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Abstract

This paper solves an stochastic growth model susceptible to the intervention of a monetary authority following a policy rule. It is shown that, if subject to investment costs and sticky inflation, the model's predictions about the patterns of the term structure fit the observed in empirical data: (i) pro-cyclical pattern of the level of nominal interest rates; (ii) counter-cyclical pattern of the term spread (as well as low sensitivity of long yields to monetary policy changes); (iii) short run stray of the real interest rate from the marginal productivity of capital; (iv) short run negative correlation between expected inflation and expected future real interest rate; and (v) low predictability of the slope of the middle of the yield. In the proposed framework, discontinuous changes of the controlled short rate play an important role and the improved ability to explain the data is incorporated into a simple intertemporal equilibrium model of the term structure of the interest rates.

1 Introduction

This paper attempt a nested answer to two apparently unrelated questions:

- How can an intertemporal equilibrium model adequately fit an arbitrary exogenous term structure of interest rates?
- What is the role of monetary policy in determining the term structure of interest rates?

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On the first issue, intertemporal equilibrium modelling of interest rates still leaves many questions unanswered. For example, the scalar time-homogenous affine equilibrium models[1] stand out for their terse description of a equilibrium economy, which provides tractable and rich analytic results. However, because of the constant level of reversion, these models are intrinsically incapable of fitting an arbitrary exogenous term structure. Worse, when tested against more general scalar specifications, they are usually rejected, suggesting either the existence non-linearities or omitted variables (Chan et al. [7] or Aït-Sahalia [1]).

On the second issue, despite the belief that changes in the monetary policy impact on asset returns in general[2] and are a major source of changes in the shape of the yield curve, micro-financial models have not accomplished to properly incorporate it yet. The neglect to deal with macro links leaves unexplained, or even contradict, certain stylized facts like the pro-cyclical nominal interest rate levels, the countercyclical term spread (Fama & French [10]), or the negative short-run correlation between expected inflation and the expected future real interest rate (Barr and Campbell [3]).

As macro links, omitted variables and constant reversion level seem to be the weak points of the scalar time-homogeneous equilibrium models, an attempt is made there to incorporate a macro monetary policy variable into a intertemporal equilibrium model.

We analyze the character of fluctuations in the term structure of nominal interest rates, the inflation and the aggregate output in an economy with staggered wage contracting and investment costs, subject to both technology and nominal interest rate shocks. Indeed, we end up solving an stochastic growth model, subject to investment costs and sticky inflation, susceptible to the intervention of an external authority. The intertemporal optimization implies a complete description of the multiperiod expected returns, and the model allows the derivation of a nominal term structure which incorporates the effects of the monetary policy. By discontinuos changes of the shortest-term nominal interest rate, the Central Bank forces the short-end of the term structure to match an exogenously specified level, implies a non-zero net supply of nominal riskless bonds, and adds the possibility of jumps of all forward-looking models' variables. Given that the monetary authority is constrained to keep inflation close to zero, future changes in the controlled rate can be forecasted by looking at the dynamics of the expected inflation and might be incorporated into the term structure shape.

Despite absence of an original hypothesis, the resulting model seems an improvement upon Balduzzi, Bertola & Foresi's [2], Rudebusch's [17] or Piazzessi's [15] analysis of the monetary policy impacts on the term structure in the sense that it explains more stylized facts, and this is done in an intertemporal optimization equilibrium framework. With a relatively simple model it is shown

[1] The univariate version of Cox, Ingersoll and Ross (1985 b) can be seen as the most important member of the class.

[2] For example, Thorbecke (1997) and Patelis (1997) document the existence of a monetary risk premium and show the role of monetary policy in the predictability of the asset returns.

that the monetary authority policy has real effects. In fact, we are able to explain: (i) the pro-cyclical pattern of the level of nominal interest rates; (ii) the countercyclical pattern of the term spread[3] (as well as low sensitivity of long yields to monetary policy changes); (iii) the strays of the expected real interest rate from the expected marginal product of capital; (iv) the short run negative correlation between expected inflation and expected future real interest rate; and (v) the low predictability of the slope of the middle of the yield curve, facts whose empirical evidence has been shown in the literature (for example, see Campbell, Loo & MacKinlay [6], Fama & French [10] and Rudebusch [17]).

In a related paper[4], implications of the here developed model are explored in a bond pricing context.

The paper has the following structure. Section 2 presents the empirical data pattern. Section 3 reviews the term structure pattern implied by the plain Real Business Cycle model and points its weaknesses in predicting nominal term structure movements. Section 4 presents the proposed model in a simplified representative agent framework. Some examples and a simulation are worked in Section 5. Section 6 shows the equivalence between the representative agent and the competitive problem and Section 7 concludes.

2 Some Stylized Facts

This section presents empirical evidences on the movements of the term structure of nominal interest rates, inflation and output, to which the numerical predictions of the theoretical models will be subsequently compared. The empirical pattern of the term structure is reproduced below by use of Fama and Bliss data set (F&B data from now on), that uses only fully taxable, non-callable, non-flower bond[5]. The monthly data contain one to five years-to-maturity bonds and cover the period from July 1952 to January 1998, providing 547 observations in total. All yields are expressed in annualized form.

The evolution of the yields-to-maturity [6] of the one and the five-year bonds are plotted in figure 1 with shades added to mark the business cycles. Every white period points one expansion cycle from trough to peak, as classified by the NBER. The gray periods mark the contraction periods from peak to trough. The (i) pro-cyclical pattern of the level of interest rates is clear: the level of the interest rates increases during expansion and decreases during contraction. Also plotted is the evolution of the slope and the curvature of the yield curve [7]. It is shown that (ii) the term spread presents a countercyclical pattern: the slope of the yield curve is big at the trough and decreases during the cycle to become

[3] The term spread is defined as the difference between the yield-to-maturities of a long and a short term bond.

[4] Another chapter of the dissertation which this paper is part of.

[5] The Fama and Bliss data set was constructed by Fama and Bliss [9] and was subsequently updated by the Center for Research in Security Prices (CRSP).

[6] The yield-to-maturity is defined as the average return on the bond held until maturity.

[7] The slope of the yield curve is nothing more than the term spread ($yr05 - yr01$). The curvature is defined as ($yr05 - 2 \cdot yr03 + yr01$).

small at the peak. Curvature does not seem to provide a clear pattern during expansions, however it seems to decreases along contractions (shades).

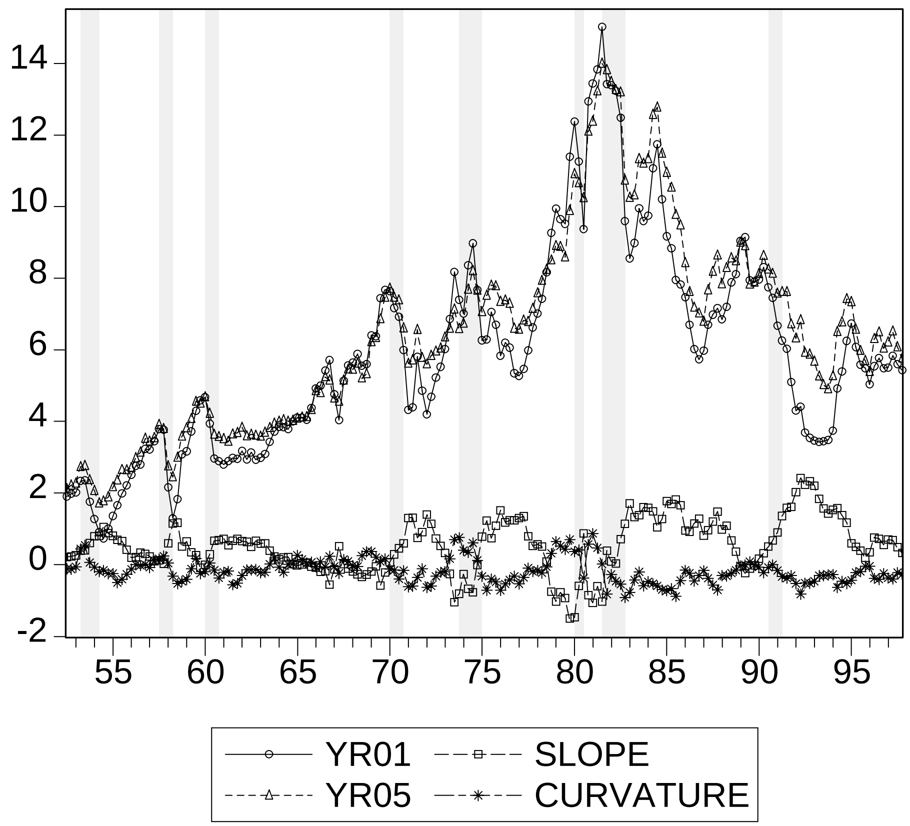

Figure 1: Evolution of U.S. nominal yields from 1952:03 to 1997:04

Other empirical fact deserving report is that (iii) expected real interest rate can stray from the expected productivity of capital for a while. More, there exist (iv) short run negative correlation between expected inflation and expected future real interest rate, as shown by Barr & Campbell [3][8].

If any version of the Expectation Hypothesis[9] holds, the slope of the yield curve has ability in forecasting interest rate moves. After some algebra, to test

[8] We are working on getting these evidences from F&B data.

[9] The Expectation Hypothesis says that the expected excesss returns on long-term bonds over short term bonds (the term premiums) are constant over time.

such a predictability reduces to check if the slope, b, of the regression:

$$\left(y_{t+l}^{l} - y_{t}^{l}\right) = a + b\left(y_{t}^{2l} - y_{t}^{l}\right) + e_{t}, \tag{1}$$

is significant. Indeed, the above hypothesis implies that $b = 1$.

We have estimated b for the holding-period l equal to one year, what means we have performed the test for the predictability of two minus one-year bond. For F&B data, the estimated b equals 0.09 with a standard error of 0.19, what means it is not significantly different from zero. Campbell, Loo & MacKinlay [6] obtain the same qualitative results in a close context, and their report provides a more comprehensive view of the empirical pattern. By use of monthly zero-coupon bond yields over the period 1952:1 to 1991:2[10], they estimate equations with meaning similar to (1) for 2 to 120 months and get the results shown in Table 1.

	2	3	6	12	24	48	120
b	.502	.467	.320	.272	.363	.442	1.402
(s.e.)	(.096)	(.148)	(.146)	(.208)	(.223)	(.384)	(.142)

Table 1: b estimates by Campbell, Loo and MacKinlay

The stylized fact that their results bring to scene is the U-shaped pattern of the coefficients: (v) the forecasting power diminishes from one month to one year and then increases up to ten years.

Besides those five evidences, principal component analysis might be useful information in the attempt to build an intertemporal optimization model that fits the term structure. By use of principal component Litterman & Scheinkman [11] identify three main factors explaining most of the term structure movements[11]. The first factor is shown to have similar impact on all bonds, and is thus interpreted as the level. The second factor is increasing in magnitude, which means it causes the changes in slope. Finally, the third factor, which has more impact at the short-end and at the long-end of the term structure, is interpreted as the curvature factor. The application of principal component analysis to F&B data has generated similar results shown in Tables 2 and 3.

Maturity	1st. Eigenvec.	2nd. Eigenvec.	3rd. Eigenvec.
1 year	0.4548	-0.7470	0.4692
2 year	0.4543	-0.2104	-0.6174
3 year	0.4465	0.1248	-0.4139
4 year	0.4428	0.3628	0.1108
5 year	0.4374	0.5005	0.4637

Table 2: Empirical Eigenvectors

[10] Campbell, Loo & MacKinlay [6] use the data from McCulloch and Know (1993).

[11] Litterman & Scheinkman [11] have used weekly observations from January 1984 to June 1988.

Maturity	Level	Steepness	Curvature
1 year	0.9754	0.0243	0.0003
2 year	0.9973	0.0020	0.0005
3 year	0.9987	0.0007	0.0002
4 year	0.9934	0.0061	0.0000
5 year	0.9875	0.0119	0.0003
Average	0.9905	0.0090	0.0003

Table 3: Relative Importance of the Empirical Factors

Slightly different is that in F&B data, the first factor explains more than in Litterman & Scheinkman [11], what might mean that the slope and the curvature factors are more important in explaining short run movements. If this is due to different frequency, length of the time series or span of the maturity is something that deserves more research.

3 A Simple Intertemporal Equilibrium Theory of the Term Structure with production

Because intertemporal optimization models imply a complete description of the multiperiod expected returns, and the term structure of interest rates is the plot of the observed multiperiod returns, these models are suitable for derivation of a microfoundated term structure.

In the Real Business Cycle (RBC) model with labor supplied inelastically, the representative agent maximizes:

$$\sum_{i=t}^{\infty} \beta^{i-t} u\left(c_i\right) \tag{2}$$

with: $u'(.) \geq 0, \quad u"(.) < 0$; subject to the budget constraint:

$$\begin{aligned} & c_t + k_{t+1} + b_{t+1}^1 + \sum_{j=2}^{\infty} b_{t+1}^j \\ = \; & \theta_t k_t^{\alpha} + (1-\delta) k_t + \frac{1}{(1+\pi_{t-1,t})} \left[(1+i_t) b_t^0 + \sum_{j=1}^{\infty} \frac{B_t^j}{B_{t-1}^{j+1}} b_t{}^j \right] - \tau_t; \end{aligned} \tag{3}$$

to the technology shock dynamics:

$$\log \theta_t = \rho \log \theta_{t-1} + \varepsilon_t, \;\; \varepsilon_t \;\tilde{}\; N\left(0, \sigma_{\varepsilon}^2\right); \tag{4}$$

and to the transversality conditions:

$$\lim_{t \to \infty} \beta^t k_t = 0; \tag{5}$$

$$\lim_{t\to\infty}\beta^t\sum_{j=1}^{\infty}\frac{B_t^j}{P_t}b_t^j=0; \tag{6}$$

where: c stands for real consumption; k is the real capital stock; θ is the productivity shock, and $0<\alpha<1$ is the capital elasticity[12]; δ is capital depreciation; $(1+\pi_{t,t-1})=\frac{P_t}{P_{t-1}}$ is the inflation between $t-1$ and t, with the price index P_t not known before t; $(1+i_t)$ is the nominal interest rate of the $one-period$ bond held between $t-1$ and t, known at $t-1$; B_t^j is the nominal price of the $j-period$ bond; b_t^j is the quantity of the bond the consumer carries from $t-1$ to t, and j is the number of periods to maturity; b_t^0 is the real quantity of the bond redeemed at t; and τ_t are the real taxes.

Although labor is not explicitly included, the above formulation couches the case of constant return-to-scale production function with labor inelastically supplied. Also, to make presentation lighter, instead of the usual normalization of nominal unit price at maturity, $B_t^0=1\ \forall\ t$, we assume that the next-to-mature bond costs one nominal unit and worthies $(1+i_{t+1})$ nominal units at redemption.

From the above, the representative agent value function can be posed as:

$$\begin{aligned} & V\left(k_t,b_t^{j\geqslant 0},\theta_t\right) \qquad (7)\\ = & \max_{c,\ k,\ b}\left\{\begin{array}{c} u\left(c_t\right)+\beta E_t V(k_{t+1},b_{t+1}^{j\geqslant 0},\theta_{t+1}) \\ -\lambda_t\left[\begin{array}{c} c_t+\tau_t+k_{t+1}+b_{t+1}^1+\sum_{j=2}^{\infty}b_{t+1}^j-\theta_t k_t^{\alpha}-\left(1-\delta\right)k_t \\ -\frac{1}{(1+\pi_{t,t-1})}\left[\left(1+i_t\right)b_t^0+\sum_{j=1}^{\infty}\frac{B_t^j}{B_{t-1}^{j+1}}{b_t}^j\right] \end{array}\right] \end{array}\right\}, \end{aligned}$$

and solved to result in the agent's optimal allocation rules:

$$u'\left(c_t\right)=\beta E_t\left\{\left[\alpha\theta_{t+1}k_{t+1}^{\alpha-1}+\left(1-\delta\right)\right]u'\left(c_{t+1}\right)\right\} \tag{8}$$

$$\frac{u'\left(c_t\right)}{\left(1+i_{t+1}\right)}=\beta E_t\left[\frac{1}{\left(1+\pi_{t+1,t}\right)}u'\left(c_{t+1}\right)\right]; \tag{9}$$

and

$$u'\left(c_t\right)\frac{B_t^j}{P_t}=\beta E_t\left[\frac{B_{t+1}^{j-1}}{P_{t+1}}u'\left(c_{t+1}\right)\right]\quad\forall j; \tag{10}$$

[12] The production function $f\left(k,\theta\right)=\theta_t k_t^{\alpha}$ present the usual conditions:

$$f_1(.)\geq 0,\ f_2\left(.\right)>0,\ f_{11}(.)\leq 0,\ f_1(0,.)=\infty,\ f_1(\infty,.)=0;$$

taking prices as given.

Recursion on (10) and the law of iterated expectations implies:

$$1 = \beta^l E_t \left[\frac{B^{j-l}_{t+l}}{B^j_t} \frac{P_t}{P_{t+l}} \frac{u'(c_{t+l})}{u'(c_t)} \right] \quad \forall j \; and \; l \geqslant 1. \tag{11}$$

Note that the real $l - period$ holding return of the $j - period$ bond can be written as:

$$\frac{B^{j-l}_{t+l}}{B^j_t} \frac{P_t}{P_{t+l}} = \frac{B^{j-l}_{t+l}}{B^j_t} \frac{1}{1 + \pi_{t+l,\ t}} = \left(1 + r^j_{t+l,\ t}\right)^l \tag{12}$$

where: $\pi_{t+l,\ t} = \left(\frac{P_{t+l}}{P_t} - 1\right)$ denotes the inflation rate between t and $t + l$, but only known at $t + l$; and $r^j_{t+l,\ t}$ stands for the $l - period$ real returns on a $j - period$ nominal bonds, from t to $t + l$.

The agent's optimal conditions allow us to define:

$$M_{lt} = \beta^l \frac{u'(c_{t+l})}{u'(c_t)} \tag{13}$$

as the stochastic discount function (or the pricing kernel); which in the present model is equivalent to the intertemporal marginal rate of substitution in consumption.

3.1 Equilibrium without external intervention: inflation and nominal interest rate indeterminacy

An equilibrium sequence is defined as a set of stochastic vector $\left(\theta_t,\ k_{t+1},\ c_t,\ i_{t+1},\ \pi_{t,t-1},\ r^j_{t+l,t},\ b^j_{t+1},\ \tau_t\right)$ satisfying the f.o.c.'s and the market clearing conditions for every t..

Without external intervention, the exogenous supply of bonds is zero:

$$b^j_t = 0 \quad \forall\ j;$$

as well as taxes: $\tau_t = 0$, and, given (4), the consumers' decision simplifies to split wealth between capital and consumption by obeying (8) and the simplified budget constraint:

$$c_t = \theta_t k^\alpha_t + (1 - \delta) k_t - k_{t+1}, \tag{14}$$

for every t.

The transversality condition (5) defines the saddle path followed by $(k\ , c)$ in the system (8) and (14). The substitution of (14) into (8) defines a stochastic difference equation in k that, given the initial capital stock, initial technology and (5), obtains the optimal capital path (k^*) and provides the inputs to obtain the optimal consumption path (c^*) by (14). The above hypotheses are enough to guarantee the distribution of optimum aggregate capital converges pointwise to a limit distribution when returns are decreasing: k is pushed to the level k_{ss} where the expected marginal productivity of capital equals the rate of time preference: $\alpha k_{ss}^{\alpha-1} - \delta = (1/\beta) - 1$. When returns are constant-to-scale, they are as well enough to guarantee that the rates of growth converge pointwise to a limit distribution[13].

The $l-period$ real returns on a $j-period$ nominal bonds, from t to $t+l$, is endogenously determined by the application of $\{c_t^*\}_{t=0}^{\infty}$ to (11):

$$1 = \beta^l E_t \left[\left(1 + r_{t+l,\ t}^j\right)^l \frac{u'\left(c_{t+l}^*\right)}{u'\left(c_t^*\right)} \right] \quad \forall\ j \geqslant l; \tag{15}$$

that gives the whole real term structure implied by the model.

When these endogenously determined rates are obtained without external intervention, like in the present section, we call them neutral values. Thus, the neutral rates are nothing more than the one for which the private sector's net demand for every maturity bond is zero: $b^j = 0\ \forall\ j$.

Equation (15) implies the expected real rate of return on any bond over l periods is the same for every $j-period$ bond, with $j \geqslant l$. That means, the "Local" version of the Expectation Hypothesis holds in this intertemporal optimizing framework: $E_t\left[r_{t+l,\ t}^j - r_{t+l,\ t}^k\right] = 0\ \forall\ j\ and\ k \geqslant l$; what gives a zero real term premium.

Inasmuch as the yield-to-maturity of every $l-period$ bond is known for certainty at t, $\frac{B_{t+l}^0}{B_t^l} = \left(1+y_t^l\right)^l$, it can be put out of the expectation operator, resulting in:

$$\frac{B_t^l}{B_{t+l}^0} = \frac{1}{\left(1+y_t^l\right)^l} = \beta^l E_t \left[\frac{1}{1+\pi_{t,\ t+l}} \frac{u'\left(c_{t+l}^*\right)}{u'\left(c_t^*\right)} \right];$$

that provides the whole nominal term structure.

Given that the shortest nominal interest rate fixed at t, i_{t+1}, is the nominal interest rate for the bond maturing next period, it is easily deduced that for $l = 1$, $y_t^1 = i_{t+1}$; and the above formula simplifies to:

$$1 = \beta E_t \left[\left(1 + r_{t+1,\ t}^1\right) \frac{u'\left(c_{t+1}^*\right)}{u'\left(c_t^*\right)} \right] = \beta E_t \left[\frac{1+i_{t+1}}{1+\pi_{t+1,\ t}} \frac{u'\left(c_{t+1}^*\right)}{u'\left(c_t^*\right)} \right]; \tag{16}$$

where i_{t+1} can be put out of the expectation if desired.

Note that because there is an stochastic shock in the production function, the neutral real spot rate fluctuates around a trend defined by the optimal capital

[13] See Brock [4] for the proof.

path. For example: if k_t is increasing along time and the production function present decreasing returns-to-scale, the productivity trend is decreasing and real neutral rate is expected to decrease as economy tends to the steady state.

Without an external intervention, the real interest rates, given by (15), are completely defined by (4), (8), (14) and (5). Equation (16) is nothing more than the Fisher relation that defines next period inflation given the spot one-period nominal interest rate, or the other way around. Because the expected spot one-period real interest rate is completely determined by the real factors and is every time the expected marginal productivity of the capital, expected inflation sensitivity to the level of the nominal interest rate is one, what means no correlation between nominal and real variables.

Although one might argue that the inflation and the nominal rate indeterminacy is a consequence of having more variables than equations, the inclusion of a cash-in-advance restriction or the fiscalist theory type of reasoning does not change the above conclusions. Given this one-to-one correspondence between i and π, there is (i) no cyclical pattern of the level of the nominal term structure, (ii) nor of the term spread. Real interest rates vary with the marginal productivity of capital, but (iii) it never strays from the expected productivity of capital. (iv) There is no correlation between expected inflation and expected future real interest rate, and (v) the predictability of the slope of the yield curve is good and equally credible for every maturity.

Summing up, system (4), (8), (14) and (5) alone does not split the changes in the nominal rate into changes in the real rate and inflation, and is not of great use in explaining how monetary policy affects real activity and inflation. Basically, it assumes neutrality (and superneutrality) and thus thwarts the possibility that nominal interest rate and inflation vary independently. Quite unrealistic, inflation reduction to zero can be done in one painless down move of the nominal rate to the expected marginal productivity level with no impact on the real activity.

Notwithstanding, there exists one degree of freedom in the above model to couch an ad hoc assumption, and this is going to be worked in conjunction with inflation stickiness in next section.

4 The Model

The proposed model is the description of a one-good closed economy[14] with firms and capital accumulation, subject to investment cost and staggered wages contracting, and susceptible to the intervention of the monetary authority. For presentation purposes, we develop the main ideas in the representative agent framework with sticky inflation. The equivalence with a more detailed economy, where consumers and firms interact in a world of staggered wage contracting, is shown at the end, in section 6.

[14] As pointed in Meltzer (1995) pp.50, in an open economy, the exchange rate would be just one more of the many relative prices in the transmission process, without altering the basic results.

4.1 The Inflation Dynamics and Monetary Authority Intervention

Let the inflation dynamics be given by:

$$\pi_t = (1 - \chi_0)\,\pi_{t-1} + \chi_0 E_t\left[\pi_{t+1}\right] + \chi_1\left(f(k,\theta) - f^*\right) + \omega_t, \tag{17}$$

where: χ_0 is the forward looking parameter, f^* is potential output and ω_t is an independent random shock. In the present equilibrium model, this dynamics will be rationalized by staggered wage contracting (in section 6).

Since we are interested on the study of term structure shape and moves, and not on the study of optimal monetary policy rule, we don't care about objective functions of the monetary authority and related issues. It is enough that the external authority (call it the monetary authority) be concerned about inflation, have funds to intervene in the bond market, and know its dynamics is given by (17). This being the case, it is prone to control the one-period spot interest rate to fight inflation. Due to operating constraints, it is assumed that it uses the rule:

$$i_{t+1} = i_t + \upsilon_t, \tag{18}$$

where:

$$\upsilon_t = \left\{ \begin{array}{l} 0,\ \textit{with probability}:\ \left(1 - \varsigma\left|\pi_{t-1}\right|\right) \\ e\frac{\pi_{t-1}}{\left|\pi_{t-1}\right|},\ \textit{with probability}:\ \varsigma\left|\pi_{t-1}\right| \end{array} \right\};$$

and e and ς are positive constants. This means that the spot rate tends to remain constant from period to period, except for jumps whose probability is an increasing function of the inflation level. If inflation is "high" the eventual jump is positive, and if inflation is "low" (deflation) the eventual jump is negative. When inflation grows, the probability of jumps increases and so the expected value of next period spot rate. Because inflation is a persistent series, policy only reverts when the inflation target has been mostly reached.

The monetary authority acts buying or selling one-period bonds that pay riskless nominal interest rate, but risky real interest rate:

$$\left(\frac{1 + i_{t+1}}{1 + \pi_{t+1,t}}\right),$$

revealed at $t + 1$.

Besides, the authority runs no deficit, what forces it to charge the individuals a lump sum tax to payoff the net interest:

$$\tau_t = \left[\frac{1 + i_t}{1 + \pi_{t,t-1}} - 1\right] b_t^a \ \ \forall\ t \geqslant 0,$$

where b_t^a stands for the per capita bond demand.

Because individuals receive the full proceeds of bonds they hold and are charged lump sum, they choose to long or short the one-period bond once its real expected return diverges from the expected neutral rate. Thus, although lending to or borrowing from the monetary authority are just simple storage in the aggregate, non-zero net demand for one-period government bonds shows up due to the non-cooperative individual behavior induced by the tax system.

Not only the above rule makes it easy to forecast tomorrow's spot rate, but it also cause the system stability as long as it guarantees that inflation does not explode and provides the long run level variables. Stability is the cause for long rates' low sensitivity to monetary policy changes: given the parameters, long run variables are implied and those are the ones that weight most in the valuation of long term bonds.

4.2 The Real Side with investment costs

The representative agent maximizes (2), subject to the budget constraint:

$$\begin{aligned} & c_t + \left[k_{t+1} + \varphi\left(\frac{k_{t+1}}{k_t} - 1\right)^2\right] + b_{t+1}^1 + \sum_{j=2}^{\infty} b_{t+1}^{\;j} \\ = \;& \theta_t k_t^\alpha + (1-\delta)\,k_t + \frac{1}{(1+\pi_{t-1,t})}\left[(1+i_t)\,b_t^0 + \sum_{j=1}^{\infty}\frac{B_t^j}{B_{t-1}^{j+1}}{b_t}^{j}\right] - \tau_t, \end{aligned} \tag{19}$$

and to (4), (5), (6); where: $\varphi\left(\frac{k_{+1}}{k} - 1\right)^2$ is a cost of adjustment, and the other variables have the previous stated meaning.

Now, the representative agent value function can be posed as:

$$\begin{aligned} & V\left(k_t, b_t^{j\geqslant 0}, \theta_t\right) \\ = \;& \max_{c,\ k,\ b}\left\{\begin{array}{c} u(c_t) + \beta E_t V(k_{t+1}, b_{t+1}^{j\geqslant 0}, \theta_{t+1}) \\ -\lambda_t\left[\begin{array}{c} c_t + \tau_t + \left[k_{t+1} + \varphi\left(\frac{k_{t+1}}{k_t} - 1\right)^2\right] + b_{t+1}^1 + \sum_{j=2}^{\infty} b_{t+1}^{\;j} \\ -\theta_t k_t^\alpha - (1-\delta)\,k_t - \frac{1}{(1+\pi_{t,t-1})}\left[(1+i_t)\,b_t^0 + \sum_{j=1}^{\infty}\frac{B_t^j}{B_{t-1}^{j+1}}{b_t}^{j}\right] \end{array}\right] \end{array}\right\}; \end{aligned} \tag{20}$$

and the solution is similar to section's 2, except that:

$$\begin{aligned} & \left[1 + 2\varphi\left(\frac{k_{t+1}}{k_t} - 1\right)\frac{1}{k_t}\right]u'(c_t) \\ = \;& \beta E_t\left\{\left[\alpha\theta_{t+1}k_{t+1}^{\alpha-1} + (1-\delta) - 2\varphi\left(\frac{k_{t+2}}{k_{t+1}} - 1\right)\frac{k_{t+2}}{k_{t+1}^2}\right]u'(c_{t+1})\right\}, \end{aligned} \tag{21}$$

substitutes (8).

4.3 Equilibrium with Intervention Possibility

Equation (19) can be simplified a bit. Since the Central Bank only intervenes in the $one - period$ bond market, only b_t^0 and b_{t+1}^1 can be different from zero and the exogenous supply of the bonds longer than one period is still zero: $b_t^j = 0 \ \forall \ j > 1$. Also, because in the representative agent world, equilibrium means:

$$b_t^a = b_t^0$$

and (19) becomes:

$$c_t = \theta_t k_t^\alpha + (1-\delta) k_t + b_t^0 - \left[k_{t+1} + \varphi \left(\frac{k_{t+1}}{k_t} - 1 \right)^2 \right] - b_{t+1}^1. \qquad (22)$$

The economy equilibrium sequence $\left(\theta_t, \ b_{t+1}^j, \ \tau_t, \ i_t, \ \pi_t, \ r_{t+l,t}^j, \ k_{t+1}, \ c_t\right)$ is now given by the system of six simultaneous equations (22), (4), (17), (18), (21) and (16), and the transversality conditions (5) and (6), given the initial values for $\theta_0, \ b_0^1, \tau_0, \ i_0, \ k_0$ and π_{-1} [15].

Although in the representative agent framework, we are able to argue in terms of the Q-theory of investment. It is possible to get the evolution of marginal Tobin's Q:

$$Q = 1 + 2\varphi \left(\frac{k_{t+1}^*}{k_t^*} - 1 \right) \frac{1}{k_t^*}; \qquad (23)$$

for the optimal capital sequence $\{k_t^*\}_{t=0}^\infty$.

4.4 The model dynamics: fitting empirical pattern

The monetary transmission mechanisms are the Tobin's Q theory of investment and the wealth effects on consumption: the spot rate change sponsors consumption and portfolio responses with real effects.

To illustrate the implications of the model, we make use of phase diagrams to look at the implied dynamics and the evolution of the term structure along time. Picture ?.1 shows the dynamics of Q and K, and Picture ?.2 shows the implied dynamics of the term structure. In Picture ?.1, Q is above unit for increasing k and is below unit for decreasing k. It is the case without intervention of an economy's growth path[16]. From equation (15) it can be inferred that the steep of the real term structure decreases as the economy comes close to the steady-state, since the ratios of two different time consumptions approach the unity

[15] The jumping variables are c, b, π.

[16] The steady state can be seen as the peak of a business cycle. In this case, at every technology shock that improves eficiency, QQ moves northeast and the term stucture dynamics begins again.

(and the real yield approach β^{-1} for every maturity). Picture ?.2 illustrates this process and partially explain why the (ii) countercyclical pattern of the term spread.

Picture ?.1 shows what happens when a temporary increase in the real spot interest rate is expected at a certain date and for a certain period[17], due to tight of the Central Bank to fight increasing inflation: once the tight becomes expected, Q jumps down and K begins to decrease up to the time when the change happens (at T). Between the effective tight and the time policy is again loosen, Q increases, while K first decreases, to increase after Q reaches unit. (Q, K) changes happen so that when policy reverts to loose again (at T'), the pair is over the original saddle and goes to the steady-state. The term structure shown in Picture ?.2 shows that consumption also jumps once the change becomes expected and decreases when interest rate is up. When policy is loosen again, consumption increases again. This implies an inverted humped yield curve.

Picture ?, on the other hand, shows what happens when the time of the target is uncertain. Once the change becomes justifiable by "high" inflation, Q jumps for an intermediary saddle path, located in accordance with the probability of change. While the change does not happen, inflation is increasing and the intermediary saddle moves southwest (due to the increasing probability), bringing together the pair (Q, K). Once the tight takes place (at T), Q jumps again to a point that depends on the expected future monetary policy.

The combination of the real spot interest rate with the inflation dynamics allows to obtain all sort of shapes for the term structure. Now, the (i) to (v) stylized facts can be explained by the above model.

Inasmuch as the expected inflation is pro-cyclical, (i) the nominal interest rates level is high in the peak and low in the trough of the business cycle.

Because the longer the bond, the more its yield is affected by moves of the future shortest nominal interest rates, (ii) the countercyclical pattern of the term spread can be explained as a "future spot-rate move risk". Due to system stability, people believe there are upper and lower bounds for the expected inflation and the probability of monetary authority action's against inflation is increasing in inflation itself. When the economy begins an expansion the levels of nominal interest rates and inflation are low and inflation is expected to grow. Eventual spot rate jumps in the near future will have positive signs, this meaning lower bond prices and capital looses for the long maturities bond holders, who charge their borrower for that. As the expansion takes place, inflation increases, followed by the spot-rate. Since there is a perceived upper limit for the inflation, the 'future spot-rate upside risk' decreases along this path, and the reduction in the term spread is consistent. The description of the recession goes along the same lines.

With the spot-rate exogenously fixed and adjustment costs, it is also evident why the Fisher hypothesis of constant real interest rate can't hold and help

[17] Although Central Bank's target changes are uncertain as well as it is how long they take, this exercise simplifies comprehension.

explain why (iii) the expected real spot interest rate can stray from the expected marginal product of capital for a while. A positive (negative) inflation shock not accompanied by a spot-rate jump accomplishes to lower (raise) real interest rate below (above) present capital productivity level and sponsors capital investment (disinvestment). In the presence of increasing investment costs, capital does not increase enough for its productivity to equal real interest rate.

The way nominal spot interest rate is modified gives rise to a (iv) negative short-run correlation between expected inflation and expected future real interest rate, since inflation innovations are not instantaneously transmitted to the nominal spot rate.

Inflation stickiness and stability seem enough to justify (v) the better predictability of the slope of the yield curve at the short- and at the long-ends (or the worse predictability of the slope of the middle of the yield curve), inasmuch as present monetary policy is supposed to last for a while and long rates are mainly affected by the stationary long run variables.

Trying an economic interpretation for the three factors presented above: the level would be a combination of the β and current inflation. The monetary policy and the growth would justify the steepness. Curvature still waiting for an interpretation.

5 Experiments and Simulation

Equations (4), (16), (17), (18), (21) and (22) form a non-linear stochastic difference system with rational expectations that can be solved according to Novales et al.[13].

Quite preliminary numerical exercises are reported below for parameter values $\sigma = 2$, $\beta = 0.99$, $\rho = 0.95$, $\alpha = 0.36$, $\delta = 0.025$, $\varphi = 380$, $\chi_0 = 0.5$, $\chi_1 = 0.2$, which are standard calibration parameter for quarterly frequency.

5.1 Experiments

The figures below illustrate the dynamics in two experiments: (i) a disinflation experiment, when capital has already reached the steady state, but the present inflation is above the steady state level (shown in figure 2), and (ii) an expansion experiment, when capital as well as inflation are below the steady state level (shown in figure 3).

As illustrated in figure 2, the level of the nominal interest rates are initially high, but short real interest rate is expected to increase and inflation to decrease. The evolution of the term structure is illustrated in the figure.

In figure 3, capital and consumption increase along time, while the real interest rate decrease.

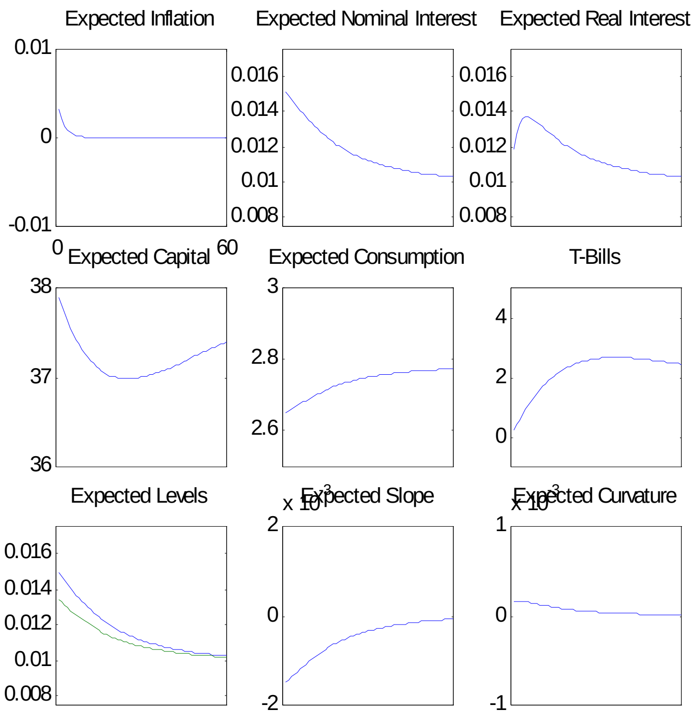

Figure 2: Contraction path

5.2 Simulation with the U.S. data

In a attempt to test if the model reproduce the data, we can feed the model with the historic shock series for $(\varepsilon,\ \omega)$ and check if the numerical predictions of the theoretical model present data pattern similar to the stylized facts from section 2. In a quite preliminary exercise, we have calculated a shock series ε in equation (4) for the U.S., and used it to generate a series of technology parameter θ for the recent history. The proxy for productivity growth used was "Output per hour of all persons, seasonally adjusted" from 1952:3 to 1997:4. provided by the U.S. Department of Labor, Bureau of Labor Statistics. The neperian logarithm of the series was taken and the trend discounted using Hodrick-Prescott filter. From this resulting series, an AR(1) with trend was estimated and the residuals were taken to be the proxy for ε [18].

Figure 4 below present the path of the variables for the simulation.

[18] The resulting residuals presented almost no correlation, standard-deviation equal to 0.008, and the Jarque-Bera test could not reject normality at 5% significance level.

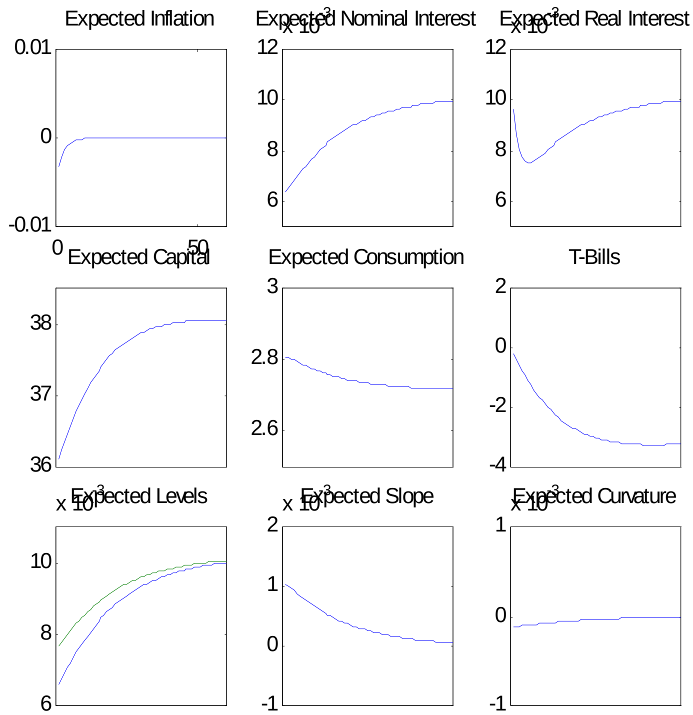

Figure 3: Expansion path

The results are not bad and are "close" to reproduce the pattern documented by Litterman and Scheinkman [11]. Tables 4 and 5 below show relative importance of the factors and the respective eingenvectors.

The simulation also present the short run negative correlation between expected inflation and expected future real interest rate. Table 6 below present the covariances between today expectation of next period inflation and expected real interest rate of this and the next three periods [19]:

6 The Competitive Problem

The equivalence of the representative consumer with a competitive economy is going to be shown below. As usual in the competitive framework, consumers and firms maximize their objective function taking prices as given. Without

[19] To have an idea of size, the variance of real interest rate is 0.0011.

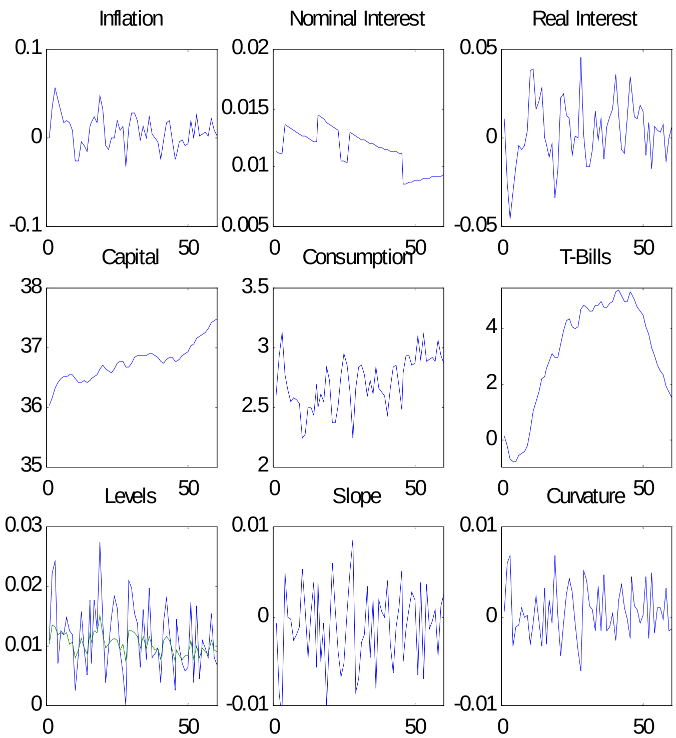

Figure 4: U.S. simulation

loss of generality, it is assumed that the firms are the owners of the capital and are all equity financed[20].

UPON REQUEST FOR LACK OF SPACE

7 Conclusion

The relatively simple macro model developed in the paper seems able to fit the empirical term structure of interest rates. It doesn't focus on the behavior of some instantaneous spot rate process derived from a particular equilibrium model to obtain the term structure as is usual in the literature. Instead, it recognizes the spot-rate is an instrument of the monetary authority, who controls it to match the goal of low price variation. This being the case, the long run levels of the state variables may be forecasted with a high degree of accuracy,

[20] For the firms decision between equity and debt in a framework similar as ours, see Brock and Turnovsky (1981). Note they deal with such decision in a perfect foresight situation.

Maturity	Level	Steepness	Curvature
1 year	0.8497	0.1475	0.0028
2 year	0.9533	0.0017	0.0417
3 year	0.9485	0.0299	0.0011
4 year	0.9150	0.0672	0.0100
5 year	0.8948	0.0756	0.0192
Average	0.9123	0.0644	0.0150

Table 4: Relative Importance of the Factors

Maturity	1st. Eigvec.	2nd. Eigvec.	3rd. Eigvec.
1 year	.5457	-.7995	.2506
2 year	.4744	.0698	-.7925
3 year	.4303	.2685	-.1169
4 year	.3935	.3749	.3282
5 year	.3702	.3785	.4333

Table 5: Eigenvectors

as well as the future changes in the spot interest rate. To obtain the term structure, people does take into account the current drift of the inflation and what future monetary policy actions it implies.

Future progress on this research may show more reliable results and how good is the proposed model to fit the empirical data.

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Covariance	$E_t\left[\pi_{t+1}\right]$
$E_t\left[r_{t+1,t}\right]$	-0.0011
$E_{t+1}\left[r_{t+2,t+1}\right]$	-0.0009
$E_{t+2}\left[r_{t+3,t+2}\right]$	-0.0009
$E_{t+3}\left[r_{t+4,t+3}\right]$	-0.0008

Table 6: Covariances

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