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Andrea Maria Accioly F. Minardi

Insper Working Paper
WPE: 059/2006

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Abstract

The equity market value can be seen as a call option on the firm's asset. The exercise price is the debt face value. We built a sample composed by the most traded stocks of Brazilian companies rated by Moody's or Standard& Poor's and estimated for each stock the default probability reflected on price using the Option Theory Model. We compared the estimated probability with the historical mortality rate published by Moody's and assigned each company a credit rating. We than compared the assigned rating to the agencies' domestic rating and concluded that in general both ratings are in accordance.

key words
option theory, credit rating, default probability

I. Introduction

"Credit Monitor", a dynamic financial service that discloses default probability estimated by Option Theory Model (OTM), has been the lead product of KMV since 1993. KMV is a credit risk software house that belongs to Moody's. McQuown (1993) has analyzed more than two thousands US industrial companies that were liquidated or went default during the last 20 years. In all cases, he observed that the default probability estimated by OTM increased drastically one or two years before the default event occurred. The shift in the OTM default probability anticipated one year or even more the downgrade in the Moody's and Standard & Poor's credit ratings.

Our objective is to estimate default probabilities of Brazilian public companies through the OTM and analyze if the results are adequate. We did so by comparing OTM results with Standard&Poor's and Moody's credit ratings in domestic currency. The investigation whether Option Theory estimates better ratings than accounting ratings, as suggested by McQuown (1993) is beyond the scope of this paper. For nine out of fifteen companies we estimated the same credit rating assigned by the agencies (at the level of the big letter). In only one company we observed OTM rating lower than the agencies' rating. In five companies we observed OTM ratings superior than the agencies' ratings, but only for Vale do Rio Doce, Petrobras and CEMIG the divergence was severe.

We structured the paper as following. In section II we explain and define credit rating; in section III we explain how to apply Option Theory to estimate default probability; in section IV we describe the sample and present the results and in section V we conclude the paper.

II. Credit Ratings

The two biggest credit risk agencies are Standard & Poor's and Moody's. According to Crouhy, Galai and Mark (2000), their market position was dominant in such an extension that the US Justice Department inquiries have investigated whether there were anticompetitive practices in the bond rating industry. The following agencies in terms of reputation and size are Duff & Phelps and Fitch. Standard & Poor's and Moody's rate issues and issuers, and anyone can have free access to their opinion. Their credit ratings are strongly correlated to interest rates.

A credit rating is not a recommendation to invest in a certain issuer or issue. Standard & Poor's (2003) defines issue credit rating as a current opinion about the credit quality of a certain issuer relative to a specific financial obligation, or to a specific class of financial obligations, or to a specific financial program. It takes into account the creditworthiness of the guarantors, insurers and any other forms of credit enhancement on the obligation.

Standard & Poor's (2003) defines issuer's credit rating as a current opinion of an obligor's overall financial capacity to pay its financial obligations. This opinion is based on the obligor's capacity and willingness to meet its financial obligations as they come due. It does not apply to any specific obligation, because it does not consider the nature and provisions of a specific obligation and neither the creditworthiness of the guarantors and insurers. Corporate issuer credit rating applies when the issuer is a company and sovereign credit rating applies when the issuer is a country.

Credit ratings are based on current information furnished by obligors or obtained from other sources that the agencies consider reliable. They may be changed, suspended or withdrawn as a result of changes in, or unavailability of such information.

The credit risk analysis considers also the country risk. The currency the payments are denominated is a key factor in this analysis. Financial obligations denominated in local currency have a higher probability of payment than obligations

denominated in foreign currency. This is due to the fact that if the obligor's country defaults, the Central Bank can retain all capital flows that would exit the country. Even though the obligor has a good credit quality and is willing to pay its foreign debt, it can be impeded of doing so by the Government, in country's default event. Sovereign credit risk is incorporated in issue's and issuer's credit ratings. Credit rating in domestic currency may be different than the credit rating in foreign currency for the same obligor.

Usually issuer credit rating in foreign currency is upward bounded by the sovereign credit rating. Issue credit rating in foreign currency is not necessarily subject to the sovereign rating roof, because the obligation may enclose collaterals, insurance and other protections that guarantee its payments even in the event of a country default.

Table I contains a description of Moody's and Standard & Poor's rating grades. According to Altman, Caouette. and Narayanan(1998), Standard&Poor's assigns grades based on business risk (industry characteristics, competitive position, management) and financial risk (financial characteristics, financial policies, profitability, capital structure, cash flow protection, financial flexibility). Industry risk (the industry competitive position and stability) is the factor that deserves more weight in the rating decision. Moody's focuses also on business fundamentals, as the nature of supply and demand, market leadership and cost positions. In analyzing financial risk, Standard&Poor's estimates a number of financial rations (interest coverage, leverage and cash flow) and track their evolution along the time. Although there are divergences, most times Standard&Poor's and Moody's grades are coincident, at least at the level of the big letter. The historical default rates by grades of both agencies are pretty similar too.

Table II shows the historical cumulative default rates by rating grades at the level of the big letter (AAA, AA, A, BBB, BB, CCC) published by Moody's (2004). Table III exhibits the cumulative default rates by gradations of the big letter, and was built from the migration matrix published by Moody's (2004). We can observe that default rates are negative correlated to rating quality and increases significantly for the speculative grades.

The financial market corroborates the rating agencies' opinions. Table IV shows data collected from Altman, Caouette and Narayanan (1998). We can observe that the interest rate or yield required by an investor is inversely proportional to the obligation's rating quality. The poorest is the rating quality, the higher is the required yield.

In Table V we can see the rating grades assigned to Brazilian companies published by Standard&Poor's and Moody's. Standard&Poor's publishes ratings in foreign and domestic currencies and Brazil. Moody's publishes ratings in foreign and domestic currencies. As we can note, the rating grades in foreign currency are always lower than the grades in domestic currency. This is due to Brazilian sovereign risk. The impact in the cost of debt of Brazilian Companies is substantial. Bloomberg builds indexes of American industrial corporate bonds by Standard& Poor's rating grades. Table VI shows yield curves of rating grade indexes at November 1st 2005. S&P graded Aracruz and Votorantim a BB- as foreign currency and AAA as Brazil rating. Using Table VI data, we can estimate roughly an increase of 2.12% in the interest rate of a 10 year financial obligation. Banco Itaú, CSM and Klabin were graded a BB- in foreign currency and AA in local currency. The same analyzes estimates roughly an increase of 2.06% in the 10 year fixed income interest rates. Petrobras was rated BBB+ in domestic currency, and downgraded to B- at foreign currency, meaning a 1.95% increase in interest rate.

III. The Use of Option Theory to Estimate Default Probability

Credit risk structural models assume that a company goes bankrupt when its asset market value (A) falls bellow the face value of its debt (B). Some academic examples can be found at Wilcox (1973), Scott (1981) and Santomero and Vinso (1977). The fundamental idea was first proposed by Merton (1974) in his Firm Theory. According to the Firm Theory, the equity market value can be priced as a call option with the underlying asset being the asset market value (A) and the exercise price being the face value of debt (B).

If the company has shares traded in the stock market, it is possible to observe the equity market value, the stock return volatility and the debt book value. If we assume that the asset value behaves according to a Geometric Brownian motion with a constant mean r, a constant standard deviation σ_A , and a lognormal distribution, it is possible to estimate the asset value through the Black & Scholes (1973) equation:

$$E = AN(h_1) - Be^{-r\tau}N(h_2) \qquad (1)$$

$$h_1 = (\ln(A/B) + (r+\sigma_A{}^2/2)\tau)/(\sigma_A(\tau)^{1/2})$$

$$h_2 = h_1 - \sigma_A(\tau)^{1/2}$$

where:

E = equity market value (call option value)

A = asset market value

B = face value of debt

r = continuously compounded risk free return rate

σ_A = standard deviation of asset return rates

N(.)= standard cumulative normal distribution

We can observe the standard deviation of equity returns, but not of asset returns. But the following expression relates asset's volatility to equity's volatility:

$$\sigma_E = (A\ (dE/dA))/E * \sigma_A \qquad (2)$$

With two equations (Black and Scholes and the relation between asset and equity volatilities) we can conduct an iterative process to estimate the two unknowns: asset current value and asset return volatility. The asset value distribution in moment t (A_t) is described as follows:

$$A_t = A_0 e^{\left\{\left(m - \frac{s_A^2}{2}\right)t + s_A\sqrt{t}Z_t\right\}} \qquad (3)$$

Where Z_t= N (0, 1) and μ is the instantaneous asset return rate. The asset expected value at moment t is equal to $A_0e^{\mu t}$.

If we assume that the firm's capital structure is composed only by equity and one zero coupon bond due in T years, with face value of B, the default can occur only if the asset value falls bellow B at debt maturity.

This model was commercialized by KMV, a credit risk software house that today belongs to Moody's. The model generates an EDF (estimated default frequency) for each company monitored by KMV. The EDF is calibrated to measure default probability in a one-year horizon. KMV defines default as the event of nonpaying interest installment, principal or any other financial commitment as they come due. The default event is different than the bankruptcy event. It occurs before the firm goes bankrupt, and therefore before the asset value falls bellow B. KMV assumes that a company enters default when

its asset value falls bellow the default point (DPT), and arbitrarily calculates it as the value of the short term debt plus half of the value of long term debt.

$$DPT = STD + \frac{LTD}{2} \quad (4)$$

Where STD is the book value of short term debt and LTD is the book value of long term debt.

The default distance (DD) equals how many times the distance between the asset expected value at T and the default point is greater than the asset return standard deviation.

$$DD = (E(A_t) - DPT) / \sigma_A \quad (5)$$

Assuming that the asset value has a lognormal distribution, we can define the default distance as follows.

$$DD = \frac{\ln(A_0 / DPT_t) + (m - (1/2)s_A^2)T}{s_A \sqrt{T}} \quad (6)$$

Where A_0 is the asset current value, DPT is the default point in T years, μ is the after tax asset return and σ_A is the asset return volatility.

EDF equals to the probability that the asset value is bellow B.

$$EDF = \text{Probability}(z < -DD) \quad (7)$$

According to McQuown (1993), models based on market information are superior than models based on accounting information, because prices are ex-ante information continuously reviewed while accounting information reflect past, and are at best quarterly reviewed. Bader and Sanvicente (1996) observed that stock prices of companies that went to *concordata* (the Brazilian process that corresponds to Chapter 11) presented negative cumulative abnormal returns three years and in some cases five years before the *concordata* event.

IV. Methodology and Results

We estimated default probabilities using Option Theory for all Brazilian companies listed on Table V (graded by Moody's and/ or S&P).

IV.1. Estimation of Default Probability using Option Theory

Initially we estimated asset value per share and asset volatility for each sample company using equations (1) and (2). We collected all required market and accounting information at Economatica (a Brazilian database). The equity market value per share (E) equals the closing price per share at December 2004 of the most traded company's stock. We calculated the default point (DPT) as the short term debt book value per share plus half of the long term debt book value per share at December 2004, as suggested in equation (4). We used consolidated financial statements. We also collected historical series of monthly closing prices of the sample stocks from December 1999 till December 2004 in order to generate equity historical return series as follows.

$$R_{it} = \ln(P_{it}/P_{i,t}-1) \qquad (8)$$

Where :

R_{it} = stock i return at month t

Ln(.) = natural logarithm

P_{it} = stock i price at month t

$P_{i,t-1}$ = stock i price at month t-1

The equity volatility (σ_e) was calculated as the standard deviation of the equity return time series.

We conducted an iterative process in three steps to calculate the asset value (A) and return volatility (σ_A). In the first step we calculate the asset value per share that solves equation (1). In the second step we calculate the asset volatility that solves equation (2). In the third step we substitute the asset volatility calculated in step 2 and run again steps 1 through 3. We stop when the round n asset volatility calculated in step 2 equals the asset volatility in round n-1.

After estimating asset value and asset return volatility, we estimate the default probability (EDF) using equations (6) and (7). One exogenous but necessary data is the asset instantaneous return rate (μ), because the expected asset value in one year equals $A_0 e^{\mu T}$.

The asset instantaneous return rate corresponds to the weighted average of expected equity return in one year and expected debt return. The weight of each component is its share in the asset financing structure.

$$m_i = \left(\frac{E_i}{A_i}\right) k_{ei} + \left(\frac{B_i}{A_i}\right) k_d \qquad (9)$$

Where:

E_i = equity market value of company i

A_i = asset market value of company i

k_{ei} = instantaneous equity expected return rate of company i

B_i = debt market value of company i

k_d = instantaneous debt expected return rate of company i

The equity expected return was estimated by the CAPM (Capital Asset Pricing Model) developed by Sharpe (1964), Lintner (1965), and Mossin (1966).

$$k_{ei} = r_f + \beta_i * (E(r_m) - r_f) \qquad (10)$$

Where:

k_{ei} = equilibrium expected equity return

r_f = risk free asset rate of return

β_i = beta coefficient of stock i, that measures the sensibility of the company i's stock returns to market portfolio returns.

$E(r_m)$ = market portfolio expected return

We used the Brazilian saving account as a proxy for the risk free asset. Its interest rate at December 31st 2004 was 9.27% per year

The equity beta of each company stock was estimated as follows.

$$R_{i,t} = \alpha_i + \beta_i * R_{m,t} + e_{i,t} \quad (11)$$

Where:

$R_{i,t}$ = historical stock i return time serie from December 1999 till December 2004

α_i = constant for stock i equation estimated by Ordinary Least Square regression model

β_i = equity beta coefficient for company i estimated by the OLS regression model

$R_{m,t}$ = IBOVESPA return time series from December 1999 till December 2004

$e_{i,t}$ = regression's error term series

We adopted the methodology suggested by Minardi and Sanvicente (2003) to estimate the market expected portfolio return used in the CAPM. Under the assumption that dividends growth at a constant rate (g), the equity intrinsic value (V_0) is calculated by the Gordon Model:

$$V_0 = \frac{D_1}{k_e - g} \quad (12)$$

where D_1 is the dividend to be paid in the beginning of period 1 and k_e is the equity equilibrium expected rate of return.

If the market is efficient, the equity intrinsic value equals the market equity value. Then, it is possible to estimate the equity equilibrium expected return as follows:

$$k_e = \frac{D_1}{P} + g \quad (13)$$

The market portfolio rate of return was estimated as the equal weighted average of all available stock returns. Assuming that $D_1 = D_0*(1+g)$, we can estimate the expected market portfolio return as:

$$E(r_m) = average\left(\frac{D_0}{P_0}\right) \times [1 + average(g)] + average(g) \quad (14)$$

Minardi and Sanvicente (2003) modeled the dividend growth rate as the growth rate the company can sustain in the long term keeping the same capital structure and the same dividend police.

$$g = ROE \times b \quad (15)$$

Where ROE is the return on equity and b is the plowback ratio. The plowback ratio equals 1 – dividend payout ratio.

In order to estimate the market portfolio expected rate of return we collected closing price at December 2004 of all public companies' most traded stock, dividend per share in December 2004 and estimated growing rates for 2002, 2003 and 2004 using the corresponding earning per share and ROE of each company. We dropped all companies that did not have shares traded in December 2004, presented accounting losses and negative ROE in any of the year 2002, 2003 and 2004. We used the 2002, 2003 and 2004 average growing rate. The final sample was composed by 90 companies and the E (r_m) was estimated in 16.60% per year.

We do not observe the instantaneous debt expected rate of return. If the company faces financial distress, this rate can be very close to zero or even negative, because debtholders do not expect that their investment position will increase in value. Besides, whenever there is a financial distress situation, debt weight in capital structure is higher than equity weight.

As the instantaneous expected asset return rate (μ) corresponds to the weighted average between equity and debt expected return rates, its value range from zero and k_e. So, we estimated two bound values for the EDF. For the superior limit, that is, the pessimistic or worst scenario, we considered that asset value return rate for the one-year horizon is zero. For the inferior limit, that is, the optimistic or best scenario, we considered that the asset value return in the one-year horizon will equal k_e.

Table VII presents the required data, the default probabilities estimated if asset return equals k_e (best scenario) or zero (worst scenario).

It is interesting to point that in all cases where the default point per share was much inferior to the stock price the convergence was very fast: two or three rounds. We had to run much more rounds in the cases where the default point per share was superior to the stock price.

We could not estimate EDF for commercial banks, Eletrobras and All America because of missing required data.

IV.2. Credit Rating Assignment and Result evaluation

We compared the estimated default probabilities (EDF) with the historical mortality rates for one-year horizon published by Moody's (Tables II and III) and assigned credit ratings at the level of the big letter.

Table VIII shows the assigned rating based on EDF for the best case and worst case scenarios, Moody's credit ratings in domestic currency and S&P credit rating Brazil. We used ratings with domestic currency to exclude Brazil country risk effect.

Consider Table VIII and compare columns 3 and 4 (S&P) or 5 and 6 (Moody's). There is no difference in assigned credit ratings in the best and worst scenarios for 12 out of 18 companies. There are three companies graded CCC or Caa in the worst scenario and a higher grades in the best scenario: Centrais Elétricas Paulista – CESP, Centrais Eléricas do Espírito Santo – Escelsa and Net Serviços de Comunicação. CESP was graded CCC by S&P, Escelsa was graded B2 in foreign currency but was not rated in domestic currency and Net was graded D by S&P.

Compare now column 1 (S&P Brazil) with columns 3 and 4 (best and worst scenarios - S&P) or column and 2 (Moody's domestic) with columns 5 and 6 (best and worst scenarios - Moody's). Although we could calculate EDF for 18 companies, we can compare only 15 of them. Although Escelsa, Cia. Brasileira de Petroleo Ipiranga and Sadia, were graded in foreign rating by one of the two agencies, they were not graded either in rating Brazil by S&P neither by domestic rating by Moody's. For nine companies, assigned ratings are coincident with the agencies. For five companies, the assigned ratings were better than the agencies' ratings. The difference of grades for Klabin (AA- by S&P versus AAA by OTM) and Usiminas (A+ by S&P versus AA by OTM) was not very high, since they were rated by the agencies one grade bellow the OTM estimated rating. It is not the case for Petrobrás (BBB by S&P versus AAA by OTM), Cia. Vale do Rio Doce (Baa2 by Moody's and AAA by OTM) and CEMIG (B1 by Moody's and AAA by OTM). We can observe that Moody's domestic

rating is usually lower than S&P Brazil rating (assigned by the local S&P team). There was only one company where the assigned OTM rating was worst than the agency's: Cosipa (A+ by S&P versus BBB by OTM). But this divergence is not very severe, because it is for neighbors' grade investment ratings.

V. Conclusion

Option Theory can be used to estimate default probabilities of companies with shares traded in stock exchanges. The results in the Brazilian sample were very reasonable and generally coincident with the agencies' opinion. Vale do Rio Doce, Petrobrás and CEMIG were the biggest divergences. Vale do Rio Doce and Petrobrás have a very strong weight in the Brazilian Economy, and the fact that the stock prices reflect a AAA rating seems reasonable. The agencies' rating can reflect though political factors considered relevant for external analysts, but not for Brazilian equity market analysts. The divergence of ratings for CEMIG seems to be more severe, but the comparison was made with Moody's rating in domestic currency, that usually is lower than S&P rating Brazil.

One difficulty in estimating EDF is the instantaneous asset rate of return, because we cannot observe it. The use of worst and best scenario is one way of dealing with the problem. In most divergences, the worst scenario assigned rating was closest to the agencies' ratings. This may reflect the conservative position of credit analysts.

Brazilian sovereign risk has a substantial impact in debt interest rate. For the cases we analyzed, it represented an average increase about 2.00% in yield.

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Table I – Issuer Credit Ratings

Investment Grade Ratings **Speculative Ratings**

S&P and other agencies	Moody's	Interpretation	S&P and other agencies	Moody's	Interpretation
AAA	Aaa	The highest credit quality. Extremely strong capacity to meet financial commitments.	BB+ BB BB-	Ba1 Ba2 Ba3	The least degree of speculation. Probably obligor will meet financial obligations, but faces major ongoing uncertainties or exposure to adverse business, financial or economic conditions.
AA+ AA AA-	Aa1 Aa2 Aa3	Very strong capacity to meet financial commitments.	B+ B B-	B1 B2 B3	Currently has the capacity to meet financial commitment, but adverse conditions will likely impair the obligor's capacity to meet financial commitment.
A+ A A-	A1 A2 A3	More susceptible to the adverse effects of changes in circumstances and economic conditions, but still strong capacity to meet financial commitments.	CCC+ CCC CCC- CC	Caa1 Caa2 Caa3	Obligor is currently vulnerable to nonpayment and its solvency is dependent upon favorable conditions. In the event of adverse conditions, the obligor is not likely to meet financial commitments.
BBB+ BBB BBB-	Baa1 Baa2 Baa3	Adequate capacity to meet financial commitments., but adverse conditions or changing circumstances are more likely to lead to a weakened capacity.	C	Ca	Reserved to "income bonds" in the event of nonpayment.
			D		Default

Table II. Historical Cumulative Default Rate per Rating Grade (1970-2004) – Issuer weighted statistics

Moody's	Years after issuance									
	1	2	3	4	5	6	7	8	9	10
Aaa	0	0	0	0.04	0.12	0.21	0.3	0.41	0.52	0.63
Aa	0	0	0.03	0.12	0.2	0.29	0.37	0.47	0.54	0.61
A	0.02	0.08	0.22	0.36	0.5	0.67	0.85	1.04	1.25	1.48
Baa	0.19	0.54	0.98	1.55	2.08	2.59	3.12	3.65	4.25	4.89
Ba	1.22	3.34	5.79	8.27	10.72	12.98	14.81	16.64	18.4	20.11
B	5.81	12.93	19.51	25.33	30.48	35.1	39.45	42.89	45.89	48.64
Caa-C	22.43	35.96	46.71	54.19	59.72	64.49	68.06	71.91	74.53	76.77

source: Moody's

Table III. Cumulative Default Rate per Rating Grade (1970-2003) – Issuer weighted statistics

Moody's	Years after issuance				
	1	2	3	4	5
Aaa	0.00%	0.00%	0.00%	0.07%	0.20%
Aa1	0.00%	0.00%	0.00%	0.25%	0.27%
Aa2	0.00%	0.00%	0.05%	0.15%	0.33%
Aa3	0.07%	0.10%	0.19%	0.29%	0.42%
A1	0.00%	0.03%	0.32%	0.52%	0.68%
A2	0.02%	0.06%	0.21%	0.43%	0.59%
A3	0.02%	0.21%	0.34%	0.41%	0.49%
Baa1	0.12%	0.42%	0.71%	0.97%	1.19%
Baa2	0.10%	0.34%	0.56%	1.07%	1.53%
Baa3	0.46%	1.09%	1.61%	2.38%	3.00%
Ba1	0.69%	2.00%	3.23%	4.65%	5.84%
Ba2	0.67%	2.35%	4.45%	6.36%	7.85%
Ba3	2.19%	5.49%	9.13%	12.47%	15.38%
B1	3.46%	8.93%	13.90%	17.65%	20.67%
B2	7.65%	14.29%	20.35%	23.61%	25.91%
B3	11.86%	20.17%	26.13%	29.66%	32.19%
Caa-C	26.05%	33.72%	37.98%	41.09%	42.48%

source: Moody's

Table IV. Corporate Bond Credit Spread over US T-Bond 30 years

Year	Ratings						
	AAA	AA	A	BBB	BB	B	CCC
1980	0.60	1.00	1.30	1.90	3.00	3.80	
1981	0.60	1.10	1.30	2.20	3.40	4.10	
1982	0.60	1.00	1.40	2.60	3.60	4.90	9.20
1983	0.70	0.75	0.90	1.57	2.70	3.60	7.60
1984	0.55	0.73	1.04	1.75	2.80	3.10	5.60
1985	0.20	0.58	1.65	1.46	3.10	4.00	6.20
1986	0.95	1.59	1.70	2.33	3.70	4.70	7.90
1987	0.58	1.02	1.18	1.82	2.43	4.03	7.23
1988	0.42	0.79	1.29	1.75	2.07	3.12	7.22
1989	1.28	1.61	2.08	2.49	3.76	4.95	11.91
1990	1.02	1.34	1.77	2.52	4.78	8.57	23.65
1991	0.82	1.13	1.61	2.71	3.92	8.93	12.95
1992	0.90	1.08	1.54	1.77	3.29	4.34	6.46
1993	0.70	0.76	1.74	1.96	2.82	4.15	6.15
1994	0.29	0.62	1.01	1.37	2.23	3.12	6.38
1995	0.43	0.60	1.17	1.49	2.45	4.25	8.75
1996	0.25	0.35	0.54	0.88	2.00	4.08	8.43
1997	0.28	0.39	0.57	0.83	1.53	3.30	7.29

source: Altman, Cauotte and Narayana (1998)

Table V. Credit Rating of Brazilian Corporate Issuers

Company	Standard& Poor's			Moody's	
	Foreign Currency	Domestic Currency	Brazil	Foreign Currency	Domestic Currency
All America			BBB+		
Ambev	BBB		AAA	B1	Baa3
Aracruz Celulose S.A.	BB-	BBB-	AAA		Baa3
Banco Bradesco S.A.	BB			B2	
Banco Itau S.A.	BB-	BB	AA	B2	
Brasil Telecom S.A.			AA+		Baa3
Braskem S.A.	BB-	BB	AA-		
Cia Energética de Minas Gerais - CEMIG					B1
Centrais Elétricas Paulista - CESP			CCC		
Cia Paranaense de Energia - COPEL					
Cia Siderurgica Nacional - CSN	BB-	BB	AA-	B1	
Cia Siderúrgica Paulista - Cosipa	BB-		A+	B1	
Eletrobrás	BB-	BB			
Embratel S.A.					B1
Centrais Elétricas do Espírito Santo - Escelsa				B2	
Gerdau S.A.			AA-		
Cia. Brasileira de Petróleo Ipiranga				B1	
Klabin S.A.	BB-	BB	AA-		
Net Serviços de Comunicação S.A.			D		Ca
Petrobrás S.A.	B-		BBB+	B1	Baa1
Sadia S.A.	BB-	BB			
Unibanco S.A.	BB-	BB		B2	
Usiminas	BB-	BB	A+	B2	
Cia Vale do Rio Doce				Ba1	Baa2
Votorantim S.A.	BB-	BBB-	AAA		

Table VI – Yields according to rating grades and maturity

	Yields						
Maturity years	US T-Strip	US Ind. AAA	US Ind. AA	US Ind. A	US Ind. BBB	US Ind. BB	US Ind. B
0.25	3.91%	4.26%	4.36%	4.49%	4.82%	5.30%	5.91%
0.5	4.26%	4.39%	4.50%	4.60%	4.91%	5.33%	6.08%
1	4.36%	4.69%	4.70%	4.77%	5.02%	5.44%	6.40%
2	4.36%	4.71%	4.74%	4.86%	5.12%	5.81%	6.75%
3	4.43%	4.72%	4.76%	4.87%	5.25%	6.16%	7.09%
4	4.46%	4.75%	4.81%	4.93%	5.35%	6.40%	7.39%
5	4.47%	4.83%	4.89%	4.99%	5.39%	6.62%	7.54%
7	4.52%	4.93%	5.00%	5.11%	5.56%	6.91%	7.73%
8	4.60%	4.99%	5.05%	5.18%	5.64%	7.05%	7.79%
9	4.63%	5.04%	5.11%	5.24%	5.72%	7.11%	7.78%
10	4.69%	5.11%	5.17%	5.30%	5.83%	7.23%	7.77%
15	4.85%	5.37%	5.40%	5.58%	6.10%	7.39%	8.08%
20	4.86%	5.47%	5.51%	5.68%	6.21%	7.40%	8.04%
25	4.79%	5.43%	5.52%	5.69%	6.20%	7.33%	8.00%
30	4.66%	5.30%	5.54%	5.70%	6.25%	7.34%	8.03%

Table VII. Default probabilities estimated by Option Theory (EDF)

Company	Stock price December 2004	DPT December 2004	Equity Volatility	Equity Beta	ke December 2004	Asset Value December 2004	Asset Volatility December 2004	EDF 1 year best ke	EDF 1 ano worst 0%
All America	15.6333	2.4818	N.A.	0.5327	13.18%	N.A.	N.A.	N.A.	N.A.
Ambev	1.3505	0.1030	31.4181%	0.4169	12.33%	1.4444	29.3758%	0.0000%	0.0000%
Aracruz Celulose S.A.	10.1200	2.3100	38.7239%	0.2698	11.25%	12.2257	32.0542%	0.0000%	0.0000%
Banco Bradesco S.A.	64.0807	N.A.	39.7834%	0.8592	15.57%			N.A.	N.A.
Banco Itau S.A.	395.6351	N.A.	38.0786%	0.9278	16.07%			N.A.	N.A.
Brasil Telecom S.A.	0.0133	N.A.	41.7518%	1.0519	16.99%			N.A.	N.A.
Braskem S.A.	0.1317	0.0430	56.9883%	1.0699	17.12%	0.1739	43.1815%	0.0316%	0.1258%
Cia Energética de Minas Gerais - CEMIG	0.0646	0.0174	40.3394%	1.0133	16.70%	0.0805	32.3967%	0.0000%	0.0002%
Centrais Elétricas Paulista - CESP	0.0130	0.0676	62.2616%	1.0843	17.22%	0.0744	11.3109%	1.0287%	21.3893%
Cia Paranaense de Energia - COPEL	0.0116	N.A.	41.4030%	1.0016	16.62%			N.A.	N.A.
Cia Siderurgica Nacional - CSN	50.7900	18.4948	45.4986%	1.0502	16.97%	67.6490	34.1602%	0.0019%	0.0144%
Cia Siderúrgica Paulista - Cosipa	1.1837	0.4761	61.5871%	0.9676	16.37%	1.6176	45.1071%	0.2196%	0.6464%
Eletrobrás	0.0392	N.A.	50.1085%	1.0287	16.82%			N.A.	N.A.
Embratel S.A.	0.0050	0.0083	75.5797%	1.5737	20.81%	0.0125	31.4218%	3.5882%	12.7520%
Centrais Elétricas do Espírito Santo - Escelsa	83.0000	281.7664	68.1440%	1.0000	16.60%	338.5707	17.5531%	2.8423%	16.8901%
Gerdau S.A.	46.4939	14.1431	45.9657%	1.0531	16.99%	59.3861	35.9871%	0.0009%	0.0070%
Cia. Brasileira de Petróleo Ipiranga	26.9493	4.7782	41.4780%	0.5760	13.49%	31.3050	35.7070%	0.0000%	0.0000%
Klabin S.A.	5.4000	1.1049	39.3779%	0.5372	13.21%	6.4072	33.1878%	0.0000%	0.0000%
Net Serviços de Comunicação S.A.	0.4287	0.7735	85.1596%	1.7362	22.01%	1.1206	34.9007%	6.4482%	18.7370%
Petrobrás S.A.	97.1500	19.4757	33.6832%	0.7845	15.02%	114.9033	28.4789%	0.0000%	0.0000%
Sadia S.A.	5.8305	3.1734	37.2069%	0.7583	14.83%	8.7233	24.8688%	0.0003%	0.0040%
Unibanco S.A.	9.0000	N.A.	22.8774%	0.1805	10.59%			N.A.	N.A.
Usiminas	51.3817	14.1862	54.5582%	1.3677	19.30%	64.3128	43.5905%	0.0111%	0.0578%
Cia Vale do Rio Doce	64.1500	6.5681	33.0168%	0.3461	11.81%	70.1372	30.1983%	0.0000%	0.0000%
Votorantim S.A.	43.3000	9.6034	36.4142%	0.3649	11.94%	52.0540	30.2904%	0.0000%	0.0000%

Table VIII. Ratings graded by S&P and Moody's versus ratings assigned based on EDF

	(1)	(2)	(3)	(4)	(5)	(6)
Company	S&P	Moody's	Assigned credit rating based on EDF (S&P)		Assigned credit rating based on EDF (Moody's)	
	Brazil	Domestic Currency	best scenario	worst scenario	best scenario	worst scenario
Ambev	AAA	Baa3	AAA	AAA	Aaa	Aaa
Aracruz Celulose S.A.	AAA	Baa3	AAA	AAA	Aaa	Aaa
Braskem S.A.	AA-		AA	BBB	Aa	Baa
Cia Energética de Minas Gerais - CEMIG		B1	AAA	AAA	Aaa	Aaa
Centrais Elétricas Paulista - CESP	CCC		BB	CCC	Ba	Caa
Cia Siderurgica Nacional - CSN	AA-		AA	A	Aa	A
Cia Siderúrgica Paulista - Cosipa	A+		BBB	BB	BBB	Ba
Embratel S.A.		B1	B	B	B	B
Centrais Elétricas do Espírito Santo - Escelsa			BB	CCC	Ba	CCC
Gerdau S.A.	AA-		AA	AA	Aa	Aa
Cia. Brasileira de Petróleo Ipiranga			AAA	AAA	Aaa	Aaa
Klabin S.A.	AA-		AAA	AAA	Aaa	Aaa
Net Serviços de Comunicação S.A.	D	Ca	B	CCC	B	Caa
Petrobrás S.A.	BBB+	Baa1	AAA	AAA	Aaa	Aaa
Sadia S.A.			AA	AA	Aa	Aa
Usiminas	A+		AA	AA	Aa	Aa
Cia Vale do Rio Doce		Baa2	AAA	AAA	Aaa	Aa
Votorantim S.A.	AAA		AAA	AAA	Aaa	Aaa