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Abstract

Two parties bargaining over a pie whose size is determined by the investment decisions of both. The bargaining rule is sensitive to the investment behavior. If a symmetric investments profile is observed, bargaining proceeds according to the Nash Demand Game; otherwise bargaining proceeds according to the Ultimatum Game. We are interested in the evolutionary emergence of both an efficient investment norm and a bargaining norm. Under some conditions we prove that these norms co-evolve; when this happens they support the efficient investment and the egalitarian distribution of the surplus. In addition, when surplus requires that at least one agent invests, then either both norms co-evolve or no norm evolves.

Key Words: evolution; norms; specific investment; hold-up problem.

Classification Codes. C78, L41.

1 Introduction

It is well known that when an economic relationship requires some individual specific investment but agents are not sure whether they will be able to catch the benefits, investments that are individually rational might not be undertaken. This is what happens in the so-called hold-up problem (Tirole 1986). This problem arises in scenarios where investments costs are sunk at the time the surplus occurs; since sunk costs are not taken into account at the bargaining stage, the ex post division of the surplus is insensitive to the level of investment. An economically inefficient outcome can then occur and an institutional arrangement is needed.

However in sequential models with a bargaining stage following a production stage some subgame perfect equilibria supporting the efficient investment profile can come to the fore even when agents bargain over the gross surplus. When only one agent makes a specific investment, followed by the Nash Demand Game, Troger (2002) and Ellingsen and Robles (2002) have shown that all the stochastically stable equilibria are efficient and a neat distributive norm evolves; when the grid of possible investments gets very fine, the evolved distributive norm virtually assigns all the surplus to the investor. When a multiplicity of subgame perfect equilibria occurs, the aforementioned papers show that evolution can solve the hold-up problem.[1] When the size of the pie is endogenous, it is thus illegitimate to separate the analysis of the bargaining stage from the prior investment stage and sunk costs matter in such a way that efficient investment incentives are provided (Troger, 2002).

However Troger (2002) and Ellingsen and Robles (2002) are concerned with the case in which the pie depends on the investment of one part only. In this paper we extend the analysis to two-sided relationship specific investment. Our purpose is to investigate under what circumstances an efficient norm of investment and a bargaining norm can endogenously evolve when the size of the pie is determined by both agents. The basic model is developed and extended in Dawid and MacLeod (2001, 2008) where two types of investments are allowed, high and low, and only high investment is costly. At the end of the first stage a surplus is realized and it depends on the investments profile. The efficient (net) surplus arises when both players have chosen the high investment. Before bargaining the surplus is perfectly observed by both agents, although it cannot be verified by a court. In this context any eductive argument is of no help in predicting the outcome of the game.

Dawid and MacLeod (2001) show that either a unique stochastically stable outcome fails to exist or, if it exists, it is not necessarily efficient. This result allows them to claim (page 161) that, in contrast to Troger (2002) and Ellingsen and Robles (2002), when both parties might invest, the requirement that norms of bargaining be stochastically stable can exacerbate the hold-up problem. Their findings however only holds under some crucial assumptions: (i) bargaining only occurs when asymmetric investments profiles are observed; (ii) agent interacts according to the Nash Demand Game; (iii) when symmetric investments profiles are observed, the total surplus is equally split. The evolutionary dynamics considered by Dawid and MacLeod (2001) is an adaptation of Young (1993) to extensive form games. However, though this extension is not problematic with one sided investment (as in Troger (2002)), it engenders a kind of cognitive bias with two sided relationship specific investment.[2] A major shortcoming of their evolutionary approach is that it prevents the study of the emergence of a bargaining norm. In fact, since bargaining only occurs when asymmetric investments profiles are observed, a bargaining norm can evolve only if the unperturbed model has some limit set supporting an heterogenous investment behavior; however under their evolutionary dynamics only symmetric investments profiles are almost always observed.[3] In order to overcome this

[1] In a similar model Andreozzi (2008) shows that this result does not extend to the case of heterogeneous investment costs.

[2] Specifically, under their evolutionary dynamics some agent can continue to believe that all the opponents make the same investment (i.e. all choose high or low investment) even when he has observed some bargaining outcome (which can happen only when high-low matches occur).

[3] In a companion paper we embed Dawid and MacLeod (2001) model into the evolutionary dynamics considered by Noldeke and Samuelson (1993); see Bagnoli and Negroni (2010).

problem, the basic model is extended in Dawid and MacLeod (2008).

In this paper we follow a different route. The two key features of our approach are the following. First we assume that bargaining occurs at each investment profile but that the bargaining rule is sensitive to the investment behavior. We think reasonable to assume that agents have the same bargaining power only when a symmetric investment profile is observed; when instead an asymmetric investment profile is observed, we deem more reasonable to give all the bargaining power to the agent who has made the high investment. Therefore a Nash Demand Game occurs when agents make the same investment and an Ultimatum Game occurs when only one agent makes the high investment. Second we use the evolutionary framework put forward by Noldeke and Samuelson (1993) which is more adequate for extensive form games since beliefs are free to change by mutations. This framework assumes two populations of agents and, in every period, every possible match between agents is allowed.

We show that, under some conditions, a bargaining norm and an investment norm coevolve. When these conditions are met then, for whatever grid of possible claims, the bargaining norm supports an egalitarian division and the investment norm supports the efficient investment profile. In order to elicit a norm based solution to the hold-up problem, the following two conditions are required: (i) each equilibrium supporting an homogenous low investment profile is Pareto dominated by some equilibria supporting different investment profiles; (ii) there exists some efficient equilibrium such that nobody has the incentive to make low investment when he believes to get almost the whole pie even when one opponent deviates by claiming more.

When only the first condition holds then neither an investment norm nor a bargaining norm evolve. When instead both conditions fail then, even if an investment norm can exist, it cannot be the efficient one. In order to give an insight of this latter result suppose that the efficient investment norm evolved and only the first condition is met. This means that since no high-low matches are observed, the beliefs concerning the bargaining outcome at high-low matches can drift. Because of drift all agents of one population might deem to capture almost the whole pie by choosing low investment in the low-high matches. Hence when a single agent of the opponent population deviates by claiming a larger demand, then by updating all agents in the first population are willing to make the low investment so that the efficient norm is overturned.

The aforementioned conditions can respectively be translated into conditions on the investment cost and the degree of investment complementarity. When investment are not complements, then for whatever investment cost both norms do not evolve. Under the same condition Dawid and MacLeod (2001) prove that no investment norm can emerge. Since, as we remarked before, they can not study the emergence of a bargaining norm, our result can be seen an extension of their findings.

Sharper results can be drawn when in order to get a surplus at least one agent has to choose high investment. In this case, by a direct application of our findings and contrarily to what emerge when the surplus is always equally split (the canonical case in which the hold-up problem is studied), we show that either the stochastically stable outcome sustains the efficient investment profile and an egalitarian distribution or no norm evolves. Contrarily to Dawid and MacLeod (2001) we can then conclude that with two sided relationship specific investment, the requirement that norms of bargaining be stochastically stable can downsize the hold-up problem.

The basic model is presented in Section 2. Section 3 describes the evolutionary dynamics and gives some preliminary results. The emergence of investment conventions is the topic of Section 4 while Section 5 is concerned with bargaining norms.

2 The model

Two risk neutral players A and B are engaged in a two stages game, Γ. In the first stage both have to simultaneously decide among two types of investment, H and L; when a player chooses H he incurs in a cost c. At the end of the first stage, a surplus is realized and it is observed; each player can then correctly estimate the opponent's investment. The value of the surplus depends on the specific investment each player has made at the beginning of the game. In particular we denote by V_H the surplus arising when both choose H; by V_M the surplus accruing when only one chooses H and lastly by V_L when both choose L. Obviously $V_H > V_M > V_L \geq 0$.

In the second stage they bargain over the available surplus. The bargaining rule depends on the investment profile. If both have chosen the same investment, they are engaged in a Nash Demand Game; if they have chosen different investment, they are engaged in an Ultimatum Game. In both cases players demand a fraction of the surplus. Let $D(V_j) = \{\delta, 2\delta, ..., V_j - \delta\}$ where $j \in \{H, M, L\}$ denote the set of feasible demands. The bargaining is always well defined if $\delta < \frac{V_L}{2}$. Moreover we assume that $\frac{V_H}{2}$ is divisible by δ.

In the Nash Demand Game both players simultaneously make a demand y and x. If the demands are compatible, each gets what he claimed; otherwise they get nothing. When both choose H, the payoff are

$$\pi_A = \begin{cases} y - c & if \quad y + x \leq V_H \\ \\ -c & if \quad y + x > V_H \end{cases}$$

and

$$\pi_B = \begin{cases} x - c & if \quad y + x \leq V_H \\ \\ -c & if \quad y + x > V_H. \end{cases}$$

When both choose L, the structure of the payoffs is similar but $c = 0$.

In the Ultimatum Game the player who has chosen H makes a demand; the opponent, after observing this demand, can either accept or reject it. Suppose HL is observed and A makes a demand y. If B accepts the payoffs are $y - c$

for A and $V_M - y$ for B; otherwise A gets $-c$ and B nothing. The rules of the Ultimatum game allows us to simplify the set of demands for the player who has chosen L. In particular, when A chooses H and makes a demand y, the set of actions for B is $\{V_M - y, V_M\}$ where the first is equivalent to accept the demand of the opponent and the second is equivalent to reject it.

In this setting, a profile of behavioral actions for player A must specify:

(i) the type of investment; (ii) the demand when both players choose H (the action at HH); (iii) the demand when both players choose L (the action at LL); (iv) the demand when A chooses H and B chooses L (the action at HL); (v) whether to accept or reject any demand made by B, when in the first stage B chooses H and A chooses L. Analogously for player B.

Γ always admits a subgame perfect equilibrium which supports the investment profile HH if $V_H - 2c > 2\delta$. Notice however that the game admits a great number of subgame perfect equilibria and that some of these support the investment profile LL. Throughout the paper we consider a class of games satisfying the condition

$$V_H - 2c > \max\left(V_M - c, V_L\right). \tag{1}$$

This means that the investment profile HH is the efficient one; this assumption, together with $V_L > 2\delta$, means that in the class of games considered some subgame perfect equilibrium supporting the efficient investment always exists.

3 Evolutionary dynamics

In this Section we apply to Γ the evolutionary dynamics put forward by Noldeke and Samuelson (1993). For each player there is a finite population of agents of size N. In every period every possible match between agents occurs. This means that each agents belonging to population A interacts with every agent of population B, one at the time. An agent is described by a *characteristic* which consists of a detailed plan of actions and a set of beliefs on the actions the opponent can take at each information set. A *state* θ is a profile of characteristics on the overall population and $z(\theta)$ is the probability distribution of outcomes generated by θ. The set of possible states, Θ, is finite.

In each period, after the state is realized, with probability μ each agent in each population independently observes the distribution of outcomes $z(\theta)$. This information allows the learning agent to update his conjectures so that, at the information set reached in θ, these are equal to the observed frequencies of actions. Afterwards, given his new beliefs, he updates his action profile by choosing, at all information sets, a best reply action.[4] However, if at some information set the learning agent has already played a best reply action, his action

[4] When the best reply contains more than one action, then one of these can be randomly chosen according to a distribution probability with full support.

does not change. With probability $1-\mu$ the single agent does not observe $z(\theta)$ so that his characteristic does not change. This learning mechanism engenders a Markov process with transition matrix P on Θ. We denote by (Θ, P) the unperturbed Markov process. We remark that under this evolutionary dynamics every subset of agents has positive probability of revising.

With a slight abuse of notation we denote by (HH, y_{HH}, x_{HH}) a terminal node in which both agents have chosen H, agent A makes a demand y_{HH} and agent B makes a demand x_{HH}. Analogously for the other terminal nodes. Let us consider a state θ_t and suppose that all agent observe $z(\theta_t)$. For every agent $i \in A$, action L is not preferred to action H if

$$p_B(\theta_t)\left(\widetilde{y}^i_{HH}(\theta_t) - \widetilde{y}^i_{LH}(\theta_t) - c\right) + (1 - p_B(\theta_t))\left(\widetilde{y}^i_{HL}(\theta_t) - \widetilde{y}^i_{LL}(\theta_t) - c\right) \geq 0;$$

analogously, for every agent $i \in B$, action L is not preferred to action H if

$$p_A(\theta_t)\left(\widetilde{x}^i_{HH}(\theta_t) - \widetilde{x}^i_{HL}(\theta_t) - c\right) + (1 - p_A(\theta_t))\left(\widetilde{x}^i_{LH}(\theta_t) - \widetilde{x}^i_{LL}(\theta_t) - c\right) \geq 0.$$

Here we denote by $p_A(\theta_t)$ (resp. $p_B(\theta_t)$) the frequency of agent A (resp. B) who played H in θ_t and by $\widetilde{y}^i_{HH}(\theta_t)$ (resp. $\widetilde{x}^i_{HH}(\theta_t)$) the expected payoffs of agent $i \in A$ (resp. $i \in B$) at the information set HH, given $z(\theta_t)$. Analogously for the other information sets.

Definition 1 *A set $\Omega \subseteq \Theta$ is called a $\omega-$limit set of the process (Θ, P) if:*

$$\begin{aligned}
\forall \theta \in \Omega, \qquad & \quad Prob\{\theta_{t+1} \in \Omega \mid \theta_t = \theta\} &= 1 \\
\forall (\theta, \theta') \in \Omega^2, \quad \exists s > 0, \quad & Prob\{\theta_{t+s} = \theta' \mid \theta_t = \theta\} &> 0.
\end{aligned}$$

When a process enters into a $\omega-$limit set (Ω), it does not exit and wanders forever in it; in this case we denote by $\rho(\Omega)$ the set of outcomes that can be observed almost always. Since Ω is not unique the unperturbed Markov process is not ergodic.

Besides updating, agents' beliefs and actions can change by mutations. In every period each agent has a probability ϵ of mutating; these mutations are independently distributed across agents. When mutating, an agent changes his characteristic according to a probability distribution assigning positive probability on each possible characteristic. This generates a new Markov process with transition matrix $P(\epsilon)$ on Θ. We denote by $(\Theta, P(\epsilon))$ the perturbed Markov process. This is now an ergodic process. It is well known that for any fixed $\epsilon > 0$, the process $(\Theta, P(\epsilon))$ has a unique invariant distribution μ_ϵ. It is standard in the literature to focus on the limit distribution μ_* defined by $\mu_* = \lim_{\epsilon \to 0} \mu_\epsilon$. A state θ is stochastically stable if $\mu_*(\theta) > 0$. We denote by Σ_S the set of stochastically stable states; this is the set of states which have a positive probability in the limit distribution. Noldeke and Samuelson (1993) proved that the stochastically stable set is contained in the union of the $\omega-$limit sets of the unperturbed process. Hence, in order to detect the stochastically stable set we have first of all to characterize the $\omega-$limit sets of our model. This is the aim of the following Proposition.

Proposition 2 *All the $\omega-$limit sets have one of the following structures:*
(a) it contains only one state and this is a self-confirming equilibrium of Γ;
(b) it contains more states and

$$\rho(\Omega) = \{(HH, y_{HH}, x_{HH}), (LL, y_{LL}, x_{LL}), (HL, y_{HL}, x_{HL}), (LH, y_{LH}, x_{LH})\}$$

where in each investment profile the demands exhaust the surplus and the payoffs satisfy the following constraints:

$$(y_{HH} - c - y_{LH})(y_{HL} - c - y_{LL}) < 0; \quad (x_{HH} - c - x_{HL})(x_{LH} - c - x_{LL}) < 0$$

$$(y_{HH} - c - y_{LH})(x_{HH} - c - x_{HL}) < 0; \quad (y_{HL} - c - y_{LL})(x_{LH} - c - x_{LL}) < 0 \tag{2}$$

Proof. See the Appendix

Example 1. Let us consider an economy populated by $N = 100$ agents, half of these are of type A. The possible surpluses are $V_H = 18$, $V_L = 3$ and $V_M = 6$. The cost of high investment is $c = 2$ and $\delta = 0.5$. Consider a state θ^* in which: (a) all agents play L, i.e. $p_A(\theta^*) = p_B(\theta^*) = 0$; (b) the expected payoff of a single agent from playing H when the opponent plays L is not larger than 3, i.e. $\widetilde{y}^i_{HL}(\theta^*) \leq 3$ for any $i \in A$ and $\widetilde{x}^i_{LH}(\theta^*) \leq 3$ for any $i \in B$; (c) in LL half of players A make a demand equal to 1 and the others make a demand equal to 2, i.e. $\widehat{\sigma}_A(\theta^*, 1) = \widehat{\sigma}_A(\theta^*, 2) = \frac{1}{2}$. The same for players B. Then θ^* is a self confirming equilibrium and the set of outcomes almost always observed is $\rho(\theta^*) = \{(LL, 1, 1); (LL, 1, 2); (LL, 2, 1); (LL, 2, 2)\}$.

Assume that the process is in some limit set Ω and a single mutation occurs which alters the characteristic of a single agent (the mutant). If this mutation does not alter the action prescribed and/or the beliefs hold by the mutant at the information set currently reached, then the mutation is called drift. Since the expected payoff of the others does not change, their characteristics do not change too. Then by a drift we move from Ω to another limit set Ω' such that for any $\theta \in \Omega$ there exists a state $\theta' \in \Omega'$ with the same distribution of outcomes, i.e. $z(\theta) = z(\theta')$.

Definition 3 *Consider a union of limit sets Ω. This set is mutation connected if for all pairs Ω and Ω' belonging to it there exists a sequence of limit sets $(\Omega_1 = \Omega, \Omega_2, ..., \Omega_n = \Omega')$ such that (a) for any $k \in \{1, ..., n-1\}$, Ω_k belongs to this set and (b) every transition from Ω_k to Ω_{k+1} needs no more than one mutation.*

Let $\Omega = \{\theta\}$ be a self confirming equilibrium and consider all the self confirming equilibria θ' such that $z(\theta) = z(\theta')$; denote the union of these equilibria by $\Sigma(\theta)$. Since any two equilibria belonging to $\Sigma(\theta)$ only differ for some beliefs (and/or actions) held in some not reached information set, then there always exists a path connecting these two equilibria and such that: (i) all the limit sets involved in the path belong to $\Sigma(\theta)$; (ii) every transition between adjacent equilibria requires one mutation. Hence the set $\Sigma(\theta)$ is mutation connected. The same argument holds for a more complex Ω satisfying point (b) of Proposition 2; in this case we denote by $\Sigma(\Omega)$ the union of these limit sets.

When the set of stochastically stable states (Σ_S) only contains equilibria supporting the same unique outcome, then we can speak of stochastically stable outcome rather than stochastically stable set.

To give an insight, recall previous Example 1 and let θ' be a state which differs from θ^* by at least one of the following: (i) some beliefs and actions at the information node HH; (ii) some beliefs and actions of some agents A at the information node LH, together with some beliefs and actions of some agents B provided that $\widetilde{x}^i_{LH}(\theta') \leq 3$; (iii) some beliefs and actions of some agents B at the information node HL together with some beliefs and actions of some agents A provided that $\widetilde{y}^i_{HL}(\theta') \leq 3$. Obviously θ' is a self confirming equilibrium with the same distribution of outcomes than θ; then in $\Sigma(\theta)$ there always exists a path from θ to θ' such that θ' can be reached from θ by a sequence of single mutations.

We know from Proposition 2 that the considered evolutionary dynamics gives rise to a large multiplicity of limit sets and that not all these are equilibria of the extensive game[5]. Differently than Dawid and Mac Leod (2001), we cannot ex ante exclude from the stochastically stable set any bargaining and investment behavior. For instance, even if we observe an homogeneous investment profile, we cannot exclude a priori any equilibrium of the Nash Demand Game. Mutatis mutandis, the same holds true for an heterogeneous investment profile. However the unperturbed dynamics admits limit sets in which both investment and bargaining behavior is uniform in each population. It is then possible for evolution to trigger an homogeneous behavior in one or both populations. When this happens we say that a norm has evolved. In general, in an evolutionary model a norm can be defined as a pattern of observable individual actions with the property that for each agent it is optimal to adhere to it when each believes that everybody else will conform. Accordingly, an investment norm has evolved if all agents belonging to the same population make the same investment and the investment behavior is correctly anticipated. Analogously a bargaining norm has evolved if, at any reached information set, there exists a pair of demands (y, x) which exhausts the gross surplus and the bargaining behavior is correctly anticipated.

Proposition 2 is then of little help to pin down which behavior is more likely to become the conventional one. Nevertheless in the next Section we shall

[5] From now on when we speak of equilibrium we refer to self-confirming equilibrium.

show that when looking for the stochastically stable outcomes, we can restrict our attention to a small subset of equilibria in which (a) a single outcome is observed and (b) each agent chooses investment H. This implies that a norm of investment emerges and it supports the efficient investment profile HH. This remarkable result stems from a direct application of Ellison (2000). In particular when his radius/modified coradius Theorem holds, then the stochastically stable set (Σ_S) must be included in a union of limit sets with the property that more than one mutation is needed to escape from it. Our first task (Lemma 4 and 5) is then to explore whether for some union of limit sets (Σ), a single mutation is enough to fly out from it; in this case when the condition for the aforementioned Theorem holds, we know that Σ does not belong to Σ_S.

Lemma 4 *Consider a set Ω such that $\rho(\Omega)$ is not a singleton. There always exists an equilibrium θ', with $\rho(\theta')$ a singleton, which can be reached from Ω by a sequence of single-mutation transitions provided that $V_M \geq V_L + c$.*

Proof. See the Appendix.

From now on we restrict our attention to the equilibrium sets supporting only one outcome. Depending on the observed investment profile we can partition these equilibrium sets into four subsets that we denote by Σ_H, Σ_L, Σ_{HL} and Σ_{LH}. Of course Σ_H includes all the equilibria supporting the outcome $\{HH, V_H - x_{HH}, x_{HH}\}$ where $x_{HH} \in D_\delta(V_H)$. Analogously for the other subsets.

Lemma 5 *Consider one equilibrium and let θ^* be a different equilibrium belonging to $\Sigma(\theta)$:*

(a) if $\theta \in \Sigma_L$ and $V_M \geq V_L + c$ then a single mutation from θ^* is sufficient to enter (with positive probability) into the basin of attraction of $\theta' \in \Sigma_H$ provided that $x_{HH} > c + \delta$ and/or $x_{HH} < V_H - c - \delta$;

(b) if $\theta \in \Sigma_L$ and $V_M \geq V_L + c$, then a single mutation from θ^* is sufficient to enter (with positive probability) into the basin of attraction of $\theta' \in \Sigma_{LH} \cup \Sigma_{HL}$ provided that at θ' the agent who has chosen H is better off and the other is not worse off than at θ;

(c) if $\theta \in \Sigma_{HL}$ (resp. Σ_{LH}), then a single mutation from θ^* is sufficient to enter (with positive probability) into the basin of attraction of $\theta' \in \Sigma_H$ provided that at θ' agent A (resp. B) is not worse off and agent B (resp. A) is better off than at θ.

Proof. See the Appendix.

These two Lemma holds when the net surplus arising when agents make different investment choices is not smaller than the surplus arising when both choose L (i.e $V_M \geq V_L + c$). From Lemma 4 we know that, when a limit set underpins a multiplicity of outcomes, then by a sequence of single mutations we can reach an equilibrium sustaining only one outcome. Lemma 5 tells us that: (i) starting from any $\theta \in \Sigma_L$, the process can reach a single equilibrium belonging either to Σ_H or to Σ_{LH} (resp. Σ_{HL}); (ii) starting from any $\theta \in \Sigma_{HL}$ (resp. Σ_{LH}), the process can reach a single equilibrium belonging to Σ_H. Hence, when Σ_S can be detected by Ellison's Theorem, both these Lemma suggest that a stochastically stable state could only support the HH investment profile.

4 Investment conventions

The conclusion of previous Section suggests to restrict our concern to the set of equilibria Σ_H. In this Section we first show that under some conditions there exists a subset (Σ_{IH}) of Σ_H such that: (i) more than one mutation is needed to escape from it and (ii) one mutation is enough to escape from the complementary set $(\Sigma_H \setminus \Sigma_{IH})$. Then we are able to prove that, when Σ_{IH} is not empty, the stochastically stable set is included in it.

In order to characterize the set Σ_{IH} some further definitions are needed. We denote by x_A^M the largest demand agent B can make at the information node HH such that agent A does not have any incentive to choose action L when A his sure that all agents B play H and make a demand x_A^M. We denote by x_A^L the largest demand agent B can make at the information node HH such that agent A does not have any incentive to choose action L when A knows that $N-1$ agents B play H and make demand x_A^L while one agent B deviates by asking a larger demand. Thus:

$$\begin{aligned} x_A^M &= \max\left\{ x \in D_\delta\left(V_H\right) \middle| V_H - x - c \geq V_M - \delta \right\} \\ x_A^L &= \max\left\{ x \in D_\delta\left(V_H\right) \middle| \left(V_H - x\right) \tfrac{N-1}{N} - c \geq V_M - \delta \right\}. \end{aligned} \tag{3}$$

We denote by x_B^M the smallest demand agent B can make at the information node HH such that this agent does not have any incentive to choose action L when he is sure that all agents A play H and make a demand $V_H - x_B^M$. We denote by x_B^L the smallest demand agent B can make at the information node HH such that this agent does not have any incentive to choose action L when he knows that $N-1$ agents A play H and make demand $V_H - x_B^L$ while one agent A deviates by asking a larger demand. Hence:

$$\begin{aligned} x_B^M &= \min\left\{ x \in D_\delta\left(V_H\right) \middle| x - c \geq V_M - \delta \right\} \\ x_B^L &= \min\left\{ x \in D_\delta\left(V_H\right) \middle| x \tfrac{N-1}{N} - c \geq V_M - \delta \right\}. \end{aligned} \tag{4}$$

We assume that the population is sufficiently large and that the least demand is sufficiently small; formally:

$$\frac{V_H}{N} < \delta \tag{5}$$

and

$$\delta < c. \tag{6}$$

From (5) it follows that either $x_A^L = x_A^M$ or $x_A^L = x_A^M - \delta$ and that either $x_B^L = x_B^M$ or $x_B^L = x_B^M + \delta$.

Our aim is now to find under what conditions the range $\left[x_B^L, x_A^L\right]$ is not empty. Consider the equilibrium $\overline{\theta} \in \Sigma_H$ in which the surplus is equally divided, i.e. $\overline{x} = \frac{V_H}{2} = V_H - \overline{x}$. Notice that when

$$\overline{x}\frac{N-1}{N} - c \geq V_M - \delta, \tag{7}$$

then $x_B^L \leq \overline{x}$ and $x_A^L \geq \overline{x}$ so that the range $\left[x_B^L, x_A^L\right]$ is not empty. When instead condition (7) does not hold, the range $\left[x_B^L, x_A^L\right]$ is empty. Therefore when condition (7) is satisfied we define the set Σ_{IH} as

$$\Sigma_{IH} = \left\{\theta \in \Sigma_H \mid x \in \left[x_B^L, x_A^L\right]\right\}.$$

By definition when $\theta \in \Sigma_{IH}$ each agent gets an equilibrium payoff not smaller that the maximum expected payoff attainable when he deviates by playing L. Then any equilibrium in Σ_{IH} dominates all the equilibria supporting other investment profiles. It is worth noticing that since $\overline{x}$ is divisible by δ then $V_H = x_A^L + x_B^L$ meaning that in Σ_{IH} the least equilibrium demand is the same for both agents.

Let $\Sigma_{CH} = \Sigma_H \diagdown \Sigma_{IH}$ be the subset of equilibria of the game in which the unique outcome is in HH but $x \notin \left[x_B^L, x_A^L\right]$. The next Lemma shows that when Σ_{IH} is not empty, then both Σ_{IH} and Σ_{CH} have some desirable features.

Lemma 6 *Let* (5)*and* (6) *hold.*

(a) When Σ_{IH} is not empty more than one mutation is needed to escape from Σ_{IH}.

(b) One mutation is enough to escape from Σ_{CH}.

(c) When Σ_{IH} is not empty, this set can be reached from $\theta \in \Sigma_{CH}$ by a sequence of single mutations provided that $V_M \geq V_L + c$.

Proof. See the Appendix.

Important informations concerning the set of stochastically stable states are obtained by collecting all the results so far derived which allow us to make use of the sufficient condition developed by Ellison (2000).

Proposition 7 *Let* (5)*and* (6) *hold. When Σ_{IH} is not empty, then the stochastically stable set is contained in Σ_{IH} provided that $V_M \geq V_L + c$.*

Proof. See the Appendix

To sum up, Proposition 7 says that in every stochastically stable state it must be true that all agents choose investment H so that the bargaining follows the rules of the Nash Demand Game. This means that choosing investment H becomes the conventional way of playing the first stage of the game thus ensuring an efficient equilibrium. Moreover, when $x_B^L = \overline{x}$ Proposition 7 claims that a stochastically stable outcome emerges and that each agent get half of the efficient surplus. When instead $x_B^L < \overline{x}$, although we do not yet know how the surplus will be distributed in the long run, in every stochastically stable state everybody gets a payoff greater than the maximum payoff attainable by playing L.

5 Bargaining conventions

In this Section we analyze the case $x_B^L < \overline{x}$. We are able to show that, when Proposition 7 holds, a stochastically stable outcome exists and the conventional bargaining rule is egalitarian for whatever very fine grid of possible claims. Also the result of this Section stems from a direct application of Ellison (2000) and exploits some results for the Nash Demand Game proved by Young (1998).

Proposition 8 *Let Σ_{IH} be not empty, $V_M \geq V_L + c$, N sufficiently large and δ sufficiently small; then a stochastically stable outcome always exists and the distributional rule is egalitarian.*

Proof. See the Appendix

Both Propositions 7 and 8 hold provided the set Σ_{IH} is not empty; this, in turn, holds when condition (7) is met, i.e.

$$V_M \leq \left(\delta - \frac{V_H}{2}\frac{1}{N} - c\right) + \frac{V_H}{2}. \tag{8}$$

According to Dawid and MacLeod (2008), the investments are complements if the marginal effect of action H when the opponent always plays H is greater than the marginal effect of action H when the opponent plays L, that is $V_M < \frac{1}{2}(V_H + V_L)$. Therefore when the set Σ_{IH} is not empty the investments are complementary in the sense of Dawid and MacLeod (2008). However in general investment can be complementary in the sense of Dawid and MacLeod (2008) also when Σ_{IH} is empty.

Example 2. We now provide one example in which we show how Proposition 8 works. Consider an economy with the following parameters: $N = 50$, $V_H = 18$, $V_L = 3$, $c = 2$ and $\delta = 0.5$. It is simple to see that conditions (5) and

(6) are met. Let $V_M = 6$. Since $V_M > V_L + c$, both Lemma 4 and 5 hold. In order to characterize the set Σ_{IH} we have to verify whether for the given value of δ the equilibrium outcome following the information node HH and such that one player gets a payoff equals to $(V_M - \delta)$ is feasible. We denote by x_B^* (resp. x_A^*) the share going to player B such that B (resp. A) gets an equilibrium payoff equals to $(V_M - \delta)$, in particular:

$$\begin{aligned} x_B^* &= V_M - \delta - c &&= 7.5 \\ x_A^* &= V_H + \delta - c - V_M &&= 10.5. \end{aligned} \tag{9}$$

Since both x_B^* and x_A^* belong to $D_\delta(V_H)$, then $x_A^M = x_A^*$ and $x_B^M = x_B^*$. Hence $x_A^L = x_A^M - \delta = 10$ and $x_B^L = x_B^M + \delta = 8$. The set Σ_{IH} is thus not empty and both Propositions 7 and 8 hold. This economy has a unique stochastically stable outcome and it is egalitarian, i.e. $(HH, 9, 9)$.

Proposition 8 is our main result. Since it requires several conditions, it is better to clear up its domain of application. To this aim we rewrite all the conditions needed in terms of the investment cost, c, and assume that V_H, V_L, δ and N are fixed. This allows us to explore how changes in the degree of investment complementarity (V_M) and the cost of investment affect the set of stochastically stable equilibria.

The efficiency condition (1) can be written as:

$$c < \min(c_1, c_2)$$

where $c_1 \equiv \frac{1}{2}(V_H - V_L)$ and $c_2 \equiv V_H - V_M$. Lemma 4, point (a) of Lemma 5 and point (c) of Lemma 6 requires that $V_M \geq V_L + c$, i.e.

$$c \leq c_3 \equiv V_M - V_L.$$

Lastly condition (7), ensuring that the set Σ_{IH} is not empty, can be written as

$$c \leq c_4 \equiv V_H \frac{N-1}{2N} - V_M + \delta.$$

When $c > c_4$, since the set Σ_{IH} is empty, an efficient investment norm can not evolve. To see this, suppose that the efficient investment norm evolved. This means that no high-low matches are observed; then the belief regarding the outcome of bargaining in high-low matches can drift. Because of drift all agents of one population might deem to capture almost the whole pie by choosing low investment in the low-high matches. Hence when a single agent of the opponent population deviates by claiming a larger demand, then by updating all agents in the first population are willing to make the low investment so that the efficient norm is overturned.

Corollary 9 *The following cases are possible.*

(I) Let $V_L < V_M < \frac{V_L}{2} + \frac{V_H}{4}\frac{N-1}{N} + \frac{\delta}{2}$*. Then Proposition 8 holds when* $c \in [0, c_3]$*. When* $c \in (c_3, c_4]$ *the behavior is indeterminate. Lastly when* $c \in (c_4, c_1)$ *if a norm of investment evolves, it cannot be the efficient one.*

(II) Let $\frac{V_L}{2} + \frac{V_H}{4}\frac{N-1}{N} + \frac{\delta}{2} \leq V_M \leq \frac{V_H}{2}\frac{N-1}{N} + \delta$*. Then Proposition 8 holds when* $c \in [0, c_4]$*. When* $c \in (c_4, c_3]$ *we can not observe neither a norm of investment nor a norm of bargaining. Lastly when* $c \in (c_3, c_1)$ *if a norm of investment evolves, it cannot be the efficient one.*

(III) Let $\frac{V_H}{2}\frac{N-1}{N} + \delta < V_M \leq \frac{1}{2}(V_H + V_L)$*. When* $c \in [0, c_3]$ *we can not observe neither a norm of investment nor a norm of bargaining. Lastly when* $c \in (c_3, c_1)$ *if a norm of investment evolves, it cannot be the efficient one.*

(IV) Let $V_M > \frac{1}{2}(V_H + V_L)$*. For any* $c \in [0, c_2)$ *we can not observe neither a norm of investment nor a norm of bargaining.*

Proof. See the Appendix.

A careful reading of Corollary 9 suggests that in the long run we can expect to observe the efficient norm of investment and a norm of distribution when investment are complementary, the investment cost is sufficiently low and the surplus arising when only one agent makes the high investment is not too large. When instead investment are not complementary, we are sure that, for whatever investment cost, we can not observe any norm. A similar conclusion, but limited to the investment norm, can be found in Dawid and MacLeod (2001); our model then allows to extend this negative result also to the bargaining norm.

5.1 A special case

A particular simple case obtains when the payoff at LL is equal to zero for both players (i..e. $V_L = 0$). This is tantamount assuming that in the first stage the two possible actions are either to invest (H) or not invest (L). In this case a surplus arises when at least one agent invests and it seems plausible to assume that $V_M > c$. We only have to take into account values of c such that $c < \min(c_1, c_2, c_3)$ where now $c_1 = \frac{V_H}{2}$ and $c_3 = V_M$. In this case we have the following result.[6]

Corollary 10 *Let* $V_L = 0$*. The following cases are possible.*

(I) Let $2\delta < V_M < \frac{V_H}{4}\frac{N-1}{N} + \frac{\delta}{2}$*. Then Proposition 8 holds when* $c \in [0, c_3]$*.*

(II) Let $\frac{V_H}{4}\frac{N-1}{N} + \frac{\delta}{2} \leq V_M \leq \frac{V_H}{2}$*. Then Proposition 8 holds when* $c \in [0, c_4]$*. When* $c \in (c_4, c_3]$ *we can not observe neither a norm of investment nor a norm of bargaining.*

(III) Let $\frac{V_H}{2} < V_M \leq V_H\frac{N-1}{2N} + \delta$*. Then Proposition 8 holds when* $c \in [0, c_4]$*. When* $c \in (c_4, c_2)$ *we can not observe neither a norm of investment nor a norm of bargaining.*

(IV) Let $V_M > V_H\frac{N-1}{2N} + \delta$*. For any* $c \in [0, c_2)$ *we can not observe neither a norm of investment nor a norm of bargaining.*

[6] We omit the proof since it relies on simple computations.

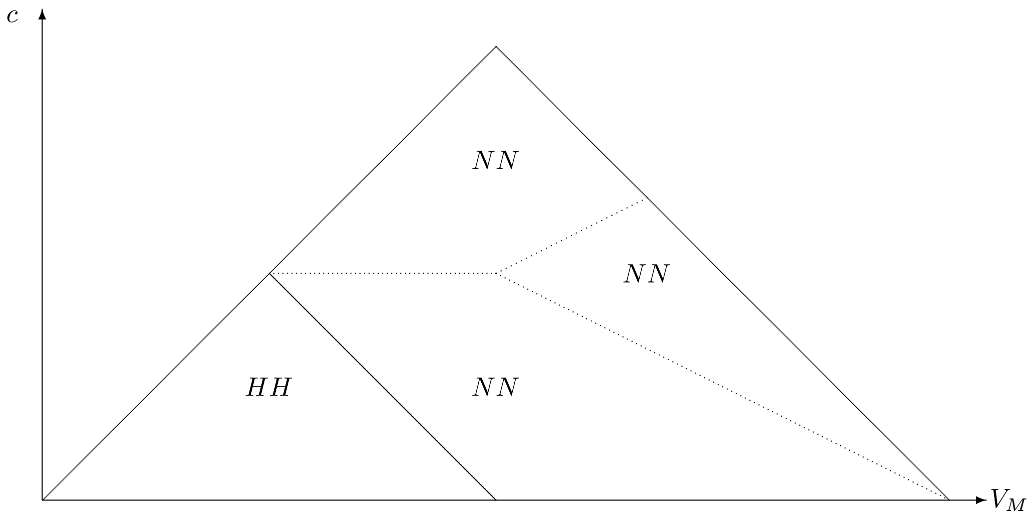

Figure 1: the model with bargaining

Two are the main consequences of Corollary 10. First, either a norm of investment and a norm of bargaining co-evolve or no norm evolves. Second, when norms co-evolve they support the efficient investment and the egalitarian distribution. Hence although it is possible that the social forces do not trigger the emergence of norms, nevertheless the traditional hold-up problem is overhauled. Corollary 10 is illustrated in Figure 1 where we denote by HH the region in which both norms coevolve and by NN the region in which no norm evolves. Figure 1 is drawn under the assumption that $\delta - \frac{V_H}{2N}$ is negligible.

It might be useful to compare these results with the canonical case in which the hold-up problem can arise. Suppose then that, for whatever investment outcome, agents do not bargain and that the pie is equally split. The resulting game can be represented by the following normal form in which we continue to assume $V_L = 0$. We still assume that the net surplus arising when both agents invest is the greatest one and that the net surplus accruing when only one invests is not negative, i.e. $c < \min(c_1, c_2, c_3)$. In this strategic framework, when there is more than one pure Nash equilibrium, the stochastically stable equilibrium coincides with the risk dominant one (Young (1993a)).

	H	L
H	$\frac{V_H}{2} - c, \frac{V_H}{2} - c$	$\frac{V_M}{2} - c, \frac{V_M}{2}$
L	$\frac{V_M}{2}, \frac{V_M}{2} - c$	$0, 0$

Few computations show that the game has either three Nash equilibria (two in pure strategies and one in mixed strategies) or only one pure symmetric Nash

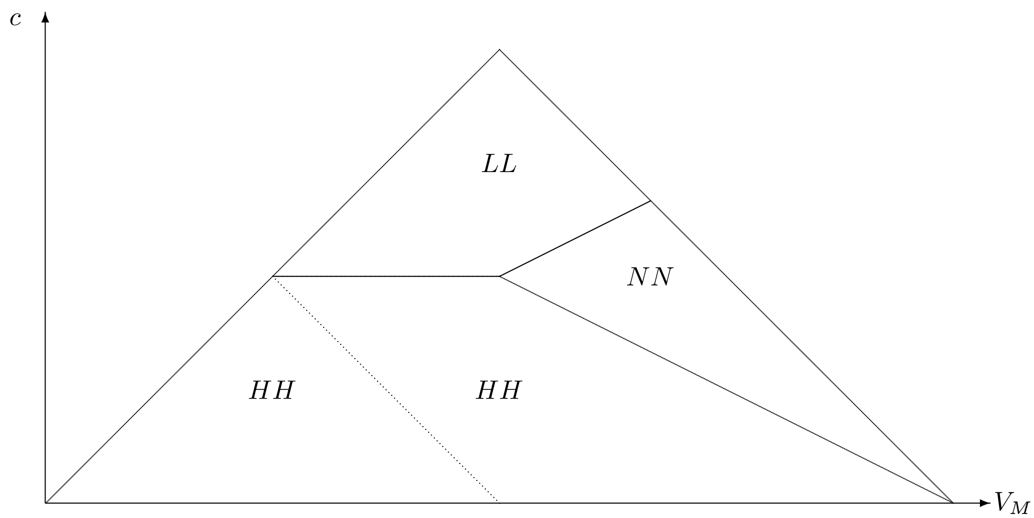

Figure 2: the model without bargaining

equilibrium. In particular, the investment profile HH is the unique stochastically stable equilibrium if $c_5 < c < \min(c_6, c_7)$ where $c_5 = \frac{V_M}{2}$, $c_6 = \frac{V_H - V_M}{2}$ and $c_7 = \frac{V_H}{4}$. The investment profile HH is the unique Nash equilibrium when $c < \min(c_5, c_6)$. Analogously, the investment profile LL is the unique stochastically stable equilibrium if $\max(c_5, c_7) < c < c_6$. The investment profile LL is the unique Nash equilibrium when $c > \max(c_5, c_6)$. Lastly, when $c_6 < c < c_5$, the game has two pure strategies asymmetric Nash equilibria, LH and HL, and both are stochastically stable.

The canonical case is illustrated in Figure 2 where HH (respectively LL) denotes the region in which the investment profile HH (respectively LL) is the unique stochastically stable equilibrium and where NN denotes the region in which the two pure strategies asymmetric Nash equilibria LH and HL are both stochastically stable. In this last game, the hold-up problem arises when the investment profile LL is the unique stochastically stable equilibrium. A direct comparison of these two Figures allows us to argue that the main consequences of adding a bargaining stage is that the region supporting the investment profile LL disappears and the region supporting the investment profile HH shrinks. Consequently the region in which no norm evolves enlarges. Contrarily to Dawid and MacLeod (2001) we can then conclude that with two sided relationship specific investment, the requirement that norms of bargaining be stochastically stable can downsize the hold-up problem.

Before concluding we further simplify the model. We still assume that $V_L = 0$ but that bargaining occurs only when HH is observed. When instead HL (or LH) is observed, only one division occurs; specifically we assume that the investing agent behave like a dictator by proposing the division $(V_M - \delta, \delta)$. The not investing agent has no actions available. In this case norms can only co-evolve; region NN in Figure 1 then disappears and it is replaced by region HH.

6 Concluding remarks

The heart of the hold-up problem is that when an economic relationship requires some individual specific investment but agents are not sure whether they will be able to catch the benefits, investments that are individually rational might not be undertaken. Ellingsen and Robles (2002) and Troger (2002) have shown that when the size of the pie is endogenous and depends on the investment of one part only, and agents are involved in a bargaining stage, then evolution can solve the hold-up problem. In this paper we have extended Ellingsen and Robles (2002) framework and we have explored under what circumstances an efficient norm of investment and an efficient norm of bargaining can endogenously evolve when the size of the pie is determined by both agents. Our model can be also seen as a variant of Dawid and MacLeod (2001) in which two main ameliorations are included. First we assume that agents always bargain but that the distribution of the bargaining power is affected by the outcome of the investment stage. Second we use the evolutionary framework put forward by Noldeke and Samuelson (1993) which is more adequate for extensive form games since it allows beliefs to drift in not observable nodes. These changes are needed in order to better understand the evolution of norms of division. We showed that, under some conditions, a bargaining norm and an investment norm coevolve. When these conditions are met then, for whatever grid of possible claims, the bargaining norm supports an egalitarian division and the investment norm supports the efficient investment profile. In a simpler model in which a surplus arises only when at least one agent chooses high investment we elicited a sharper result. In this case, contrarily to what emerge when the surplus is always equally split (the canonical case in which the hold-up problem is studied), we showed that either the stochastically stable outcome sustain the efficient investment and an egalitarian distribution or no norm evolves. Contrarily to Dawid and MacLeod (2001) we can then conclude that with two sided relationship specific investment, the requirement that norms of bargaining be stochastically stable can downsize the hold-up problem.

7 Appendix

The following preliminary results are needed in order to prove Proposition 2.

Lemma 11 *Let* $\{x_{LL,l}(\theta)\}_{l=1}^{k}$ *or* $\{x_{HH,l}(\theta)\}_{l=1}^{k}$*) be the ordered sets of demands made by* B *such that* $x_{LL,l}(\theta) < x_{LL,l+1}(\theta)$ *or* $x_{HH,l}(\theta) < x_{HH,l+1}(\theta)$*. These are the demands which can be observed by* A *in state* θ*. Let* $\{y_{LL,l}(\theta)\}_{l=1}^{k}$ *or* $\{y_{HH,l}(\theta)\}_{l=1}^{k}$ *be the sets of best (behavioral) responses of* A*. Then:*

$$\{y_{LL,l}(\theta)\}_{l=1}^{k} \subseteq \{V_L - x_{LL,l}(\theta)\}_{l=k}^{1}$$

$$\{y_{HH,l}(\theta)\}_{l=1}^{k} \subseteq \{V_H - x_{HH,l}(\theta)\}_{l=k}^{1}$$

The same argument applies to the other population of agents.

Proof. See Lemma A.1 in Ellingsen and Robles (2002)).

Lemma 12 *Let* Ω *be a* ω*-limit set of* (Θ, P)*. If* $(HL, y_{HL}, x_{HL}) \in \rho(\Omega)$ *and/or* $(LH, y_{LH}, x_{LH}) \in \rho(\Omega)$ *then:*

(i) $x_{HL} = V_M - y_{HL}$ *or* $y_{LH} = V_M - x_{LH}$*;*

(ii) (HL, y_{HL}, x_{HL}) *(resp.* (LH, y_{LH}, x_{LH})*) is the unique outcome which supports the investment profile* HL *(resp.* LH*) in* $\rho(\Omega)$*.*

Proof. We only consider the profile HL. The same holds true for LH.

Point (i). Let θ be a state such that: (a) $\theta \in \Omega$; (b) (HL, y_{HL}, x_{HL}) occurs with positive probability in $z(\theta)$ and $x_{HL} \neq V_M - y_{HL}$. Let us suppose that only B agents update their characteristics: they all will accept y_{HL}. For whatever belief on the behavior of the opponents, this action is a best reply for any individual agent B; then it is impossible to return to the original state θ. This contradicts the assumption that $\theta \in \Omega$.

Point (ii). First we show that Ω can not include a state θ in which multiple demands are made at HL; then we show that Ω can not include two different states supporting different outcomes following HL.

Let θ be a state such that: (a) $\theta \in \Omega$ and (b) multiple demands are made be agents A at HL. We already know from point (i) that at θ all agent B accept all the demands made by opponents. Suppose now that all agents A (but only them) revise their characteristics. Then in HL any A will choose the maximum demand observed. Hence it is impossible to return to the original state θ. This contradicts the assumption that $\theta \in \Omega$.

Now let θ and θ' be two states such that: (a) both states belong to Ω and (b) HL is observed, a single demand is made by A but $y_{HL}(\theta') > y_{HL}(\theta)$. Since from θ' is impossible to return to θ then the assumption $\theta \in \Omega$ is contradicted. ■

Lemma 13 *Let Ω be a $\omega-$limit set of (Θ, P). If at least one of the following statement is true:*

(i) $\{(HH, y, x); (HH, y', x')\} \in \rho(\Omega)$ *and either* $x \neq x'$ *or* $y \neq y'$

(ii) $\{(LL, y, x); (LL, y', x')\} \in \rho(\Omega)$ *and either* $x \neq x'$ *or* $y \neq y'$

then Ω is a singleton so that Ω is a self-confirming equilibrium of Γ.

Proof. Consider a set Ω satisfying statement (i) and let $\theta \in \Omega$ be a state in which at least one population (i.e. B) made multiple demands and suppose that at least one of these demands (x^*) is not a best reply to $z(\theta)$. Suppose that, after observing $z(\theta)$, all agents who demanded x^* revise; as a consequence x^* disappears. A new state $\overline{\theta} \in \Omega$ is then reached in which the profile HH is still observed. Suppose now that all A update; then, by Lemma 11, nobody will make the demand $\{V_H - x^*\}$. These two demands have thus disappeared and it is impossible to return to the original state θ. This contradicts the assumption that $\theta \in \Omega$. Therefore, if multiple demands are made, each must be a best reply to $z(\theta)$.

Consider now an agent belonging to population A who has played H in θ and suppose this agent has the incentive to change his investment should he know $z(\theta)$. When this agent updates the distribution of the demands made by population A in subgame HH differs from the original one. This implies that at least one demand made by some opponent (i.e. B) is no longer a best reply when B updates. By applying the argument made in the paragraph above we conclude that at least a pair of demands have disappeared and cannot reappear; this contradicts the assumption that $\theta \in \Omega$.

If Ω satisfies assertion (ii) we can draw the same conclusion for subgame LL. By Lemma 12, since the set $\rho(\Omega)$ can include at most one outcome following the profile HL or LH, then the state θ must be a self-confirming equilibrium of the game Γ. ■

Proof of Proposition 2

Assume that Ω is not singleton. We know from Lemmas 12 and 13 that if a bargaining subgame is reached, then only one of its terminal node is observed almost always.

First we show that $\rho(\Omega)$ must contain one outcome for every bargaining subgame. Of course $\rho(\Omega)$ must differ from $\{(HH, y_{HH}, V_H - y_{HH}), (LL, y_{LL}, V_L - y_{LL})\}$. Suppose $\rho(\Omega)$ includes only the following outcomes: (a) (HH, y_{HH}, x_{HH}) with $y_{HH} + x_{HH} = V_H$; (b) (HL, y_{HL}, x_{HL}) with $y_{HL} + x_{HL} = V_M$. In Ω a state θ in which both outcomes are observed must exist and it can not be an equilibrium. We show that from θ is possible to reach either the basin of attraction of one equilibrium of the game or a state in which all bargaining nodes are observed. Suppose some agents B update. If $x_{HH} - c > x_{HL}$ than the updating agents will choose H so that at the new state θ' the frequency of this action in population B will increase.

If at least one agent A has beliefs $\left(\widetilde{y}_{LH}^{i}\right)$ leading him not to prefer H to L when all agent B play H, then from θ we can reach a state in which all investment profiles are realized. This contradicts the assumption that $\rho(\Omega) = \{(HH, y_{HH}, x_{HH});(HL, y_{HL}, x_{HL})\}$. Otherwise from state θ it is possible to reach the basin of attraction of one equilibrium of the game. If $x_{HH} - c \leq x_{HL}$ we get the same conclusion by using a similar argument. It is simple to see that the same conclusion holds when $\rho(\Omega)$ includes any two different outcomes. Therefore if Ω is a not a singleton all the bargaining nodes are visited almost always meaning that $\rho(\Omega)$ includes four outcomes each of these is a subgame equilibrium.

We now have to show that the payoffs must satisfy the constraints (2). Notice that a state $\theta \in \Omega$ in which all the investment profiles are observed must exist. Moreover when we allow all agents to update, all agents A will choose H:

$$p_B(\theta)(y_{HH} - y_{LH} - c) + (1 - p_B(\theta))(y_{HL} - y_{LL} - c) > 0, \tag{10}$$

and all agents B will choose H:

$$p_A(\theta)(x_{HH} - x_{HL} - c) + (1 - p_A(\theta))(x_{LH} - x_{LL} - c) > 0. \tag{11}$$

We can rewrite these conditions as

$$\begin{aligned} p_B(\theta) A_1 + (1 - p_B(\theta)) A_2 &> 0 \\ p_A(\theta) B_1 + (1 - p_A(\theta)) B_2 &> 0. \end{aligned}$$

If all expressions are null, then Ω is a singleton. Furthermore, when for some population both expressions are either not negative or not positive, and at least one is not null, then from θ the process can arrive to a new state which is a self-confirming equilibrium.

Consider the case in which for one population (i.e. A) both expressions are null. When B_1 is strictly positive and B_2 is strictly negative all B prefers H if $p_A(\theta) > p_A^*$ where:

$$p_A^* = \frac{c + x_{LL} - x_{LH}}{(x_{HH} - x_{HL}) - (x_{LH} - x_{LL})}. \tag{12}$$

Otherwise when B_1 is strictly negative and B_2 is strictly positive all B prefers H if $p_A(\theta) < p_A^*$. In both cases when all B agents update they will choose the same investment. Hence a state which is an equilibrium of the game can be reached from θ.

A similar argument is applied when both expressions B_1 and B_2 are null. In this case the threshold value of $p_B(\theta)$ is p_B^* which is now given by:

$$p_B^* = \frac{c + y_{LL} - y_{HL}}{(y_{HH} - y_{LH}) - (y_{HL} - y_{LL})}. \tag{13}$$

We are left with the case in which for each population the product of the corresponding two expressions is strictly negative. However, when A_1 and B_1 have the same sign, a similar argument allows us to reach the same conclusion. Indeed, suppose that both A_1 and B_1 are strictly positive: this implies that all B prefer H if $p_A(\theta) > p_A^*$ and all A prefer H if $p_B(\theta) > p_B^*$. Hence, for whatever values of $p_A(\theta)$ and $p_B(\theta)$, starting from θ the process can reach an equilibrium when one population at the time revises.

The remaining possible case occurs when (a) $B_1 B_2 < 0$ and $A_1 A_2 < 0$ and (b) $A_1 B_1 < 0$. This last case coincides with (2) in the main text. ■

Proof of Lemma 4

In this proof when multiple demands are observed at some homogenous profile, we denote respectively by $\{x_{JJ,l}(\theta)\}_{l=1}^{k}$ and $\{y_{JJ,l}(\theta)\}_{l=1}^{k}$ the ordered sets of demands made by B and A (in these cases $J \in \{L, H\}$). In all other cases we denote the pair of demands by $(y_{JJ,1}; V_J - y_{JJ,1})$.

I) Consider an equilibrium θ in which only one profile is realized and multiple demands are made. Suppose, for instance, that only LL is observed at θ. Let a single agent B switch from $x_{LL,k}(\theta)$ to $x_{LL,1}(\theta)$. Let all agents A update; then they will make a demand $y_{LL,k}(\theta) = V_H - x_{LL,1}(\theta)$. Hence we arrive at a new equilibrium θ' in which only LL is observed and only the pair of demands $(V_L - x_{LL,1}(\theta), x_{LL,1}(\theta))$ occurs. We get a similar conclusion when we assume that at θ only HH is realized and multiple demands are made.

II) Suppose now that at the equilibrium θ two profiles are observed. This implies that in one population the same type of investment is made. We give the proof only when HH and HL are observed. The other remaining cases are similar.

II.1) Consider first the case in which multiple demands are made following HH. Since θ is an equilibrium, the following conditions must be always met:

$$p_B(\theta)\left(y_{HH,1} - \widetilde{y}_{LH}^{i}(\theta) - c\right) + (1 - p_B(\theta))\left(y_{HL} - \widetilde{y}_{LL}^{i}(\theta) - c\right) \geq 0, \forall i \in A$$

$$(V_H - y_{HH,k}) - c = V_M - y_{HL}, \forall i \in B.$$

Consider an equilibrium $\theta_1 \in \Sigma(\theta)$ in which $\left(y_{HH,1} - \widetilde{y}_{LH}^{i}(\theta_1) - c\right) > 0$ for all A. When $y_{HH,1} - c > \delta$, the population can get from θ to $\theta_1 \in \Sigma(\theta)$ through a sequence of single-mutations. At θ_1 let a single agent A mutate from $y_{HH,k}(\theta_1)$ to $y_{HH,1}(\theta_1)$ and let all agents B revise; as a consequence they all will choose H. Therefore the process enters into a new equilibrium θ' where $\rho(\theta') = \{HH, y_{HH,1}(\theta), V_H - y_{HH,1}(\theta)\}$. When instead $y_{HH,1} - c \leq \delta$, the inequality $y_{HL} - \widetilde{y}_{LL}^{i}(\theta_1) - c \geq 0$ must hold for all A. Suppose a single A mutates from $y_{HH,k}(\theta_1)$ to $\overline{y}$ where $\overline{y} > y_{HH,k}(\theta_1)$ and let all agents B update: as a consequence they all will choose L. Therefore the process arrives at a new equilibrium θ' where $\rho(\theta') = \{HL, y_{HL}, V_M - y_{HL}\}$.

II.2) Consider now the case in which a single demand is made following HH.

Consider an equilibrium $\theta_1 \in \Sigma(\theta_1)$ in which $y_{HL} - \widetilde{y}^i_{LL}(\theta_1) - c \geq 0$ for all A. When $y_{HL} - c \geq \delta$ the population can get from θ to $\theta_1 \in \Sigma(\theta)$ through a sequence of single-mutations. At θ_1 let a single agent A mutates from $y_{HH,1}(\theta_1)$ to $\overline{y}$ where $\overline{y} > y_{HH,1}$ and let all agents B revise; as a consequence they will choose L. Hence the process arrive at a new equilibrium θ' where $\rho(\theta') = \{HL, y_{HL}, V_M - y_{HL}\}$. When instead $y_{HL} - c < \delta$, then: (a) under the asumption that $V_M \geq V_L + c$, the subgame $(HL, V_M - \delta)$ at θ is not reached; (b) $y_{HH,1} - \widetilde{y}^i_{LH}(\theta_1) - c \geq 0$ for every A. By drifting all agents B are led to accept the maximum feasible demand made by A in HL so that a new equilibrium θ_1 is reached. Sure enough $\theta_1 \in \Sigma(\theta)$. Suppose now that a single agent A changes his demand from y_{HL} to $(V_M - \delta)$. When all agents A update, they observe that all B have accepted the demand $(V_M - \delta)$; therefore in HL their best response is $y_{HL} = V_M - \delta$. When all agent B update they will choose H being $x_{HL} = \delta$. Hence the process arrive at the equilibrium θ' where $\rho(\theta') = \{HH, y_{HH,1}, V_H - y_{H,1}\}$.

III) Suppose now that at the equilibrium θ all investment profiles are observed. Since θ is an equilibrium the following conditions must be satisfied:

$$p_B(\theta)(y_{HH,1} - y_{LH} - c) + (1 - p_B(\theta))(y_{HL} - y_{LL,1} - c) = 0$$

$$p_A(\theta)(x_{HH,1} - x_{HL} - c) + (1 - p_A(\theta))(x_{LH} - x_{LL,1} - c) = 0.$$

We may rewrite these conditions as

$$\begin{aligned} p_B(\theta) A'_1 + (1 - p_B(\theta)) A'_2 &= 0 \\ p_A(\theta) B'_1 + (1 - p_A(\theta)) B'_2 &= 0 \end{aligned}$$

We argue that when the second expression (A'_2 or B'_2) is not positive for at least one population then the process, through a sequence of single-mutations, can arrive at one equilibrium in which a smaller number of investment profiles are realized. In order to see this suppose, for instance, that $A'_2 < 0$; then, when $V_M \geq V_L + c$, we get $y_{HL} < V_M - \delta$. Therefore in θ the subgame $(HL, V_M - \delta)$ is not reached. A drift can lead all agents B to accept the maximum feasible demand of the opponent at HL. A new $\theta_1 \in \Sigma(\theta)$ is then reached. Suppose now that at this new equilibrium a single agent A mutates his demand from y_{HL} to $V_M - \delta$. When all agents A revise they will play H and will make a demand $y_{HL} = V_M - \delta$. Let now all agents B update. Since each agent B knows that $x_{HL} = \delta$ and that all A have played H, then his best reply depends on the sign of $(x_{HH,1} - \delta - c)$. However it is simple to see that whatever is the value of $(x_{HH,1} - \delta - c)$, the process can arrive at a new equilibrium in which a smaller number of investment profiles is realized. If at this new equilibrium two investment profiles are realized, then by a further sequence of single transition (see the arguments above) the process can arrive at an equilibrium which supports a unique outcome.

When both A'_2 and B'_2 are positive, a single mutation occurring in population A is enough to move the process from θ to a new equilibrium θ' where:

$$\rho\left(\theta'\right) = \{LH, y_{LH}, V_M - y_{LH}\}. \tag{14}$$

The mutation needed depends on how many demands are observed at HH and at LL. In particular:

(i) when multiple demands are made at HH, it is enough that one agent A mutate from $y_{HH,k}(\theta)$ to $y_{HH,1}$;

(ii) when only one demand is made at HH but multiple demands are observed at LL, it is enough that one agent A mutate from H to L and make a demand $y_{LL,k}(\theta)$ in LL;

(iii) when at both profiles HH and LL only one demand is observed, it is enough that one agent A mutate from H to L and make a demand $y_{LL,1}(\theta)$ in LL;

IV) The remaining case occurs when Ω is not singleton. Under the assumption $V_M > V_L + c$, at least one of the following two subgames $(LH, V_M - \delta)$ and $(HL, V_M - \delta)$ is never reached. The same argument used above implies that the population can get from Ω to θ' through a sequence of single-mutations. ■

Proof of Lemma 5

(a) Let θ be an equilibrium belonging to Σ_L and suppose $V_M \geq V_L + c$ and $x_{LL}(\theta) < V_L - \delta$ (for at least one population this is always true). From θ, by a sequence of single mutations, the process can arrive at a new equilibrium $\theta^* \in \Sigma(\theta)$ in which for every agent A and B it is true that: (i) $\widetilde{y}^i_{HH}(\theta^*) = y_{HH}$ and $y_{HH} > c + \delta$; (ii) at the subgame $(LH, V_M - \delta)$ each agent A accepts (i.e. he chooses δ); (iii) $\widetilde{x}^i_{HH}(\theta^*) = V_H - y_{HH}$ and $(V_H - y_{HH}) - \widetilde{x}^i_{HL}(\theta^*) - c \geq 0$. Suppose now that an agent B mutates by playing H and makes a demand $V_M - \delta$ in LH. When all agents B update they will choose H since all agents A have accepted the demand $V_M - \delta$.Suppose now that all agents A revise. Since $y_{HH} > c + \delta$, they will play H. Hence the process arrive at a new equilibrium $\theta' \in \Sigma_H$ where $\rho\left(\theta'\right) = \{HH, y_{HH}, V_H - y_{HH}\}$.

(b) Let θ be an equilibrium belonging to Σ_L and suppose $V_M \geq V_L + c$ and $x_{LL}(\theta) < V_L - \delta$. From θ, by a sequence of single mutations, the process can arrive at a new equilibrium $\theta^* \in \Sigma(\theta)$ in which for every agent A it is true that: $\widetilde{y}^i_{HH}(\theta^*) - \widetilde{y}^i_{LH}(\theta^*) - c < 0$ and $\widetilde{y}^i_{LH}(\theta^*) = y_{LH} \geq y_{LL}$ but $(V_M - y_{LH}) - c > x_{LL}$; at the subgame $(LH, V_M - y_{LH})$ each agent A accepts (i.e. he chooses y_{LH}). Suppose now that an agent B mutates by playing H and makes a demand $V_M - y_{LH}$ in LH. When all agents B update, they will choose H since population A have accepted the demand $V_M - y_{LH}$. Suppose now that all agents A revise. Since $\widetilde{y}^i_{HH}(\theta^*) - y_{LH} - c < 0$ they will continue to play L. Hence the process arrive at a new equilibrium $\theta' \in \Sigma_{LH}$ in which the pair of demand is $(y_{LH}, V_M - y_{LH})$. Assume now $V_M = V_L + c$ and $x_{LL}(\theta) = V_L - \delta$. From θ, by a sequence of single mutations, the process can arrive at a new equilibrium $\theta^* \in \Sigma(\theta)$ in which for every agent B it is true that:

$\widetilde{x}^i_{HH}(\theta^*) - \widetilde{x}^i_{HL}(\theta^*) - c < 0$ and $\widetilde{x}^i_{HL}(\theta^*) = x_{HL}$ but $(V_M - x_{HL}) - c > \delta$. At subgame $(HL, V_M - x_{HL})$ each agent B accepts (i.e. he chooses x_{HL}). Suppose now that an agent A mutates by playing H and makes a demand $(V_M - x_{HL})$ in HL. When all agents A update they will choose H since population B have accepted the demand $V_M - x_{HL}$. Suppose now that all agents B revise. Since $\widetilde{x}^i_{HH}(\theta^*) - x_{HL} - c < 0$ they will continue to play L. Hence the process arrives at a new equilibrium $\theta' \in \Sigma_{HL}$ in which the pair of demand is $(V_M - x_{HL}, x_{HL})$.

(c) Let θ be an equilibrium belonging to Σ_{HL}. At θ the pair of demands $(y_{HL}, V_M - y_{HL})$ is observed. Let θ' be an equilibrium where only the profile HH is reached and the pair of demands $(y_{HH}, V_H - y_{HH})$ are made where $y_{HL} \leq y_{HH} < (V_H - V_M + y_{HL}) - c$. From θ, by a sequence of single mutations, the process can arrive at an equilibrium $\theta^* \in \Sigma(\theta)$ in which all agent A have beliefs such that $\widetilde{y}^i_{HH}(\theta^*) = y_{HH}$ and $y_{HH} - \widetilde{y}^i_{LH}(\theta^*) - c > 0$. Suppose now that an agent B mutates by playing H and making a demand $(V_H - y_{HH})$ in HH. Let all agents B revise; they will choose H. When agents A update the process arrives at a new equilibrium $\theta' \in \Sigma_H$ in which the pair of demand is $(y_{HH}, V_M - y_{HH})$. ■

Proof of Lemma 6

Point (a). Consider some $\theta \in \Sigma_{IH}$ and let $\{V_H - x, x\}$ be the observed pair of demands. We show that a single mutations transition is not enough to push the process into the basin of attraction of a different equilibrium which does not belong to Σ_{IH}.

I) First of all we show that a single mutation from H to L does not enable the process to enter into the basin of attraction of a different equilibrium even if each agent expects to get: (i) the maximum payoff at LL; (ii) the maximum payoff when he plays L but the opponent still plays H; (iii) the minimum payoff when he plays H but the opponent shifts to L.

From the definition of Σ_{IH} and the assumption that $V_M > V_L$, when a single mutation occurs in one population, updating would not cause other agents in the same population to imitate. Moreover this single mutation does not lead agents in the other population to play L since for any $x \in \left[x^L_B; x^L_A\right]$, it is true that the condition

$$\frac{N-1}{N}\left[(V_H - x - c) - (V_M - \delta)\right] + \frac{1}{N}\left[(\delta - c) - (V_L - \delta)\right] > 0 \qquad (15)$$

holds for any A and the condition

$$\frac{N-1}{N}\left[(x - c) - (V_M - \delta)\right] + \frac{1}{N}\left[(\delta - c) - (V_L - \delta)\right] > 0 \qquad (16)$$

holds for any B. Hence a single mutation from H to L does not trigger a transition to a different equilibrium.

II) We now show that a single mutation from x to x'(resp. from $V_H - x$ to y') does not enable the process to enter into the basin of attraction of a different equilibrium. Suppose each agent expects to get the maximum payoff when he plays L and the opponent chooses H. Let an agent B change only his demand to x'. Obviously no agents B imitate the mutant when revising. Consider the population A and allow them to update. By Lemma (11) their best response is either $(V_H - x)$ or $(V_H - x')$.

If $x' > x$ agent A expects to get $(V_H - x)\frac{N-1}{N} - c$ when he demands $(V_H - x)$ and $(V_H - x') - c$ when he demands $(V_H - x')$. It is simple to see that under Assumption (5) the former payoff is greater than the latter one. Hence agents A will not change their demand when updating. Moreover, since $(V_H - x)\frac{N-1}{N} - c \geq (V_M - \delta)$, then updating will not cause agents A to play action L.

If $x' < x$ agent A expects to get $(V_H - x) - c$ when he demands $(V_H - x)$ and $\frac{1}{N}(V_H - x') - c$ when he demands $(V_H - x')$. It is simple to see that under Assumption (5) the former payoff is greater than the latter one. Hence agents A will not change their demand when updating. Moreover, since $(V_H - x) - c > (V_M - \delta)$, then updating will not cause agents A to play action L. The case in which an agent A mutates from $V_H - x$ to y' is symmetric. Hence a single mutation from x to x'(resp. from $V_H - x$ to y') does not trigger a transition to a different equilibrium.

Taken together, these results imply that whatever single mutation we consider, this does not trigger a transition to a different equilibrium. Moreover the population returns to an equilibrium $\theta' \in \Sigma(\theta)$ as soon as the mutating agent revises.

Point (b). Consider some $\theta \in \Sigma_{CH}$ and let $\{V_H - x, x\}$ be the observed pair of demands. We show that a single mutation transition is enough to enter into the basin of attraction of an equilibrium θ' in which $\rho(\theta') = \{LL, V_L - x_{LL}, x_{LL}\}$. Point (c) then follows from Lemma 5.

In order to full describe the transition from θ to θ' we have to take into account four cases: (1) $x > x_A^M$; (2) $x = x_A^M$ and $x_A^L = x_A^M - \delta$; (3) $x < x_B^M$; (4) $x = x_B^M$ and $x_B^L = x_B^M + \delta$. Since case (3) and case (4) are respectively symmetric with respect to case (1) and case (2), we give the proof for these latter cases only.

Case (1): $x > x_A^M$.
At θ the following inequality must hold:

Population A	Population B	
$(V_H - x - c) - \widetilde{y}^i_{LH}(\theta) \geq 0$	$x - c - \widetilde{x}^i_{HL}(\theta) \geq 0$	(17)
$V_H - x - c < V_M - \delta$	$x - c > V_M - \delta.$	

From θ, by a sequence of single mutations, the process can arrive at a new equilibrium $\theta_1 \in \Sigma(\theta)$ in which for every agent it is true that: (i) $\widetilde{x}^i_{LL}(\theta_1) = x_{LL}$ and $\widetilde{x}^i_{LH}(\theta_1) = \delta$; (ii) $\widetilde{y}^i_{LL}(\theta_1) = V_L - x_{LL}$ and $\widetilde{y}^i_{HL}(\theta_1) - c - (V_L - x_{LL}) < 0$.

Suppose an agent A mutates by playing L and accepting the demand made by his opponent at LH. Let all agents A update. Since the mutating agent gets $V_M - \delta$, all A imitate and play L. When agents B revise they will play L and demand x_{LL}. The process then arrives at a new equilibrium θ' where $\rho(\theta') = \{LL, V_L - x_{LL}, x_{LL}\}$.

Case (2): $x = x_A^M$ and $x_A^L = x_A^M - \delta$.
At θ the following inequality must hold for agents A :

$$\left(V_H - x_A^M - c\right) \geq V_M - \delta \qquad \left(V_H - x_A^M\right) \tfrac{N-1}{N} - c < V_M - \delta. \tag{18}$$

From θ, by a sequence of single mutations, the process can arrive at new equilibrium $\theta_1 \in \Sigma(\theta)$ in which for every agent it is true that: (i) $\widetilde{x}_{LL}^i(\theta_1) = x_{LL}$ and $\widetilde{x}_{LH}^i(\theta_1) = x_{LH}$; (ii) $x_{LH} - x_{LL} - c < 0$; (iii) $\widetilde{y}_{LL}^i(\theta_1) = V_L - x_{LL}$ and $\widetilde{y}_{LH}^i(\theta_1) = (V_M - \delta)$; (iv) $\widetilde{y}_{HL}^i(\theta_1) - (V_L - x_{LL}) - c < 0$. Suppose an agent B mutates by demanding $x' > x_A^M$ at HH. When agents A update all them will choose L since, for whatever best action at HH, the expected payoff by playing H is now smaller that $V_M - \delta$. When all agents B revise they will play L and demand x_{LL}. The process then arrive at a new equilibrium $\theta' \in \Sigma_L$.

Lastly point (c) follows by a direct application of point (a) of Lemma 5. ■

Proof of Proposition 7

Before giving the proof we briefly introduce the radius modified coradius criterion (Ellison (2000)). Let Σ be a union of limit sets (Ω); these sets can be mutation connected or not. The radius $R(\Sigma)$ is the minimum number of mutations needed to exit with positive probability from the basin of attraction of Σ. Consider an arbitrary state $\theta \notin \Sigma$ and let $(z_1, z_2, .., z_T)$ be a path from θ to Σ where $\Omega_1, \Omega_2, ..\Omega_r$ is the sequence of limit sets through which the path passes consecutively. Obviously $\Omega_i \notin \Sigma$ for $i < r$ and $\Omega_r \subset \Sigma$. Furthermore it may be that a limit set can appear several time but not consecutively. The modified cost of this path is defined by:

$$c^*(z_1, ..., z_T) = c(z_1, .., z_T) - \sum_{i=2}^{r-1} R(\Omega_i)$$

where $c(z_1, .., z_T)$ is the total number of mutations over the path $(\theta, z_1, z_2, .., z_T)$. Let $c^*(\theta, \Sigma)$ be the minimal modified costs for all paths from θ to Σ. The modified coradius of the basin of attraction of Σ is then:

$$CR^*(\Sigma) = \max_{\theta \notin \Sigma} \; c^*(\theta, \Sigma).$$

Theorem 2 of Ellison (2000) shows that every union of limit sets Σ with $R(\Sigma) > CR^*(\Sigma)$ encompasses all stochastically stable states.

We are now ready to give the proof of Preposition 7. By Lemmas 4, 5 and 6, points (b) and (c), we can deduce that for any $\theta \notin \Sigma_{IH}$ the minimal modified costs for all paths from θ to Σ_{IH} is equal to one, whatever is the number of limit sets the path goes through. Therefore $CR^*(\Sigma_{IH}) = 1$. Since we know from point (a) of Lemma 6 that $R(\Sigma_{IH}) > 1$, by a direct application of Ellison's result it follows that all stochastically stable states are included in Σ_{IH}. ■

Also Proposition 8 follows by a direct application of Ellison (2000). To this end we need: (i) the radius of $\Sigma(\theta)$, i.e. the smallest number of mutations required to destabilize the outcome supported by θ, $\forall\theta \in \Sigma_{IH}$; (ii) to find an equilibrium belonging to Σ_{IH} such that $R(\Sigma(\theta)) > CR^*(\Sigma(\theta))$. From now on write θ_x as shorthand for an equilibrium belonging to Σ_{IH} and in which the distribution $(V_H - x; x)$ is realized. The following two Lemmas gives the relevant details for detecting $R(\Sigma(\theta_x))$ for every $x \in \left[x_B^L; x_A^L\right]$.

Lemma 14 *The minimum number of mutations required to get from $\Sigma(\theta_x)$ to an equilibrium which supports a different investment profile is:*

$$\begin{aligned} \overline{r}_A(x) &= \left\lfloor N\left(1 - \tfrac{V_A - \delta + c}{x}\right)\right\rceil \quad \text{if} \quad x < \tfrac{V_H}{2} \\ \overline{r}_B(x) &= \left\lfloor N\left(1 - \tfrac{V_A - \delta + c}{V_H - x}\right)\right\rceil \quad \text{if} \quad x > \tfrac{V_H}{2} \end{aligned} \tag{19}$$

where $\lfloor s\rceil$ denote the least integer greater than s when s is not an integer and $(s+1)$ otherwise.

Proof. Suppose p_1 agents B mutate by playing L and p_2 agents B mutate by claiming $x' > x_A^M$. For a given pair (p_1, p_2) agents A have the largest incentive to change into L if their belief are such that: (i) they expect to get the maximum payoff both in a match LL and in a match LH; (ii) they expect to obtain the minimum payoff in a match HL. Consider the equilibrium $\widetilde{\theta}_x \in \Sigma(\theta_x)$ in which for all agents:(i) $\widetilde{y}_{LL}^i = V_L - \delta$; $\widetilde{y}_{LH}^i = V_M - \delta$ and $\widetilde{y}_{HL}^i = \delta$; (ii) $\widetilde{x}_{LL}^i = \widetilde{x}_{LH}^i = \delta$ and in the subgame $\{HL, \delta\}$ all agents B accept. When at $\widetilde{\theta}_x$ some agents B mutate and these mutations induce all agents A to play L, then with positive probability the process enters into the basin of attraction of the equilibrium θ' such that $\rho(\theta') = \{LL, V_L - \delta, \delta\}$. Sure enough after updating all agents A decide to play L if

$$\frac{N - p_1}{N}(V_M - \delta) + \frac{p_1}{N}(V_L - \delta) > \mu_H\left(\widetilde{\theta}_x, p_1, p_2\right) \tag{20}$$

where the LHS is the expected payoff by playing L and the RHS is the expected payoff by playing H. However $\mu_H\left(\widetilde{\theta}_x, p_1, p_2\right)$ depends on which is the best demand in a match HH. In particular

$$\mu_H(.) = \begin{cases} \frac{N - p_2 - p_1}{N}(V_H - x) + \frac{p_1}{N}\delta - c & if \quad \frac{N - p_2 - p_1}{N - p_1}(V_H - x) \geq (V_H - x') \\ \\ \frac{N - p_1}{N}(V_H - x') + \frac{p_1}{N}\delta - c & if \quad \frac{N - p_2 - p_1}{N - p_1}(V_H - x) < (V_H - x') \end{cases} \tag{21}$$

The minimum number of mutations in population B comes from the comparison between the solutions of two constraint minimization problems. In both problem the objective function is $p_1 + p_2$. In the first (resp. second) problem we contemplate the case in which the best action in HH is $V_H - x'$ (resp. $V_H - x$). Both problems require $p_1 = 0$ as a solution; moreover $p_2^{M1} = N\left(\frac{x'-x}{V_H - x}\right)$ is the solution of the first problem and $p_2^{M2} = N\left(1 - \frac{V_M - \delta + c}{V_H - x}\right)$ is the solution of the second one. Since $p_2^{M1} > p_2^{M2}$, the minimum number of mutations in the population B involve that: (i) mutating agents only change their demands in the HH profile; (ii) these mutations cause agent A to shift to action L when the best action in the match HH continues to be $(V_H - x)$. Hence:

$$\overline{r}_B(x) = \left\lfloor N\left(1 - \frac{V_M - \delta + c}{V_H - x}\right)\right\rfloor \tag{22}$$

and

$$\overline{r}_B = \min_x \overline{r}_B(x) = \overline{r}_B\left(x_A^L\right). \tag{23}$$

We now suppose that some agents A mutate. As before two kind of mutations must be considered: p_1 agents A mutate by playing L and p_2 agents A mutate by demanding $(V_H - x')$ where $x' < x_B^L$. In this case we look for an equilibrium $\widehat{\theta}_x \in \Sigma(\theta_x)$ in which for all agents:(i) $\widetilde{x}_{LL}^i = V_L - \delta$; $\widetilde{x}_{LH}^i = \delta$ and $\widetilde{x}_{HL}^i = V_M - \delta$; (ii) $\widetilde{y}_{LL}^i = \widetilde{y}_{HL}^i = \delta$ and in the subgame $\{LH, \delta\}$ all agents A accept. It is easy to see that if some mutations of agents A occurs at $\widehat{\theta}_x$ and these mutations induce all agents B to play L, then with positive probability the process enters into the basin of attraction of the equilibrium θ' such that $\rho\left(\theta'\right) = \{LL, \delta, V_L - \delta\}$.

After updating all agents B decide to play L if

$$\frac{N - p_1}{N}(V_M - \delta) + \frac{p_1}{N}(V_L - \delta) > \mu_H\left(\widehat{\theta}_x, p_1, p_2\right) \tag{24}$$

where

$$\mu_H(.) = \begin{cases} \frac{N-p_2-p_1}{N}x + \frac{p_1}{N}\delta - c & if \quad \frac{N-p_2-p_1}{N-p_1}x \geq x' \\ \frac{N-p_1}{N}x' + \frac{p_1}{N}\delta - c & if \quad \frac{N-p_2-p_1}{N-p_1}x < x'. \end{cases}$$

Proceeding as before, the minimum number of mutations in the population A is

$$\overline{r}_A(x) = \left\lfloor N\left(1 - \frac{V_M - \delta + c}{x}\right)\right\rfloor \tag{25}$$

and

$$\overline{r}_A = \min_x \overline{r}_A(x) = \overline{r}_A\left(x_B^L\right) \tag{26}$$

By comparing (22) and (25) we get $\overline{r}_B(x) < \overline{r}_A(x)$ if $x > \frac{V_H}{2}$. ■

The next Lemma provides the minimum number of mutations required to make a transition from an equilibrium with outcome $\{HH, V_H - x, x\}$ and $x \in \left[x_B^L, x_A^L\right]$ to one equilibrium with outcome $\{HH, V_H - x', x'\}$. Along this transition the investment does not change.

Lemma 15 *For δ sufficiently small, the minimum number of mutations needed to get from $\Sigma(\theta_x)$ to an equilibrium with the same investment profile but different demands is:*

$$\begin{aligned} r_B^+(x) &= \left\lfloor N\left(\frac{\delta}{V_H - x}\right)\right\rceil \quad \text{if} \quad x < \frac{V_H}{2} \\ r_A^-(x) &= \left\lfloor N\left(\frac{\delta}{x}\right)\right\rceil \quad \text{if} \quad x > \frac{V_H}{2} \end{aligned} \tag{27}$$

where $r_B^+(x)$ is the number of mutations needed for the transition from $\Sigma(\theta_x)$ to $\Sigma(\theta_{x+\delta})$whereas $r_A^-(x)$ is the number of mutations needed for the transition from $\Sigma(\theta_x)$ to $\Sigma(\theta_{x-\delta})$. Moreover $r_B^+(x)$ is a strictly increasing function of x and $r_A^-(x)$ is a strictly decreasing function of x.

Proof: By a direct application of Young (1993). ■

Collecting these last results we can derive the radius of the set $\Sigma(\theta_x)$ for any $x \in \left[x_B^L; x_A^L\right]$. These informations will be used in the Proof of Proposition 8. Recall that for any $x \in \left(x_B^L; x_A^L\right)$ it is always true that

$$(V_H - (x + \delta)) - c \geq V_M - \delta \tag{28}$$

and

$$(x - \delta) - c \geq V_M - \delta. \tag{29a}$$

Thus from (28) and (29a) we infer respectively that:

$$\begin{aligned} r_B^+(x) &\leq \overline{r}_B(x) \\ r_A^-(x) &\leq \overline{r}_A(x). \end{aligned} \tag{30}$$

For any $x \in \left(x_B^L; x_A^L\right)$, it follows from Lemma (14) and Lemma (**??**) that

$$\begin{aligned} R(\Sigma(\theta_x)) &= r_B^+(x) \quad if \quad x < \frac{V_H}{2} \\ R(\Sigma(\theta_x)) &= r_A^-(x) \quad if \quad x > \frac{V_H}{2}. \end{aligned} \tag{31}$$

These conclusions hold true also when either $x = x_B^L = x_B^M + \delta$ or $x = x_A^L = x_A^M - \delta$ (remember that $\frac{V_H}{2} \in \left[x_B^L; x_A^L\right]$). Hence we can not generally say what is the easiest transition when either $x = x_B^L = x_B^M$ or $x = x_A^L = x_A^M$. However since $\overline{r}_B\left(x_A^M\right) < r_B^+\left(x_A^M\right)$ and $\overline{r}_A\left(x_B^M\right) < r_A^-\left(x_B^M\right)$, then:

$$
\begin{aligned}
R\left(\Sigma\left(\theta_{x_B^M}\right)\right) &= \min\left(r_B^+\left(x_B^M\right);\overline{r}_A\left(x_B^M\right)\right) \\
R\left(\Sigma\left(\theta_{x_A^M}\right)\right) &= \min\left(r_A^-\left(x_A^M\right);\overline{r}_B\left(x_A^M\right)\right).
\end{aligned} \tag{32}
$$

Moreover the monotonicity of $r_A^-(x)$ and $r_B^+(x)$ ensures that $R\left(\Sigma\left(\theta_{x_B^M}\right)\right) \leq r_B^+\left(\frac{V_H}{2}-\delta\right)$ and $R\left(\Sigma\left(\theta_{x_A^M}\right)\right) \leq r_A^-\left(\frac{V_H}{2}+\delta\right)$ when $\frac{V_H}{2} \in \left(x_B^L; x_A^L\right)$. We are now ready to give the Proof of Proposition **8.**

Proof of Proposition 8

Let $\overline{x} \equiv \frac{V_H}{2}$ and consider the set of equilibria $\Sigma(\theta_{\overline{x}})$. Our aim is to detect the stochastically stable outcome when $\overline{x} \in \left(x_B^L; x_A^L\right)$.

Suppose $x_B^L \neq x_B^M$ (resp. $x_A^L \neq x_A^M$). Let $\theta_x \in \Sigma_{IH}$ be an equilibrium. When $x < \overline{x}$ then the minimal modified costs from θ_x to $\Sigma(\theta_{\overline{x}})$ is associated with the path $\theta_x \rightarrow \theta_{x+\delta} \rightarrow ... \rightarrow \theta_{\overline{x}-\delta} \rightarrow \Sigma(\theta_{\overline{x}})$. Conversely, when $x > \overline{x}$ the minimal modified costs is associated with the path $\theta_x \rightarrow \theta_{x-\delta} \rightarrow ... \rightarrow \theta_{\overline{x}+\delta} \rightarrow \Sigma(\theta_{\overline{x}})$. Hence

$$
\begin{aligned}
c^*\left(\theta_x; \Sigma(\theta_{\overline{x}})\right) &= r_B^+(x) \quad \text{if} \quad x < \tfrac{V_H}{2} \\
c^*\left(\theta_x; \Sigma(\theta_{\overline{x}})\right) &= r_A^-(x) \quad \text{if} \quad x > \tfrac{V_H}{2}.
\end{aligned} \tag{33}
$$

By the monotonicity of $r_B^+(x)$ and $r_A^-(x)$ we obtain

$$
CR^*\left(\Sigma(\theta_{\overline{x}})\right) = \max\left(r_B^+(\overline{x}-\delta); r_A^-(\overline{x}+\delta)\right).
$$

Since

$$
R\left(\Sigma(\theta_{\overline{x}})\right) = r_B^+(\overline{x}) = r_A^-(\overline{x}) > CR^*\left(\Sigma(\theta_{\overline{x}})\right) \tag{34}
$$

it follows from Ellison (2000) that the only stochastically stable states belong to $\Sigma(\theta_{\overline{x}})$ and thus the unique stochastically stable outcome is $\left\{HH, \frac{V_H}{2}, \frac{V_H}{2}\right\}$.

Suppose now $x_B^L = x_B^M$ (resp. $x_A^L = x_A^M$) Since for whatever value of $R\left(\Sigma\left(\theta_{x_B^M}\right)\right)$ (resp. $R\left(\Sigma\left(\theta_{x_A^M}\right)\right)$) $CR^*\left(\Sigma(\theta_{\overline{x}})\right)$ does not change, the unique stochastically stable outcome continues to be$\left\{HH, \frac{V_H}{2}, \frac{V_H}{2}\right\}$. ■

The following Theorem is needed in order to prove Corollary 9.

Theorem 16 *(Ellison (2000)) Let $(Z, P, P(\varepsilon))$ be a model of evolution with noise. If for some limit set Ω and some state $\theta \notin \Omega$ we have $R(\Omega) = c^*(\theta, \Omega)$ then $\mu_*(\theta) > 0$ implies $\mu_*(\Omega) > 0$.*

Before giving the proof of Corollary 9 it is worth recalling that in our model, for any limit set Ω, $R(\Omega) = 1$. Moreover the support of the limit distribution (μ_*) is contained in the union of all limit set of the unperturbed process. Hence, if for two limit sets Ω and Ω' we have $c^*(\Omega, \Omega') = 1$, then $\mu_*(\Omega) > 0$ implies that $\mu_*(\Omega') > 0$.

Proof of Corollary 9.

We give the proof for point I and point II only.

Point I). When $V_M < \frac{V_L}{2} + \frac{V_H}{4}\frac{N-1}{N} + \frac{\delta}{2}$ then $0 < c_3 < c_4 < c_1 < c_2$. Hence for any $c \in [0, c_3]$ Proposition 8 holds. When $c \in (c_3, c_4]$, although some results continue to hold, we are not able to say which pattern of behavior is most probable and/or least probable to be observed in the long run. This vagueness mainly comes from the fact that Lemma 4 is not longer true. Therefore, in order to apply the radius/modified coradius Theorem, we have to compute $R(\Omega)$ for any limit set Ω not singleton. However since we have several limit sets not singleton and since many kinds of mutations are possible, this task is too demanding for the purpose of the present paper. Hence we say that in this range the long run behavior is indeterminate.

When $c \in (c_4, c_1)$ we know that $\Sigma_{IH} = \emptyset$. It follows from point (b) of Lemma 6 that for any $\theta \in \Sigma_H$ there exists at least one equilibrium $\theta' \in \Sigma_L$ such that $c^*(\theta, \theta') = 1$. Since $R(\theta') = 1$ then, by appealing to the aforementioned Ellison's Theorem, if a norm of investment evolves it cannot be the efficient one because if $\mu_*(\theta) > 0$ then also $\mu_*(\theta') > 0$.

Point II). When $\frac{V_L}{2} + \frac{V_H}{4}\frac{N-1}{N} + \frac{\delta}{2} \leq V_M \leq \frac{V_H}{2}\frac{N-1}{N} + \delta$ then $0 < c_4 < c_3 < c_1 < c_2$. Hence for any $c \in [0, c_4]$ Proposition 8 holds and for any $c \in (c_3, c_1)$ if a norm of investment evolves it cannot be the efficient one (see argument above). The only remaining case is when $c \in (c_4, c_3]$. In this case we know that for any limit set Ω:

(i) if $\rho(\Omega)$ is not singleton there exists at least one equilibrium θ', with $\rho(\theta')$ singleton, such that $c^*(\Omega, \theta') = 1$ (Lemma 4);

(ii) if $\Omega \in \Sigma_L$ there exist at least two equilibria, θ' and θ^*, with different distributional rule but both belonging to either Σ_H or $(\Sigma_{HL} \cup \Sigma_{LH})$ and such that $c^*(\Omega, \theta') = c^*(\Omega, \theta^*) = 1$ (points (a) and (b) of Lemma 5);

(iii) if $\Omega \in \Sigma_{HL}$ (resp. Σ_{LH}) there exist at least two equilibria, θ' and θ^*, with different distributional rule but both belonging to Σ_H such that $c^*(\Omega, \theta') = c^*(\Omega, \theta^*) = 1$ (point (c) of Lemma 5);

(iv) if $\Omega \in \Sigma_H$ there exist at least two equilibria, θ' and θ^*, with different distributional rule but both belong to either Σ_L or Σ_{HL} (resp. Σ_{LH}) and such that $c^*(\Omega, \theta') = c^*(\Omega, \theta^*) = 1$ (point (b) of Lemma 6 and point (b) of Lemma 5);

Let Ω be a limit set such that $\mu_*(\Omega) > 0$. By collecting previous informations we conclude that: (i) if $\rho(\Omega)$ is not singleton, then $\mu_*(\theta') > 0$ where $\rho(\theta')$ is singleton; (ii) if $\Omega \in \Sigma_L$, then $\mu_*(\theta') > 0$ and $\mu_*(\theta^*) > 0$ where θ' and θ^* both belong to either Σ_H or $\Sigma_{HL} \cup \Sigma_{LH}$; (iii) if $\Omega \in \Sigma_H$ then $\mu_*(\theta') > 0$ and $\mu_*(\theta^*) > 0$ where both θ' and θ^* belong to either Σ_L or $\Sigma_{HL} \cup \Sigma_{LH}$; (iv) if $\Omega \in \Sigma_{HL}$ (resp. Σ_{LH}) then $\mu_*(\theta') > 0$ and $\mu_*(\theta^*) > 0$ where both θ' and θ^* belong to either Σ_H or Σ_L. Hence when $c \in (c_4, c_3]$ a investment norm and bargaining norm cannot evolve in the long run.

The remaining two points use similar arguments by noticing that $c_4 < 0 < c_3 < c_1 < c_2$ at Point III) and $c_4 < 0 < c_2 < c_1 < c_3$ at Point IV). ■

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