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São Paulo

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Insper Working Paper
WPE: 044/2003

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First Version: September 10th, 2002
This Version: February 10th, 2003
JEL Classification: H0; H41; I22; I28

Abstract

In Brazil, there have been some initiatives to introduce a system of affirmative action quotas in public universities, to benefit either black students or individuals who have studied in public schools. In this paper, I develop a simple model to analyze theoretically the effects of the introduction of such a system on the efficiency of expenditures in higher education. Efficiency of public expenditures in universities inevitably suffers a reduction. However, the effects on the overall efficiency of the total investments (public and private) can be beneficial, nonexistent, or detrimental, depending on the degree of the liquidity constraint of the low-income families and the quality of the public university vis-à-vis private ones. Finally, these same variables influence the impact of quotas on the likelihood that the individuals with greatest academic abilities will reach a university, independently of their income's family background.

*I would like to thank Danniel Cardoso Oliva and Heitor Hiroaki Hirota for their research assistance.

1 Introduction

The Brazilian educational system is characterized by a great disparity in terms of the qualities of its schools and universities. On the one hand, in the primary and secondary levels, the best schools are in general the privates ones. Families who can afford do send their children to private schools, even though they have the option of public education free of charge. The difference between the two types of school's quality is significant. The offspring's expected greater future income related to the better school compensates the families' foregone consumption today in paying the school tuition. In general, only low-income families opt to send their children to public schools: either because they can not afford the private option or because their utility cost of sending their children to the private school is greater than the corresponding cost for high-income families.

On the other hand, however, the best universities are in general the public ones.[1] All candidates take the admissions test, and those who achieve the greatest scores obtain the right to attend the public universities. It is no surprise, then, that those individuals who attended private primary and secondary schools are better prepared and perform better in these exams. In general, the private universities of higher quality are not superior enough to justify families, whose children have the right to attend public universities, in paying the private university tuition. Hence, most of the students who attend public universities come from high-income families.

In order to counterbalance the advantage of the individuals from the high-income families in the admission tests, there have been some initiatives to introduce some form of affirmative action into the Brazilian public universities' entrance system. The intended beneficiaries of these policies are either black individuals, or individuals who have studied in public schools. They are the target because they in general come from low-income families.[2] The state of Rio de Janeiro has already introduced a system of quotas, under which 50% of all students of the state public universities must have attended public schools and 40% of them must be black students. The state of Bahia also has a system under which 40% of all students of the state university students must be black students. Other states are in the process of adopting a similar scheme. On the federal level, there are plans to establish quotas directed to black students in the federal universities. These plans would guarantee the admission of a certain number of individuals from the target groups who attended public universities, in particular those who obtained the highest scores, relative to the target group,

[1] All undergraduate students have to take a national exam (called 'Provão') in order to obtain their university degree. Based on this exam, the government provides ranks of the universities in different courses. In 2002, the number of public universities among the top 20 universities in different fields is the following: Business (13), Economics (13), Law (17), Medicine (17), Pedagogy (16), and Physics (16).

[2] One could argue that the government should subsidize education, partially or fully, only for those individuals coming from low-income families, and eliminate the system of free tuition for all individuals, independently of their family background, who pass the admission test for the public universities. However, there are no movements in this direction.

in the admission exams. Hence, individuals not from the target groups who have performed better in these tests can lose their right to attend public universities.

The objective of this paper is to analyze theoretically the effects of the introduction of the system of quotas into the Brazilian public universities' entrance system, assuming that the government maintains free tuition for all individuals who pass the public university admissions test.[3] This type of policy represents an income transfer to its beneficiaries, as they can attend better universities in general without incurring the tuition cost. However, this impact is not the focus of the analysis. This paper concentrates on the effects of quotas on the efficiency of the expenditures in education.

Efficiency is measured in three different ways. It can be measured by the quality of the labor force produced either by the public or the overall (sum of public and private) investments in education. Moreover, it can be measured by whether the educational system guarantees that individuals with the greatest academic abilities[4] are reaching the university, regardless of their income's family background. The main contribution of this paper is to indicate under which circumstances the introduction of quotas can be beneficial or detrimental to the efficiency of the investments in education at the higher level.

In this paper I develop a two-period model in order to mimic the family's problem of deciding whether or not to send a child to university. A parent who opts to send a child to university has two options: either the funded public university, if the child passes the admissions test, or the private university, with the cost of paying tuition. There is a probability that a child who is sent to university becomes an individual with a high level of human capital, which depends positively on his or her ability. The introduction of quotas alters the allocation of human resources between the two types of universities in the ways described above.

This paper's model suggests the following results. With quotas, individuals with lower ability on average attend public universities, as the average grades on the admission tests are lower.[5] This reduces the efficiency of public investments in higher education. When the quality of both types of universities (private and public) is the same, the effect on the total efficiency of the system depends in great part on the degree of the liquidity constraint of the beneficiaries of the system of quotas. In the case that this constraint is not very tight, the beneficiaries would have been able to afford a private university anyway. Quotas simply move individuals with greater and lower ability, respectively, to private and public universities, without any impact on overall efficiency. In the case of

[3]The system of quotas is irrelevant if the government finances higher education, partially or fully, for only those individuals who can not afford it.

[4]In this paper, when the term ability is used alone, it always means academic ability.

[5]The first admissions test with quotas at the University of the State of Rio de Janeiro (UERJ) took place in 2003. There was an important difference between the grades of the student with the lowest score from the non-target group and from the target group who was accepted in the university in its different programs, with the former obtaining higher grades. Here are some examples (with all grades out of 100): dentistry school (77.5 for the non-target group versus 6.25 for the target group), history department (71.25 versus 63.25), and medical school (92.5 versus 64).

a tight constraint, individuals previously deprived of the possibility of attending any university go to a public university. Quotas reduce the link between income's family background and the capacity of an individual to attend a university. The consequence is a reduction in the skill premium and a weaker economic incentive for the least able individuals to continue attending private universities. The overall ability of the university students and the efficiency of the system increases.

When the quality of private universities is greater, quotas move individuals with greater abilities to them, making more efficient the allocation of human resources between both types of universities. This does not occur without quotas, when individuals opt to attend public universities because they are free, even though they are of lower quality than the private universities. Quotas correct this distortion and increase the overall efficiency of the system. Finally, when the quality of public universities is greater, the shift in the allocation of human resources between both types of universities caused by the introduction of quotas is not the most efficient one. Individuals with the greatest abilities are taken away from the universities with greater quality, which implies a less efficient system.

The model in this paper is similar to the theoretical framework developed in Caucutt and Kumar (2001). Their focus, however, is on the effects of subsidies on efficiency. As they do not analyze the impact of quotas, they do not have to make any distinction between the two types of universities, private and public, as I do here. Other related literature is Holzer and Neumark (2000), who make a very extensive survey of the theoretical and empirical literature of effects of affirmative action policies in the United States, including their impacts on efficiency.

The rest of this paper is organized as follows. The next section presents the structure of the model. Section 3 discusses the competitive equilibrium without quotas. In section 4, the effects of quotas on efficiency are analyzed, and the paper ends with a last, concluding section.

2 Model

The model employs a two-period economy. There are two types of families, which differ by the parent's level of human capital. At time $t = 0$, parent has either a high or a low level of human capital. Each parent works, receives wages, decides whether to send his child to university, consumes, and dies. At time $t = 1$, the child becomes a worker with high or low level of human capital, depending on the educational decision of his parent, receives a wage, and consumes. The measure of each generation (and the number of workers in each period) is constant and is normalized to one. Let $n_{h,t}$ and $n_{l,t}$ be the fraction of individuals, respectively, with high and low levels of human capital at time t. Note that $n_{h,0} = n^*_{h,0}$and $n_{l,0} = n^*_{l,0}$ are exogenously given, and $n^*_{h,0} + n^*_{l,0} = 1$.

As I do not model the schooling decision, it is assumed that all children

have a school degree and are able to attend university. However, they differ in their ability to perform well at university and become an individual with a high level of human capital. A child with ability a who attends a private university becomes an individual with high or low level of human capital, respectively, with probability $\pi_{pr}(a)$ and $(1-\pi_{pr}(a))$. In the case of attending a public university, the equivalent probability can differ and is denoted by $\pi_{pu}(a)$. If the quality of the public university is greater than the private, then $\pi_{pu}(a) > \pi_{pr}(a)$, $\forall a$. The inequality reverses if the opposite holds, that is, if the private university has a higher quality. This feature of the model limits the heterogeneity to two levels of human capital, and therefore to two income levels, which simplifies the analysis.[6] If a child does not attend university, he has the lower level of human capital with probability equal to one. Let $F(.)$ be the distribution function for ability on the support $[0,1]$, and $f(.)$ be the corresponding density function. As in Caucutt and Kumar (2001), the distribution is identical across types and within families of the same type, and all ability draws are independent of each other.[7]

Assumption 1: $a \in [0,1]$, $0 \leq \pi_j(a) \leq 1$, $\pi_j'(a) > 0$, $\pi_j''(a) < 0$, $\forall a$, and $\pi_j(0) = 0$, $j = pr, pu$.

There are important additional differences between the private and public universities, besides the possible differences in quality mentioned above. The public university is free of charge, and the number of students who can attend it is exogenously determined and equal to v. As the number of applicants to fill these places is greater than v, it is necessary to have some form of rationing. The students with the right to attend the public university are selected using a ranking of the individuals' abilities, which is assumed to be public information; they are the v individuals with greater ability who choose to attend the public university. The cost of each student is equal to E units of consumption, which the government finances.[8] As will be pointed out below, the government imposes an income tax at time $t = 0$ to finance these expenditures in education. The

[6] See Andrade (1998) for an example of the use of this framework.

[7] The ability of an individual to attend university and acquire a high level of human capital is determined in great part by two factors. The first is the individual's innate or cognitive ability, and is independent of the social environment where the individual grows up. There is no reason to assume that the distribution of this innate ability differ between groups of individuals who differ by family income status (see Heckman (1995)). The second factor is related to expenditures in education received by the individual before entering university. This second factor is greatly affected by the income of the individual's family. For example, high-income families are capable of paying for extra courses and better quality schools. See discussion in Becker (1993). Combining these two factors, one could imagine that the distribution function for a child's ability differ across types of families, but is identical within the same type of family. Nonetheless, the results in this paper would not undergo any change assuming this difference in distribution across types of families. Hence, I assume the same distribution for simplicity.

[8] This feature of the model tries to mimic the current system of students' selection to the Brazilian public universities. All applicants must take the same exam (called "vestibular") prepared by the universities. The accepted students are those who achieve the highest scores. In the model, it is assumed that the students who achieve the highest scores are the ones with greater abilities.

private university cost is also equal to E units of consumption, and every family who is willing to pay this fee can send a child to a private university.

The parent whose child has ability a and is one of the selected students to attend the public university has three options: sending the child to attend the public or the private university, or alternatively, letting the child remain solely with a school degree. The family's problem can be written in the following way:

$$\max_{pu,pr,s} \{u\left((1-\tau)w_{i,0}\right) + \beta\left[\pi_{pu}(a)u\left(w_{h,1}\right) + \left(1-\pi_{pu}(a)\right)u\left(w_{l,1}\right)\right]; \quad (1)$$
$$u\left((1-\tau)w_{i,0} - E\right) + \beta\left[\pi_{pr}(a)u\left(w_{h,1}\right) + \left(1-\pi_{pr}(a)\right)u\left(w_{l,1}\right)\right];$$
$$u\left((1-\tau)w_{i,0}\right) + \beta u\left(w_{l,1}\right)\}, \; i = h, l.$$

where τ is the income tax, $w_{i,t}$ is the wage of individual with i level of human capital at time t, and the utility function has the usual properties. The first term within the parenthesis in the family's problem indicates the public university option. At $t = 0$, the family consumes its wage net of taxes and sends its child to a free public university. At $t = 1$, the child who becomes a worker either with high or low level of human capital earns his or her wage. The second term indicates the private university option. The difference with respect to the first option is that the family pays both taxes and the cost of the private education (E) at $t = 0$, and the probability of the child becoming a worker is now based on the quality of the private university. The last term is the option of not sending the child to university at all. The family pays taxes and the child becomes a worker with low level of human capital.[9] The other families, whose children are not among those selected to attend the public university, have a similar problem, with the difference that they do not have the first option of sending their children to public university.

Two comments are worth noting before proceeding to describe production in this economy. First, there is no capital market to finance education. This assumption is commonly used and widely accepted.[10] Second, the only type of bequest allowed in this model is through investments in education. Other types of bequests are not empirically relevant for most families, even in the most developed countries.[11]

There is a single non-storable good in the economy produced by firms operating in a competitive market. The inputs in the production process are the two types of labor, individuals with high and low levels of human capital. Therefore, there is no physical capital in this economy. The production function presents constant returns to scale and is as follows:

$$Y_t = A\left[\theta\left(N_{h,t} + \gamma N_{l,t}\right)^d + \left(1-\theta\right)\left(N_{l,t} + \varepsilon N_{h,t}\right)^d\right]^{\frac{1}{d}},$$

where $0 < \varepsilon < 1$, $d < 1$, $\gamma \ll \varepsilon$. $N_{h,t}$ and $N_{l,t}$ are, respectively, the number of individuals with high and low levels of human capital employed in the production

[9] It should be clear that this last option is never chosen by the parent whose child is selected as one of the students to attend public university for free.

[10] See Becker (1991) for an extensive discussion of this topic.

[11] See Stokey (1998) for an extensive discussion of this topic.

process at time t. Note that the possible values for the variables $N_{l,t}$ and $N_{h,t}$ are in the interval $[0,1]$, as the number of workers in each period is constant and is normalized to one. Hence, total output can be expressed as a function of the fraction of individuals with a high level of human capital in each period. As in Stokey (1996) and Caucutt and Kumar (2001), the first and second terms within the square brackets can be thought of, respectively, as "brain" and "brawn." There are two important ideas behind this production function. First, as γ is assumed to be close to zero, the individuals with high level of human capital are the main providers of "brain". Second, as $\varepsilon < 1$, the individuals with a high level of human capital can perform (almost) all the tasks that the other type of labor can, and more. The following assumption simplifies the analysis and guarantees that the wages paid to the individuals with a high level of human capital are always greater than the ones for the individuals with a low level of human capital, for all possible values for $N_{h,t}$ and $N_{l,t}$.

Assumption 2: $\varepsilon > \left(\frac{1-\theta}{(1-\gamma)\theta}\right)^{\frac{1}{1-\sigma}}$.[12]

The results in this paper do not depend on this assumption, which is made just for convenience and to simplify the analysis. In any equilibrium with private investment in education, since it is costly (cost of E), families finance their children's education only if they anticipate a positive skill premium (a positive difference between the wage paid to the individuals with high and low levels of human capital). If this skill premium is negative, there is no private investment in education. As a result, the wage premium would increase, inducing private university enrollment.[13]

The government's budget constraint has to be in equilibrium. The number of students in the public university is fixed and equal to v, with the cost of E units of consumption per student. The only source of revenues are the income tax imposed at $t = 0$. Therefore:

$$\tau \left(N_{h,0} w_{h,0} + N_{l,0} w_{l,0}\right) = vE.$$

3 Competitive Equilibrium

This section discusses the equilibrium of the model. It is of interest to analyze the equilibrium with two characteristics that are true of the current Brazilian educational system. The first characteristic is the existence of private expenditures in education from both types of families, those having both low and high level of human capital. It could be the case that the expected skill premium resulting only from the public expenditures in education would not be large enough to create economic incentives for individuals to invest their own resources in private education. Let $w'_{i,1}$ $(i = h, l)$ be the wage if the v individuals

[12] This assumption indicates that ε (the coefficient of the individual with a high level of human capital in the production of "brawn") is large enough to guarantee that $w_{h,t} > w_{l,t}$.

[13] See assumption 4 below for a necessary condition of the existence of positive expenditures in private education, which is the type of equilibrium of interest in this analysis.

with greatest ability attend public university and if there is no investment in private education and a_{pu} be the ability level of the individual with the lowest ability who has the right to attend the public university. The following assumption eliminates the possibility of existing only public investment in education:

Assumption 3:

$$\beta\pi_{pr}(a_{pu})\left[u\left(w'_{h,1}\right)-u\left(w'_{l,1}\right)\right]>u\left((1-\tau)w_{i,0}\right)-u\left((1-\tau)w_{i,0}-E\right).$$

The above assumption indicates that there are some parents from both types of families whose children do not have the right to attend public universities, and who are made better off by investing in private education instead of not sending their children to university. This condition holds even if the individuals with the greatest abilities attend public universities, which would lead to the lowest possible wage premium with only public expenditures in education.

The second characteristic that is true of the current Brazilian educational system is the fact that individuals in general opt to attend public universities, because they are free, even if a private university has a higher quality. That is, even if $\pi_{pr}(a)>\pi_{pu}(a)$, $\forall a$. In order to restrict the analysis to an equilibrium with this characteristic, the following assumption is made:[14]

Assumption 4: For any a, $a\in[1,a_{pu}]$,

$$u\left((1-\tau)w_{i,0}\right)-u\left((1-\tau)w_{i,0}-E\right)>\beta\left(\pi_{pr}(a)-\pi_{pu}(a)\right)\left(u\left(w'_{h,1}\right)-u\left(w'_{l,1}\right)\right).$$

The above assumption indicates that the differences in quality between both types of university $(\pi_{pr}(a)-\pi_{pu}(a))$ are not large enough to justify the investment E in private education.

Before proceeding to define the competitive equilibrium in this economy, it is worth characterizing the behavior of both types of parents with respect to the decision to send their child to private university.

Proposition 1: For any given wage differential: (i) there exists an unique $a^*_{pr,i}$ which is a function of the wage differential $(w_{h,1}-w_{l,1})$ such that $a^*_{pr,i}\left(w_{h,1}-w_{l,1}\right)\in(a_{pu},0)$, $i=h,l$, and a parent with i level of human capital sends his child to university if $a\geq a^*_{pr,i}\left(w_{h,1}-w_{l,1}\right)$, and does not otherwise; and (ii) $a^*_{pr,h}\left(w_{h,1}-w_{l,1}\right)<a^*_{pr,l}\left(w_{h,1}-w_{l,1}\right)$.[15]

The first part of the above proposition indicates that there is threshold ability behavior for both types of families. Parents with high and low levels of human capital send their children to private university only if their children's ability is greater than, respectively, the threshold values $a^*_{pr,h}$ and $a^*_{pr,l}$. If a child has an ability lower than this threshold value, it is not profitable to pay for a private education and obtain a lower immediate utility. This is the case because the expected future gain in utility, as a result of the expenditure E in education today, is very small, as the probability of this child becoming an individual with a high level of human capital is very low.

[14] With this assumption, the family whose child has the right to attend the public university always chooses to send its child to public university. The other two options become redundant.

[15] See in the appendix the proof of all propositions in this paper.

The second part of the above proposition shows that parents with a high level of human capital send children to university with lower ability vis-à-vis parents with a low level of human capital. Under assumption 2, the former receive greater wages and are richer than the latter. As a consequence, their utility cost of paying the cost of education and sending children to private university is lower, lowering the ability required to compensate for that cost.

I now turn to the definition and the proof of existence and uniqueness of the equilibrium:

Definition 1: Given v, E, and $n^*_{i,0}$, a competitive equilibrium is characterized by $n^*_{i,1}$, $w^*_{i,0}$, $w^*_{i,1}$, a^*_{pu} and $a^*_{pr,i}$ ($a^*_{pr,i} \in (a_{pu}, 0)$) such that ($i = h, l$): (i) given wages, firms maximize profits; (ii) the labor market clears, that is, $N_{h,t} = n^*_{h,t}$ and $N_{l,t} = n^*_{l,t}$ ($t = 0, 1$); (iii) a^*_{pu} is obtained from the following equation for v, $v = 2\int_{a^*_{pu}}^{1} f(a)daa$; (iv) given wages and E, a^*_{pu} and $a^*_{pr,i}$ solves the university decision problem of family i in (1); (v) the government budget constraint is in equilibrium; and (v) the law of motion for the variable n_h, which is given by:

$$\begin{aligned} n^*_{h,1} &= n^*_{h,0}\left[\int_{a^*_{pu}}^{1} \pi_{pu}(a)f(a)da + \int_{a^*_{pr,h}}^{a^*_{pu}} \pi_{pr}(a)f(a)da\right] + \\ &+ n^*_{l,0}\left[\int_{a^*_{pu}}^{1} \pi_{pu}(a)f(a)da + \int_{a^*_{pr,l}}^{a^*_{pu}} \pi_{pr}(a)f(a)da\right]. \end{aligned} \tag{2}$$

Proposition 2: There exists a competitive equilibrium and it is unique.

This unique equilibrium has the following characteristics. First, all families whose children have the right to attend a public university send their children to a public university. Second, there are children coming from both types of families who attend private universities. Finally, the skill premium and the fraction of individuals with a high level of human capital at $t = 1$ are positive.

Implicit in the analysis up to this point is the assumption that $(1-\tau)w_{l,0} > E$. This assumption means that the net wages of parents with a low level of human capital are greater than the cost of private education, which implies that these individuals have enough resources to finance their children's private education. As there is no capital market to finance education, this assumption is a necessary one to have an equilibrium in which some children whose parents have a low level of human capital attend private universities. It is easy to show that there is also a unique competitive equilibrium in the special case when $(1-\tau)w_{l,0} < E$. In the next section, the effects of quotas are also analyzed for this special case.

4 Effects of Quotas

This section analyzes the effects of quotas in public universities according to three variables: (i) the efficiency of public expenditures in education; (ii) the

efficiency of the total, private and public, expenditures in education; and (iii) the modulus of the difference in the threshold abilities behavior of both types of families, that is, the value of $DIF = \left| a^*_{pr,h} - a^*_{pr,l} \right|$.

Before discussing how the system of quotas can be introduced into the model, it is important to define formally the efficiency of expenditures in education. Following Caucutt and Kumar (2001), I use the ratio of the number (measure) of students who turned out to be individuals with a high level of human capital to the public or total resources expended in educating them. Hence, the efficiency of the public (EFF_{pu}) and total (EFF_{to}) expenditures in education are, respectively, equal to:

$$EFF_{pu} = \frac{n^*_{h,0} \left[\int_{a^*_{pu}}^{1} \pi_{pu}(a) f(a) da \right] + n^*_{l,0} \left[\int_{a^*_{pu}}^{1} \pi_{pu}(a) f(a) da \right]}{vE},$$

and

$$EFF_{to} = \frac{n^*_{h,1}}{\left(v + \int_{a^*_{pr,h}}^{a^*_{pu}} f(a) da + \int_{a^*_{pr,l}}^{a^*_{pu}} f(a) da \right) E}.$$

Note that the more efficient the total investments in education are, ceteris paribus, the greater is the total level of output at time $t = 1$. This is the case because output in this model is a positive function of the fraction of individuals with a high level of human capital, and this fraction increases with the efficiency of the investments in education. Moreover, there is nothing to preclude the efficiency in the public sector from dropping at the same time that efficiency in the private sector increases, or vice-versa. Obviously, the efficiency of the whole educational sector is the more relevant variable.

One can consider that an efficient educational system is one in which individuals with the greatest abilities are capable of reaching the university, independently of their income's family background. Hence, the lower the variable DIF, the more efficient is the educational system. Without quotas, proposition 1 indicates that parents with a high level of human capital (and greater income) send children to university with lower ability vis-à-vis parents with a low level of human capital (and lower income). It is interesting to analyze whether the introduction of quotas into the system affects this feature of the competitive equilibrium.

Without the system of quotas and under assumption 4, the v parents whose children possess the greatest ability opt to send their children to public universities. The introduction of quotas into the system has the effect of changing the individuals who have the right to attend public universities. This change occurs formally in the following way. The q individuals with the lowest ability levels compared to other individuals of their background, who might have had the right to attend the public university and whose parents have a high level of human capital, lose their right to attend public universities. In their place, the q individuals with the highest ability levels among individuals of their back-

grounds, who would not have had the right to attend a public university and whose parents have a low level of human capital, acquire that right.

Hence, the ability range of the individuals who attend public universities changes with the introduction of quotas into the system. With quotas, the range of individuals whose parents have high and low levels of human capital changes, respectively, from $[1, a^*_{pu}]$ to $[1, a^*_{pu,h}]$ and $[1, a^*_{pu}]$ to $[1, a^*_{pu,l}]$. Note that $a^*_{pu,h} > a^*_{pu} > a^*_{pu,l}$. The abilities $a^*_{pu,h}$ and $a^*_{pu,l}$ are obtained, respectively, from the following equations: $q = \int_{a^*_{pu}}^{a^*_{pu,h}} f(a)da = \int_{a^*_{pu,l}}^{a^*_{pu}} f(a)da$.[16]

The way the system of quotas is modeled in this paper tries to mimic the types of quotas that have been or are in the process of being implemented in the Brazilian educational system. These quotas are intended to benefit black students and individuals who have attended public schools, who come in general from lower-income families. This is the reason for directing the quotas in the model to families whose parents have a low level of human capital. Moreover, the choice of the beneficiaries of the quotas is based on the admissions test. Those students whose low-income background qualifies them for quota admissions, who achieve the highest scores in this exam compared to other students of their backgrounds, and who would not gain the right to study in the public universities without quotas, are the ones who acquire the right to attend the public university. They replace students who achieve the lowest scores on the exam compared to other students not eligible to benefit from the quotas, and who would have the right to attend the public university without a quota system. The change in the ability range of the individuals who attend public university with the introduction of the system of quotas mentioned above seeks to mimic this choice of the beneficiaries of the quotas. Recall that in this model it is assumed that there is a direct link between ability and performance on the admissions test.

The remainder of this section analyses the effects of the introduction of the system of quotas on the measures of efficiency defined above. I divide the analysis into two parts. First, it is assumed that the quality of both types of universities, private and public, is the same. Formally, this means that $\pi_{pr}(a) = \pi_{pu}(a)$, $\forall a$. Second, it is considered that either the quality of the public university is greater ($\pi_{pr}(a) < \pi_{pu}(a)$, $\forall a$) or the inverse holds ($\pi_{pr}(a) > \pi_{pu}(a)$, $\forall a$).

4.1 Same Quality in Private and Public Universities

In this subsection, I consider the effects of the introduction of the system of quotas on the efficiency of the public and total expenditures in education, and the difference in the threshold ability behavior of both types of families. I analyze two possible cases in which the quality of both universities, public and private, are the same. In the first case, the condition $(1 - \tau)w_{l,0} > E$ holds, that is, the net wages of the individuals with a low level of human capital are greater than the cost of private education. In the second case, the sign of this inequality is reversed.

[16] It is assumed that the number of quotas in the public university is such that $a^*_{pu,l} > a^*_{pr,l}$.

Proposition 3: With $(1-\tau)w_{l,0} > E$ and $\pi_{pr}(a) = \pi_{pu}(a)$, $\forall a$, the introduction of the system of quotas reduces EFF_{pu} and does not affect $a_{pr,l}$, $a_{pr,h}$, EFF_{to} and DIF.

The above proposition indicates that the introduction of the system of quotas does not affect either the efficiency of the total expenditures in education (EFF_{to}), or the difference in the threshold ability behavior of both types of parents (DIF). However, it reduces the efficiency of the public expenditures in education (EFF_{pu}).

The economic intuition behind these results runs as follows. With quotas, individuals with lower ability on average attend public universities. With the same amount of public expenditures in education (vE), the fraction of individuals who acquire a high level of human capital is lower. This fact explains the reduction in EFF_{pu}.

Moreover, on the one hand, individuals who benefit from the quotas simply move from the private university to the public one. The quality of the universities is the same, so families opt for the one free of charge. On the other hand, individuals who lose their right to attend public universities move to private universities. This increases the average ability of those individuals attending private universities. Combining these two effects and with the assumption that the quality in both types of universities are the same, the variable EFF_{to} does not undergo any change. That is, the loss in efficiency of the public investments is offset exactly by the gain in efficiency of the private ones.

As there is no change in the private incentives to invest in education, the expected skill premium is the same and $a^*_{pr,l}$ and $a^*_{pr,h}$ do not change. Hence, total investments in education and the fraction of individuals who acquire a high level of human capital are equal to those figures in the equilibrium without quotas. Finally, the system of quotas works as a way of transferring income from families who lose their right to send their children to the public university (the high-income families) to the ones who acquire the right (the low-income families).

I now turn to the analysis of the situation in which parents with a low level of human capital do not have enough resources to pay for private education, that is, when $(1-\tau)w_{l,0} < E$.[17]

Proposition 4: With $(1-\tau)w_{l,0} < E$ and $\pi_{pr}(a) = \pi_{pu}(a)$, $\forall a$, the introduction of the system of quotas reduces EFF_{pu} and DIF, and increases $a_{pr,h}$, and EFF_{to}.

The above proposition asserts that the introduction of the system of quotas is beneficial when $(1-\tau)w_{l,0} < E$. In spite of the reduction in EFF_{pu}, the other indicators of efficiency of the educational sector, DIF and EFF_{to}, improve.

These results can be explained in the following way. As in the previous case, with quotas, individuals with lower ability on average attend public universities,

[17] In the special case when $(1-\tau)\,w_{l,0} < E$, there is no private investment in education from families whose parents have a low level of human capital. In this case, the variable DIF is defined in the following way: $DIF = \left| a^*_{pr,h} - a^*_{pu,l} \right|$.

leading to a reduction in EFF_{pu}. The difference now is that the beneficiaries of the quotas would not have been attending private universities under the system without quotas, as their families could not afford them. With this change, the number of individuals from low-income families who attend public universities increases and the ability threshold of those individuals is lower, moving from a^*_{pu} to $a^*_{pu,l}$. Ceteris paribus, the expected number of individuals with a high level of human capital at time $t = 1$ increases and the expected skill premium drops. Hence, the economic incentive to invest in private education diminishes. The implications are that, at the margin, the number of individuals from the high-income class who attend private universities diminishes and the ability threshold of those individuals is greater. Combining the effects on the ability threshold of the individuals from both types of families who attend university, the result is a lower value for the variable DIF.

Finally, the total amount invested in education is lower in this new equilibrium with quotas. Public investments remain unaltered, but the amount of private investment is lower. Nonetheless, the fraction of individuals with a high level of human capital at time $t = 1$ increases because individuals with a greater ability on average are attending university. This fact explains a greater EFF_{to}. The lower efficiency of the public investments in education is more than offset by the greater efficiency of the private investments.

The two above propositions show very different effects of the system of quotas in terms of the efficiency of the educational sector. The results are very dependent on whether individuals with a low level of human capital (the poorer individuals) can or cannot afford to pay for their children to attend a private university. In reality, one might think that many beneficiaries of the system of quotas would be those who could actually pay for a private education. In this case, the quotas would simply be a way of transferring income to poorer families. However, it is not unlikely that some beneficiaries would not be able to attend a university without the quotas, allowing the quotas to make the educational system more efficient. With the same quality in both types of universities, the likely result of the system of quotas would probably be a mix of those obtained in the two above propositions.

4.2 Different Quality in Private and Public Universities

The objective of this subsection is to analyze the effects of the system of quotas on the different variables that measure the efficiency of the educational sector when the quality of both types of universities, public and private, are different.[18] Assuming that the public university has a higher quality, the following result is obtained:

Proposition 5: With $\pi_{pu}(a) > \pi_{pr}(a)$, $\forall a$, the introduction of the system of quotas reduces EFF_{pu}, EFF_{to}, $a_{pr,l}$, $a_{pr,h}$ and DIF.

As in the previous subsection, with quotas, public expenditures in education become less efficient. As before, the reason is that the ability of individuals who

[18] Throughout this subsection, it is assumed for simplicity that $(1 - \tau)w_{l,0} > E$.

attend the public university is on average lower.

The explanation for the reduction in EFF_{to} runs as follows. If the ability threshold for both types of individuals attending private universities ($a^*_{pr,l}$ and $a^*_{pr,h}$) did not change with the introduction of quotas, the number of individuals with a high level of human capital at time $t = 1$ would be lower and the skill premium greater. The reason is that some of the individuals with the greatest abilities would be taken away from the universities of higher quality, the public ones. The allocation of human resources between the two types of universities would thus not be the most efficient one. Hence, individuals would have a greater economic incentive to invest in education and the ability thresholds ($a^*_{pr,l}$ and $a^*_{pr,h}$) could not stay the same. The new equilibrium is one with a lower ability threshold for both types of individuals. With more individuals with lower ability attending the university and the inefficient allocation of human resources between both types of universities, EFF_{to} is necessarily lower. Due to the concavity of the probability function, the reduction in $a_{pr,l}$ is greater than the one in $a_{pr,h}$, for a given increase in the skill premium. Hence, DIF is lower in the new equilibrium. The ideal reduction in the variable DIF should come concomitantly with the increase in the efficiency of total expenditures in education, which is not the case here.

The following proposition presents the effect of quotas when the private university has a greater quality.

Proposition 6: With $\pi_{pu}(a) < \pi_{pr}(a)$, $\forall a$, the introduction of the system of quotas reduces EFF_{pu}, and it increases EFF_{to}, $a_{pr,l}$, $a_{pr,h}$ and DIF.

The explanation of the results in the above proposition is analogous to the one provided for proposition 5. For the same reason as before, the public investments in education are less efficient. With quotas, if the ability threshold for both types of individuals attending private universities ($a^*_{pr,l}$ and $a^*_{pr,h}$) did not change, the number of individuals with a high level of human capital at time $t = 1$ would be greater and the wage premium would be lower. The reason is that some of the individuals with the greatest abilities would be moved to the university of higher quality, the private one. They would had previously opted for the public university, because it is free of charge.[19] The allocation of human resources between the two types of universities becomes more efficient. Hence, the individuals would have less economic incentive to invest in education and the ability thresholds ($a^*_{pr,l}$ and $a^*_{pr,h}$) could not stay the same. The new equilibrium is one with a higher ability threshold for both types of individuals. With a greater ability of individuals who attend university on average and the more efficient allocation of human resources between both types of universities, EFF_{to} is necessarily greater. Due to the concavity of the probability function, the increase in $a_{pr,l}$ is greater than the one in $a_{pr,h}$, for a given reduction in the skill premium. Hence, DIF is greater. This is the side effect of the imposition of quotas when the quality of the private university is greater.

[19] See assumption 4 above.

5 Conclusion

There are have been some initiatives to introduce some form of affirmative action in the Brazilian public universities. The intended beneficiaries of these policies are either black individuals or individuals who have studied in public schools. The analysis in this paper suggests that, if one concern of policymakers is the efficiency of the educational system, the introduction of the system of quotas should be adopted with caution.

It could be the case that many beneficiaries of quotas would be families who could actually pay for a private education. Quotas would then simply be a way of transferring income to low-income families. However, it is not unlikely that some beneficiaries would not be able to attend a university without the help of quotas. The reduction in the link between income's family background and the capacity of an individual to attend a university leads to an overall efficiency of the system.

If adopted, quotas should not be implemented across public universities and courses within the same university indiscriminately. With quotas, some individuals who have performed better on the admissions exams lose their right to attend public universities. If the relationship between the performance in these exams and academic ability is strong, inevitably the efficiency of the public expenditures in higher education is reduced. In other words, there is a decrease in the quality of the labor force produced by these public investments. However, the main concern of the policymakers should be the overall efficiency of the educational system.

In this regard, quotas could make the system more efficient when the quality of courses in private universities is greater. Without quotas, individuals may opt to attend public universities because they are free, when the difference in quality between both types of universities (public and private) is not great enough to justify incurring the cost of private tuitions. Quotas would correct this distortion and allocate human resources in a way that pushes individuals with greater abilities to attend the more qualified universities. The overall efficiency of the system increases. However, the opposite occurs when the quality of courses in public universities is higher. With quotas, there is a shift in the allocation of human resources between both types of universities, which turns out not to be very efficient. Individuals with the greatest abilities are taken away from the universities with greater quality, which implies a less efficient system.

The Ministry of Education carries out regular evaluation of the Brazilian universities in various ways. One of these forms is through a national exam, called "Provão", which all undergraduate students have to take in order to obtain a university degree. Based on this exam, the government ranks universities in different courses of study. One possibility is to use these ranks to structure the system of quotas in a way suggested by the analysis in this paper, in order to improve the overall efficiency of the system. That is, quotas should be less used (or even non-existent) for the most highly ranked public universities or courses.

The structure of the model in this paper does not include some features

discussed in the literature that analyzes the effects of affirmative action policies. Three of them are worth mentioning. First, students could benefit from the presence of individuals from different races, ethnicities, and income's family backgrounds on universities campuses. An environment characterized by diversity, which allows the exchange of different experiences of life among students, could have important impacts on the students' learning experiences, including a reduction in discrimination over time. Second, quotas can make less rare the presence of role models from different races, ethnicities and income's family backgrounds. These individuals who are well successful in their careers, could serve as "examples" to younger individuals from less favorable groups in society, signalling to them that they can also succeed in their lives, and serving as an incentive for them to pursue their objectives. Third, the average academic ability of the students in a classroom affects the performance of each of its members. This externality factor is commonly referecend and widely accepted in the literature. Quotas, by altering the allocation of human resources among the universities, could certainly affect this externality factor.

However, the empirical evidence in the United States, the country that has adopted more aggressively affirmative action policies, does not indicate that the first two factors (the diversity and the "role model" factors) can impact in a significant way the educational quality of higher education. The last factor, that of externality, would actually reinforce the theoretical results obtained in this paper.[20]

Finally, the system of quotas represents a way of tranferring income to its beneficiaries, black individuals or individuals who have studied in public schools. They can in general attend better universities without incurring the tuition cost. Policymakers who have income transfers to target groups as one of their objectives could use quotas as one instrument to reach their goals. This paper emphasizes the potential trade-off of this policy as the resulting reduction in efficiency. There is an alternative, however, that can accomplish both objectives, of making transfers to target groups and increasing efficiency. It is to eliminate the system of free tuition for all individuals, independently of their family background, who pass the admissions test for the public universities. The system of quotas becomes an irrelevant option if the government finances, partially or fully, higher education only for those individuals who can not afford it.

6 Appendix

Proposition 1: For any given wage differential: (i) there exists an unique $a^*_{pr,i}$ which is a function of the wage differential $(w_{h,1} - w_{l,1})$ such that $a^*_{pr,i}(w_{h,1} - w_{l,1}) \in (a_{pu}, 0)$, $i = h, l$, and a parent with i level of human capital sends his child to university if $a \geq a^*_{pr,i}(w_{h,1} - w_{l,1})$, and does not otherwise; and (ii) $a^*_{pr,h}(w_{h,1} - w_{l,1}) < a^*_{pr,l}(w_{h,1} - w_{l,1})$.

[20] See Holzer and Neumark (2000) for the review of the literature on these and other aspects of the effects of affirmative action policies.

Proof. With assumption 3, $a^*_{pr,i} < a_{pu}$, $i = h, l$. Using the fact that $\pi_{pr}(0) = 0$ from assumption 1, one obtains:

$$\begin{aligned} & u\left((1-\tau)w_{i,0} - E\right) + \beta\left[\pi_{pr}(0)u\left(w_{h,1}\right) + \left(1-\pi_{pr}(0)\right)u\left(w_{l,1}\right)\right] = \\ = \quad & u\left((1-\tau)w_{i,0} - E\right) + \beta u\left(w_{l,1}\right) < u\left((1-\tau)w_{i,0}\right) + \beta u\left(w_{l,1}\right), \end{aligned}$$

for any possible value for $w_{l,1}$. Hence, $a^*_{pr,i} > 0$, $i = h, l$. Combining both results, there exist threshold abilities $a^*_{pr,i} \in (a_{pu}, 0)$, $i = h, l$, such that a parent with i level of human capital is better off sending his child to private university if his child's ability is greater or equal to $a^*_{pr,i}$.

For any given wage differential, parent i sends his child to private university if and only if:

$$\beta\pi_{pr}(a)[u\left(w_{h,1}\right) - u\left(w_{l,1}\right)] > u\left((1-\tau)w_{i,0}\right) - u\left((1-\tau)w_{i,0} - E\right) = A_i.$$

With the usual properties of the utility function, $A_h < A_l$ which implies that $a^*_{pr,h} < a^*_{pr,l}$ for any wage differential. ■

Proposition 2: There exists a competitve equilibrium and it is unique.

Proof. The equilibrium is characterized by five unknowns ($w_{h,1}$, $w_{l,1}$, $n_{h,1}$, $a_{pr,h}$, $a_{pr,l}$) and five equations (two first-order conditions of the firm's problem, the law of motion for the variable $n_{h,t}$ and the threshold ability for both types of individuals).

From the first-order conditions for the firm's problem, it is easy to see that $w_{h,1}$ decreases with $n_{h,1}$ and $w_{l,1}$ increases with $n_{h,1}$. Departing from the situation in which there is only public investment in education and using assumptions 3 and 4, one finds that:

$$\beta\pi_{pr}(a_{pu})\left[u\left(w'_{h,1}\right) - u\left(w'_{l,1}\right)\right] > u\left((1-\tau)w_{i,0}\right) - u\left((1-\tau)w_{i,0} - E\right) = A_i, \tag{3}$$

where the v individuals with greatest abilities attend the public university and $w'_{i,1}(i = h, l)$ is the wage of individual type i at time 1 if there is no investment in private education. The RHS is constant. As the LHS is greater than the RHS in equation (3), individuals from both types of families will invest in private education. They have incentives to reduce the threshold ability (from equation (3)). Hence, $n_{h,1}$(from equation (2)) and $w_{l,1}$ (from the firm's first-order condition) increase, and $w_{h,1}$ (from the firm's first-order condition) decreases . All these changes lead to a reduction in the LHS. As $\pi_{pr}(0) = 0$, there is one and only one fraction of individuals with high level of human capital (and then unique wages and threshold abilities) that equates the LHS and the RHS in equation (3). ■

Proposition 3: With $(1-\tau)w_{l,0} > E$ and $\pi_{pr}(a) = \pi_{pu}(a)$, $\forall a$, the introduction of the system of quotas reduces EFF_{pu} and does not affect $a_{pr,l}$, $a_{pr,h}$, EFF_{to} and DIF.

Proof. In the competitive equilibrium without quotas, the following condition holds:

$$\beta\pi_{pr}(a^*_{pr,i})\left[u\left(w^*_{h,1}\right) - u\left(w^*_{l,1}\right)\right] = u\left((1-\tau)w_{i,0}\right) - u\left((1-\tau)w_{i,0} - E\right) = A_i.$$

With quotas, ceteris paribus, the shift in the allocation of human resources does not change the fraction of individuals with high level of human capital. Hence, it does not affect $w^*_{h,1}$ and $w^*_{l,1}$. Thus, there is no change in the incentive for the indiviuals to change their expenditures in private education and $a^*_{pr,i}$ ($i = h, l$) remains unaltered. The above equality still holds with the same $a^*_{pr,i}$, $w^*_{h,1}$ and $w^*_{l,1}$. Hence, DIF and EFF_{to} do not change.

EFF_{pu} reduces because the numerator with quotas is lower. That is, $\left(n^*_{h,0}\left[\int_{a^*_{pu}}^{1}\pi_{pu}(a)f(a)da\right]+n^*_{l,0}\left[\int_{a^*_{pu}}^{1}\pi_{pu}(a)f(a)da\right]\right)$ is greater than $\left(n^*_{h,0}\left[\int_{a^*_{pu,h}}^{1}\pi_{pu}(a)f(a)da\right]+n^*_{l,0}\left[\int_{a^*_{pu,l}}^{1}\pi_{pu}(a)f(a)da\right]\right)$. ■

Proposition 4: With $(1-\tau)w_{l,0} < E$ and $\pi_{pr}(a) = \pi_{pu}(a)$, $\forall a$, the introduction of the system of quotas reduces EFF_{pu} and DIF, and increases $a_{pr,h}$, and EFF_{to}.

Proof. In the competitive equilibrium without quotas, the following condition holds:

$$\beta\pi_{pr}(a^*_{pr,h})\left[u\left(w^*_{h,1}\right)-u\left(w^*_{l,1}\right)\right]=u\left((1-\tau)w_{h,0}\right)-u\left((1-\tau)w_{h,0}-E\right)=A_h.$$

With quotas, ceteris paribus, the shift in the allocation of human resources does change the fraction of individuals with high level of human capital, increasing it. Hence, the above equality does not hold anymore with the threshold ability $a^*_{pr,h}$, as the skill premium reduces. As a consequence, the new equilibrium is characterized by a greater value for $a_{pr,h}$. As $a_{pu} > a_{pu,l}$, then DIF reduces. For the same reason as in proposition 3, EFF_{pu} is lower. EFF_{to} increases because the average ability of the individuals attending university is greater. ■

Proposition 5: With $\pi_{pu}(a) > \pi_{pr}(a)$, $\forall a$, the introduction of the system of quotas reduces EFF_{pu}, EFF_{to}, $a_{pr,l}$, $a_{pr,h}$ and DIF.

Proof. In the competitive equilibrium without quotas, the following condition holds:

$$\beta\pi_{pr}(a^*_{pr,i})\left[u\left(w^*_{h,1}\right)-u\left(w^*_{l,1}\right)\right]=u\left((1-\tau)w_{i,0}\right)-u\left((1-\tau)w_{i,0}-E\right)=A_i.$$

With quotas, ceteris paribus, the shift in the allocation of human resources does change the fraction of individuals with high level of human capital, reducing it. Hence, the above equality does not hold anymore with the threshold ability $a^*_{pr,h}$, as the skill premium increases. As a consequence, the new equilibrium is characterized by a lower value for $a_{pr,h}$ and $a_{pr,l}$. For the same reason as in proposition 3, EFF_{pu} is lower. EFF_{to} reduces because the average ability of the individuals attending university is lower. Due to the concavity of the probability function (from assumption 1), the reduction in $a_{pr,l}$ is greater than the one in $a_{pr,h}$, for a given increase in the skill premium. Hence, DIF is lower. ■

Proposition 6: With $\pi_{pu}(a) < \pi_{pr}(a)$, $\forall a$, the introduction of the system of quotas reduces EFF_{pu}, and it increases EFF_{to}, $a_{pr,l}$, $a_{pr,h}$ and DIF.

Proof. In the competitive equilibrium without quotas, the following condition holds:

$$\beta\pi_{pr}(a^*_{pr,i})\left[u\left(w^*_{h,1}\right)-u\left(w^*_{l,1}\right)\right]=u\left((1-\tau)w_{i,0}\right)-u\left((1-\tau)w_{i,0}-E\right)=A_i.$$

With quotas, ceteris paribus, the shift in the allocation of human resources does change the fraction of individuals with high level of human capital, increasing it. Hence, the above equality does not hold anymore with the threshold ability $a^*_{pr,h}$, as the skill premium decreases. As a consequence, the new equilibrium is characterized by a greater value for $a_{pr,h}$ and $a_{pr,l}$. For the same reason as in proposition 3, EFF_{pu} is lower. EFF_{to} increases because the average ability of the individuals attending university is greater. Due to the concavity of the probability function (from assumption 1), the increase in $a_{pr,l}$ is greater than the one in $a_{pr,h}$, for a given reduction in the skill premium. Hence, DIF is greater. ■

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