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Carsten Trenkler*, Enzo Weber**

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* Carsten Trenkler is Professor of Empirical Economics at the Department of Economics at the University of Mannheim, L7, 3-5, 68131 Mannheim, Germany.
Phone: +49-621-181-1852, E-mail: trenkler@uni-mannheim.de
** Enzo Weber is Juniorprofessor of Economics at the Department of Economics and Econometrics at the University of Regensburg, 93040 Regensburg, Germany.
Phone: +49-941-943-1952, E-mail: enzo.weber@wiwi.uni-regensburg.de

Codependence and Cointegration*

Carsten Trenkler[†], Enzo Weber[‡]

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Abstract

We introduce the idea of common serial correlation features among non-stationary, cointegrated variables. That is, the time series do not only trend together in the long run, but adjustment restores equilibrium immediately in the period following a deviation. Allowing for delayed re-equilibration, we extend the framework to codependence. The restrictions derived for VECMs exhibiting the common feature are checked by LR and GMM-type tests. Alongside, we provide corrected maximum codependence orders and discuss identification. The concept is applied to US and European interest rate data, examining the capability of the Fed and ECB to control overnight money market rates.

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†University of Mannheim, L7, 3-5, D-68131 Mannheim, Germany, trenkler@uni-mannheim.de, phone: +49 (0)621 181-1852, fax: +49 (0)621 181-1931

‡University of Regensburg, D-93040 Regensburg, Germany, enzo.weber@wiwi.uni-regensburg.de, phone: +49 (0)941 943-1952, fax: +49 (0)941 943-2735

1 Introduction

In this paper we discuss common serial dependence of non-stationary variables. In particular, we analyze the imposition of serial correlation common features (SCCFs) and codependence restrictions on the levels of cointegrated variables. As a special case of a common feature, Engle & Kozicki (1993) introduced the concept of SCCF. A SCCF exists if a linear combination of serially correlated variables cannot be predicted by the history of the variables. Hence, the variables contain a common factor such that the linear combination does not exhibit any serial correlation. As a consequence, the impulse responses of the variables are collinear. Based on Gourieroux & Peaucelle (1988, 1992), Vahid & Engle (1997) generalize SCCF to the concept of codependence. Codependence of order q is present if the (nonzero) impulse responses of the variables are collinear after the first q periods. Thus, the linear combination has a moving average representation of order q, which is lower than the order of the individual variables. Obviously, SCCF implies $q = 0$ and is, therefore, a special case of codependence.

In relation to variables that are integrated of order one, $I(1)$, the existing literature has imposed SCCF and codependence only on the first differences. Vahid & Engle (1993) show that if the first differences of $I(1)$ variables exhibit a SCCF, then the corresponding linear combination of the levels completely eliminates the cyclical parts of the variables in the multivariate Beveridge-Nelson decomposition. In other words, the variables have a common cycle. This property is analogous to a common trend, which is eliminated by the cointegration vector. Not surprisingly, Vahid & Engle (1997) demonstrate that codependence of order q in the first differences of $I(1)$ variables implies codependence of order $q-1$ in their cycles. Therefore, they speak of codependent cycles. Based on the work of Vahid & Engle (1993, 1997), Schleicher (2007) discusses in detail SCCF and codependence in relation to cointegrated variables within the vector error correction model (VECM) framework.

In contrast to the previous papers, we allow the *levels* of $I(1)$ variables to be codependent, including the special case of SCCF. Since codependence implies that a linear combination of the variables has a (stationary) finite-order MA representation, the variables must be cointegrated; thereby, the weights of the relevant linear combination are given by the cointegration vector. Hence, a cointegration vector does not only eliminate the common trend but also the common cyclical movements after q lags. This possibility has not been discussed in the literature so far. To be precise, it has been ignored that a codependence structure in first differences can be restricted in such a way that codependence is also present in the levels of cointegrated variables. Even if formally, this represents only a special case, economic interpretations and applications differ considerably. We will show one example in Section 3.

Because of the implied serial correlation structure in the cointegration error, codependence has direct implications for the dynamics of the adjustment toward the cointegration equilibrium. If cointegrated variables are codependent, then a deviation from the cointegration relation due to a shock is completely eliminated after $q+1$ periods. Thus, the codependence framework for cointegrated variables is well suited to analyze empirical setups for which a very fast, or even immediate, adjustment to an equilibrium is expected. Important examples refer to market-driven arbitrage processes or policy-driven control of certain variables using specific instruments. The adjustment property links the concept of codependent cointegrated variables to the framework of Pesaran & Shin (1996). They introduced so-called persistence profiles of the cointegrating relations. These profiles can be interpreted as the square of impulse-responses of the cointegration relation to a system-wide shock and thereby allow to analyze how quickly the convergence to the cointegration equilibrium occurs. Codependence of order q implies that all persistence profiles are zero after horizon q.

Our analysis will be based on the VECM framework since all relevant testing procedures suggested in the literature are either directly or indirectly linked to this framework. Moreover, it allows to impose codependence restrictions while using the VECM for forecasts or structural analysis. In fact, the results of Vahid & Issler (2002) indicate that imposing such constraints may lead to higher accuracy of forecasts and of estimates of impulse-response functions.

We will make two contributions. First, we characterize the concept of codependence for cointegrated variables based on VEC models, relate our framework to the ones existing in the literature, and discuss three testing approaches.

We provide corrected upper bounds on the codependence order within a VECM and argue that codependent VECMs are not generally identified, a fact that has been overseen in the literature. The identification problems are not specific to the case of $I(1)$ variables but also apply to stationary model setups. To test for codependence in identified setups, we employ the likelihood ratio (LR) test principle based on a nonlinear maximum likelihood (ML) estimation of the underlying VECM. This full information testing can provide clear efficiency gains compared to other approaches, compare e.g. Schleicher (2007). Nevertheless, we also consider a test for a cut-off in the serial correlation of the cointegration error. This test is motivated by a GMM estimation approach that has been proposed by Vahid & Engle (1997). The main advantage of the GMM-type test is that it can be applied if a codependent VECM cannot be identified. Moreover, we relate the GMM test to a Wald test for nonlinear restrictions in terms of the VEC model parameters and discuss the scope of both methods.

Second, using the codependence framework for cointegrated variables we analyze whether central banks can control overnight interest rates. In particular, monetary authorities like the Federal Reserve Bank (Fed) or the European Central Bank (ECB) try to control short-term interest rates in the sense of keeping them close to the announced target values. Hence, if central banks can sufficiently control overnight rates, then overnight and target rates should be cointegrated and deviations of the overnight rate from the target should be relatively short-lived. In other words, the deviations may ideally be white noise or have a low-order MA(q) representation such that they are completely eliminated after few periods. Evidently, this implies codependence between the levels of the interest rates. We will argue that the Fed was much more successful in controlling overnight rates than the ECB in the recent decade.

The plan for the rest of paper is as follows. In the next section we present the methodology by first describing the model framework and characterizing the codependence restrictions. Then we relate our framework to the ones existing in the literature and explain the testing procedures. Using the codependence approach we explore in Section 3 whether the Fed and the ECB could control overnight rates. Finally, the last section concludes. A proof is deferred to the appendix.

2 Methodology

2.1 Model Framework

The starting point is the following model for the n-dimensional time series $y_t = (y_{1t}, \ldots, y_{nt})'$, $t = 1, \ldots, T$,

$$y_t = \mu_0 + \mu_1 t + x_t, \quad t = 1, 2, \ldots,$$

where μ_0 and μ_1 are $(n \times 1)$ parameter vectors. To simplify the exposition in the following we set $\mu_i = 0$, $i = 0, 1$, without loss of generality such that $y_t = x_t$. The stochastic component x_t follows a vector autoregression of order p, VAR(p),

$$x_t = A_1 x_{t-1} + \cdots + A_p x_{t-p} + \varepsilon_t, \quad t = 1, 2, \ldots, \tag{2.1}$$

where A_j are $(n \times n)$ coefficient matrices and the initial values $x_0, \ldots, x_{-p+1}$ are taken as given. The error terms ε_t are i.i.d.$(0, \Omega)$ with positive definite covariance matrix Ω and finite fourth moments. Defining $\Pi = -(I_n - A_1 - \cdots - A_p)$ and $\Gamma_j = -(A_{j+1} + \cdots + A_p)$, $j = 1, \ldots, p-1$, we can re-write (2.1) in the vector error correction form

$$\Delta x_t = \Pi x_{t-1} + \sum_{j=1}^{p-1} \Gamma_j \Delta x_{t-j} + \varepsilon_t, \quad t = 1, 2, \ldots.$$

The relationship of the VAR and VECM representations can be compactly described by $A(L) = I_n - A_1 L - \cdots - A_p L^p = I_n \Delta - \Pi L - \Gamma_1 \Delta L - \cdots - \Gamma_{p-1} \Delta L^{p-1} = \Pi(L)$ with $\Delta = 1 - L$ and L being the lag operator.

To assure the applicability of the Granger's representation theorem, we make the following assumption, compare e.g. Hansen (2005).

Assumption 1.

(a) If $\det \Pi(z) = 0$, then $|z| > 1$ or $z = 1$.

(b) The matrix Π has reduced rank $r < n$, i.e. the matrix Π can be written as $\Pi = \alpha\beta'$, where α and β are $n \times r$ matrices with rk$(\alpha) =$ rk$(\beta) = r$.

(c) The number of unit roots, $z = 1$, in $\det \Pi(z) = 0$ is exactly $n - r$.

Hence, the cointegrating rank is equal to r. It follows from Granger's representation theorem that the vector of cointegration errors $\beta' x_t$ and Δx_t are zero mean $I(0)$ processes (compare Hansen 2005, Corollary 1). In particular, the theorem implies that the cointegration error $\beta' x_t$ can be interpreted as a set of r linear transformations of a well-defined vector MA(∞) process with an absolutely summable sequence of coefficient matrices even though x_t does not possess such a stationary MA representation.

Hansen (2005) provides a closed-form expression for $\beta' x_t$ in terms of the VECM parameters. For our purpose it is useful to work with

$$\beta' x_t = \sum_{i=0}^{\infty} \beta' \Theta_i \varepsilon_{t-i} = \beta' \Theta(L) \varepsilon_t, \tag{2.2}$$

compare Remark 1 in Hansen (2005). The coefficients of $\Theta(L)$ are given by the recursion

$$\Delta\Theta_i = \Pi\Theta_{i-1} + \sum_{j=1}^{p-1} \Gamma_j \Delta\Theta_{i-j}, \quad i = 1, 2, \ldots, \tag{2.3}$$

or equivalently by

$$\Delta\Theta_i = \Theta_{i-1}\Pi + \sum_{j=1}^{p-1} \Delta\Theta_{i-j}\Gamma_j, \quad i = 1, 2, \ldots, \tag{2.4}$$

with $\Theta_0 = I_n$ and $\Theta_i = 0$ for $i < 0$. Three remarks are in order. First, $\Theta(L)$ is not an absolutely summable polynomial. However, we have $\Theta_i = C + C_i$, $i = 1, 2, \ldots$, with $C = \beta_\perp(\alpha_\perp' \Gamma \beta_\perp)^{-1}\alpha_\perp'$ and $\Gamma = I_n - \sum_{i=1}^{p-1} \Gamma_i$. Morever, $C(L) = I_n + C_1 L + C_2 L^2 + \cdots$ is absolutely summable. Hence, $\beta'\Theta(L) = \beta' C(L)$ such that we can regard $\beta'\Theta(L)$ as a set of r linear combinations of an absolutely summable lag polynomial.

Second, the equivalence of the recursions (2.3) and (2.4) can be shown as follows. The parameter matrices Θ_i, $i = 1, 2, \ldots$, in (2.3) and (2.4) satisfy $\Pi(L)\Theta(L) = I_n$ and $\Theta(L)\Pi(L) = I_n$, respectively. The inverse of $\Pi(L)$ exists since $\Pi_0 = I_n$ is nonsingular. Hence, we obtain from both relationships $\Theta(L) = \Pi(L)^{-1}$. Third, the recursions can also be written in terms of the VAR parameters yielding $\Theta_i = \Theta_{i-1}A_1 + \Theta_{i-2}A_2 + \cdots + \Theta_{i-p}A_p = A_1\Theta_{i-1} + A_2\Theta_{i-2} + \cdots + A_p\Theta_{i-p}$, $i = 1, 2, \ldots$, with $\Theta_0 = I_n$ and $\Theta_i = 0$ for $i < 0$. These recursions have been already applied in the literature.[1]

[1]The former recursion is the usual one for MA coefficient matrices obtained from the VAR coefficient matrices

2.2 Codependence Restrictions

We can now characterize the parameter restrictions that are implied when the components of x_t are codependent. In the following, we focus on the case of a single cointegration vector and comment on setups with several cointegration relations later on. Moreover, we assume that the first element of the cointegration vector is normalized to 1.

Codependence of order q is present in the levels if $\beta' x_t$ has a finite-order MA(q) representation. This requires $\beta'\Theta_i = 0$ for all $i > q$ and $\beta'\Theta_q \neq 0$. The latter restriction means that $\beta' x_t$ can be regarded as a linear combination of a multivariate MA(q). Hence, $\beta' x_t$ has a univariate MA(q) representation according to Lütkepohl (2005, Proposition 11.1).[2] The cointegration vector, thus, is also a codependence vector in this setup.[3] To distinguish codependence in levels from codependence in first differences of $I(1)$ variables, we introduce the terminology of level codependence of order q, abbreviated as LCO(q). Accordingly, if $q = 0$, a level serial correlation common feature (LSCCF) is present.

In light of recursion (2.4), it is clear that the restrictions $\beta'\Theta_i = 0$ for $i = q{+}1, q{+}2, \ldots, q{+}p$ and $\beta'\Theta_q \neq 0$ are sufficient to assure that $\beta' x_t$ has a MA(q) representation. In case of LSCCF, one can easily deduce from the recursion (2.3) that this implies $\beta'\Pi = -\beta'$, i.e. $\beta'\alpha = -1$, and $\beta'\Gamma_i = 0$ for $i = 1, \ldots, p-1$. In general, however, it is more convenient to use a state-space representation based on the companion form of the VECM to describe the restrictions on the corresponding model parameters, compare Schleicher (2007). It is useful to work with the following state-space form that is due to Hansen (2005):

$$
\begin{aligned}
\Delta x_t &= J_\Delta X_t \\
X_t &= F X_{t-1} + U_t,
\end{aligned}
\tag{2.5}
$$

in a stationary framework. Lütkepohl (2005, Ch. 6) uses this recursion to determine impulse responses for $I(1)$ VAR processes. The representation (2.2) for $\beta' x_t$ in conjunction with the latter recursion has also been derived by Pesaran & Shin (1996).

[2]Given the assumptions on ε_t, the error term of the univariate MA representation is an innovation. Then, future values of $\beta' x_t$ are not predictable from $x_{t-q}, x_{t-q-1}, \ldots$, although the components in x_t are. The latter is the case because the variables are $I(1)$ such that they have individual infinite-order MA representations. Hence, the formal definition of codependence as a common feature applies, compare Engle & Kozicki (1993) and Vahid & Engle (1997).

[3]The label 'codependence vector' has been introduced by Gourieroux & Peaucelle (1988, 1992) within the stationary framework.

where

$$J_\Delta = [0_{n\times 1}\ I_n\ 0_{n\times n(p-2)}], \tag{2.6}$$

$$X_t = [x_t'\beta, \Delta x_t', \Delta x_{t-1}', \dots, \Delta x_{t-p+2}']', \quad U_t = [\varepsilon_t'\beta,\ \varepsilon_t',\ 0_{n(p-2)\times 1}]', \tag{2.7}$$

and (2.8)

$$F = \begin{bmatrix} (1+\beta'\alpha) & \beta'\Gamma_1 & \beta'\Gamma_2 & \cdots & \beta'\Gamma_{p-2} & \beta'\Gamma_{p-1} \\ \alpha & \Gamma_1 & \Gamma_2 & \cdots & \Gamma_{p-2} & \Gamma_{p-1} \\ 0 & I_n & 0 & \cdots & 0 & 0 \\ 0 & 0 & I_n & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \cdots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & I_n & 0 \end{bmatrix} \tag{2.9}$$

is a $(n(p-1)+1)\times(n(p-1)+1)$ companion matrix of which all eigenvalues are smaller than 1, see Hansen (2005, Lemma A.2). For the presentation, we regard the cointegration vector β as given. This is a common assumption in the literature on codependence related to VECMs, compare e.g. Schleicher (2007). In Subsection 2.4 we comment on testing strategies in which the assumption of a given β can be skipped.

By iterative substitution we obtain

$$\begin{aligned} X_t &= FX_{t-1} + U_t \\ &= F^2X_{t-2} + U_t + FU_{t-1} \\ &\vdots \\ &= F^{q+1}X_{t-q-1} + \sum_{j=0}^{q} F^j U_{t-j}. \end{aligned} \tag{2.10}$$

Hence, LCO(q) is given if

$$\gamma_0' F^q \neq 0 \text{ and} \tag{2.11}$$

$$\gamma_0' F^{q+1} = 0, \tag{2.12}$$

where $\gamma_0 = (1\ 0_{1\times n(p-1)})'$. Clearly, (2.12) implies that $\gamma_0'F^i = 0$ for all $i > q+1$. Hence, further restrictions on F^i for $i > q+1$ are not necessary. Defining $\gamma_k' = \gamma_0'F^k$ and following Schleicher (2007), we can write the restrictions (2.11)-(2.12) as a set of linear restrictions regarding F

$$\begin{aligned} &\gamma_j'F = \gamma_{j+1}', \quad 0 \le j < q-1 \\ &\gamma_q'F = 0. \end{aligned} \tag{2.13}$$

Note that the vectors γ_j, $j = 0, 1, \ldots, q$, are linearly independent (see Schleicher 2007, Lemma 1). Thus, (2.13) translates the nonlinear LCO(q) constraints on the VECM parameters into a set of linear restrictions regarding the companion form parameters in F.

We now address two important issues in turn. First, we derive an upper bound for q, say q_{max}. Second, we discuss under what conditions a unique so-called pseudo-structural form for the VECM can be obtained.

The result on the upper bound q_{max} is summarized in the following theorem, of which the proof is given in the Appendix.

Theorem 1. Let F be a companion matrix as defined in (2.5) and let $\gamma_0 = (1\ 0_{1\times(n-1)p})'$ for which the restrictions $\gamma_0' F^q \neq 0$ and $\gamma_0' F^{q+1} = 0$ hold. Then, it must be that $q \leq q_{max}$, where $q_{max} = (n-1)(p-1)$.

Some remarks are in order. An upper bound for q is due to the recursive relationship between the VECM and MA parameter matrices given in (2.4). The crucial point is that $\beta'\Theta_q$ has to be nonzero to identify the MA(q) process. For a sufficiently large q, the restriction $\beta'\Theta_q \neq 0$ rules out that $\beta'\Theta_i = 0$ for $i > q$, given that $\beta'\Theta_j \neq 0$ also has to hold for at least some $j < q$.

Theorem 1 provides a correction of Lemma 1 in Schleicher (2007) which implied an upper bound of $q^*_{max} = n - 1$ for our setup. The difference between q^*_{max} and q_{max} is due to the fact that the codependence and the companion restrictions do not need to be jointly independent as assumed by Schleicher (2007). Joint independence is not automatically given, although each codependence restriction, i.e. each γ_i, $i = 0, 1, \ldots, q$, is independent of the companion restrictions which are described below. Examples with linear dependence can be easily constructed.[4] Thus, once the independence assumption is dropped, a rather large upper bound for the codependence order is obtained.

The proof in the Appendix shows that the result of Theorem 1 is eventually based on the structure of *VAR-type* companion restrictions. This means, on the one hand, that we cannot cover general restrictions as addressed in Schleicher (2007, Lemma 1). On the other hand, the upper bound of the order is also higher than the one implied by Schleicher (2007, Lemma 1) for codependence among stationary variables or with respect to the first differences of $I(1)$ variables. The corresponding results are summarized in Lemma 1 in the Appendix.

To set up a pseudo-structural form representation, we first summarize the restrictions in

[4]The problem in Schleicher (2007, Lemma 1) is the following. Let Γ be a $(n \times q)$ matrix and R be a $(n \times m)$ matrix with full column rank. Then, a full column rank of Γ together with linear independence of one of the columns in Γ of R does not imply that the matrix $M = [\Gamma : R]$ has full column rank $m + q$. In other words, the columns are not necessarily jointly independent.

(2.13) by $B'F = C'$ with $B = [\gamma_0,\ \gamma_1,\ \cdots,\ \gamma_q]$ and $C = [\gamma_1,\ \gamma_2,\ \cdots,\ 0_{(n(p-1)+1)\times 1}]$ being $(n(p-1)+1) \times (q+1)$-dimensional matrices. Moreover, F has to satisfy the companion form restrictions $R'F = Q'$ with $R = [0_{n(p-2)\times(n+1)} : I_{n(p-2)}]'$ and $Q = [0_{n(p-2)\times 1} : I_{n(p-2)} : 0_{n(p-2)\times n}]'$ and $B'_\beta F = \gamma'_1 + \gamma'_0$ with $B_\beta = J'_\Delta \beta$. Although B_β and γ_0 are linearly independent, they both generate γ_1 which is used to define the sequence of restrictions in (2.13). Clearly, it is more convenient to use γ_0 from a conceptual point of view and to regard $B'_\beta F = \gamma'_1 + \gamma'_0$ as a part of the companion restrictions.

Schleicher (2007) has suggested to add, if necessary, equations representing free parameters in F to B, B_β, and R. Thereby, a system like $\Psi F = \Gamma$ can be obtained so that the reduced form parameters in F can be recovered from the structural form parameters in Ψ and Γ if Ψ is invertible. A unique and invertible Ψ, i.e. identification, is always guaranteed if the companion and codependence restrictions are jointly independent as was assumed by Schleicher (2007). However, as pointed out above, the columns of $M = [B : B_\beta : R]$ do not need to be linearly independent although each column of B is independent from $M_R = [B_\beta : R]$. Hence, the vectors γ_j, $j = 0, \ldots, q$, in B together with a subset of the companion restrictions may generate some of the remaining companion restrictions. This result is not specific to our companion structure but holds in general. A consequence is that the pseudo-structural representation is not generally unique. Both, the specific structure and the number of involved restrictions could be unknown.

However, if $q < n$, the columns in M can be independent. In that case, the free parameters can be represented by $R'_P F = P'$, where $R_P = [0_{1\times(n-q-1)} : I_{(n-q-1)} : 0_{(n(p-2)+q+1)\times(n-q-1)}]'$ and P is a $(n(p-1)+1) \times (n-q-1)$ matrix that contains the free parameters. Then, we are able to define $\Psi = [B : R_P : M_R]'$ and $\Gamma = [C : P : Q_R]'$, where $Q_R = [\gamma_0 + \gamma_1 : Q]$. This gives the pseudo-structural form representation for the state-space system (2.5)

$$x_t = J_\Delta X_t$$

$$\Psi X_t = \Gamma X_{t-1} + \Psi U_t.$$

The reduced form parameters are then obtained via $F = \Psi^{-1}\Gamma$ from the structural form parameters in Ψ and Γ since Ψ has full rank if the columns in M are linearly independent. We also see that there are $n(p-1)+1$ restrictions underlying the pseudo-structural model because we have $n(n(p-1)+1)$ reduced form parameters in F but only $[q + (n-q-1)](n(p-1)+1)$ structural form parameters in B, C, and P. Note that the number of parameters does not include the parameters in β, since we regard the cointegration vector as given. If we treat β as unknown, the number of restrictions would not change since β enters both the structural and reduced form representations. Compared to Schleicher (2007), the reduction in degrees of freedom is larger by $n-1$. The reason is that the potential codependence vector, the cointegration vector, is already included in the model. Therefore, no new vector has to be determined. Hence, the structural form needs $n-1$ parameters less to capture the codependence restrictions given the normalization of β.

The columns in M are always linearly independent if $q=0$ or $q=1$. Linear independence is easy to verify for LSCCF ($q=0$) since $B=\gamma_0=(1\ 0_{1\times(n-1)p})'$ in this case. Moreover, the LSCCF restrictions imposed on $\alpha, \Gamma_1, \ldots, \Gamma_{p-1}$ by the cointegration vector are linear. In case of LCO(1) we have $B=[\gamma_0 : \gamma_1]'$, so that dependency between B and M_R would only be present if $\beta'\Gamma_1 = c\beta'$ for some $c \in \mathbb{R}$. Note that $\gamma_1 = [(1+\beta'\alpha) : \beta'\Gamma_1 : \cdots : \beta'\Gamma_{p-1}]'$ and $\gamma_1' F = 0$. Using $\beta'\Gamma_1 = c\beta'$, the latter zero constraints can be written as $(1+\beta'\alpha+c)\beta'\alpha + (1+\beta'\alpha) = (1+\beta'\alpha+c)\beta'\Gamma_1 + \beta'\Gamma_2 = (1+\beta'\alpha+c)\beta'\Gamma_2 + \beta'\Gamma_3 = \cdots = (1+\beta'\alpha+c)\beta'\Gamma_{p-1} + \beta'\Gamma_p = (1+\beta'\alpha+c)\beta'\Gamma_p = 0$. From here it is easy to see that this leads to a LSCCF setup, what contradicts the LCO(1) assumption $\gamma_0' F \neq 0$.

For $1 < q < n$, scenarios with both linear dependence and independence of the columns in M, i.e. of the companion and codependence restrictions, are possible. This induces the following three comments.

First, there can exist LCO(q) representations with and without dependency. Second, if companion and codependence restrictions are dependent, then blocks of the γ_j, $j=0,\ldots,q$, have to linearly depend on each other because of the structure of the companion restrictions. Each vector γ_j can be decomposed into a first element and $(p-1)$ blocks of size $1\times n$. However, these blocks are nonlinear functions of the VEC model parameters. Thus, it is not possible to express the blocks as linear combinations of the companion matrix. Therefore, the dependencies between the blocks of the γ-vectors (re-)introduce nonlinear constraints on the companion matrix. Accordingly, the advantage of the companion form, which lies in translating the nonlinear parameter restrictions implied by LCO(q) into linear restrictions on the companion matrix, disappears. This, in turn, creates ambiguities regarding the specific type and the number of restrictions underlying the LCO(q) setup. To be precise, it is in general not possible to determine which and how many blocks are dependent, how many parameter restrictions are related to the block dependencies, and which columns in R are redundant. As a consequence, a pseudo-structural representation cannot be setup in case of dependency in M.

Third, as mentioned above, identification is still given under the maintained assumption of independence of the columns in M. Hence, in the case of independence a unique pseudo-structural form also exist for $1 < q < n$. Thus, LR testing of a joint hypothesis of independence and LCO(q) would be possible.

Finally, if $q > n$, the vectors γ_j, $j=0,\ldots,q$, and the columns in R have to be jointly dependent. According to foregoing comments identification of the pseudo-structural form cannot be achieved in general.

We now discuss two remaining issues: the case of several cointegration vectors and the inclusion of deterministic terms.

If several cointegration vectors exist and LCO(q) is tested only with respect to one vector the foregoing applies accordingly. To be precise, if no restrictions are imposed on the other cointegration vectors, then the additional parameters in F enter the pseudo-structural form as free parameters. Hence, the number of restrictions associated with LCO(q) does not change compared to the case of a single cointegration vector.

If LCO(q) restrictions regarding several cointegration vectors are imposed, then for each cointegration vector a sequence of γ-vectors expressing the LCO(q) constraints exists. However, these systems of vectors does not need to be linearly independent as claimed by Schleicher (2007) for the case of several codependence vectors. Given that the last vector in each of these systems represents zero constraints, the systems that linearly depend on each other have to merge. That means, the corresponding last vectors in the sequences have to be equal or multiples of each other. As a consequence, the upper bound of Theorem 1 is equal to the rank of the stacked systems rather than to the sum of all vectors involved. Hence, the identification of certain LCO order combinations has to be evaluated on a case-by-case basis. E.g. identification is given if all cointegration vectors generate LSCCF.

The inclusion of deterministic terms like a constant and a linear trend is harmless since their coefficients can be treated as free parameters within the pseudo-structural form. Hence, the consideration of deterministic terms increases the number of reduced and structural form parameters in the same way.

2.3 Comparison to Existing Approaches

To relate LSCCF and LCO(q) to existing frameworks in the literature, we first describe the scalar component model (SCM) introduced by Tiao & Tsay (1989) and the Beveridge-Nelson-Stock-Watson (BNSW) decomposition used by Vahid & Engle (1993, 1997).

A non-zero linear combination $v_0' x_t$ of a n-dimensional process x_t follows an SCM(p,q) structure, if one can write

$$v_0' x_t + \sum_{j=1}^{p} v_j' x_{t-j} = v_0' \varepsilon_t + \sum_{j=1}^{q} w_j' \varepsilon_{t-j}$$

for a set of n-dimensional vectors $\{v_j\}_{j=1}^{p}$ and $\{w_j\}_{j=1}^{q}$ with $v_p \neq 0$ and $w_q \neq 0$, see Schleicher (2007). Thus, SCCF and codependence of order q with respect to x_t are then in line with SCM(0,0) and SCM(0,q) models, respectively.

For our purpose it is sufficient to state the final version of the BNSW decomposition used by Vahid & Engle (1993) for cointegrated variables. Based on a Wold representation for Δx_t we have

$$x_t = \gamma \tau_t + c_t,$$

where γ is a $n \times (n-r)$ parameter matrix, τ_t is a linear combination of $n-r$ random walks, the trend part, and c_t is a stationary infinite-order MA polynomial, the cyclical part. As pointed out by Vahid & Engle (1993), a cointegration vector eliminates the trend component and can therefore be seen as a linear combination of the cyclical part: $\beta' x_t = \beta' c_t$. Moreover, they show that a SCCF with respect to Δx_t leads to a common cycle, i.e. a SCCF vector v_0 with $v_0' \Delta x_t = v_0' \varepsilon_t$ eliminates the cycles such that $v_0' c_t = 0$.[5] Vahid & Engle (1997) generalize this result. If Δx_t is codependent of order q with a codependence vector v_0' so that $v_0' \Delta x_t$ is a SCM$(0, q)$, then $v_0' c_t$ is a SCM$(0, q-1)$. Thus, there exists a codependent cycle of order $q-1$.

In contrast to Vahid & Engle (1993, 1997) we impose a SCCF and codependence on the level of x_t but not on Δx_t directly. Thus, $v_0 = \beta$ is required in our setup. This means first, that a level SCCF cannot produce a common cycle since a common cycle vector has to be orthogonal to the cointegration space, see Vahid & Engle (1993). Obviously, if x_t satisfies the LCO(q) constraints such that $\beta' x_t$ is a SCM(0,q), then $\beta' \Delta x_t = \beta' x_t - \beta' x_{t-1}$ follows a SCM(0,$q+1$). But we know from Vahid & Engle (1997), that a SCM(0,$q+1$) for the first differences of an $I(1)$ variable implies that $\beta' c_t$ is a SCM(0,q). Hence, the cointegration vector also produces a codependent cycle of order q. Thus, a codependence vector with respect to the first differences need not only be related to codependence structures in the cycles but also to codependence structures in the levels.

Obviously, a cointegration vector that is a codependence vector has to satisfy additional restrictions compared to the type of codependence vectors analyzed in Vahid & Engle (1997) or Schleicher (2007). It must produce a specific SCM structure with respect to Δx_t given by

$$\beta' \Delta x_t = \beta' \varepsilon_t + \beta'(\Theta_1 - I_n)\varepsilon_{t-1} + \beta'(\Theta_2 - \Theta_1)\varepsilon_{t-2} + \cdots + \beta'(\Theta_q - \Theta_{q-1})\varepsilon_{t-q} - \beta' \Theta_q \varepsilon_{t-q-1}.$$

This constrained SCM can be rewritten as $\beta' \Delta x_t = \beta' \Delta \varepsilon_t + \beta' \Theta_1 \Delta \varepsilon_{t-1} + \cdots + \beta' \Theta_q \Delta \varepsilon_{t-q}$, which is a linear combination of a vector MA(q) model in terms of $\Delta \varepsilon_t$. Such restricted SCM versions for Δx_t have not been addressed in the literature so far. Admittedly, it is rather difficult

[5] This implies $v_0' \alpha = 0$ in the VECM. Hence, in typical model setups at least one time series is not adjusting towards the cointegrating relationship, but deviating from it. Consider e.g. the bivariate case, where the first element of v_0 is normalized to one. Then, the second element is negative if the peaks and troughs of the two cycles coincide respectively. Logically, both coefficients in α must have the same sign to assure $v_0' \alpha = 0$. Thus, one time series is not adjusting towards the cointegration equilibrium such that the other one has to compensate this non-adjusting behaviour. A common cycle with equal weights in both series, i.e. $v_0 = (1, -1)'$, would even exclude long-run re-equilibration.

to identify such structures if common feature restrictions are imposed on the first differences of cointegrated variables. Yet, we want to point out that $v_0'x_t$ is not necessarily an $I(1)$ variable if v_0 is a codependence vector with respect to Δx_t in contrast to the statement in Schleicher (2007, Appendix A). This also means that cointegration and codependence vectors (with respect to the first differences) are not necessarily orthogonal and, thus, the number of codependence vectors could exceed $n - r$.

Constraints in terms of levels of $I(1)$ variables have already been discussed in Cubadda (2007). He considers a weak form SCCF with respect to a VECM which implies $\delta'(\Delta x_t - \alpha\beta' x_{t-1}) = \delta'\varepsilon_t$ for some nonzero vector δ because $\delta'\Gamma_i = 0, i = 1, \ldots, p-1$, is assumed. This results in a polynomial SCCF (PSCCF) of order $p = 1$ for x_t. As pointed out by Schleicher (2007), a PSCCF can be interpreted as a SCM(p,0) structure which imposes a reduced rank restriction on the autoregressive lag structure rather than on the moving average structure as in the case of codependence. However, if δ is set equal to β and $\beta'\alpha = -1$ is assumed, we obtain a LSCCF with respect to x_t. Yet, no simple relationship between LCO and higher order PSCCF in terms of x_t exists.

Finally, we comment on the relation of LSCCF, weak form SCCF and codependence of order 1 with respect to Δx_t. The latter implies that the impulse responses of the components of Δx_t are collinear after lag 1, while weak form SCCF requires that Δx_t and $\alpha\beta' x_{t-1}$ have collinear impulse response functions, see Hecq, Palm & Urbain (2006). Besides these properties, LSCCF implies in addition collinearity of the impulse responses of the components of x_t. Hence, it is clearly the most restrictive structure.

As mentioned in the introduction, the adjustment properties implied by level codependence relate our framework to Pesaran & Shin (1996). They propose the persistence profile approach to analyze the dynamics of adjustment toward the cointegration equilibrium. Persistence profiles represent the effect of systems-wide shocks on the cointegration relations over time. For the case of a single cointegration relation we can define the persistence profile of $z_t = \beta' x_t$ in our notation as

$$H_z(j) = \frac{\beta'\Theta_j\Omega\Theta_j'\beta}{\beta'\Omega\beta}, \quad \text{for } j = 0, 1, 2, \ldots. \tag{2.14}$$

Since only a single cointegration vector is assumed, $H_z(j)$ can be interpreted as the square of the impulse response function of z_t to a unit composite shock in the error $u_t = \beta'\varepsilon_t$. From (2.14) it is clear that a LCO(q) constraint requires the persistence profiles $H_z(j)$ to be zero for $j > q$. Hence, if an identified pseudo-structural form can be found, one can test the joint hypothesis $H_z(j) = 0$ for all $j > q$. Such a test also takes the correlation of the estimators of $H_z(j)$ into account. By contrast, Pesaran & Shin (1996) have only provided the limiting distributions of the ML estimators of $H_z(j)$ for each j individually. Hence, significance testing could only be done pointwise as it is standard in the impulse response literature.

Similar comments can be made regarding the serial correlations of z_t. As shown by Pesaran & Shin (1996), those are given by

$$\rho_z(j) = \frac{\sum_{i=0}^{\infty} \beta' \Theta_j \Omega \Theta_{j+s}' \beta}{\sum_{i=0}^{\infty} \beta' \Theta_j \Omega \Theta_j' \beta}, \quad \text{for } j = 0, 1, 2, \ldots.$$

Trivially, LCO(q) implies that $\rho_z(j) = 0$ for all $j > q$. Hence, a VECM-based test on LCO(q) allows to jointly check whether all relevant serial correlations are zero.

2.4 Testing Approaches

2.4.1 LR Test

If an identified pseudo-structural form exists we can apply the LR principle to test the null of LCO(q). That is, we estimate the unrestricted and restricted model by ML and take twice the log-likelihood difference. The LR test statistic is asymptotically χ^2 distributed with $n(p-1)+1$ degrees of freedom according to the discussion in Section 2.1. Unfortunately, only VEC models with LSCCF or LCO(1) constraints are uniquely identified. However, the applicability of the LR test can be extended to LCO(q) setups where $1 < q < n$ if we are willing to assume that codependence and companion restrictions are jointly independent. However, this leads to a joint restriction test in the sense that rejection of the null hypothesis could be both due to a wrong LCO(q) assumption and due to an inappropriate imposition of independence.

So far we have regarded the cointegration vector as given as it is usual in the existing literature on SCCF and codependence, see e.g. Schleicher (2007) or Vahid & Engle (1993).[6] Such an assumption is typically justified in cases of strong (economic) priors and statistical evidence for a particular cointegration vector. This applies e.g. to situations of strong arbitrage that would imply a cointegration vector $\beta = (1, -1)'$ if a bivariate setup is considered. Another example is the analysis of controllability of overnight interest rates by central banks as discussed in the next section.

Of course, we may treat the cointegration vector as unknown and estimate it both under the null and the alternative hypotheses. Since the cointegration vector then enters both the pseudo-structural and the reduced form, the number of degrees of freedom does not change. Moreover, we may jointly test for codependence and restrictions on the cointegration vector. Then, the constrained vector is imposed under the null hypothesis but estimated under the alternative. However, in this case the number of restrictions increases by $n - 1$ to np.

[6] In this case, the LSCCF restrictions regarding the VECM parameters could be also tested by F- or WALD-tests, which have their usual asymptotic distributions.

In the next subsection we consider a GMM-type test that is applicable if a codependent VECM cannot be identified. In particular, the GMM test alleviates the sensitivity to model misspecification. Furthermore, it circumvents the potentially demanding numerical optimisation under the LR null hypothesis, see Schleicher (2007). Nevertheless, we have a preference for applying the LR test for identified model setups. This is, first, due to problems regarding the GMM test that can be uncovered by relating it to a Wald test for nonlinear restrictions and, second, due to the results of the simulation study in Schleicher (2007). The latter indicates an obvious advantage of the LR over the GMM test in terms of small sample power.

2.4.2 GMM-type and Wald Tests

If x_t is level codependent of order q, then $z_t = \beta' x_t$ should be uncorrelated with all its lags beyond q. Assuming that Δx_t follows a VECM with lag length $p-1$, it is sufficient to focus on zero correlations between z_t and $X_{t-q-1} = (z_{t-q-1}, \Delta x'_{t-q-1}, \Delta x'_{t-q-2}, \ldots, \Delta x'_{t-q-p+1})'$ according to the state-space setup (2.5)-(2.9). In their corresponding frameworks, Vahid & Engle (1997) and Schleicher (2007) have used such zero correlations as moment conditions for GMM estimation of the codependence vector. Based on the GMM approach, overidentifying restrictions can then be tested.[7]

We do not apply the GMM principle to estimate β since it is involved in both z_t and X_{t-q-1} but use the corresponding test approach. Following Vahid & Engle (1997) and Schleicher (2007), we test the null hypothesis

$$H_0 : g(\beta) = \mathrm{E}(z_t X_{t-q-1}) = 0_{[(n(p-1)+1)\times 1]} \tag{2.15}$$

by considering the statistic

$$Z_T = g_T(\beta)' P_T(\beta) g_T(\beta), \tag{2.16}$$

with $g_T(\beta) = \frac{1}{\sqrt{T-p-q}} \sum_{t=p+q+1}^{T} z_t X_{t-q-1}$ and $P_T = \Big(\hat{\sigma}^2 \Big(\frac{1}{T-p-q} \sum_{t=p+q+1}^{T} X_{t-q-1} X'_{t-q-1}\Big)$ $+ \sum_{i=1}^{q} \hat{\gamma}_i \Big(\frac{1}{T-p-q} \sum_{t=p+q+i+1}^{T} (X_{t-q-1} X'_{t-q-1-i} + X_{t-q-1-i} X'_{t-q-1})\Big)\Big)^{-1}$, which is the weighting matrix with $\hat{\sigma}^2$ and $\hat{\gamma}_i$ being consistent estimators of the variance and autocovariances of z_t. Note that we are testing for a cut-off in the serial correlation of the cointegration errors after q lags by applying the statistic (2.16).

As already pointed out by Schleicher (2007), the choice of the instrument set w_t makes the GMM test depend on the VECM framework. In other words, this approach can only be

[7] Vahid & Engle (1997) describe in detail the link between the GMM approach and a test based on canonical correlations that was suggested by Tiao & Tsay (1989). The latter procedure does not depend on the normalization of the codependence vector and is more convenient when testing for the number of codependence vectors. However, these issues are not relevant in regard of typical applications of our setup. Therefore, we do not consider the procedure of Tiao & Tsay (1989).

interpreted as a test for LCO(q) or, to be more precise, the null hypothesis (2.15) only represents the LCO(q) constraints if the VECM provides a correct representation of Δx_t. This link has two important implications. First, the upper bound for the codependence order should also be applied with respect to the GMM test. Second, the covariances considered in (2.15) are actually nonlinear functions of the VECM parameters. Therefore, we may express the moment conditions also in terms of $\theta = \text{vec}(J_\Delta F)$ by explicitly writing $g(\beta, \theta)$. Note in this respect that we have $g(\beta, \theta) = \text{E}(z_t X_{t-q-1}) = (\gamma_0' F^{q+1} \Gamma_X(0))'$, where $\Gamma_X(0) = \text{E}(X_t X_t')$. Thus, on the one hand, it is easily seen that the LCO(q) constraints (2.11)-(2.12) imply a zero covariance between z_t and X_{t-q-1}. On the other hand, if the usual assumption is made that $\Gamma_X(0)$ is nonsingular, then a zero covariance between z_t and X_{t-q-1} results in $\gamma_0' F^{q+1} = 0$, which is the set of zero parameter constraints in (2.12) that underlie LCO(q).

It is of course also possible to directly test $r(\theta) = (\gamma_0' F^{q+1})' = 0$ via a Wald test for nonlinear restrictions. Let $\hat{\theta}$ be an estimator of θ conditional on β or on a superconsistent estimator of β with $\sqrt{T}(\hat{\theta} - \theta) \xrightarrow{d} N(0, \Sigma_\theta)$. Then the usual Wald statistic is given by $W_T = Tr(\hat{\theta})' \left(\widehat{\frac{\partial r(\theta)}{\partial \theta'}} \widehat{\Sigma}_\theta \widehat{\frac{\partial r(\theta)'}{\partial \theta}} \right)^{-1} r(\hat{\theta})$, where $\widehat{\partial r(\theta)}/\partial \theta'$ and $\widehat{\Sigma}_\theta$ are consistent estimators of $\partial r(\theta)/\partial \theta'$ and Σ_θ, respectively. However, the statistic W_T has an asymptotic χ^2-distribution only if the restrictions in $r(\theta)$ are functionally independent, requiring the Jacobian matrix $\partial r(\theta)/\partial \theta'$ to be of full row rank, see e.g. Andrews (1987). Obviously, if the Jacobian matrix is not of full row rank, then $\Sigma_{r(\theta)} = \frac{\partial r(\theta)}{\partial \theta'} \Sigma_\theta \frac{\partial r(\theta)}{\partial \theta}$ will be singular and the asymptotic distribution of W_T is nonstandard.

A rank deficit regarding $\partial r(\theta)/\partial \theta'$ can also occur in our setup given the results of Lütkepohl & Burda (1997). They consider the case of multi-step Granger causality within a VAR. The nonlinear parameter restrictions involved can in fact induce a singularity in the relevant covariance matrix in part of the parameter space. The restrictions we are testing are of the same kind as the ones underlying multi-step Granger causality, compare also Dufour & Renault (1998). Thus, it cannot be ruled out that $\Sigma_{r(\theta)}$ is singular.

It logically follows from the foregoing discussion that functional independence of the components in $g(\beta, \theta)$ cannot be guaranteed for the whole parameter space. Thus, the GMM test may be similarly affected by a singularity problem as the Wald test. As a consequence, the statistic Z_T may not have an asymptotic χ^2-distribution.

Even though the GMM test is not without drawbacks, its usefulness is emphasized by the following argumentation. Consider the case that the VECM is regarded only as an approximation of the process Δx_t. Then, no mapping between the VECM parameters and the considered covariances exists so that the preceding discussion does not apply. However, the null hypothesis $g(\beta) = 0_{[(n(p-1)+1)\times 1]}$ is only covering a subset of the restrictions implied by a

LCO(q) setup. Nevertheless, evidence *against* LCO(q) can still be collected since rejection of $g(\beta) = 0_{[(n(p-1)+1)\times 1]}$ implies also rejection of the LCO(q) constraints. In this respect, the GMM-type test is a useful procedure, in particular in situations where the LR test cannot be applied due to nonidentification of the pseudo-structural VECM.

Obviously, the GMM statistic (2.16) has an asymptotic χ^2 distribution with $n(p-1)+1$ degrees if no link to the VECM is given and Δx_t is assumed to have a Wold representation in terms of its innovation vector, compare Vahid & Engle (1997). To compute g_T and P_T, we can either use a pre-specified vector for β or a superconsistent estimator $\hat{\beta}$. The use of the latter is justified by its superconsistency and the fact that $P_T(\hat{\beta})$ is a consistent estimator of $(\lim_{T\to\infty} \mathrm{E}(g_T(\beta)g_T(\beta)'/T))^{-1}$, compare also Brüggemann, Lütkepohl & Saikkonen (2006).

Lütkepohl & Burda (1997) have suggested modifications of the Wald test that address the potential singularity of the relevant covariance matrix $\Sigma_{r(\theta)}$. These modifications assure that the modified statistics have an asymptotic χ^2-distribution. They propose e.g. to draw a random vector from a multivariate normal distribution and add it to the restriction vector $r(\hat{\theta})$. Thereby, a nonsingular covariance matrix for the modified restriction vector is obtained. The modification, however, requires the specification of a scaling parameter with respect to the random noise vector. Although the simulation study in Lütkepohl & Burda (1997) provides some evidence for a range of reasonable values for this parameter, no clear guideline for choosing them in applied work exists. We have applied different values for the scaling parameter, but the resulting p-values strongly differ in the empirical applications of the next section. That is why we decided not to present the outcomes of the modified Wald test. Accordingly, we refer to Lütkepohl & Burda (1997) for further details.

3 Can Central Banks Control Overnight Rates?

Herein, we provide a useful application of the new LSCCF and LCO concept. I.e., we examine to which extent overnight money market rates are controllable by monetary policy makers. We look in turn at the world's most prominent central banks, the Fed and the ECB.

3.1 Federal Reserve Bank

Over decades, the Fed has developed an institutional framework for effectuating its monetary policy. Since February 1994, the Fed has announced changes in the federal funds target rate immediately after the decision. Such transparency is likely to contribute to low persistence of deviations of the federal funds rate from its target, see Nautz & Schmidt (2008). The same presumably holds true for the forward-looking assessment of inflationary pressure and economic slowdown, which complements the FOMC statements since January 2000.

The Fed requires commercial banks to hold a certain average amount of reserves during each maintenance period of two weeks. Therefore, we argue that a natural frequency for the empirical analysis is provided by biweekly data. In this, while the maintenance periods end on the so-called Settlement Wednesdays, we measure the interest rates on the Wednesdays in between. Doing so has the further advantage to avoid dealing with predictable day-of-the-week effects or the Settlement Wednesday tightness (see Hamilton 1996), which is rendered innocuous by sampling at the midpoints of the maintenance periods. The sample is chosen as 06/28/2000-12/03/2008, where the starting point ensures consistency with the European case discussed below. The end date is determined by the fact that the Fed replaced its target rate by a target *range* (initially from 0 to 0.25) on 16 December 2008. In total, we have 221 observations. Figure 1 shows the federal funds and the target rate as well as the policy spread.

Figure 1. Federal funds rate, target rate and spread

Evidently, the overnight rate closely follows the target, so that the spread reveals no long-lasting swings. The correlogram of the spread can be seen in Figure 2. The spread yields mostly small serial correlations that would be judged insignificant applying the asymptotic standard error $1/\sqrt{T}$. However, the first two autocorrelations do not seem to be necessarily negligible. Therefore, as discussed above, testing within the underlying VECM is preferred in order to jointly consider all relevant lags.

The VECM for the federal funds rate i_t and the target rate i_t^* is specified with a restricted constant and four lags in first differences, as suggested by the AIC and HQ criteria. The Johansen trace test easily confirms cointegration with a test statistic of 35.27 (p-value = 0.02%). The Portmanteau test for non-autocorrelated residuals, compare Lütkepohl (2005), is clearly

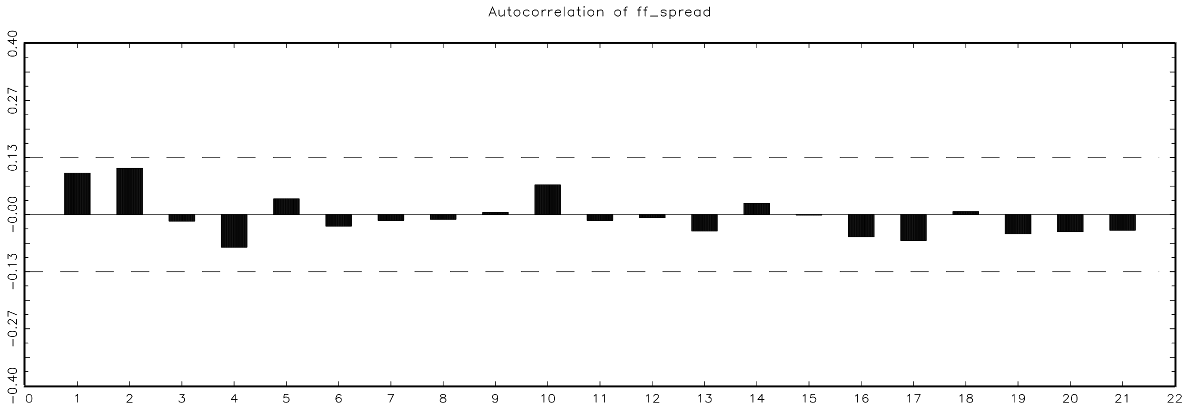


Figure 2. Autocorrelations of federal funds spread

insignificant at all lags, so that the model seems to be adequate in the sense of picking up the complete dynamics from the data. In contrast, reducing the lag length would leave pronounced autocorrelation in the residuals. Concerning the cointegration vector, we have a strong theoretical prior for $\beta = (1 \;\; -1)'$. Empirically, this restriction is not rejected given a LR p-value of 27.6%.[8] The estimated VECM takes the following form:

$$\begin{pmatrix} \Delta i_t \\ \Delta i_t^* \end{pmatrix} = \begin{pmatrix} \underset{(0.225)}{-0.766} \\ \underset{(0.140)}{0.168} \end{pmatrix} (i_{t-1} - i_{t-1}^* + \underset{(0.014)}{0.023}) + \begin{pmatrix} \underset{(0.204)}{0.042} & \underset{(0.215)}{-0.036} \\ \underset{(0.127)}{0.025} & \underset{(0.134)}{-0.126} \end{pmatrix} \begin{pmatrix} \Delta i_{t-1} \\ \Delta i_{t-1}^* \end{pmatrix} + \begin{pmatrix} \underset{(0.177)}{0.047} & \underset{(0.189)}{0.213} \\ \underset{(0.110)}{-0.070} & \underset{(0.118)}{0.170} \end{pmatrix} \begin{pmatrix} \Delta i_{t-2} \\ \Delta i_{t-2}^* \end{pmatrix}$$
$$+ \begin{pmatrix} \underset{(0.148)}{0.072} & \underset{(0.168)}{0.236} \\ \underset{(0.092)}{0.021} & \underset{(0.105)}{0.295} \end{pmatrix} \begin{pmatrix} \Delta i_{t-3} \\ \Delta i_{t-3}^* \end{pmatrix} + \begin{pmatrix} \underset{(0.108)}{0.060} & \underset{(0.137)}{0.147} \\ \underset{(0.067)}{0.113} & \underset{(0.085)}{0.195} \end{pmatrix} \begin{pmatrix} \Delta i_{t-4} \\ \Delta i_{t-4}^* \end{pmatrix} + \begin{pmatrix} \hat{u}_{1t} \\ \hat{u}_{2t} \end{pmatrix}$$

The difference of the adjustment coefficients, $\alpha_2 - \alpha_1$, lies near one. Among the parameters in the short-run dynamics, there are pairs that are more and some that are less equal. Even though most single coefficients are estimated relatively imprecisely, the model has been carefully specified and is unlikely to provide an incorrect reflection of the true data generating process (as far as any model can be correct, of course). Rather, the large standard errors are more a consequence of natural multicollinearity in VARs than signs of true expendability. In sum, the impression from institutional facts, visual inspection and preliminary statistical analysis suggests a close controllability of the federal funds rate by the Fed. Indeed, applying the nine LSCCF restrictions $\beta'\alpha = 0$ and $\beta'\Gamma_i = 0$, $i = 1, \ldots, 4$, to the VECM, the likelihood does not shrink dramatically. The p-value of the according LR test with nine degrees of freedom amounts to 34.1%. Moreover, the corresponding GMM test results in a p-value of 17.9%. Therefore, we can conclude that on average, deviations of the federal funds rate from the target

[8] This test was repeated after the LSCCF restrictions had been imposed, with the same result.

are corrected at least within one maintenance period.

3.2 European Central Bank

We use the European example to provide some discussion of potential problems connected to our testing procedure. In this, we allude to critical points in the model specification, where special care is advised.

The ECB provides liquidity to the European banking sector through weekly main refinancing operations (MROs). The relevant market and target rates are the Euro Overnight Index Average (Eonia) and the minimum bid rate (MBR). Since June 2000, the date chosen as our starting point, the MROs are conducted as variable rate tenders, see Hassler & Nautz (2008). Furthermore, the ECB shortened the MRO maturity from two weeks to one week in March 2004. The considerable rise in spread persistence, as established by Hassler & Nautz (2008), could then be explained by higher costs and risk of refinancing. Consequently, we split the European data into two sub-samples in order to accommodate a potential structural break. These sub-samples have 97 (June 2000 - February 2004) and 124 (March 2004 - December 2008) observations, respectively. We keep the frequency of the US data. Figure 3 plots the Eonia and the MBR as well as the European spread.

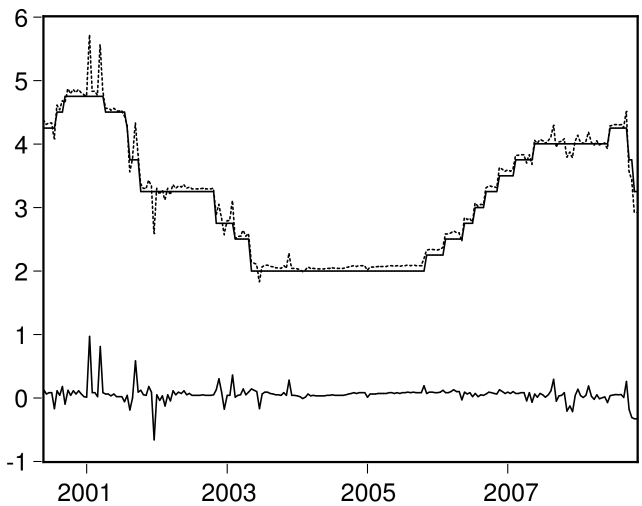
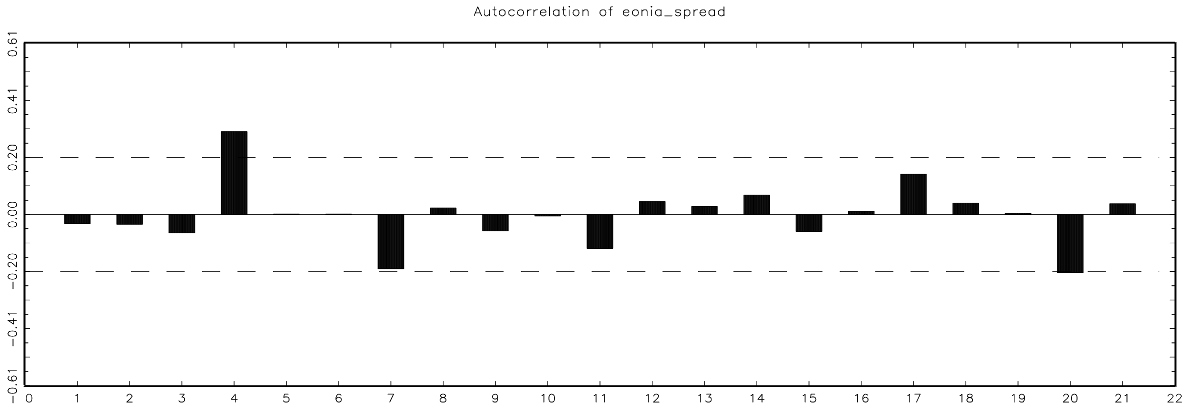


Figure 3. Eonia, minimum bid rate and spread

Even though the spreads are still small compared to the level of the interest rates, the deviations do not feature the white-noise character from the US case. Backing the visual impression, Figure 4 presents the autocorrelations for both sub-samples.

Most serial correlations of the spread are rather negligible. However, lag four in the first

Panel A: 1st sub-sample: June 2000 - February 2004



Panel B: 2nd sub-sample: March 2000 - December 2008

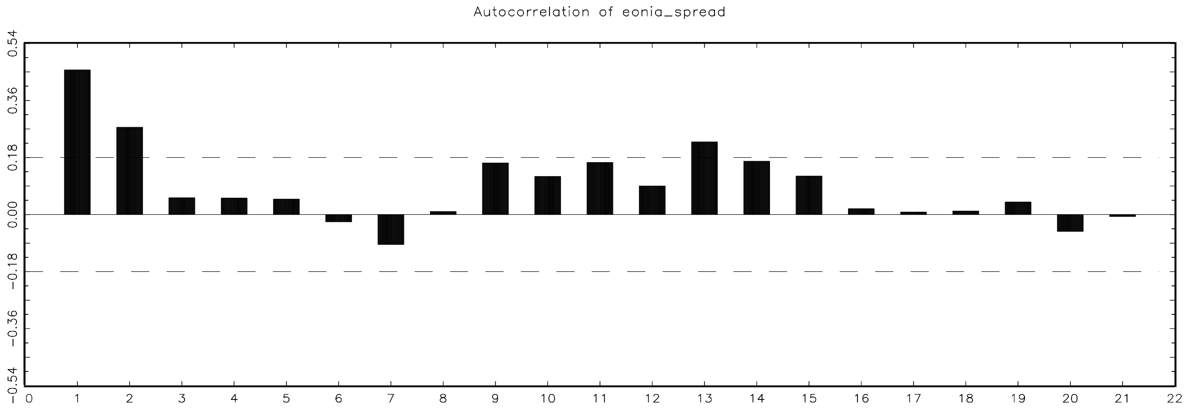


Figure 4. Autocorrelations of Eonia Spread

and various lags in the second sub-period cast the LSCCF hypothesis into doubt. In the second period, Hassler & Nautz (2008) have established long-memory for the spread using daily data. In general, long-memory behaviour should not change when sampling at different frequencies (e.g. Chambers 1998). Indeed, Panel B of Figure 4 reveals a typical pattern of persistent serial correlations, even if most of them individually do not reach significance due to the relatively low number of observations.

For the first sub-period VECM, all information criteria suggest a lag length of zero. One cointegrating relation is significant with a trace statistic of 74.90, and the $\beta = (1 \ -1)$ restriction passes with a p-value of 25.8%. The resulting model is

$$\begin{pmatrix} \Delta i_t \\ \Delta i_t^* \end{pmatrix} = \begin{pmatrix} \underset{(0.125)}{-1.059} \\ \underset{(0.070)}{-0.026} \end{pmatrix} (i_t - i_t^* - \underset{(0.017)}{0.068}) + \begin{pmatrix} \hat{u}_{1t} \\ \hat{u}_{2t} \end{pmatrix} .$$

One can assert at first sight that the LSCCF restriction $\alpha_1 - \alpha_2 = -1$ is empirically acceptable. Indeed, the LR and GMM tests produce p-values of 75.6% and 75.2%, respectively. However, the Portmanteau test is significant from lag nine onwards. That is, despite the unanimous decision of all information criteria, the model seems to be misspecified. Presumably, the VECM(0) has not taken into account the 30% autocorrelation on the fourth lag of the spread (Figure 4, Panel A).

This last conjecture can be supported when estimating a VECM(3), equivalent to VAR(4), which yields good Portmanteau results. Now, the seven LSCCF constraints cannot be rejected with p-values of 19.2% for the LR test and 25.7% for the GMM test. Nevertheless, the bulk of the coefficients introduced by the higher model order is insignificant and superfluous, as it is but the fourth VAR lag that matters for the Eonia dynamics. Indeed, the second and third lag can be excluded from the VAR by conventional Wald tests. This reduces the number of restrictions coming from LSCCF by four - two for each 2×2 matrix. When applying the LR test with $7 - 4 = 3$ restrictions to the VECM derived from the accordingly restricted VAR(4), we obtain a p-value of 2.8%.[9] This suggests that the fully general VECM(3) had inflated the number of degrees of freedom, lowering the power of the LR test. In conclusion, it is an advantage to have a closed modelling framework and a clear-cut test at hand, but correct model specification and power issues are to be carefully dealt with.

In the second sub-period, both AIC and HQ choose two lags. The Portmanteau tests are quite favourable until lag 22, but for a few tens of lags from 23 upwards, the p-values do not reach more than 3% to 4%. The trace statistic of 31.67 is clearly significant, whereas the evidence against $\beta = (1, -1)$ is somewhat stronger than before with a p-value of 1.5%. Nonetheless, we proceed with $\beta_2 = -1$, because restricting the freely estimated parameter of -0.975 is not going to affect the LSCCF test outcome. The VECM results as

$$\begin{pmatrix} \Delta i_t \\ \Delta i_t^* \end{pmatrix} = \begin{pmatrix} \underset{(0.227)}{-0.531} \\ \underset{(0.153)}{0.036} \end{pmatrix} (i_t - i_t^* - \underset{(0.011)}{0.029}) + \begin{pmatrix} \underset{(0.219)}{-0.259} & \underset{(0.256)}{0.598} \\ \underset{(0.147)}{-0.119} & \underset{(0.172)}{0.166} \end{pmatrix} \begin{pmatrix} \Delta i_{t-1} \\ \Delta i_{t-1}^* \end{pmatrix} + \begin{pmatrix} \underset{(0.191)}{0.275} & \underset{(0.280)}{0.148} \\ \underset{(0.128)}{0.200} & \underset{(0.188)}{0.029} \end{pmatrix} \begin{pmatrix} \Delta i_{t-2} \\ \Delta i_{t-2}^* \end{pmatrix} + \begin{pmatrix} \hat{u}_{1t} \\ \hat{u}_{2t} \end{pmatrix} .$$

As might be suspected in view of the estimates for the adjustment coefficients, both the LSCCF-LR and LSCCF-GMM tests reject the null hypothesis, implying five degrees of freedom, with p-values close to zero. However, Figure 4, Panel B might suggest that this rejection is primarily triggered by the significant autocorrelations at lag one and two. In other words, the adjustment process would be finished in the third period. Theoretically, an LCO(2) setup can exist since the order $q = 2$ equals the maximum order $q_{max} = (p - 1)(n - 1)$. However, we have $q = n$ in this case such that the pseudo-structural form is not identified. Accordingly, an LR test cannot be applied. Applying a LR test for LCO(1) instead leads to a clear rejection due

[9]The GMM test cannot be adjusted to the restricted VAR model.

to a p-value close to zero. The same result is obtained using the GMM-type test. The latter test can also be used to test for level codependence order of two. Again we have to reject the null hypothesis due to a p-value of 2.4%.

Hence, the tests seem to pick up the later non-negligible autocorrelations in the EONIA spread, judging them as evidence against MA(1) or MA(2) processes. Indeed, exactly this decision was to be expected, recalling the long-memory result of Hassler & Nautz (2008). Thus, our test succeeds in discriminating between different degrees of interest rate controllability both through different countries and time periods.

4 Conclusions

While cointegration denotes the commonality of non-stationary components among different variables, we combine it with the concept of common serial correlation. Time series obeying the according restrictions move in parallel in the sense that a specific linear combination is free of any autocorrelation structure. Concerning cointegration adjustment, this implies that any deviation from the equilibrium is corrected within a single period. In order to accommodate delayed adjustment, we extend the framework to codependence, which describes constellations where equilibrium ist restored after a lag of q periods.

For both LSCCF and codependence, we derive the constraints to be fulfilled in VECMs. Thereby, maximum codependence orders and identification issues are discussed, correcting several results from the literature. For statistical inference, we propose ML estimation as well as LR and GMM testing.

Important applications of the developed framework arise whenever economic reasoning suggests that variables stay in close contact over time. Such development may be generated by processes of financial arbitrage. In our empirical section, we examine the question of controllability of interest rates by central banks. In particular, we examine whether the Fed and the ECB succeeded in making overnight money market rates closely follow their target rates. Results for the US are quite favourable in this regard, since LR and GMM tests yield no evidence against the LSCCF hypothesis. The European case delivers contrary results, even though in the 2000-2004 sub-period, a LSCCF might be present. However, since a change in the operational monetary policy framework in 2004, neither LSCCF nor the weaker concept of codependence can be empirically confirmed.

In conclusion, this paper offers both an innovative and a cautious perspective: On the one hand, common serial correlation in levels provides an intuitive and useful enhancement of the literatures of common cycles, cointegration and adjustment speed. On the other hand, we critically evaluate the scope of VECM-based common serial correlation analyses, pointing at conceptual and empirical problems. Nevertheless, we believe that an appreciable potential of the underlying methodology could be exploited, if the crucial econometric issues are carefully dealt with.

Appendix: Proof of Theorem 1

First, consider the structure of the companion matrix F. It consists of a first row containing the parameters in the equation for $\beta' x_t$, then a first block of n rows containing the parameters in the equation for Δx_t, and a set of $p-2$ blocks of n rows consisting of identity and zero matrices. The latter blocks are numbered from 2 to $p-1$.

Let us assume that there exists level codependence of order q in x_t. Then, we have $\gamma_0' F^{q+1} = \gamma_0' F^{q+2} = \cdots = \gamma_0' F^{q+p} = 0$. Since LCO($q$) implies codependence of order $q+1$ in Δx_t, it follows that $B_\beta' F^{q+2} = B_\beta' F^{q+3} = \cdots = B_\beta' F^{q+p} = 0$. The structure of identity and zero matrices in blocks 2 to $p-1$ assures that the first block of F^{q+2} is the second block in F^{q+3}, the third block in F^{q+4} and so forth so that the first block of F^{q+2} is the last block in F^{q+p}. Define for processes with $p \geq 3$, $\gamma_{\beta,3} = (0 : 0_{1\times n} : \beta' : 0_{1\times n(p-3)})'$, $\gamma_{\beta,4} = (0 : 0_{1\times 2n} : \beta' : 0_{1\times n(p-4)})'$, $\ldots$, $\gamma_{\beta,p} = (0 : 0_{1\times(p-2)n} : \beta')'$. In line with the foregoing, we obtain $\gamma_{\beta,3}' F^{q+3} = \gamma_{\beta,3}' F^{q+4} = \cdots = \gamma_{\beta,3}' F^{q+p} = \gamma_{\beta,4}' F^{q+4} = \gamma_{\beta,4}' F^{q+5} = \cdots = \gamma_{\beta,4}' F^{q+p} = \cdots = \gamma_{\beta,p}' F^{q+p} = 0$.

Since γ_0, B_β, $\gamma_{\beta,3}$, $\gamma_{\beta,4}$, $\ldots$, $\gamma_{\beta,p}$ are linearly independent as a system and rk$(F^i) \leq$ rk(F) for $i \in \mathbb{N}$ by Lütkepohl (1996, Section 3.7), rk$(F^{q+i}) <$ rk(F^{q+i-1}) for $i = 1, 2, \ldots, p$. Otherwise, the codependence order would be larger than q. Moreover, note the rule rk$(F^i) =$ rk(F^{i+1}) for some $i \in \mathbb{N} \Rightarrow$ rk$(F^i) =$ rk(F^j) for all $j \geq i$, compare Lütkepohl (1996, Section 4.3.1). As a consequence, the ranks of increasing powers of F must have fallen throughout starting from F to F^2. Because of rk$(F^{q+p}) \geq 0$, the maximum order q is obtained if $q+p$ is equal to the dimension of F. Thus, $q_{max} = n(p-1) + 1 - p = (n-1)(p-1)$ is obtained. This completes the proof. ■

It is easy to see that in case of a general cointegration rank r, the maximum order for a single codependence vector is given by $q_{max} = n(p-1) + r - p = (n-1)p - (n-r) = (n-1)(p-1) + (r-1)$. Based on the idea of the proof of Theorem 1, that is related to the structure of *VAR-type* companion matrices, we can also derive upper bounds for various other setups. This refers to codependence in stationary VAR processes as well as to codependence in the first differences of $I(1)$ variables that follow either VECM processes or non-cointegrated VARs. The results are summarized in the following lemma.

Lemma 1. (1) Let x_t be a n-dimensional stationary VAR(p) process. Then, the maximum codependence order with respect to x_t is given by $q^l_{max} = (n-1)p$ in case of a single codependence vector. (2) Let x_t be a n-dimensional vector of $I(1)$ variables that follows a VAR(p) with a cointegrating rank r, where $0 \leq r \leq n-1$. Then, the maximum codependence order with respect to Δx_t is given by $q^d_{max} = (n-1)(p-1) + r$ in case of a single codependence vector.

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