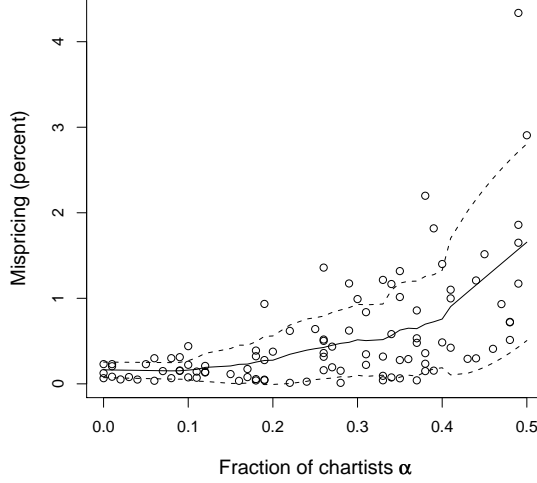


FIGURE 2. Percent mispricing as a function of the fraction of chartist α . The circles show the point values for each simulation and the average, estimated non parametrically, is shown with a solid line. The dashed lines show the mean plus (minus) one standard deviation.

It is clear that the worst-case mispricing is much bigger than suggested in Table 3: if $\alpha = 0.3$, say, it is not uncommon for the mispricing to reach roughly 1% despite an average value that is half this amount; if the number of chartists is approaching 50% of the agents, the mispricing can rather frequently exceed 2% or higher values. This evidence is showing that large proportions of chartists can produce a substantial mispricing and related episodes of inefficient allocation. The amount of the loss in wealth terms that is borne by the rational agents is still to be quantified and the next subsection provides the details.

3.3. Wealth. The effect of the presence of some M-traders or chartists on the wealth of the remaining part of rational agents can be assessed by the fraction of the fair share $m(i, T)/m_i^*$ they are able to retain, see Footnote 3. Table 4 reports the (averaged) percentage of fair share of the rational traders in different markets⁷. In general, agents get slightly more than 100% of their fair share, with the exception of situations where some chartist traders are present.

⁷We display in Table 4 the values of $100\hat{e}(\alpha)$, where

$$\hat{e}(\alpha) = \frac{1}{100} \sum_{r=1}^{100} \omega(|_r\alpha - \alpha|) \frac{1}{|_rR|} \sum_{i \in {}_rR} m(i, T)/m_i^*,$$

and ${}_rR$ is the the set of indexes of Rational agents in the r -th simulation.

		α					
		λ	0.1	0.2	0.3	0.4	0.5
Mtr	0.50	100.01 (0.0071)	100.02 (0.011)	100.03 (0.012)	100.04 (0.016)	100.06 (0.022)	
	1.00	100.02 (0.011)	100.06 (0.020)	100.10 (0.024)	100.16 (0.034)	100.23 (0.023)	
	1.50	100.04 (0.018)	100.11 (0.036)	100.19 (0.047)	100.30 (0.072)	100.47 (0.048)	
Cha		99.96 (0.014)	99.95 (0.016)	99.97 (0.047)	100.00 (0.088)	100.15 (0.28)	

TABLE 4. Average percentage of fair share gained by rational traders (standard deviation in parentheses).

There is no evidence that the action of M-traders produces a reduction of the average wealth of the rational agents in all the markets, regardless of the values of α and λ . Indeed, the first three rows suggest surprisingly that an increment of α is beneficial. Only in the presence of chartists, the wealth drops below 100%. Again, average values do not allow a deeper understanding of the wealth dynamics. Figure 3 depicts the mean wealth (with standard deviations) in the chartist market.

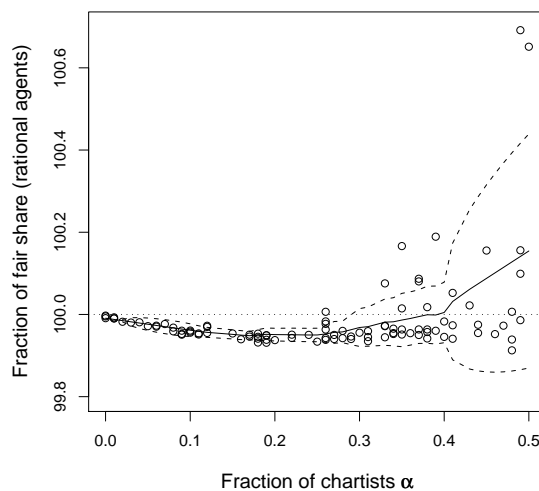


FIGURE 3. Average wealth of rational traders as a function of α (solid line) in a market populated by chartist traders. The dashed lines show a confidence interval (one standard deviation away from the mean).

The graph shows a visible increase of the volatility around $\alpha = 0.4$, see also the parenthesized numbers in Table 4. Most of the simulations

yield wealth figures that are below 100% of the fair share (exactly in 84/100 cases) and some entries in Table 4 are bigger than 100 only because of a few outlying samples. The fanning-out of simulated wealth's fractions is consistent with worst cases (for $\alpha \geq 0.4$) where the wealth could drop to values close to 99%.

Overall, these results suggest that the wealth of rational traders is not diminished by M-traders but can be compromised by sizable levels of chartists. Both kinds of traders display some irrational traits, in the peculiar sense given to the term in this paper. However, while the former group is incorporating unbiased information in the evaluation of the risky asset with limited contribution to the mispricing, the latter set of traders is exploiting the common signal given by the price and, if many chartists are active, their herding can produce notable deviations from the equilibrium price with relevant reverberation on wealth.

3.4. Rational vs irrational. It is of interest to investigate whether irrational traders can survive in the long term. Standard arguments suggest that if subrational traders had steadily worse results than rational agents, they would evaporate given enough time, say because they learn to correctly evaluate the asset. A comparison is performed by looking at the difference of the average fractions of fair share that is gained by the two groups and is shown in the top panel⁸ of Table 5. Positive entries mean that rational traders get a bigger portion of their fair share than the subrational agents. As all the entries are positive in the top panel, we can conclude that rational agents always get “more” than irrational ones (even if they may get less than 100% as seen before).

A perhaps fairer comparison, however, should take into account that subrational agents coherently compute their certainty equivalents using the (possibly) wrong parameter $\mu(i, T)$ instead of the correct μ . The lower part of Table 5 shows the (averaged) difference of the fraction of fair share of the rational traders⁹ with the same quantity *as perceived* by subrational agents. As expected, little is changing for M-traders,

⁸In the top panel of Table 5 we display the values of $100\hat{e}(\alpha)$,

$$\hat{e}(\alpha) = \frac{1}{100} \sum_{r=1}^{100} \omega(|_r\alpha - \alpha|) \left(\frac{1}{|_rR|} \sum_{i \in {}_rR} m(i, T)/m_i^* - \frac{1}{|_rS|} \sum_{i \in {}_rS} m(i, T)/m_i^* \right),$$

where ${}_rR$ and ${}_rS$ denote the set of indexes of Rational and Subrational agents, respectively.

In the bottom panel of the same table we report $100\hat{e}(\alpha)$,

$$\hat{e}(\alpha) = \frac{1}{100} \sum_{r=1}^{100} \omega(|_r\alpha - \alpha|) \left(\frac{1}{|_rR|} \sum_{i \in {}_rR} m(i, T)/m_i^* - \frac{1}{|_rS|} \sum_{i \in {}_rS} \pi(i, T)/m_i^* \right)$$

⁹It might be worth noticing that, being $\mu(i, T) \equiv \mu$ for rational traders, we have that $\pi(i, T) \equiv m(i, T)$.

		Difference of fair shares (rational-subrational)				
		α				
	λ	0.1	0.2	0.3	0.4	0.5
Mtr	0.50	0.11	0.11	0.11	0.11	0.12
	1.00	0.27	0.32	0.35	0.4	0.47
	1.50	0.46	0.55	0.64	0.76	0.94
Cha		1.65	1.65	1.75	1.87	2.22

		Difference of perceived fair shares				
		α				
	λ	0.1	0.2	0.3	0.4	0.5
Mtr	0.50	0.12	0.12	0.10	0.10	0.11
	1.00	0.31	0.33	0.35	0.40	0.45
	1.50	0.53	0.57	0.65	0.75	0.91
Cha		-2.70	-2.07	-1.78	-1.19	-0.11

TABLE 5. Top panel: difference of average fractions of fair share gained by rational and irrational traders (percent values). Bottom panel: difference of average fractions of fair share gained by rational agents and fair share perceived by subrational traders (percent values).

whose perceived certainty equivalent is on average equal to final one, being $E[\mu(i, t)] = \mu$.

More interestingly, as chartist agents have often vastly incorrect estimates of the fundamental value, i.e. $\mu(i, t) \neq \mu$, their perceived certainty equivalent can be different from m_i^* . The last row of the bottom panel of Table 5, in fact, shows that all populations of chartists perceive a certainty equivalent that is often considerably bigger than the fair share of the rational traders. As seen in Figure 4, moreover, the fewer the chartists the bigger is the perceived advantage. A stylized learning argument in a repeated game setting may then be used to argue that if a handful of irrational chartists are initially present, more agents could be attracted to this particular dark side of subrational behavior. The process should naturally extinguish itself when α has reached a value exceeding 0.4.

4. CONCLUSION

This paper presents a model of an exchange economy (with a continuous double auction) populated by some rational CARA-normal agents and subrational traders that have the same form of demand function but do not know one relevant parameter. We define two ways to build beliefs, based on private informative signals (M-traders) and on the use of the publicly observable price (chartists).

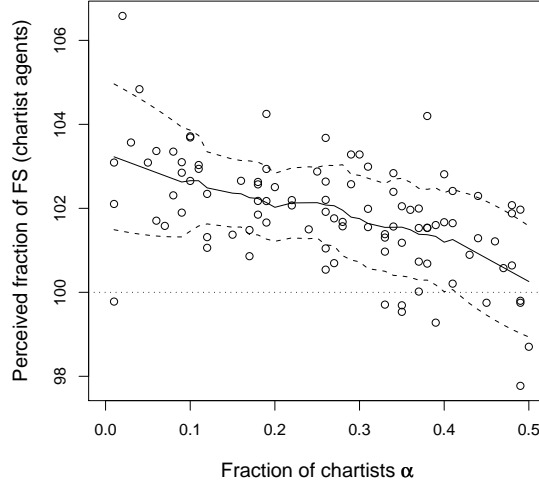


FIGURE 4. Perceived (average) fraction of fair shared earned by the chartist agents, solid line. The dashed lines show a confidence interval (one standard deviation away from the mean).

Aiming to assess the effects of varying proportions α of subrational traders on the the remaining part of rational agents, we compare the fraction of fair share that is conserved by them despite the presence of potentially disruptive subrational partners.

We show that M-traders do not diminish the fair share of the rational agents (that is, the optimal risk-adjusted wealth that a rational trader should be given for his capability to bear risk is unchanged or even slightly increased). M-traders incorporate private unbiased information in their limit prices which, albeit noisy, offer valuable liquidity at correct (average) levels to the rational buyers/sellers. On the contrary, chartists take the extreme approach to use the price as the unique determinant of fundamental value. Even if the estimate they compute would be correct in equilibrium (ex-post), they can push the price to the wrong level and they implicitly herd on a common signal (the observed price is unique so everyone is using the same piece of information). This produces at times considerable mispricings and wealth losses as big as 1% can be borne by the rational agents.

Moreover, chartists keep using subrational behavior because their perceived wealth is bigger than the one owned by the rational group (a similar argument is in [De Long et al., 1990]). Hence, in an evolving setting we can conjecture that the number of chartists might increase to reach 40 or 50% of the total number of agents. Disappointingly, these quotas are close to the ones where the toll they impose on the others is greater.

The model has not yet reached the level of sophistication that would suggest a serious calibration exercise but, if we steer this conclusion toward more perilous horizons, some research avenues could be suggested.

First, chartist agents differ from M-traders because they huddle together but it is conceivable that private signals as well may be correlated or common. If this is the case, M-traders too would behave in a more or less coordinated fashion whose effect should be clarified in future work.

Second, we could assert that there is no rational agent in the real markets (at least, not as rational as needed in order to know the parameters and functional form of the density of the risky asset and the preferences of the other agents). Put in another way, the rational agents should be replaced in the model by subrational M-traders who still has to deal with nasty chartists. The former use (rather rationally?) the available information to construct their own belief on the fundamental value, while the latter exploit the pseudo-content of the prices.

In third place, there is no reason why subrational agents should adjust their $\mu(i, t)$ alone. In actual markets, the dispersion of profits may be much more difficult to know than the average. It is plausible that subrational agents engaged in simultaneous estimation of both parameters μ and τ could fuel much bigger market sparkles and provide better fit to realistic stylized facts.

The policy implications of this work are, at this stage, rather grim. Eradication of chartism is difficult due to the perceived high levels of certainty equivalent that results from subrational behaviour. Together with the practical difficulties in detecting “deviant” conducts, this makes it difficult for institutions to devise painless solutions.

REFERENCES

- [Arthur et al., 1997] Arthur, W., Holland, J., LeBaron, B., and Taylor, P. (1997). Asset pricing under endogenous expectations in an artificial stock market. In Arthur, W., Lane, D., and Durlauf, S., editors, *The economy as an evolving complex system II*. Addison-Wesley.
- [Brock and Hommes, 1998] Brock, W. and Hommes, C. (1998). Heterogeneous beliefs and routes to chaos in a simple asset pricing model. *Journal of Economic Dynamics and Control*, 22:1235–1274.
- [Chan et al., 1996] Chan, L., Jegadeesh, N., and Lakonishok, J. (1996). Momentum strategies. *The Journal of Finance*, 51(5):1681–1713.
- [Chiarella and He, 2001] Chiarella, C. and He, T. (2001). Asset price and wealth dynamics under heterogeneous expectations. *Quantitative Finance*, 1(5):509–526.
- [Day and Huang, 1990] Day, R. and Huang, W. (1990). Bulls, bears and market sheep. *Journal of Economic Behavior and Organization*, 14(3):299–329.
- [De Long et al., 1990] De Long, J., Shleifer, A., Summers, L., and Waldmann, R. (1990). Noise trader risk in financial markets. *The Journal of Political Economy*, 98(4):703–738.

- [Friedman and Stuetzle, 1981] Friedman, J. and Stuetzle, W. (1981). Projection pursuit regression. *Journal of the American Statistical Society*, 76:817–823.
- [Gode and Sunder, 1993] Gode, D. and Sunder, S. (1993). Allocative efficiency of markets with zero intelligence traders. *Journal of Political Economy*, 101:119–137.
- [Grinblatt et al., 1995] Grinblatt, M., Titman, S., and Wermers, R. (1995). Momentum investment strategies, portfolio performance, and herding: A study of mutual fund behavior. *The American Economic Review*, 85(5):1088–1105.
- [Hommes, 2006] Hommes, C. (2006). Heterogeneous agent models in economics and finance. In *Handbook of computational economics Vol. 2: Agent-based computational economics*, page 1107. North-Holland.
- [Hong and Stein, 1999] Hong, H. and Stein, J. (1999). A unified theory of underreaction, momentum trading, and overreaction in asset markets. *The Journal of Finance*, 54(6):2143–2184.
- [LeBaron, 2006] LeBaron, B. (2006). Agent-based computational finance. In Tesfatsion, L. and Judd, K., editors, *Handbook of computational economics Vol. 2: Agent-based computational economics*, pages 1187–1233. North-Holland.
- [LiCalzi and Pellizzari, 2007] LiCalzi, M. and Pellizzari, P. (2007). Simple market protocols for efficient risk sharing. *Journal of Economic Dynamics and Control*, 31(11):3568–3590.
- [Wilson, 1968] Wilson, R. (1968). The theory of syndicates. *Econometrica*, 36:119–132.

CA' FOSCARI UNIVERSITY, DEPT. OF APPLIED MATHEMATICS, DORSUDURO
3825/E, 30123 VENICE - ITALY
E-mail address: paolop@unive.it