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Abstract

This paper analyzes consumption and savings decisions in a two-period consumption setting, supposing that future income is uncertain in the sense of Knight (1921). The results imply that uncertainty-averse agents save more than risk-averse agents.

Key-words: consumption, savings, uncertainty aversion.

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1 Introduction

Consumption decision is a central subject for economists and, since the development of Friedman's (1957) Permanent Income Hypothesis (PIH), economists have agreed that consumption decisions depend on random variables such as future income, existing risk or uncertainty[1]. According to Knight (1921), risk is a situation in which a single additive probability measure on the states of nature is available to conduct choice. However, under uncertainty, information is too imprecise to be summarized by any additive probability measure. In general, economic models assume the rational expectations hypothesis, which means that individuals know the objective probability law, or the Bayesian approach in which they have a (single) prior subjective probability distribution. In both cases uncertainty is doffed.

Recently, Manski (2004, p.1330) has extensively argued that "observed choice may be consistent with many alternative specifications of preferences and expectations". However, the prevailing practice is to assume the rational expectations hypothesis, which implies that other possible explanations for economics phenomena are discarded and applied works are reduced to inference about preferences alone. Manski advocates that researchers must use data on expectations formation to test assumptions about expectations. Following the advice, the first difficulty emerges immediately; in general, survey respondents have only the option to report expectations in probabilistic form because the questions asked are about point prediction of random events or verbal assessments of likelihood.[2] However, Das and Soest's (1997, 1999) results generate indirect evidence in favor of uncertainty. Studying the Dutch Socio-Economic Panel from 1984 to 1985, Das and Soest (1997) found that a large fraction of households (34.9%) underestimated their future income growth while only a small

[1] Some authors use ambiguity instead of uncertainty.
[2] To enable persons to express uncertainty, survey researchers could elicit ranges of probabilities rather than precise probabilities for events of interest (Manski, 2004).

proportion (15,5%) overestimated their income change. According to the authors, a possible explanation is that some people are simply too pessimistic, on average. Das and Soest (1999) extended the sample until 1989, finding a similar pattern. Additionally, using a formal test, they rejected the rational expectations hypothesis.

A generalization of these results seems to be very premature. However, it is worth investigating what effects income uncertainty has on consumer behavior. In order to implement this analysis, this paper employs a two-period consumption model using the Choquet Expected Utility (CEU) approach, an axiomatic treatment of uncertainty, developed by Schmeidler (1989), in which the agent's belief is represented by a convex non-additive probability function. Uncertainty aversion is introduced by means of a uniform contraction of any additive probability measure.

The paper proceeds as follows. Section 2 presents useful results of the CEU model. Section 3 develops and solves the model. Final section summarizes the conclusions in light of other departures from PIH.

2 Uncertainty Aversion[3]

First of all, define Ω as a set of the *states of nature* and Λ as an algebra from its subsets. Thus, *i)* $\Omega \in \Lambda$; *ii)* $A, B \in \Lambda \Rightarrow A \cup B \in \Lambda$, and *iii)* $A \in \Lambda \Rightarrow A^c \in \Lambda$, where A^c is the set of elements of Ω not in *A*. The elements of Λ are the *events*. A function $P : \Lambda \rightarrow [0,1]$ is a non-additive probability if *i)* $P(\phi) = 0$, where ϕ is the empty set; *ii)* $P(\Omega) = 1$, and *iii)* $A, B \in \Lambda,\ P(A) \leq P(B)$ if $A \subset B$. Imposing the additional restriction: *iv)* $A, B \in \Lambda, P(A \cup B) + P(A \cap B) \geq P(A) + P(B)$, we obtain a convex non-additive probability function *P*.

[3] Proofs are omitted and the reader is referred to Schmeidler (1986, 1989), Gilboa (1987), Gilboa and Schmeidler (1989) and Dow and Werlang (1992).

In the presence of uncertainty, information is too scarce for the agent to discard the additive probabilities until there is only one left. For each action, an uncertainty averse agent considers the additive measure that accentuated the probability weights associated with the least favorable outcome (Mukerji and Tallon, 2001). The core of *P*, $C(P)$, identified these additive probability:

$$C(P)=\{Q\in\Delta(\Omega)\,|\,Q(X)\geq P(X),\ \text{for all } X\subset\Omega\}, \qquad (1)$$

where $\Delta(\Omega)$ is the set of all additive probability measure on Ω and *X* is a random variable.

The Choquet expected value of a random variable *X* is defined as:

$$E_P[X]=\int_{\Omega} X\,dP=\int_{-\infty}^{0}[P(X\geq\alpha)-1]d\alpha+\int_{0}^{\infty}P(X\geq\alpha)d\alpha \qquad (2)$$

whenever these integrals exist (in the improper Riemann sense) and are finite. Alternatively, the expected value of X can be defined based on $C(P)$:

$$E_P[X]=\min_{Q\in C(P)}\{E_Q[X]\} \qquad (3)$$

Thus, the uncertainty averse agent evaluates an uncertain act by the minimum expected value that may be associated with it. Two useful results are: $\forall\ a\geq 0,\ b\in\Re,\ E_P[aX+b]=aE_P[X]+b$ and if $u:\Re\to\Re$ is a concave function, then $E_P[u(X)]$ is concave.

Definition [Dow and Werlang (1992)]: Let P be a probability and $A\subset\Omega$ *an event. The uncertainty aversion of P at A is defined by*

$$\theta(P,A)=1-P(A)-P(A^c) \qquad (4)$$

Thus, the degree of uncertainty aversion is proportional to the amount of probability "lost". As Dow and Werlang (1992) showed, *P* can be generated by increasing the uncertainty aversion of an additive probability *Q*: fix $\theta\in[0,1]$, let

$P(\Omega)=1$ and $P(A)=(1-\theta)Q(A)$ for $A \neq \Omega$. Then, for all $A \neq \Omega$, $\theta(P,A)=\theta$, the uncertainty aversion is constant and identical to the probability "lost" by the uniform contraction. Although this result seems to be very restrictive, it is not the case. The degree of uncertainty aversion may be interpreted as an individual parameter and, in general, economists assume that the agents' parameters are constant for all events. Lastly, assuming that $X^L = \inf_{w \in \Omega} X(w) \geq 0$, it is possible to show that $E_P[X] = \theta X^L + (1-\theta)E_Q[X]$. As anticipated, a CEU model gives an extra weight to the worst event, leading to a pessimistic decision criterion.

3 Consumption Model

Consider a consumption model in which individuals live for two periods. In the first period, the consumer has an income w_1 and chooses consumption, c_1, and savings, s. In the second period, the consumer picks consumption, c_2, taking into account income, w_2, and financial wealth, Rs, where R is the gross rate of return. Suppose that w_2 is uncertain.

Assume that utility function u is C^2 and $u'>0$, $u''<0$. Monotonic preferences imply that budget constraints are binding and the consumer's problem becomes[4]

$$\max_{c_1, c_2, s_1} E_P[u(c_1) + \beta u(c_2)]$$

$$\text{st. } c_1 = w_1 - s \text{ and } c_2 = w_2 + sR$$

where $0 < \beta < 1$ is the intertemporal discount factor and $E_P(\cdot)$ is the expected value on the convex non-additive probability *P*. Substituting the restrictions on the objective function, $U(s) = u(w_1 - s) + \beta E_P u(w_2 + sR)$ is obtained and the consumer chooses only

[4] Inada's conditions are assumed to guarantee interior solution.

current savings. Furthermore, the objective function is concave and, consequently, *i)* $s > 0$ if the right side derivative of $U(s)$, evaluated in $s = 0$, is greater than zero, $U_+^{'}(0) > 0$; *ii)* $s < 0$ if the left side derivative of $U(s)$, evaluated in $s = 0$, is lesser than zero, $U_-^{'}(0) < 0$ and, *iii)* $s = 0$ if $U_+^{'}(0) \leq 0 \leq U_-^{'}(0)$.

Suppose that P is generated from a uniform contraction of an additive probability, *Q*, using an uncertainty aversion measure $\theta \in [0,1]$, thereby

$$U(s) = u(w_1 - s) + \beta\theta \min_{w_2} u(w_2 + sR) + \beta(1-\theta)E_Q u(w_2 + sR) \qquad (5)$$

In practice, the lower bound of income is zero, so define $w^L = \inf_{w \in \Omega} w \geq 0$. Thus, $\min_{w_2} u(w_2 + sR) = u(w^L + sR)$ and, accordingly

i) $s > 0$ if $u'(w_1) < \beta\theta u'(w^L)R + \beta(1-\theta)E_Q u'(w_2)R$;

ii) $s < 0$ if $u'(w_1) > \beta\theta u'(w^L)R + \beta(1-\theta)E_Q u'(w_2)R$;

iii) $s = 0$ if $u'(w_1) = \beta\theta u'(w^L)R + \beta(1-\theta)E_Q u'(w_2)R$.

In the first case, in order to increase the current marginal utility, consumer decreases consumption. In the second case, the opposite occurs. And the third case does not require lending or borrowing. Notice that, if the agent is risk averse ($\theta = 0$), savings have a similar pattern, except that future marginal utility is replaced by $E_Q u'(w_2)R$. Graph 1 shows hypothetical savings functions for $\theta > 0$ and $\theta = 0$, where $a = \beta E_Q u'(w_2)R$ and $b = \beta\theta u'(w^L)R + \beta(1-\theta)E_Q u'(w_2)R$. Because $w^L \leq w_2$ and u is concave, $a < b$. Once the uncertainty averse agents are more pessimist to forecast future income than risk averse agents, when the latter group starts to borrow, the former is still lending in order to smooth consumption path.

< Insert Graph 1 >

The following proposition discusses the impact of θ on savings.

Proposition: If future income w_2 is uncertain and its probability function is generated by a uniform contraction of any additive probability measure, then the consumer's savings increases with the uncertainty aversion measure.

Proof: Objective function

$$U(s) = u(w_1 - s) + \beta\theta u(w^L + sR) + \beta(1-\theta)E_Q u(w_2 + sR)$$

FOC:

$$-u'(w_1 - s) + \beta\theta u'(w^L + sR)R + \beta(1-\theta)E_Q u'(w_2 + sR)R = 0$$

Using the implicit function theorem,

$$\frac{ds}{d\theta} = \frac{-\beta R\left[u'(w^L + sR) - E_Q u'(w_2 + sR)\right]}{u''(w_1 - s) + \beta R^2\left[\theta u''(w^L + sR) + (1-\theta)E_Q u''(w_2 + sR)\right]} > 0$$

Once $u'' < 0$, the denominator is negative. Furthermore, $w^L + sR \leq w_2 + sR, \forall w$, as a result $u'(w^L + sR) > E_Q u'(w_2 + sR)$ and the numerator is negative. □

When θ increases, both the expected value of future income, $E_P w_2 = \theta w^L + (1-\theta)E_Q w_2$, and the uncertainty aversion measure are affected. Thereby, perhaps the impact of θ on *s* may be driven only by the variation in expected income. The following example sheds light on this question.

Example: Consider a quadratic utility: $u(c) = -a(b-c)^2/2$, $a > 0$ and $\mathrm{b} > c$. Then,

$$s = \frac{\beta bR - b + w_1 - \beta RE_P w_2}{1 + \beta R^2}$$

The uncertainty aversion measure affects s only via the *expected income. Thus, as in the case of risk aversion ($\theta = 0$), the solution of the model exhibits certainty equivalence when the quadratic utility is used (there is no precautionary savings). In addition, contrary to Hall's (1978) results, when $\beta R = 1$ is assumed, instead of the consumption random walk hypothesis, the following process is obtained:*

$$c_2 = \theta\left(w_2 - w^L\right) + c_1 + (1-\theta)\varepsilon_2$$

where $E_Q[\varepsilon_2] = 0$. *The pessimist behavior generates an excess of resource in the second period, the difference of* w_2 and w^L. *When* $\theta = 0$ *the random walk hypothesis is restored.*

4 Conclusions

In the literature there are, at least, two important departures from PIH. Firstly, the hyperbolic discounting models, which predict that consumers have both a short-run preference for instantaneous gratification and a long-run preference for acting patiently and, as a consequence from this self-control problem, consumers are likely to increase consumption to obtain a higher instant gratification. This approach is able to explain the evidence that many consumers save too little (Laibson, 1997; Angeletos et al., 2001). On the other hand, as Gourinchas and Parker (2001, p. 406) stress "one of the basic motives for saving is the accumulation of wealth to ensure future welfare." More precisely, if $u'''(\cdot) > 0$, then introducing risk in future income made current savings rise due to a precautionary motive. Despite this prediction, this model is able to explain a lower saving behavior if agents have a high discount rate, once the last factor induces current consumption but individuals tend to keep a small amount of savings to use in case a large fall in income takes place (Carrol, 1992; Carrol, 1997).

Motivated by Das and Soest's (1997, 1999) results, this paper investigated another departure from PIH, the effect of future income uncertainty on savings and concluded that an uncertainty averse agent saves more than a risk aversion agent and this gap increases with the degree of uncertainty aversion. This result is also compatible with low savings, if the discount rate is sufficiently high. Indeed, the unique implication of the model is: people save more when future is uncertain.

References

Angeletos G., Laibson, D., Repetto, A., Tobacman, J. and Weinberg, S., 2001, The Hyperbolic Consumption Model: Calibration, Simulation and Empirical Evaluation, Journal of Economic Perspective 15 (3), 47-68.

Carroll, C., 1992, The Buffer Stock Theory of Saving: Some Macroeconomic Evidence, Brookings Papers on Economic Activity 2, 61-135.

Carroll, C., 1997, Buffer Stock Saving and the Life Cycle/Permanent Income Hypothesis, The Quarterly Journal of Economics 112, 1-57.

Das, V. and A. v. Soest, 1997, Expected and realized income changes: Evidence from the Dutch socio-economic panel, Journal of Economic Behavior and Organization 32, 137-154.

Das, V. and A. v. Soest, 1999, A panel data model for subjective information on household income growth, Journal of Economic Behavior and Organization 40, 409-426.

Dow, J. and S. Werlang, 1992, Uncertainty aversion, risk aversion, and the optimal choice of portfolio, Econometrica, 60, 197–204.

Friedman, M. 1957, A theory of the consumption function (Princeton, Princeton University Press).

Gilboa, I. 1987, Expected utility with purely subjective non-additive probabilities, Journal of Mathematical Economics, 16, 65–88.

Gilboa, I. and D. Schmeidler, 1989, Maxmin expected utility with a non-unique prior, Journal of Mathematical Economics, 18, 141–153.

Gourinchas, P. and Parker, J., 2001, The Empirical Importance of Precautionary Saving, American Economic Review, Papers and Proceedings 91(2), 406-412.

Hall, R. 1978, Stochastic implications of the life cycle-permanent income hypothesis: theory and evidence, Journal of Political Economy, 91, 249-265.

Knight, F. 1921, Risk, uncertainty, and profit (Boston: Houghton Mi²in)

Laibson, D., 1997, Golden Eggs and Hyperbolic Discounting, Quarterly Journal of Economics 112, 443-477.

Manski, C. 2004, Measuring Expectations, Econometrica 72, 1329-1376.

Mukerji, S. and J. Tallon, 2001, Ambiguity Aversion and Incompleteness of Financial Markets, The Review of Economics Studies 68, 883-904.

Schmeidler, D. 1986, Integral representation without additivity, Proceedings of Am Math Society 97, 255-261.

Schmeidler, D. 1989, Subjective probability and expected utility without additivity, Econometrica 57, 571–587.

Graph

Graph 1 – Saving Functions