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A Direct Proof of the First Welfare Theorem

Alexandre B. Cunha

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Does It Really Matter Whether the Exchange Rate Floats or Not?*

Alexandre B. Cunha†
IBMEC Business School – Rio de Janeiro

Abstract: We show in this paper that the composition of government liabilities between domestic and foreign bonds is undetermined in a competitive equilibrium. Hence, the choice of exchange rate regime is irrelevant, in the sense that a floating exchange regime is consistent with any competitive equilibrium. This finding suggests that the theoretical study of the properties of exchange rate regimes should depart from the competitive framework. Thus, we study the optimal monetary policy without commitment in a small open economy and adopt the Markov sustainable equilibrium concept. In such a context, the aforementioned indeterminacy does not arise.

Keywords: exchange rate regime, indeterminacy, time consistency, Markov sustainable equilibrium.
JEL classification: E42, E58, F31, F41.

1 Introduction

The implications of adopting an exchange rate regime constitute a long standing debate in economics. Many theoretical and empirical papers have discussed the consequences of adopting a specific regime.

Friedman [15] strongly advocated the adoption of a floating regime. Many years later, economists still debate the advantages and disadvantages of exchange rate regimes. Relatively recent papers with a theoretical flavor on the subject are Calvo [8], Edwards and Levy-Yeyati [13], Fischer [14], Obstfeld and Rogoff [25], and Sopraseuth [29].

The empirical regularities associated with each type of exchange rate regime are also the focus of a large body of literature. Baxter and Stockman [7], Husain, Moody and Rogoff [16] and Levy-Yeyati and Sturzenegger [20] are good examples of this line of investigation.

*The author acknowledges financial support from the Brazilian Council of Science and Technology (CNPq).

†E-mail: abcunha@ibmecrj.br. Web site: http://professores.ibmecrj.br/abcunha.

There are also many papers focused on classifying a country's exchange rate policy. Calvo and Reinhart [9], Levy-Yeyati and Sturzenegger [21] and Reinhart and Rogoff [27] are typical examples of this literature.

In this paper we study whether adopting a floating exchange rate regime has clear-cut implications for equilibrium prices and allocations. We adopt a small open-economy deterministic version of the cash-credit model of Lucas and Stokey [22]. There exists a single consumption good. Consumers face a cash-in-advance constraint on a fraction of their purchases of that good. Labor is the only input. There is free financial capital mobility. Government consumption and tax rates on labor income are exogenous.

Contrary to the conventional wisdom on exchange rate regimes, we show that competitive equilibrium prices and allocations are consistent with any path for the government's foreign assets. In this sense, it is not relevant whether the exchange rate floats or not.

A simple way to provide intuition for the result mentioned in the last paragraph is to relate our finding to Modigliani and Miller [23]. These authors showed that in a competitive environment, it is irrelevant whether a firm finances its investment projects with equity or debt. Similarly, it is irrelevant whether a government meets its financing needs by issuing domestic bonds or reducing its foreign assets. In other words, a competitive equilibrium pins down only the total government debt and not its composition.

Several macroeconomic papers have derived some type of indeterminacy. Barro [3] is a well-known one. Recently, Bassetto and Kocherlakota [6] managed to extend Barro's finding to a model with distorting taxes. Wallace [30] obtained a Modigliani-Miller result for open-market operations. The indeterminacy result we present is in line of the findings of these authors.

The indeterminacy of a government's debt composition in a competitive equilibrium suggests that the theoretical study of exchange rate regimes requires a departure from the competitive framework. Therefore, we study the problem of selecting an optimal monetary policy in the same small open-economy model in which we derived that indeterminacy. Following Chari and Kehoe [10], we assume that the government cannot commit to a sequence of policies and adopt the sustainable equilibrium concept.

As Alvarez, Kehoe and Neumeyer [2], Lucas and Stokey [22] and Persson, Persson and Svensson [26] pointed out, the composition and the maturity of government debt matters for the time consistency of monetary policy. We build on these authors' works to show that the best sustainable outcomes are not invariant to the path of government foreign assets.[1] That is, the indeterminacy present in a competitive equilibrium does not show up when a government selects policies in a sequential way.

This paper is organized as follows. Section 2 describes the model. Section 3 studies some features of the competitive equilibrium. Section 4 discusses the properties of optimal policies without commitment. Section 5 presents our

[1] As Albanesi, Chari and Christiano [1] pointed out, a game without commitment may have many equilibria. Therefore, we focus on a best outcome.

concluding remarks. The Appendix contains the proof of a proposition.

2 The economy

Consider a small country populated by a continuum of identical infinitely lived households with Lebesgue measure one and a government. A household is composed of a shopper and a worker, who is endowed with one unit of time.

This country produces a single good. This good is consumed by households (c) and government (g). It can also be exported (x) or imported ($-x$).

Transactions take place in this economy in a particular way. At a first stage of each date t, spot markets for goods and labor services operate. At a second stage, security and currency markets operate.[2]

A domestic currency M circulates in this economy. Two types of securities are traded: a claim B, with maturity of one period, to one unit of M and a claim A, with the same maturity, to one unit of some foreign currency. Foreigners do not sell or buy claims to the domestic currency. Government and residents can purchase and/or sell the claims A^* at a price, in terms of the foreign currency, q_t^*.

Workers cannot sell their services outside the country. Shoppers face a cash-in-advance constraint. A fraction of their purchases of the consumption good must be paid for with the domestic currency. Except for these cash purchases, all other transactions are liquidated during the securities and currency trading session. The date t price, in terms of the foreign currency, of the tradable good is constant and equal to 1.

Technology is described by $0 \leq y \leq l$, where y is the output of the sole good and l is the amount of labor allocated to its production. The good is produced by a single competitive firm. As usual, the index t denotes time. Feasibility requires

$$c_{1t} + c_{2t} + g_t + x_t = l_t \ , \tag{1}$$

where c_{1t} denotes people's purchase of the consumption good that is paid cash and c_{2t} denotes their remaining purchase of the consumption good.

The government finances the sequence $\{g_t\}_{t=0}^{\infty}$ by issuing and withdrawing domestic currency; by issuing and redeeming claims B of maturity of one period to one unit of the domestic currency; by purchasing and selling B^*; and by taxing labor income at a proportional tax τ.

The sequence $\{g_t, \tau_t, q_t^*\}_{t=0}^{\infty}$ is exogenous. For each t, the vector (g_t, τ_t, q_t^*) belongs to a finite set contained in $[0,1)^2 \times (0,1)$.

The government budget constraint is

$$S_t g_t + M_t + B_t + S_t q_t^* A_{G,t+1} = \tau_t w_t l_t + M_{t+1} + q_t B_{t+1} + S_t A_{G,t} \ , \tag{2}$$

[2] We adopted the Svensson [28] timing. In this context, unexpected inflation does not act as a pure lump sum tax. Therefore, the problem of selecting an optimal policy will have a well defined solution even if the government has some outstanding debt at date zero. See Nicolini [24], especially section 3, for further details.

where w_t and q_t are the respective date t monetary prices (in terms of the domestic currency) of labor services and the domestic claim; S_t is the nominal exchange rate; $A_{G,t+1}$ stands for the foreign assets held by the government at the end of date t; and M_{t+1} and B_{t+1} are the amounts of domestic currency and public debt held by the households at the end of date t. A negative value for $A_{G,t+1}$ means that the government is borrowing abroad, while a negative value for B_{t+1} means that the government is lending to domestic residents. At $t = 0$ the government holds an initial amount $\bar{A}_G$ of foreign assets.

Let $A_{H,t+1}$stand for the foreign assets held by the household at the end of date t. To avoid Ponzi schemes, we impose the borrowing constraints

$$\left|\frac{B_{t+1}}{S_{t+1}}\right|, |A_{H,t+1}|, |A_{G,t+1}| \leq K < \infty \tag{3}$$

on asset holdings. As usual, K is some real number large enough so that these constraints never bind in a competitive equilibrium.

The function $u : \mathbb{R}^2_+ \times [0,1] \to \mathbb{R} \cup \{-\infty\}$, $u = u(c_1, c_2, 1-l)$ is the typical household period utility function. This function displays local non-satiability and satisfies standard differentiability and Inada conditions. Intertemporal preferences are described by

$$\sum_{t=0}^{\infty} \beta^t u(c_{1t}, c_{2t}, 1-l_t) \ , \tag{4}$$

where $\beta \in (0,1)$. The date t budget constraint of the typical household is

$$S_t(c_{1t}+c_{2t})+M_{t+1}+q_t B_{t+1}+S_t q_t^* A_{H,t+1} \leq (1-\tau_t)w_t l_t+M_t+B_t+S_t A_{H,t} \ , \tag{5}$$

where $A_{H,t+1}$stands for the foreign assets held by the household at the end of date t. People face the cash-in-advance constraint

$$S_t c_{1t} \leq M_t \ . \tag{6}$$

At date zero, given initial asset holdings $(\bar{M}, \bar{B}, \bar{A}_H)$, a household chooses a sequence $\{c_{1t}, c_{2t}, l_t, M_{t+1}, B_{t+1}, A_{H,t+1}\}_{t=0}^{\infty}$ to maximize (4) subject to the constraints (5), (6), (3) and $l_t \leq 1$. Except for B_{t+1} and $A_{H,t+1}$, all these variables must be non-negative. Additionally, the sequences $\{c_{1t}\}_{t=0}^{\infty}$, $\{c_{2t}\}_{t=0}^{\infty}$ and $\{M_{t+1}/S_{t+1}\}_{t=0}^{\infty}$ have to be bounded. At each period t, the firm chooses l_t to maximize $S_t l_t - w_t l_t$.

3 Competitive equilibrium

We start this section by establishing some notation. We denote a date t price vector (S_t, w_t, q_t) by ψ_t and a date t bundle (c_{1t}, c_{2t}, l_t) by χ_t, while φ_{t+1} stands for people's end of period t asset holding $(M_{t+1}, B_{t+1}, A_{Ht+1})$. Additionally, $(\psi, \chi, \varphi) = \{\psi_t, \chi_t, \varphi_{t+1}\}_{t=0}^{\infty}$.

Definition 1 *A* competitive equilibrium *is an object* (ψ, χ, φ) *plus sequences* $\{A_{G,t+1}\}_{t=0}^{\infty}$ *and* $\{x_t\}_{t=0}^{\infty}$ *that satisfies: (i) given* ψ*,* (χ, φ) *provides a solution to the typical household problem; (ii)* $w_t = S_t$*; (iii) (1) and (2) hold. Sequences* ψ*,* χ*,* φ*,* $\{A_{G,t+1}\}_{t=0}^{\infty}$ *and* $\{x_t\}_{t=0}^{\infty}$ *are* attainable *if they are part of some competitive equilibrium.*

A balance-of-payment condition was not spelled out in definition 1. It is not necessary to do so. Observe that adding the zero-profit condition $w_t l_t = S_t(c_{1t} + c_{2t} + g_t + x_t)$ to (2) and (5) taken as equality, one obtains

$$x_t + A_{G,t} + A_{H,t} - q_t^*(A_{G,t+1} + A_{H,t+1}) = 0 \ , \tag{7}$$

which is the balance-of-payments identity of this model economy.

As usual in small open-economy models, a competitive equilibrium must satisfy a condition that rules out arbitrage between domestic and foreign assets. Namely, the nominal exchange rate and domestic and foreign bond prices must satisfy the parity condition

$$S_t q_t^* = S_{t+1} q_t \ . \tag{8}$$

We are now in a position to establish that the composition of the government debt between domestic and foreign bonds is irrelevant in a competitive equilibrium.

Proposition 2 *Let* $(\psi, \chi, \varphi, \{A_{G,t+1}\}_{t=0}^{\infty}, \{x_t\}_{t=0}^{\infty})$ *be a competitive equilibrium. If the sequence* $\{A'_{G,t+1}\}_{t=0}^{\infty}$ *is bounded, then there exists a portfolio* φ' *such that* $(\psi, \chi, \varphi', \{A'_{G,t+1}\}_{t=0}^{\infty}, \{x_t\}_{t=0}^{\infty})$ *is a competitive equilibrium.*

Proof. We start by constructing the sequence φ'. For each t, set $M'_{t+1} = M_{t+1}$ and $A'_{H,t+1} = A_{H,t+1} + A_{G,t+1} - A'_{G,t+1}$. Recall that the initial assets are still equal to $(\bar{M}, \bar{B}, \bar{A}_H, \bar{A}_G)$. This allows us to construct $\{B'_{t+1}\}_{t=0}^{\infty}$ in a recursive fashion. Given B'_t, define B'_{t+1} so that (5) holds with equality.

We will now show that $(\psi, \chi, \varphi', \{A'_{G,t+1}\}_{t=0}^{\infty}, \{x_t\}_{t=0}^{\infty})$ is a competitive equilibrium. The first step in this process consists of showing that φ' satisfies the borrowing bounds in (3). The sequence $\{A'_{G,t+1}\}_{t=0}^{\infty}$ respects that constraint by assumption. The boundedness of $\{A'_{H,t+1}\}_{t=0}^{\infty}$ follows from the inequality $|A'_{H,t+1}| \leq |A_{H,t+1}| + |A_{G,t+1}| + |A'_{G,t+1}|$.

It takes a little longer to prove that $\{B'_{t+1}/S_{t+1}\}_{t=0}^{\infty}$ is bounded. Recall that $B'_0 + S_0 A'_{H,0} = \bar{B} + S_0 \bar{A}_H = B_0 + S_0 A_{H,0}$. Moreover, both φ and φ' satisfy (5) with equality. Then, $q_0 B'_1 + S_0 q_0^* A'_{H,1} = q_0 B_1 + S_0 q_0^* A_{H,1}$. Combine this equality with (8) to conclude that $B'_1 + S_1 A'_{H,1} = B_1 + S_1 A_{H,1}$. Now, assume that

$$B'_{t+1} + S_{t+1} A'_{H,t+1} = B_{t+1} + S_{t+1} A_{H,t+1} \tag{9}$$

holds for a generic date t. As before, we use (5) and (8) to obtain (9) forwarded by one period. Thus, induction establishes that (9) holds for for all t. Hence, $|B'_{t+1}/S_{t+1}| \leq |B_{t+1}/S_{t+1}| + |A_{H,t+1}| + |A'_{H,t+1}|$ and $\{B'_{t+1}/S_{t+1}\}_{t=0}^{\infty}$ is bounded.

We are now able to conclude the proof. Concerning item (i) of definition 1, the pair (χ, φ') yields the same lifetime utility as (χ, φ). So, the former is an optimal choice for the household when the prevailing price system is ψ. Clearly, ψ satisfies (ii). With respect to item (iii), χ and $\{x_t\}_{t=0}^{\infty}$ obviously satisfy (1). Moreover, we constructed φ' so that $A'_{H,t+1} + A'_{G,t+1} = A_{H,t+1} + A_{G,t+1}$. Thus, $\{x_t\}_{t=0}^{\infty}$ and φ' satisfy (7). We combine that condition with (1) and (5) with equality to conclude that (2) holds. □

We now turn to the task of providing some intuition to the above result. A possible way to interpret Proposition 2 consists of seeing it as equivalent to the Modigliani-Miller theorem of corporate finance. In a perfectly competitive environment with full information, whether a firm finances its investment projects with equity or debt is irrelevant. In a similar fashion, it does not matter whether the government finances its temporary deficits by issuing domestic or foreign bonds.

There is a second way to view Proposition 2. Lucas and Stokey [22] studied optimal fiscal policies in a one-sector closed economy. They allowed the government to issue debt of all maturities. They showed that any competitive equilibrium pins down only the present value of the public debt, but not its composition. Chari and Kehoe [11] reached the same conclusion. Proposition 2 shows that only the total value of the public debt matters. Its composition between domestic and foreign bonds is irrelevant.

Arbitrage opportunities are ruled out in a competitive equilibrium. This fact provides an alternative interpretation to Proposition 2. (8) ensures that people are indifferent between domestic and foreign bonds. This allows the government to change the composition of its debt $B_t - S_t A_{Gt}$ without affecting its value. For instance, the government can sell abroad Δ units of foreign currency denominated bonds. Simultaneously, people sell to the government $S_t q_t^* \Delta / q_t$ units of domestic debt and use the proceedings to buy exactly Δ units of foreign bonds. This type of financial operation does not change the wealth of either the government or people or the external sector. Therefore, the composition of government debt is undetermined and many sequences $\{A_{G,t+1}\}_{t=0}^{\infty}$ can decentralize competitive equilibrium prices and allocations.

We want to emphasize that our results have a nature distinct from that of Kareken and Wallace [17]. These authors showed that in a two-country model, if the two national currencies are perfect substitutes, then the exchange rate path is undetermined. Clearly, the indeterminacy we found in Proposition 2 is of a different type.

Proposition 2 is a consequence of the absence of arbitrage opportunities in a competitive equilibrium. Therefore, similar results can arise in many distinct models. This finding would survive even if we assumed less-than-perfect capital mobility or that people and the government faced different interest rates in the international market. For the case of stochastic economies, we could also obtain a similar result, provided that there is a sufficiently rich set of contingent assets.

Usually the exchange rate is said to float if the government does not intervene in the foreign exchange market. That is, the government carries a constant amount of foreign assets. The next definition follows this tradition.

Definition 3 *The exchange rate* floats at date t *in a competitive equilibrium if* $A_{Gt+1} = A_{Gt}$.

As Krugman [18] pointed out, if the government will intervene in the foreign exchange market sometime in the future, that prospective intervention may affect the price of the exchange rate today. Hence, it may be convenient to distinguish permanent from temporary floating.

Definition 4 *The exchange rate* permanently floats *in a competitive equilibrium if* $A_{Gt+1} = \bar{A}_G$ *for all* t.

The last definition requires the government never to intervene in the foreign exchange market. This requirement is clearly stronger than the one stated in Definition 3.

We are now in a position to apply Proposition 2 to establish a result concerning floating exchange-rate regimes.

Corollary 5 *Let* $(\psi, \chi, \varphi, \{A_{G,t+1}\}_{t=0}^{\infty}, \{x_t\}_{t=0}^{\infty})$ *be a competitive equilibrium. Then, there exist sequences* $\{A'_{G,t+1}\}_{t=0}^{\infty}$, φ', $\{A''_{G,t+1}\}_{t=0}^{\infty}$ *and* φ'' *that satisfy: (1)* $(\psi, \chi, \varphi', \{A'_{G,t+1}\}_{t=0}^{\infty}, \{x_t\}_{t=0}^{\infty})$ *is a competitive equilibrium in which the exchange rate permanently floats, and (2)* $(\psi, \chi, \varphi'', \{A''_{G,t+1}\}_{t=0}^{\infty}, \{x_t\}_{t=0}^{\infty})$ *is a competitive equilibrium in which the exchange rate does not float at any date* t.

Proof. We start with the first statement. Define $A'_{G,t+1} = \bar{A}_G$. Apply Proposition 2 to conclude that $(\psi, \chi, \varphi', \{A'_{G,t+1}\}_{t=0}^{\infty}, \{x_t\}_{t=0}^{\infty})$ is a competitive equilibrium for some φ'. Trivially, the exchange rate permanently floats in that competitive equilibrium. For the second statement, let δ be any real number. Then, set $A''_{G,t+1} = \bar{A}_G + (-1)^{t+1}\delta$ and use the previous reasoning to finish the proof. □

As we have already discussed, the key point underlying Proposition 2 and Corollary 5 is the fact that in a competitive equilibrium the composition of the government debt is irrelevant. However, as Lucas and Stokey [22], Chari and Kehoe [11], Alvarez, Kehoe and Neumeyer [2] and Persson, Persson and Svensson [26] pointed out, the same is not true when it comes to the time consistency of macroeconomic policies. We will further explore this point in the next section.

4 Optimal policies

In the previous section we established that the path of foreign assets and external debt are not uniquely determined in a competitive equilibrium. Two features of the competitive environment are crucial for this result. First, the government is not an active player in a Walrasian setting. This prevents this agent from reacting to people's portfolio selection. Second, as Bassetto [4] pointed out, the competitive equilibrium concept is mute when it comes to out-of-equilibrium actions.

So that we can consider out-of-equilibrium actions and assess the impact of household actions on the government decisions, we consider a game built on the structure we considered in Section 3. This game will be similar to the ones without commitment in Chari and Kehoe [10] and [11]. Just as those authors did, we assume that the government cannot commit to a sequence of policies. Instead, policies are selected on a period-by-period basis.[3]

We now start describing the game we consider. Given the purposes of this paper, the behavior of the fiscal variables is not relevant. Therefore, we will retain the assumption that $\{g_t\}_{t=0}^{\infty}$ and $\{\tau_t\}_{t=0}^{\infty}$ are exogenous.

Following Bassetto [4], we require the government to have a balanced budget at all possible nodes of the game. However, as Bassetto [5] showed, it is impossible to meet this requirement if government expenditures are exogenous. We take this problem into consideration while setting the game up.

At the beginning of every period t, before markets open, the government chooses a policy. Then markets open and private agents and government trade. Note that markets work as described in Section 3. That is, markets for goods and labor services operate and close before securities and currency markets open.

A monopolistic firm cannot select a combination of price and quantity outside of the demand curve. Similarly, the government cannot freely select exchange rate, interest rate, foreign assets, public debt and nominal balances. We assume that the government selects S_t, q_t and w_t.[4] Accordingly, we call the vector $\psi_t = (S_t, w_t, q_t)$ of a date t policy.

At each date t, the government is the first player to act. Given the vector $(\varphi_t, A_{G,t})$, the government selects a date t policy ψ_t and a contingency plan for future choices. Denote this former choice by $\sigma_t(\varphi_t, A_{G,t})$. As in Chari and Kehoe [10] and [11], we say that the sequence $\sigma = \{\sigma_t\}_{t=0}^{\infty}$ is *policy plan* and the sequence $\sigma^t = \{\sigma_k\}_{k=t}^{\infty}$ is the *continuation* of σ from date t onwards. The government is benevolent. Hence, its payoff is

$$\sum_{k=t}^{\infty} \beta^{k-t} u(c_{1k}, c_{2k}, 1 - l_k) \ . \tag{10}$$

So far we have not required the government's actions to respect either the feasibility (1) or the balance of payment (7). However, one of them must constrain the government's choices. Otherwise, the government could simply pick a policy that would induce households to consume an infinite amount. Recall that with (5) holding with equality, (2), (1) and (7) are linearly dependent. So,

[3] We do not consider in this paper the case in which the government can commit to a policy. We do so because it is a straightforward exercise to show that results equivalent to Proposition 2 and Corollary 5 hold in such a context.

[4] By selecting which variables the government can choose, we are defining a strategy set for that player and consequently selecting a particular game to study. The resulting outcomes are not invariant to such a choice. However, for the purposes of this paper it is enough to consider a game in which the neutrality results obtained in Section 3 do not hold.

without loss of generality we assume that (1) constrains the government's actions and let this agent select x_t. We denote by $\eta_t(\varphi_t, A_{G,t}, \psi_t)$ the government trade-balance strategy as a function of $(\varphi_t, A_{G,t}, \psi_t)$, by η the sequence $\{\eta_t\}_{t=0}^{\infty}$ and by η^t its continuation.

The next agents to act are the households. When a typical household takes its date t decisions, it already knows both $(\varphi_t, A_{G,t})$ and ψ_t. Given $(\varphi_t, A_{G,t}, \psi_t)$, a household chooses χ_t and φ_{t+1} and contingency plans for future dates. We denote the date t decisions (χ_t, φ_{t+1}) by $\xi_t(\varphi_t, A_{G,t}, \psi_t)$. The sequence $\{\xi_t\}_{t=0}^{\infty}$ is denoted by ξ and it is called a *decision plan*. The *continuation* ξ^t of ξ is the sequence $\{\xi_k\}_{k=t}^{\infty}$. A typical household payoff is given by (10).

The firm acts as in the previous section. Namely, it observes ψ_t and then chooses l_t to maximize its date t profit.

After learning of households' and the firm's actions, the government will select $A_{G,t+1}$. Since this agent is required to balance its budget under all contingencies, the only possible choice for this variable is the one consistent with (2).[5]

As we have previously mentioned, Bassetto [5] pointed out that for some conceivable household actions, the government may fail to meet its budget constraint. To deal with this problem, we provide the government with a shutdown option. At each date t, while the securities and currency markets are open, the government can default on all of its debt and change the fiscal variables g and τ so that they both will equal zero at t and afterwards. No government bond will ever be issued again and the economy will operate with a constant stock of money thereafter. If the government is ever shut down, its payoff will be $u(0, 0, 0)$ at t and all future dates. Clearly, the government will always prefer not to exercise this option. A shutdown will take place only if the government fails to balance its budget without resorting to it.

Before proceeding with the game description, it is convenient to emphasize two points. First, by introducing the shutdown option we allowed the government policy to be contingent on households' actions, as Bassetto [5] suggested. Second, as that author pointed out, allowing the policies to depend on people's actions enlarges the set of equilibrium outcomes. For the particular game we are considering, it is possible to have an equilibrium with a government shutdown. However, we are not interested in this type of equilibrium and we will solely focus on equilibria without this feature. Therefore, from now on we simply disregard the shutdown option.

At this point it is possible to verify how the vector $(\varphi_t, A_{G,t})$ evolves. The law-of-motion for households' asset holdings is simply

$$\varphi_{t+1} = \xi_{\varphi,t}(\varphi_t, A_{G,t}, \sigma_t(\varphi_t, A_{G,t})) \ , \tag{11}$$

where $\xi_{\varphi,t}$ denotes the coordinates of ξ_t that are associated with the asset holdings $(M_{t+1}, B_{t+1}, A_{H,t+1})$. The government's foreign assets are then obtained from its budget constraint (2).

[5] Observe that out of equilibrium, a household and firm's choice of l_t may differ. So, we assume that the tax on labor income is always evaluated taking the household choice as the basis. Hence, $A_{G,t+1}$ is well defined.

Consider the situation of a household at date t. Given φ_t, $A_{G,t}$, ψ_t and a policy plan σ^t, a household chooses $\{c_{1k}, c_{2k}, l_k\}_{k=t}^{\infty}$ and $\{M_{k+1}, B_{k+1}, A_{H,k+1}\}_{k=t}^{\infty}$ to maximize (10) subject to (5) and (6). A household takes into consideration that the future policies are induced by σ^t.

Consider now the government situation at date t. Given φ_t, $A_{G,t}$ and a decision plan ξ^t, the government has to choose a policy plan σ^t and a trade-balance rule η^t to maximize (10) subject to (1) and (2). The government considers that the sequence of future allocations is given by the best responses ξ^t to futures policies, which in turn are induced by σ^t.

Definition 6 *A* Markov sustainable equilibrium *is an array* (σ, η, ξ) *satisfying: (i) given the policy plan* σ*,* ξ^t *provides solutions for both households' and firms' problems at every period* t *and all vectors* $(\varphi_t, A_{G,t})$*; (ii) given the decision plan* ξ*,* σ^t *and* η^t *solve the government's problem at every period* t *and all vectors* $(\varphi_t, A_{G,t})$*. An array* $(\psi^m, \chi^m, \varphi^m, \{A^m_{G,t+1}\}_{t=0}^{\infty}, \{x^m_t\}_{t=0}^{\infty})$ *is a* Markov sustainable outcome *if there exists a Markov sustainable equilibrium* (σ, η, ξ) *such that* $\sigma_t(\varphi^m_t, A^m_{G,t}) = \psi^m_t$*,* $\eta_t(\varphi^m_t, A^m_{G,t}, \psi^m_t) = x^m_t$ *and* $\xi_t(\varphi^m_t, A^m_{G,t}, \psi^m_t) = (\chi^m_t, \varphi^m_{t+1})$*.*

It is worth emphasizing three points in the above definition. First, the Markov sustainable equilibrium builds on the sustainable equilibrium introduced in Chari and Kehoe [10]. Second, this equilibrium concept requires optimal behavior even for out-of-equilibrium assets $(\varphi_t, A_{G,t})$ and policies ψ_t. Third, a Markov sustainable outcome is attainable (i.e., it is a competitive equilibrium).

We now turn to the task of characterizing the best Markov sustainable outcomes.[6] We first characterize the attainable set. To simplify the notation, $u(t)$, $u_1(t)$, $u_2(t)$, and $u_3(t)$ denote, respectively, the value of u and its partial derivatives evaluated at the point (c_{1t}, c_{2t}, l_t). The sum $u_1(t)c_{1t} + u_2(t)c_{2t} - u_3(t)l_t$ is denoted by $W(t)$.

We use techniques similar to those of Lucas and Stokey [22] and Chari and Kehoe [12] to characterize the competitive equilibrium set in terms of a few equalities and inequalities. The first condition is the feasibility condition (1). The second one is

$$\sum_{k=t}^{\infty} \beta^{k-t} W(k) = u_1(t)c_{1t} + u_2(t)\left[A_{H,t} + \frac{B_t + M_t}{S_t} - c_{1t}\right] , \qquad (12)$$

which consolidates households' period budget constraints from date t onwards. The third condition is a balance-of-payment constraint

$$-\sum_{k=t}^{\infty} \frac{Q^*_s}{Q^*_t} x_k = A_{H,t} + A_{G,t} \ . \qquad (13)$$

A fourth requirement, ensuring that people's intertemporal marginal rate of substitution is consistent with q^*_t, is

[6] In the type of game we are considering the equilibrium set can be large. We focus on a best (i.e, one that yields the higher date zero utility) outcome.

$$q_t^* = \beta \frac{u_2(t+1)}{u_2(t)} \ . \tag{14}$$

A fifth condition,

$$(1-\tau_t) = \frac{u_3(t)}{u_2(t)} \ , \tag{15}$$

is an implementability constraint for labor income taxation. The inequality

$$S_t c_{1t} \leq M_t \tag{16}$$

ensures that the cash-in-advance constraints hold, while

$$u_2(t) \leq u_1(t) \tag{17}$$

ensures $q_t \leq 1$. Conditions (1) and (12)-(17) characterize the set of attainable sequences.

We now start characterizing a best Markov sustainable outcome. In the discussion that follows, we assume that the set of sequences that satisfies (1) and (12)-(17) is not empty. That is, there is at least one conceivable competitive equilibrium.

Consider the following family of problems: at each date t, choose $\{\psi_k\}_{k=t}^{\infty}$, $\{\chi_k\}_{k=t}^{\infty}$, $\{\varphi_{k+1}\}_{k=t}^{\infty}$, $\{A_{G,k+1}\}_{k=t}^{\infty}$, and $\{x_k\}_{k=t}^{\infty}$ to solve

$$V_t(\varphi_t, A_{G,t}) = \max \sum_{k=t}^{\infty} \beta^{k-t} u(c_{1k}, c_{2k}, l_k) \tag{18}$$

subject to (1), (12)-(17), and

$$\sum_{k=T}^{\infty} \beta^{k-T} u(c_{1k}, c_{2k}, l_k) \geq V_T(\varphi_T, A_{G,T}) \ , \ T \geq t+1. \tag{19}$$

Arguments similar to those in Chari and Kehoe [11] establish that from the solution of the above problems it is possible to construct plans σ, η and ξ that constitute a Markov sustainable equilibrium. By construction, this equilibrium yields the highest date zero utility in the sustainable set. We summarize this discussion in the next proposition.

Proposition 7 *The solution of problem (18) for* $t = 0$ *constitutes a best Markov sustainable outcome.*

Proof. See the Appendix.

We are now in a position to establish that a best Markov sustainable allocation and policy cannot be decentralized by many sequences φ and $\{A_{G,t+1}\}_{t=0}^{\infty}$. In other words, the counterparts of Proposition 2 and Corollary 5 are generally not true in the particular game we are considering. We carry out this task by means of an example.

Example 8 Alvarez, Kehoe, and Neumeyer [2] showed that time consistent outcomes are Markov sustainable. We use this fact repeatedly in this example.

Assume that $u(c_{1t}, c_{2t}, 1 - l_t) = \log c_{1t} + \log c_{2t} + \log(1 - l_t)$, $\bar{M} > 0$, $\bar{B} = \bar{A}_H = \bar{A}_G = 0$, $q_t^* = \beta$, $g_t = g > 0$ and $\tau_t = 0$ for all t. Since the government has a permanent deficit, it will be willing to use all available lump-sum revenue at date zero. Hence, a Ramsey outcome (i.e., a best competitive equilibrium) will surely require that (6) hold as equality at date zero. So, we can write the date zero constraint (12) as

$$\sum_{t=0}^{\infty} \beta^t \left[2 - \frac{l_t}{1 - l_t} \right] = 1 \ . \tag{20}$$

Consider now the problem of maximizing (4) subject to (20), $c_{1t}+c_{2t}+g+x_t = l_t$, $\sum_{t=0}^{\infty} \beta^t x_t = 0$, $c_{2t} = c_{2t+1}$ and $c_{2t} = 1 - l_t$ Note that the last four constraints correspond to (1), (13), (14) and (15). A little work shows that the solution of this problem specifies constant values for c_{1t}, c_{2t}, l_t and x_t. Moreover, the need to satisfy the government budget constraint will ensure that there is a positive inflation rate and consequently (17) holds.

Let ψ^r, χ^r and $\{x_t^r\}_{t=0}^{\infty}$ denote the Ramsey policy and allocation. From Proposition 2 and Corollary 5 there are many cash and asset sequences that can implement $(\psi^r, \chi^r, \{x_t^r\}_{t=0}^{\infty})$ as a competitive equilibrium. However, as Lucas and Stokey [22], Alvarez, Kehoe and Neumeyer [2] and Persson, Persson and Svensson [26] showed, there is usually one debt structure that will make the Ramsey outcome time consistent. We define $A_{G,t+1}^r = A_{H,t+1}^r = B_{t+1}^r = 0$ and $M_{t+1}^r = S_{t+1}^r c_{1t}^r$. With these assets, if the Ramsey problem is solved again at some date $T > 0$, this new problem will have exactly the same constraints as the date zero problem. Hence, the continuation of the date zero solution will also solve the date T Ramsey problem.

The Ramsey outcome is attainable. The fact that it is time consistent ensures that $\sum_{k=T}^{\infty} \beta^{k-T} u(c_{1k}^r, c_{2k}^r, l_k^r) \geq V_T(\varphi_T^r, A_{G,T}^r)$ for all T. Thus, the Ramsey outcome is also Markov sustainable. In this outcome, $A_{G,t+1}^r = 0 = \bar{A}_G$. Hence, in this particular example, the Markov sustainability of the best attainable allocation calls for a permanently floating regime.

Suppose now that at some date $T - 1 > 0$, households and government deviate from the assets φ_T^r to an alternative array φ_T'. More specifically, M_T' is equal to M_T^r, but B_T' satisfies $B_T' + M_T' < 0$. Of course, to balance the households' budgets and the country's external account, it is necessary that $A_{H,T}' > 0$ and $A_{G,T}' < 0$. After date $T-1$, people and government revert to the Ramsey cash and bond holdings. Denote these alternative sequences by φ' and $\{A_{G,t+1}'\}_{t=0}^{\infty}$.

Had a statement equivalent to Proposition 2 been true for Markov sustainable outcomes, $(\psi^r, \chi^r, \varphi', \{A_{G,t+1}'\}_{t=0}^{\infty}, \{x_t^r\}_{t=0}^{\infty})$ would be a best Markov sustainable outcome. However, this is not the case. Consider the situation of the government at date T. Observe that $B_T' + M_T' < 0$ and households are in debt to the government. Therefore, the government can pick an exchange rate S_T'' that is low enough to make the real value of the debt sufficiently large to cover its future consumption expenditures and any future interest it owes abroad (recall that $A'_{G,T} < 0$). We can go even further and conclude the government can implement the Friedman rule $q_t = 1$. Since $\tau_t = 0$ for all t, the government will be able to implement a Pareto efficient allocation as a competitive equilibrium. Denote that allocation by $\{\chi''_t, x''_t\}_{t=T}^{\infty}$ and the underlying policy by $\{\psi''_t\}_{t=T}^{\infty}$. Clearly, if future assets are selected to satisfy $(B''_t + M''_t)/S''_t = (B'_T + M'_T)/S''_T$, we ensure that $\{\chi''_t, x''_t\}_{t=T}^{\infty}$ is time consistent. Therefore, $(\psi^r, \chi^r, \varphi', \{A'_{G,t+1}\}_{t=0}^{\infty}, \{x^r_t\}_{t=0}^{\infty})$ cannot be a best Markov sustainable outcome.

5 Conclusion

Economists have long argued about the advantages and disadvantages of floating exchange rate regimes. In this paper we showed that this type of regime is consistent with any competitive equilibrium.

The intuition for this result is simple. Arbitrage opportunities are absent in a competitive equilibrium. Thus, it is always possible to change the composition of people's portfolios between foreign and government issued bonds. As a consequence of this indeterminacy of people's portfolios, the composition of the government's debt between domestic and foreign claims is also undetermined. Hence, any path for the foreign assets or debt is possible at the competitive equilibrium prices. One can easily relate this result to the Modigliani-Miller Theorem.

It is well known that the composition and maturity of government debt matter for the time consistency of monetary policy. We used this fact to establish that in a game in which the government selects policies in a sequential fashion, the outcomes are not invariant to the path of the foreign assets. Therefore, the study of the implications of adopting a specific exchange rate regime should rely on more sophisticated equilibrium concepts than the competitive one.

6 Appendix

Proof of Proposition 7. Let $(\psi^m, \chi^m, \varphi^m, \{A^m_{G,t+1}\}_{t=0}^{\infty}, \{x^m_t\}_{t=0}^{\infty})$ be a solution for (18). We need to show that there exists a Markov sustainable equilibrium (σ, η, ξ) that satisfies $\sigma_t(\varphi^m_t, A^m_{G,t}) = \psi^m_t$, $\eta_t(\varphi^m_t, A^m_{G,t}, \psi^m_t) = x^m_t$ and $\xi_t(\varphi^m_t, A^m_{G,t}, \psi^m_t) = (\chi^m_t, \varphi^m_{t+1})$.

We carry out the proof in four steps: (1) construct the plans σ, η and ξ; (2) show that these plans constitute a Markov sustainable equilibrium; (3) show that (σ, η, ξ) induces $(\psi^m, \chi^m, \varphi^m, \{A^m_{G,t+1}\}_{t=0}^{\infty}, \{x^m_t\}_{t=0}^{\infty})$ and (4) show that (σ, η, ξ) induces the highest attainable date zero utility.

Step 1: For each t, set $\sigma_t(\varphi_t, A_{G,t})$ as the ψ_t solution of problem (18). To construct ξ we use another maximization problem. Select $\{\psi_k\}_{k=t+1}^{\infty}$, $\{\chi_k\}_{k=t}^{\infty}$,

$\{\varphi_{k+1}\}_{k=t}^{\infty}$, and $\{x_k\}_{k=t}^{\infty}$ to solve

$$V_{H,t}(\varphi_t, A_{G,t}, \psi_t) = \max \sum_{k=t}^{\infty} \beta^{k-t} u(c_{1k}, c_{2k}, l_k) \tag{21}$$

subject to date t (5) and (6); dates $k \geq t+1$ (1), (12)-(17), and (19). The above problem does not impose the date t government budget constraint (2), (1) and other competitive equilibrium conditions. We proceed in this way because, as Chari and Kehoe [11] pointed out, the function ξ must specify the behavior of the consumer even for out-of-equilibrium policies. Additionally, we construct problem (21) in such a way that people consider that the government will pick policies according to the solution of problem (18) for all dates $s \geq t+1$. We define $\xi_t(\varphi_t, A_{G,t}, \psi_t)$ as a (χ_t, φ_{t+1}) solution to (21). We then define $\eta_t(\varphi_t, A_{G,t}, \psi_t)$ to satisfy (1).

Step 2: To show that ξ_t is an optimal choice for households, observe that constraint (19) ensures that people take into consideration that future policies are given by the solution of problem (18) while solving (21). Thus, given $\{\sigma_k\}_{k=t+1}^{\infty}$, ξ_t is an optimal choice for households. To show that given $\{\xi_k\}_{k=t}^{\infty}$ σ_t is an optimal choice for the government, consider the problem

$$V_{G,t}(\varphi_t, A_{G,t}) = \max_{\psi_t} V_{H,t}(\varphi_t, A_{G,t}, \psi_t)$$

subject to date t (1), (2), (14), (15), and (17). Its solution is the best the government can do, given the decision rule ξ^t. But the constraints of the above problem are exactly those of problem (18) and consequently $V_t(\varphi_t, A_{G,t}) = V_{G,t}(\varphi_t, A_{G,t})$. Thus, σ_t^F is an optimal choice for the government.

Step 3: We now apply induction. Since $(\psi^m, \chi^m, \varphi^m, \{A_{G,t+1}^m\}_{t=0}^{\infty}, \{x_t^m\}_{t=0}^{\infty})$ solves (18) for $t=0$, it follows that $\sigma_0(\bar{\varphi}, \bar{A}_G) = \psi_0^m$, $\xi_0(\bar{\varphi}, \bar{A}_G, \psi_0^m) = (\chi_0^m, \varphi_1^m)$ and $\eta_0(\bar{\varphi}, \bar{A}_G, \psi_0^m) = x_0^m$. Assume that $\sigma_t(\varphi_t^m, A_{G,t}^m) = \psi_t^m$, $\xi_t(\varphi_t^m, A_{G,t}^m, \psi_t^m) = (\chi_t^m, \varphi_{t+1}^m)$ and $\eta_t(\varphi_t^m, A_{G,t}^m, \psi_t^m) = x_t^m$. At date $t+1$, given assets φ_{t+1}^m, constraint (19) ensures that the solution of problem (18) will specify the policy ψ_{t+1}^m. Given $(\varphi_{t+1}^m, \psi_{t+1}^m)$, the argument of Step 2 shows that

$$V_H(\varphi_{t+1}^m, A_{G,t+1}^m, \psi_{t+1}^m) = V_{t+1}(\varphi_{t+1}^m, A_{G,t+1}^m) = \sum_{k=t+1}^{\infty} \beta^{s-(t+1)} u(c_{1k}^m, c_{2k}^m, l_k^m) \ .$$

Thus, $(\chi_{t+1}^m, \varphi_{t+2}^m)$ constitutes an optimal choice for a typical household. Trivially, $\eta_{t+1}(\varphi_{t+1}^m, A_{G,t+1}^m, \psi_{t+1}^m) = x_{t+1}^m$.

Step 4: Any competitive equilibrium $(\psi, \chi, \varphi, \{A_{G,t+1}\}_{t=0}^{\infty}, \{x_t\}_{t=0}^{\infty})$ that yields a higher date zero utility than $(\psi^m, \chi^m, \varphi^m, \{A_{G,t+1}^m\}_{t=0}^{\infty}, \{x_t^m\}_{t=0}^{\infty})$ must violate some constraint of the family (19). Therefore, such a competitive equilibrium cannot be a Markov sustainable outcome. □

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