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Abstract

This paper investigates conditions for equilibrium determinacy under inflation-targeting interest-rate rules when the basic new Keynesian model is extended to incorporate real wage rigidity. I show that the introduction of real wage rigidity increases the determinacy region under forward-looking rules. Moreover, the Taylor principle continues to ensure equilibrium uniqueness under current-looking rules.

Keywords: real wage rigidity, determinacy, inflation targeting

JEL Classification: E43, E52, E58

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1 Introduction

Equilibrium determinacy is an important issue for monetary policy design. Monetary policy rules can be destabilizing, leading to multiple equilibria in which non-fundamental shocks can influence aggregate dynamics. To avoid this outcome, the Taylor principle, which prescribes that the interest rate should increase by more than the increase in inflation, is considered an important feature of a sound monetary policy. Lubik and Marzo (2007) provide additional determinacy conditions for the basic new Keynesian dynamic model under different monetary policy rules.

The basic model has been extended to incorporate additional features, but real rigidity was not systematically studied until recently. Blanchard and Galí (2007) consider a model with an admittedly ad hoc form of real imperfection in which real wages respond sluggishly to economic conditions. They show that real wage rigidity generates a natural trade-off between stabilizing inflation and the output gap, and thus having important implications for inflation persistence and the propagation of shocks.

This paper examines determinacy conditions in the Blanchard and Galí (2007) extension of a basic new Keynesian model. The analysis focus on inflation-targeting interest-rate rules, without a feedback on the output gap. I show that the Taylor principle is a necessary condition for determinacy. If the central bank responds to current inflation, it is also a sufficient condition. The presence of real wage rigidity imposes an upper bound for the reactiveness of the interest rate under forward-looking rules. This upper bound is an increasing function of the index of real wage rigidity, allowing the central bank to react more strongly to expected inflation as real wage rigidity becomes more important. A lower bound is still given by the Taylor principle.

2 The Model

The model features nominal price rigidity and a partial adjustment mechanism for real wages.

Households with a time-separable utility and a discount factor β maximize their expected lifetime utility given a sequence of budget constraints. The period utility is given by:

$$U(C_t, N_t) = \frac{C_t^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\varphi}}{1+\varphi},$$

where C_t and N_t are consumption and employment, respectively.

After imposing market clearing conditions, the log-linear form of the Euler equation is:

$$y_t = E_t(y_{t+1}) - \frac{1}{\sigma}[i_t - E_t(\pi_{t+1})]. \tag{1}$$

Firms in a monopolistic competitive environment produce differentiated goods with a linear technology using only labor. Price decisions are described by the Calvo mechanism, where θ is the fraction of firms not adjusting their price in a given period. In the neighborhood of a zero-inflation steady state, inflation dynamics is described by the new Keynesian Phillips curve:

$$\pi_t = E_t(\pi_{t+1}) + kw_t, \tag{2}$$

where $k = \frac{(1-\beta\theta)(1-\theta)}{\theta}$.

The variables y_t, i_t, π_t and w_t are output, the nominal interest rate, inflation and real wages, respectively, all expressed in log-deviations from their steady states. Note that, owing to the technology used, real marginal costs are equivalent to real wages in log-linear form.

A partial adjustment mechanism for real wages is incorporated to introduce real wage rigidity:

$$\frac{W_t}{P_t} = \left(\frac{W_{t-1}}{P_{t-1}}\right)^{\gamma} (MRS_t)^{1-\gamma},$$

where W_t and P_t are the nominal wage and price. MRS_t is the marginal rate of substitution between consumption and employment. The parameter γ is the index of real wage rigidity, summarizing the effects of friction in the labor market that are not explicitly modeled.

In log-linear form, after imposing market clearing conditions, real wage dynamics is given by:

$$w_t = \gamma w_{t-1} + (1-\gamma)(\sigma + \varphi) y_t. \tag{3}$$

The parameters k, σ and φ are strictly positive, with $0 < \beta < 1$ and $0 < \gamma < 1$.

Equations (1), (2) and (3) constitute a log-linear approximation of the model. To close the system, a monetary policy rule is needed.

3 Inflation targeting and determinacy

In the analysis of equilibrium determinacy, I focus on pure inflation-targeting rules for two basic reasons. First, it is important to study such rules since they are simple and transparent, acting as a benchmark for more sophisticated interest-rate rules. Second, there are problems associated with the choice of an appropriate measure for the output gap in Taylor rules. Some of these problems are discussed in Orphanides(2003).

It is assumed that the central bank follows two types of interest-rate rules:

$$i_t = \alpha_\pi \pi_t, \tag{4}$$

$$i_t = \alpha_\pi E_t \pi_{t+1}, \tag{5}$$

where α_π is strictly positive.

Equations (4) and (5) describe a current-looking and a forward-looking rule, respectively,.

Equilibrium determinacy conditions are summarized in the following propositions.

Proposition 1 *Under current-looking rules, the necessary and sufficient condition for a rational-expectations equilibrium to be determinate is that:*

$$\alpha_\pi > 1. \tag{6}$$

Proof. First, I eliminate the interest rate using (4) and rewrite the log-linear approximation of the model in matrix form:

$$E_t(x_{t+1}) = Ax_t,$$

where

$$x_t = \begin{bmatrix} \pi_t \\ y_t \\ w_{t-1} \end{bmatrix},$$

$$A = \begin{bmatrix} \frac{1}{\beta} & -(1-\gamma)(\sigma+\varphi)\left(\frac{k}{\beta}\right) & -\frac{\gamma k}{\beta} \\ -\frac{1}{\sigma\beta} + \frac{\alpha_\pi}{\sigma} & 1 + (1-\gamma)(\sigma+\varphi)\left(\frac{k}{\sigma\beta}\right) & \frac{\gamma k}{\sigma\beta} \\ 0 & (1-\gamma)(\sigma+\varphi) & \gamma \end{bmatrix}.$$

The characteristic equation of A is:

$$P(\lambda) = \lambda^3 + A_2\lambda^2 + A_1\lambda + A_0 = 0.$$

Determinacy requires that one root of the equation is inside the unit circle and two roots are outside. Following Woodford (2003), this occurs if and only if:

either (case I)

$$1 + A_2 + A_1 + A_0 < 0$$

and

$$-1 + A_2 - A_1 + A_0 > 0,$$

or (case II)

$$1 + A_2 + A_1 + A_0 > 0,$$

$$-1 + A_2 - A_1 + A_0 < 0,$$

and

$$A_0^2 - A_0 A_2 + A_1 - 1 > 0,$$

or (case III)

$$1 + A_2 + A_1 + A_0 > 0,$$

$$-1 + A_2 - A_1 + A_0 < 0,$$

$$A_0^2 - A_0 A_2 + A_1 - 1 < 0$$

and

$$|A_2| < 3.$$

The coefficients of the characteristic equation are:

$$A_2 = -\left(\frac{1}{\beta} + 1 + \frac{k}{\sigma\beta}(1-\gamma)(\sigma+\varphi) + \gamma\right),$$

$$A_1 = \gamma + \frac{\gamma}{\beta} + \frac{1}{\beta} + \frac{k}{\sigma\beta}(1-\gamma)(\sigma+\varphi)\alpha_\pi,$$

$$A_0 = -\frac{\gamma}{\beta}.$$

Since $-1 + A_2 - A_1 + A_0 < 0$, case I can be ruled out. In cases II and III, $1 + A_2 + A_1 + A_0 > 0$. This condition can be reduced to (6), which is a necessary condition for equilibrium determinacy.

The additional conditions for determinacy are $A_0^2 - A_0 A_2 + A_1 - 1 > 0$ or $|A_3| < 3$. These lead to the following expressions:

$$\left(\frac{\gamma}{\beta}-1\right)\left(\frac{\gamma}{\beta}-\gamma-\frac{1}{\beta}+1\right)+\frac{k}{\sigma\beta}(1-\gamma)(\sigma+\varphi)\left(\alpha_{\pi}-\frac{\gamma}{\beta}\right)>0, \qquad (7)$$

$$\left(\frac{1}{\beta}+\beta-2\right)+\frac{k}{\sigma\beta}(1-\gamma)(\sigma+\varphi)+(\gamma-\beta)>0. \qquad (8)$$

Equilibrium is determinate if and only if the parameters of the model satisfy (6) and either (7) or (8). I show that (6) is both necessary and sufficient for determinacy. This claim is proved by showing that any set of parameter values satisfying (6) but not (8) must necessarily satisfy (7).

Since $\frac{1}{\beta}+\beta-2>0$ and the second term is strictly positive, (8) will not hold only if $\beta>\gamma$.

If $\beta>\gamma$, then $\frac{\gamma}{\beta}-1<0$ and $\frac{\gamma}{\beta}-\gamma-\frac{1}{\beta}+1<0$ imply:

$$\left(\frac{\gamma}{\beta}-1\right)\left(\frac{\gamma}{\beta}-\gamma-\frac{1}{\beta}+1\right)>0.$$

Therefore, condition (7) will hold if $\alpha_{\pi}-\frac{\gamma}{\beta}>0$. This is the case since, $\alpha_{\pi}>1$ and $\frac{\gamma}{\beta}<1$ imply that $\alpha_{\pi}>1>\frac{\gamma}{\beta}$. This last step completes the proof and (6) is a necessary and sufficient condition for determinacy. ■

Proposition 2 *Under forward-looking rules, the necessary and sufficient condition for a rational-expectations equilibrium to be determinate is that:*

$$1 < \alpha_\pi < 1 + \frac{2(1+\beta)}{k(\sigma+\varphi)} \left(\frac{1+\gamma}{1-\gamma} \right). \tag{9}$$

Proof. Under forward-looking rules, A becomes:

$$A = \begin{bmatrix} \frac{1}{\beta} & -(1-\gamma)(\sigma+\varphi)\left(\frac{k}{\beta}\right) & -\frac{\gamma k}{\beta} \\ -\frac{(1-\alpha_\pi)}{\sigma\beta} & 1+(1-\gamma)(1-\alpha_\pi)(\sigma+\varphi)\left(\frac{k}{\sigma\beta}\right) & \frac{\gamma(1-\alpha_\pi)k}{\sigma\beta} \\ 0 & (1-\gamma)(\sigma+\varphi) & \gamma \end{bmatrix}.$$

The coefficients of the characteristic equation are:

$$A_2 = -\left(\gamma + \frac{1}{\beta} + 1 + \frac{k}{\sigma\beta}(1-\gamma)(1-\alpha_\pi)(\sigma+\varphi) \right),$$

$$A_1 = \gamma + \frac{\gamma}{\beta} + \frac{1}{\beta},$$

$$A_0 = -\frac{\gamma}{\beta}.$$

Again, case I can be ruled out, since conditions $1 + A_2 + A_1 + A_0 < 0$ and $-1 + A_2 - A_1 + A_0 > 0$ lead to the following contradiction:

$$\frac{2(1+\gamma)(1+\beta)}{\beta} < (1-\gamma)(\alpha_\pi - 1)(\sigma + \varphi)\left(\frac{k}{\sigma\beta}\right) < 0,$$

but $\frac{2(1+\gamma)(1+\beta)}{\beta} > 0$.

In cases II and III, $1 + A_2 + A_1 + A_0 > 0$. This condition can be reduced to (9), which is a necessary condition for equilibrium determinacy.

The additional conditions for determinacy are $A_0^2 - A_0 A_2 + A_1 - 1 > 0$ or $|A_2| < 3$. These lead to the following expressions:

$$\left(\frac{\gamma}{\beta} - 1\right)\left(\frac{\gamma}{\beta} - \gamma - \frac{1}{\beta} + 1\right) + \frac{\gamma}{\beta}(1-\gamma)(\alpha_\pi - 1)(\sigma + \varphi)\left(\frac{k}{\sigma\beta}\right) > 0, \quad (10)$$

$$|(1-\gamma)(\alpha_\pi - 1)(\sigma + \varphi)(\frac{k}{\sigma\beta}) - \left(\gamma + \frac{1}{\beta} + 1\right)| < 3. \quad (11)$$

Equilibrium is determinate if and only if the parameters of the model satisfy (9) and either (10) or (11). I show that (9) is both necessary and sufficient for determinacy. This claim is proved by showing that any set of parameter values satisfying (9) but not (11) must necessarily satisfy (10).

Suppose (11) does not hold; then

$$\left(\gamma + \frac{1}{\beta} + 1\right) - 3 < (1-\gamma)(\alpha_\pi - 1)(\sigma + \varphi)(\frac{k}{\sigma\beta}) < \left(\gamma + \frac{1}{\beta} + 1\right) + 3.$$

If $\left(\gamma + \frac{1}{\beta} + 1\right) - 3 < 0$, then $\frac{1}{\beta} + \beta - 2 + \gamma - \beta < 0$. This inequality holds only if $\beta > \gamma$, given that $\frac{1}{\beta} + \beta - 2 > 0$.

If $\beta > \gamma$, $\left(\frac{\gamma}{\beta} - 1\right)\left(\frac{\gamma}{\beta} - \gamma - \frac{1}{\beta} + 1\right) = -\left(\frac{\gamma}{\beta} - 1\right)(1-\gamma)(\frac{1}{\beta} - 1) > 0$.

Since (9) is satisfied,

$$0 < (1-\gamma)(\alpha_\pi - 1)(\sigma + \varphi)(\frac{k}{\sigma\beta}) < \frac{2(1+\gamma)(1+\beta)}{\beta}$$

implies $\frac{\gamma}{\beta}(1-\gamma)(\alpha_\pi - 1)(\sigma + \varphi)(\frac{k}{\sigma\beta}) > 0$. Therefore, (10) is necessarily satisfied.

If $\left(\gamma + \frac{1}{\beta} + 1\right) - 3 > 0$, then $(1-\gamma)(\alpha_\pi - 1)(\sigma + \varphi)(\frac{k}{\sigma\beta}) > \left(\gamma + \frac{1}{\beta} + 1\right) - 3$. Using this last inequality, condition (10) becomes:

$$\begin{aligned} & \left(\frac{\gamma}{\beta} - 1\right)\left(\frac{\gamma}{\beta} - \gamma - \frac{1}{\beta} + 1\right) + \frac{\gamma}{\beta}(1-\gamma)(\alpha_\pi - 1)(\sigma + \varphi)\left(\frac{k}{\sigma\beta}\right) \\ > \; & \left(\frac{\gamma}{\beta} - 1\right)\left(\frac{\gamma}{\beta} - \gamma - \frac{1}{\beta} + 1\right) + \frac{\gamma}{\beta}\left(\gamma + \frac{1}{\beta} + 1 - 3\right). \end{aligned}$$

After some algebraic manipulations, the last term in the expression can be reduced to:

$$\left(\frac{\gamma}{\beta}-1\right)\left(\frac{\gamma}{\beta}-\gamma-\frac{1}{\beta}+1\right)+\frac{\gamma}{\beta}\left(\gamma+\frac{1}{\beta}+1-3\right)=\left(\frac{\gamma}{\beta}-1\right)^{2}+\left(\gamma+\frac{1}{\beta}\right)^{2}>0.$$

The last inequality shows that condition (10) holds if $\left(\gamma+\frac{1}{\beta}+1\right)-3>0$. This last step completes the proof. ■

According to Propositions 1 and 2, monetary policy should be active when real wage rigidity is present, following the Taylor principle. Under forward-looking rules, a monetary policy should not be excessively active. This result is also true for the basic new Keynesian model in which $\gamma=0$. Since $\frac{1+\gamma}{1-\gamma}>1$ and increases in γ, the central bank can be more aggressive as real wage rigidity becomes more important, without destabilizing aggregate dynamics. If $\gamma\to 1$, the Taylor principle, restriction (3), is restored as a necessary and sufficient condition for determinacy.

In the basic new Keynesian model, an excessive response to inflation expectations can lead to a deep recession and cause deflation, destabilizing the economy. In a model with real wage rigidity, π_t is less sensitive to y_t and a much stronger recession is needed to induce deflation. Therefore, the determinacy region is increased and monetary authorities are less restricted in taking an active approach.

4 Conclusion

In a new Keynesian model with real wage rigidity, I show that an active monetary policy is a necessary condition for equilibrium determinacy under current-looking and forward-looking inflation-targeting rules. Therefore, the Taylor principle continues to be an important recommendation for the design of simple monetary policy rules after the introduction of real imperfections. Furthermore, the determinacy region is increased under forward-looking rules, allowing the central bank to respond more strongly to inflation expectations as the importance of real wage rigidity increases.

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