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Identification of Lagged Duration Dependence in Multiple Spells Competing Risks Models *

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Abstract

We show non-parametric identification of lagged duration dependence in mixed proportional hazard models for duration data, in the presence of competing risks and consecutive spells. We extend the results to the case in which data provide repeated realizations of the consecutive spells competing risks structure for each subject.

Keywords: lagged duration dependence, competing risks, MPH models, identification.
JEL classification codes: C14; C41; J64

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1 Introduction

Time spent in a previous state can affect the duration of sojourn in the current state. Multiple causes of transitions are furthermore possible for each sojourn. Modelling is in a competing risks framework, where random variables measure the duration until a risk materialization, and only the smallest of all these durations are observed along with the corresponding exit destination. The joint distribution of all the durations, observed and censored, is not non-parametrically identified in a single spell competing risks framework (Cox, 1962; Tsiatis, 1975). Identification requires more structure, such as independent risks, parametric failure times joint distribution (see van den Berg (2001) for a survey), or variation in the explanatory variables (Heckman and Honoré, 1989; Abbring and van den Berg, 2003a; Lee, 2006).

We show the identification of mixed proportional hazard (MPH) models with lagged duration dependence in a multiple spells competing risks framework. We consider the simplest case where two consecutive spells are observed per unit, and each spell can terminate because of two competing risks. Our identification result can be easily extended to more than two spells or two destination states. We thus generalize the single risk results of Honoré (1993) and Frijters (2002) and, in contrast to Omori (1998), we do not use exclusion restrictions across risks of failure and across spells. Moreover, we extend the identification analysis to the case in which repeated realizations of the lagged durations are observed for each unit. We show that covariates are not required for identification and can enter the model in a general way.

Finally we establish a link between our identification analysis and Abbring and van den Berg's (2003b) identification result for treatment effects in duration models. Abbring (2008) rephrased Abbring and van den Berg's (2003b) type of model in terms of an event-history competing risks model with state dependence. On the basis of this reformulation, we show that the identification of Abbring and van den Berg's (2003b) models with heterogeneous treatment effects[1] can be extended to allow the treatment effect to also depend on pre-treatment duration.

Applications include the study of repeated temporary jobs (Gagliarducci, 2005), youth job stability after early unemployment events (Doiron and Gørgens, 2008; Gaure et al., 2008; Cockx and Picchio, 2009), the impact for the unemployed of different training programs on subsequent labour market performance (Gritz, 1993; Bonnal et al., 1997).

[1] We are referring to Abbring and van den Berg's (2003b) Models 1B and 2B, pp. 1507 and pp. 1510, respectively.

Section 2 describes the model. Section 3 and Section 4 derive the identification result when data provide single and repeated realizations of lagged duration for each subject, respectively. Section 5 establishes a link between our identification result and Abbring and van den Berg's (2003b) analysis. Finally, Section 6 concludes.

2 Lagged Duration Dependence in MPH Competing Risks Models

Let $t = 0$ be the start of the process and $\{Z(t), t \in \Re_+\}$ be a finite state point process. $Z(t)$ indicates the state occupied by each unit at time t and takes values in $\{o, a, b, c, d, e, f\}$. $Z(t)$ is generated by the following sequence:

(i) The state space is $\{o, a, b\}$. State o is the origin state of the first spell, for all the units under study. Every unit can experience at most a unique transition to a state in $\{a, b\}$. The observed outcome of the first spell is:

$$\begin{aligned} T_1 &= \min(T^*_{oa}, T^*_{ob}), \\ \Delta_1 &= \arg\min_{\{a,b\}}(T^*_{oa}, T^*_{ob}), \\ T^*_{ok} &= \inf\{t|Z(t) = k\},\ \forall\, k \in \{a, b\}. \end{aligned}$$

The T^*_{ok}'s are latent origin-destination-specific durations. We only observe their minimum and the destination state of the first spell. Assume ties have zero probability and define the latent duration distributions by the following mixed proportional hazard (MPH) rates:

$$\theta_{ok}(t|x, v_{ok}) = \lambda_{ok}(t)\phi_{ok}(x)v_{ok},\ \forall\, k \in \{a, b\}, \tag{1}$$

where the functions $\lambda_{ok}(\cdot)$ are the baseline hazards, $\phi_{ok}(\cdot)$ the systematic parts, x a vector of regressors and v_{ok}, for all $k \in \{a, b\}$, a vector of unobserved non-negative specific random variables. Dependence between T^*_{oa} and T^*_{ob} is assumed to be captured by observed and unobserved characteristics.

(ii) For all $k \in \{a, b\}$, let us denote by t_{ok} the observed duration of a first spell ending up in k, that is $t_{ok} = T^*_{ok}$ when $T_1 = T^*_{ok}$. New state spaces are available: a transition to a leads to a new state space where the only possible further transitions are toward $\{c, d\}$, whereas a transition to b leads to a new state space with transitions toward $\{e, f\}$. Consider a first transition toward a. We have:

$$\begin{aligned} T_2 &= \min(T^*_{ac}, T^*_{ad}), \\ \Delta_2 &= \arg\min_{\{c,d\}}(T^*_{ac}, T^*_{ad}), \\ T^*_{ak} &= \inf\{t - t_{oa} | Z(t) = k\}, \ \forall\, k \in \{c, d\}. \end{aligned}$$

Distributions the of T^*_{ak} are characterized by the origin-destination-specific hazard functions:

$$\theta_{ak}(t|x, t_{oa}, v_{ak}) = \lambda_{ak}(t)\phi_{ak}(x)h_{ak}(t_{oa})v_{ak}, \ \forall\, k \in \{c, d\}, \tag{2}$$

where $h_{ak}(\cdot)$ captures the effect of lagged duration in state o, t_{oa}, on the current transition intensity. Dependence between T^*_{ac} and T^*_{ad} is assumed to be captured by observed characteristics, unobservables, and lagged duration t_{oa}. Duration of a sojourn in b are defined in a symmetric way. The joint cumulative distribution of the unobserved heterogeneity vector $v \equiv (v_{oa}, v_{ob}, v_{ac}, v_{ad}, v_{be}, v_{bf})$ is G. G is allowed to be such that the unobserved heterogeneity components may have mass points at 0, with $\Pr(v > 0) > 0$.

At the end of the second spell, we observe $(T_1, \Delta_1, T_2, \Delta_2)$, and possible trajectories are in $\{oac, oad, obe, obf\}$. Denote by $\mathcal{D}_1 = \{oa, ob\}$ the set of the possible transitions during the first spell and by $\mathcal{D}_2 = \{ac, ad, be, bf\}$ the set of transitions during the second spell. The joint survival function is:

$$\begin{aligned} &\Pr\{\cap_{j \in (\mathcal{D}_1 \cup \mathcal{D}_2)}(T^*_j > t_j), |x\} = S(t_{oa}, t_{ob}, t_{ac}, t_{ad}, t_{be}, t_{bf}|x) \\ &= \int_{\Re^6_+} \exp\Big[- \sum_{k \in \mathcal{D}_1} \Lambda_k(t_k)\phi_k(x)v_k - \sum_{\substack{l \in \mathcal{D}_2 \\ m(l) \in \mathcal{D}_1}} \Lambda_l(t_l)\phi_l(x)h_l(t_{m(l)})v_l \Big] dG(v), \end{aligned} \tag{3}$$

where $\Lambda_k(t_k) = \int_0^{t_k} \lambda_k(u)du$, for $k \in (\mathcal{D}_1 \cup \mathcal{D}_2)$. It is equal to:

$$\begin{aligned} &\mathscr{L}_G\big\{\Lambda_{oa}(t_{oa})\phi_{oa}(x), \Lambda_{ob}(t_{ob})\phi_{ob}(x), \Lambda_{ac}(t_{ac})\phi_{ac}(x)h_{ac}(t_{oa}), \\ &\quad \Lambda_{ad}(t_{ad})\phi_{ad}(x)h_{ad}(t_{oa}), \Lambda_{be}(t_{be})\phi_{be}(x)h_{be}(t_{ob}), \Lambda_{bf}(t_{bf})\phi_{bf}(x)h_{bf}(t_{ob})\big\}, \end{aligned}$$

where $\mathscr{L}_G$ is the Laplace transform of G.[2]

Denote by $Q_l(t_1, t_2|x)$, for $l \in \mathcal{D}_2$, the subsurvival probability function, that is probability to survive t_1 time periods in the origin state o and t_2 time periods in a second state. Data provide information on these subdensities,

[2]See, e.g., Lancaster (1990, appendix 2) for properties of the Laplace transform.

i.e. we can compute:

$$\begin{aligned}
Q_{ac}(t_1,t_2|x) &\equiv \Pr(T^*_{oa} > t_1, T^*_{ob} > T^*_{oa}, T^*_{ac} > t_2, T^*_{ad} > T^*_{ac}|x),\\
Q_{ad}(t_1,t_2|x) &\equiv \Pr(T^*_{oa} > t_1, T^*_{ob} > T^*_{oa}, T^*_{ac} > T^*_{ad}, T^*_{ad} > t_2|x),\\
Q_{be}(t_1,t_2|x) &\equiv \Pr(T^*_{oa} > T^*_{ob}, T^*_{ob} > t_1, T^*_{be} > t_2, T^*_{bf} > T^*_{be}|x),\\
Q_{bf}(t_1,t_2|x) &\equiv \Pr(T^*_{oa} > T^*_{ob}, T^*_{ob} > t_1, T^*_{be} > T^*_{bf}, T^*_{bf} > t_2|x).
\end{aligned}$$

Applications of a competing risks model with lagged duration dependence may embrace assessment of the participation to the labour market. Suppose state o denotes unemployment, a employment, and b inactivity. An employment spell can be terminated by a transition either to a second unemployment event $c = o$ or to inactivity $(d = b)$. Inactivity can end because of a transition either to a second unemployment event $(e = c = o)$ or to employment $(f = a)$. Such a model is estimated in Doiron and Gørgens (2008) and Cockx and Picchio (2009) to understand the effect of the previous labour outcome on the subsequent labour market performance. Another example is the analysis of the effect of different training programs durations on subsequent job stability. Our theoretical framework is more general than what is assumed in these examples.

3 Identification with Single Realization Data

Theorem 1 *Assume that the joint survivor function of* $(T^*_{oa}, T^*_{ob}, T^*_{ac}, T^*_{ad}, T^*_{be}, T^*_{bf})$ *conditional on* x *is given by (3). Functions* $\mathscr{L}_G$, (Λ_j, ϕ_j), $\forall j \in \mathcal{D}_1 \cup \mathcal{D}_2$, *and* h_l, $\forall l \in \mathcal{D}_2$, *are identified from the distribution of* $(T_1, \Delta_1, T_2, \Delta_2)|x$ *under the following assumptions:*

A1 The support χ *of* x *is an open set in* $\Re^n$. *For all* $j \in \mathcal{D}_1 \cup \mathcal{D}_2$, *the* ϕ_j*'s are continuous functions such that* $\{\phi_{oa}(x), \phi_{ob}(x), \phi_{ac}(x), \phi_{ad}(x), \phi_{be}(x), \phi_{bf}(x)\}$ *contains a non-empty open set in* $\Re^6_+$.

A2 $\Lambda_j(t) < \infty$ *are non-negative, differentiable, and strictly increasing* $\forall j \in \mathcal{D}_1 \cup \mathcal{D}_2$ *and* $\forall t \in \Re_+$.

A3 Vector v *has non-negative components with distribution function* G *independent of* x, $E[v_{oj}v_{jk}] < \infty$, *with* $jk \in \mathcal{D}_2$, *and* $E[v] < \infty$.

A4 For all $j \in \mathcal{D}_1 \cup \mathcal{D}_2$, $\phi_j(x^0)$=1 *for some fixed* $x^0 \in \chi$. $\forall j \in \mathcal{D}_1 \cup \mathcal{D}_2$, $\Lambda_j(t^0) = 1$ *for some fixed* $t^0 \in \Re_+$.

A5 The h_l*'s are non-negative on* $\Re_+$ *and* $h_l(t^{00}_{m(l)}) = 1$ *for some fixed* $t^{00}_{m(l)} \in \Re_+$, *for all* $(l \in \mathcal{D}_2) \cap (m(l) \in \mathcal{D}_1)$.

Proof: 1: Under Assumptions A1-A4 and from the marginal distribution of $(T_1, \Delta_1)|x$, we can identify (Λ_k, ϕ_k) and the marginal distribution of v_k, $\forall k \in \mathcal{D}_1$ (Abbring and van den Berg, 2003a).

Conditional on (T^*_{oa}, T^*_{ob}), x is no longer independent on the unobserved heterogeneity $(v_{ac}, v_{ad}, v_{be}, v_{bf})$ and we can not iteratively apply Heckman and Honoré's (1989) or Abbring and van den Berg's (2003a) single spell identification results. A specific approach is required to identify the second-spell functions. From now on the proof proceeds in steps. In step **(a)**, identification of the second-spell systematic parts is shown. Step **(b)** deals with the identification of the unobserved heterogeneity distribution. In step **(c)** the lagged dependence functions are identified. Finally, step **(d)** shows identification of the second-spell baseline hazards.

(a) From the data we can compute the densities:

$$\begin{aligned} Q''_{ac}(t_1, t_2|x) &= \left[\frac{\partial^2 S}{\partial t_{oa}\partial t_{ac}}\right]_{\substack{t_{oa}=t_{ob}=t_1\\ t_{ac}=t_{ad}=t_2}}, \; Q''_{ad}(t_1, t_2|x) = \left[\frac{\partial^2 S}{\partial t_{oa}\partial t_{ad}}\right]_{\substack{t_{oa}=t_{ob}=t_1\\ t_{ad}=t_{ac}=t_2}}, \\ Q''_{be}(t_1, t_2|x) &= \left[\frac{\partial^2 S}{\partial t_{ob}\partial t_{be}}\right]_{\substack{t_{ob}=t_{oa}=t_1\\ t_{be}=t_{bf}=t_2}}, \; Q''_{bf}(t_1, t_2|x) = \left[\frac{\partial^2 S}{\partial t_{ob}\partial t_{bf}}\right]_{\substack{t_{ob}=t_{oa}=t_1\\ t_{bf}=t_{be}=t_2}}. \end{aligned}$$

Consider for instance $Q''_{ac}(t_1, t_2|x)$. It is the observed probability distribution function of first spell o terminating in a after t_1 time periods and second spell a terminating in c after t_2 time periods. Formally, it is equal to

$$\begin{aligned} Q''_{ac}(t_1, t_2|x) &= \lambda_{oa}(t_1)\phi_{oa}(x)\lambda_{ac}(t_2)\phi_{ac}(x)h_{ac}(t_1) \\ &\quad \times D_{ac}\mathscr{L}_G\Big\{\Lambda_{oa}(t_{oa})\phi_{oa}(x), \Lambda_{ob}(t_1)\phi_{ob}(x), \Lambda_{ac}(t_2)\phi_{ac}(x)h_{ac}(t_1), \\ &\qquad \Lambda_{ad}(t_2)\phi_{ad}(x)h_{ad}(t_1), \Lambda_{be}(t_2)\phi_{be}(x)h_{be}(t_1), \Lambda_{bf}(t_2)\phi_{bf}(x)h_{bf}(t_1)\Big\}, \end{aligned}$$

where $D_{ac}\mathscr{L}_G() \equiv \partial^2\mathscr{L}_G\{s_{oa}, s_{ob}, s_{ac}, s_{ad}, s_{be}, s_{bf}\}/\partial s_{oa}\partial s_{ac}$.

Consider Q''_{ac} and fix $(x, x^0) \in \chi^2$. As $t_2 \to 0$,

$$\begin{aligned} \frac{Q''_{ac}(t_1, t_2|x)}{Q''_{ac}(t_1, t_2|x^0)} \to \frac{\phi_{oa}(x)\phi_{ac}(x)}{\phi_{oa}(x^0)\phi_{ac}(x^0)} \\ \times \frac{D_{ac}\mathscr{L}_G\left[\Lambda_{oa}(t_1)\phi_{oa}(x), \Lambda_{ob}(t_1)\phi_{ob}(x), 0, 0, 0, 0\right]}{D_{ac}\mathscr{L}_G\left[\Lambda_{oa}(t_1)\phi_{oa}(x^0), \Lambda_{ob}(t_1)\phi_{ob}(x^0), 0, 0, 0, 0\right]}. \end{aligned} \quad (4)$$

As $t_1 \to 0$, $D_{ac}\mathscr{L}_G(.) \to \mathrm{E}(v_{oa}v_{ac}) < \infty$. Since ϕ_{oa} has already been identified, identification of ϕ_{ac} is obtained up to a constant. Analogously working on Q''_l, $\forall l \in \mathcal{D}_2 - \{ac\}$, yields the identification of ϕ_{ad}, ϕ_{be}, and ϕ_{bf}.

(b) After imposing $t_{oa} = t_{oa}^{00}$ and $t_{ob} = t_{ob}^{00}$, evaluate the joint survivor function (3) at $t_l = t^0$, $\forall l \in \mathcal{D}_2$. We obtain:

$$S(t_{oa}^{00}, t_{ob}^{00}, t^0, t^0, t^0, t^0) = \mathscr{L}_G\big[\Lambda_{oa}(t_{oa}^{00})\phi_{oa}(x), \Lambda_{ob}(t_{ob}^{00})\phi_{ob}(x), \phi_{ac}(x), \phi_{ad}(x), \phi_{be}(x), \phi_{bf}(x)\big]. \quad (5)$$

The left-hand side of (5) is observed from the data. By exploiting Assumption A1, we can trace the completely monotone function $\mathscr{L}_G$ on a non-empty open subset of $\Re_+^6$ by appropriately varying x in (5).[3] This uniquely identifies it on an non-empty open subset of $\Re_+^6$ by Proposition 1 of Abbring and van den Berg (2003a). As $\mathscr{L}_G$ is real analytic, it can be extended to all of $\Re_+^6$ and uniqueness of the Laplace transform concludes the identification of G.

(c) Consider Q''_{ac} and fix $(t_1, t_{oa}^{00}) \in \Re_+^2$ and $x \in \chi$. As $t_2 \to 0$,

$$\frac{Q''_{ac}(t_1, t_2|x)}{Q''_{ac}(t_{oa}^{00}, t_2|x)} \to \frac{\lambda_{oa}(t_1)}{\lambda_{oa}(t_{oa}^{00})}\frac{h_{ac}(t_1)}{h_{ac}(t_{oa}^{00})} \times \frac{D_{ac}\mathscr{L}_G\left[\Lambda_{oa}(t_1)\phi_{oa}(x), \Lambda_{ob}(t_1)\phi_{ob}(x), 0, 0, 0, 0\right]}{D_{ac}\mathscr{L}_G\left[\Lambda_{oa}(t_{oa}^{00})\phi_{oa}(x), \Lambda_{ob}(t_{oa}^{00})\phi_{ob}(x), 0, 0, 0, 0\right]}. \quad (6)$$

Since $\mathscr{L}_G$, the first-spell baseline hazards, and the first-spell systematic parts have already been identified, by letting t_1 vary over $\Re_+$ we identify h_{ac} up to a constant. Identification of h_{ad}, h_{be}, and h_{bf} on all of $\Re_+$ is analogous.

(d) To identify Λ_{jk}, $\forall jk \in \mathcal{D}_2$, compute for given t_1 and x the Q''_{jk}'s and solve in λ_{jk}'s. One gets a system of differential equations with initial conditions $\Lambda_{jk}(t^0) = 1$, $\forall jk \in \mathcal{D}_2$, made up of:

$$\lambda_{jk}\Big(t_2, \Lambda_{ac}(t_2), \Lambda_{ad}(t_2), \Lambda_{be}(t_2), \Lambda_{bf}(t_2)\Big) = \frac{Q''_{jk}(t_1, t_2|x)}{\lambda_{oj}(t_1)\phi_{oj}(x)\phi_{jk}(x)h_{jk}(t_1)M_{jk}}, \quad (7)$$

where:

$$\begin{aligned} M_{jk} = & D_{jk}\mathscr{L}_G\big[\Lambda_{oa}(t_1)\phi_{oa}(x), \Lambda_{ob}(t_1)\phi_{ob}(x), \Lambda_{ac}(t_2)\phi_{ac}(x)h_{ac}(t_1), \\ & \Lambda_{ad}(t_2)\phi_{ad}(x)h_{ad}(t_1), \Lambda_{be}(t_2)\phi_{be}(x)h_{be}(t_1), \\ & \Lambda_{bf}(t_2)\phi_{bf}(x)h_{bf}(t_1)\big]. \end{aligned}$$

Set $t_2 = t^0$. The numerators are observed in (7) and $\mathscr{L}_G$, Λ_k, h_l and ϕ_j have

[3] Complete monotonicity of the Laplace transform is ensured by the Hausdorff-Bernstein-Widder Theorem, in Widder (1941, pp. 160).

already been identified, $\forall k \in \mathcal{D}_1, \forall l \in \mathcal{D}_2, \forall j \in \mathcal{D}_1 \cup \mathcal{D}_2$. We can compute, for all $jk \in \mathcal{D}_2$, the $\lambda_{jk}(t^0)$'s using the normalization in assumption A4. We can also compute the $\Lambda_{jk}(t^0 + \varepsilon)$'s for a sufficiently small ε, and deduce the marginal changes λ_{jk}. Plugging them into the system of differential equations (7) and solving iteratively, we can trace out the Λ_{jk}'s on all of $\Re_+$.[4] This completes the proof. ■

Our assumptions are in line with Honoré's (1993) and Abbring and van den Berg's (2003a) assumptions. In contrast to Omori (1998), neither exclusion restrictions across spells and/or risks nor time-variation of the covariates from one spell to another are required.[5] Moreover, we do not need the systematic parts to take on every value in the set of the positive real numbers. A non-empty open set of the positive real numbers suffices. This is a condition more likely to be satisfied in empirical applications, in particular if spell- and time-varying explanatory variables are available. Variability in the explanatory variables, assumed in A1, is required in step **(b)** to identify the unobserved heterogeneity distribution. Assumption A2 is a regularity requirement on the integrated baseline hazards which is standard in the literature. Assumption A3 normalizes the unobserved heterogeneity component by restricting the mean to be finite. This is required in step **(a)** to identify the systematic parts. Note that, as in Abbring and van den Berg (2003a), the model is allowed to be defective in the distribution of the latent failure times since the individual heterogeneity distribution is allowed to have mass points at zero. The hazard rates are proportional and a way to identify their components is to normalize them. Assumptions A4 and A5 are innocuous normalizations of the integrated baseline hazards, systematic parts, and the lagged duration dependence functions.

4 Identification with Repeated Realizations

In this section, the identification analysis is extended to the case in which data cover repeated realizations of the first and the subsequent second spells for each subject.[6] For the sake of simplicity, we focus on two repeated ob-

[4]Satisfaction of the generalized smoothness Lipschitz continuity ensures the uniqueness of the traced out Λ_l's (Abbring and van den Berg, 2003a).

[5]However, in applications, spell- and time-varying explanatory variables would help to achieve identification by making it easier to satisfy Assumption A1.

[6]Applications that used such data focused, for example, on multiple "new" unemployment spells followed by either employment or inactivity spells (Doiron and Gørgens, 2008; Cockx and Picchio, 2009), by different types of training (Gaure et al., 2008), and by different types of job contract (Bonnal et al., 1997; Gagliarducci, 2005).

servations of the first and second spells. The identification result can be trivially extended to more that two repeated realizations. For each individual we observe a random drawn identified minima $(T_1^1, \Delta_1^1, T_2^1, \Delta_2^1)$ from the first realization and a random drawn identified minima $(T_1^2, \Delta_1^2, T_2^2, \Delta_2^2)$ from the second realization. The identified minima are defined as in Section 2. Hyperscripts 1 and 2 refer to the order of the realization. Therefore, (T_1^1, Δ_1^1) is the identified minimum at the end of the first spell of the first realization and (T_1^2, Δ_1^2) is the identified minimum at the end of the first spell of the second realization. We assume that $(T_1^1, \Delta_1^1, T_2^1, \Delta_2^1)$ and $(T_1^2, \Delta_1^2, T_2^2, \Delta_2^2)$ are independent conditional on v. This means that repeated trajectories and timings are treated as causally unrelated, though dependent through the unobserved determinants.

Similarly to Section 2, the distribution of the latent failure times of the first spells, T_{ok}^{*r} with $k \in \{a, b\}$, is characterized by the following MPH rates

$$\theta_{ok}^r(t^r|v_{ok}) = \lambda_{ok}^k(t^r)v_{ok}, \ \forall\, k \in \{a, b\}, \ r = 1, 2. \tag{8}$$

The distribution of the latent failure times of the subsequent spells, T_{jk}^{*r} with $jk \in \mathcal{D}_2$, is characterized by the following MPH rates

$$\theta_{jk}^r(t^r|t_{oj}^r, v_{jk}) = \lambda_{jk}^r(t^r)h_{jk}^r(t_{oj}^r)v_{jk}, \ \forall\, jk \in \mathcal{D}_2, \ r = 1, 2. \tag{9}$$

Note that we suppress the covariates x. The following identification results does not indeed require regressors variation. As in Honoré (1993) and Abbring and van den Berg (2003a; 2003b), the analysis can be thought of being conditional on x, that can enter the model in a general way. This means that the identification analysis in this section can be extended to cover a model where the baseline hazards, lagged duration dependence functions, and individual heterogeneity distribution depend on x. Note also that, while the baseline hazards and lagged duration dependence functions are allowed to vary between the first and the second realization of the process, the individual heterogeneity components are kept fixed. Variation between spells and within individual will be exploited to identify the baseline hazards and the lagged duration dependence.

The joint survival function of $\cap_{r=1,2}(T_1^r, \Delta_1^r, T_2^r, \Delta_2^r)$ is

$$\begin{aligned}
\Pr\{\cap_{\substack{j\in(\mathcal{D}_1\cup\mathcal{D}_2)\\ r=1,2}}(T_j^{*r} > t_j^r)\} &= S(t_{oa}^1, t_{ob}^1, t_{ac}^1, t_{ad}^1, t_{be}^1, t_{bf}^1, t_{oa}^2, t_{ob}^2, t_{ac}^2, t_{ad}^2, t_{be}^2, t_{bf}^2) \\
&= \int_{\Re_+^6} \prod_{r=1,2} \exp\Big[-\sum_{k\in\mathcal{D}_1} \Lambda_k^r(t_k^r)v_k - \sum_{\substack{l\in\mathcal{D}_2 \\ m(l)\in\mathcal{D}_1}} \Lambda_l^r(t_l^r)h_l^r(t_{m(l)}^r)v_l\Big] dG(v) \\
&= \mathscr{L}_G\Big\{ \sum_{r=1,2} \Lambda_{oa}^r(t_{oa}^r)\ ,\ \sum_{r=1,2} \Lambda_{ob}^r(t_{ob}^r)\ ,\ \sum_{r=1,2} \Lambda_{ac}^r(t_{ac}^r)h_{ac}^r(t_{oa}^r)\ , \\
&\sum_{r=1,2} \Lambda_{ad}^r(t_{ad}^r)h_{ad}^r(t_{oa}^r)\ ,\ \sum_{r=1,2} \Lambda_{be}^r(t_{be}^r)h_{be}^r(t_{ob}^r)\ ,\ \sum_{r=1,2} \Lambda_{bf}^r(t_{bf}^r)h_{bf}^r(t_{ob}^r)\Big\}, \quad (10)
\end{aligned}$$

where $\mathscr{L}_G = \{s_{oa}, s_{ob}, s_{ac}, s_{ad}, s_{be}, s_{bf}\}$ is the six-variate Laplace transform of G.

Before moving on to the identification result, consider that from large data we can compute, exploiting information on those individuals that experience, for example, a trajectory of type oac in the first realization, the subsurvival probability function

$$\begin{aligned}
Q_{ac}^1(t_1^1, t_2^1, t_1^2, t_2^2) \equiv \Pr\big(&T_{oa}^{*1} > t_1^1, T_{ob}^{*1} > T_{oa}^{*1}, T_{ac}^{*1} > t_2^1, T_{ad}^{*1} > T_{ac}^{*1}, \\
&T_{oa}^2 > t_1^2, T_{ob}^2 > t_1^2, T_{ac}^2 > t_2^2, T_{ad}^2 > t_2^2, T_{be}^2 > t_2^2, T_{bf}^2 > t_2^2\big). \quad (11)
\end{aligned}$$

Similarly, exploiting information on those individuals that experience a trajectory of type oac in the second realization, we can compute

$$\begin{aligned}
Q_{ac}^2(t_1^1, t_2^1, t_1^2, t_2^2) \equiv \Pr\big(&T_{oa}^{*1} > t_1^1, T_{ob}^{*1} > t_1^1, T_{ac}^{*1} > t_2^1, T_{ad}^{*1} > t_2^1, T_{be}^{*1} > t_2^1, T_{bf}^{*1} > t_2^1 \\
&T_{oa}^2 > t_1^2, T_{ob}^2 > T_{oa}^2, T_{ac}^2 > t_2^2, T_{ad}^2 > T_{ac}^2\big). \quad (12)
\end{aligned}$$

Subsurvival probabilities like those in (11) and (12) will be used to prove the following theorem.

Theorem 2 *Assume that the joint survivor function of $\cap_{r=1,2}(T_{oa}^{*r}, T_{ob}^{*r}, T_{ac}^{*r}, T_{ad}^{*r}, T_{be}^{*r}, T_{bf}^{*r})$ is given by (10). Functions $\mathscr{L}_G$, Λ_j^r, $\forall j \in \mathcal{D}_1 \cup \mathcal{D}_2$ and $r = 1, 2$, and h_l^r, $\forall l \in \mathcal{D}_2$ and $r = 1, 2$, are identified from the distribution of $\cap_{r=1,2}(T_1^r, \Delta_1^r, T_2^r, \Delta_2^r)$ under the following assumptions:*

- *B1 $\Lambda_j^r(t)$, $\forall j \in \mathcal{D}_1 \cup \mathcal{D}_2$ and $r = 1, 2$, are non-negative, differentiable, strictly increasing, and not allowed to be ∞, $\forall t \in \Re_+$. $\forall j \in \mathcal{D}_1 \cup \mathcal{D}_2$, $\Lambda_j^1(t^0) = 1$ for some fixed $t^0 \in \Re_+$.*
- *B2 The h_l^r's are non-negative on $\Re_+$ and $h_l(t_{m(l)}^{0r}) = 1$ for some fixed $t_{m(l)}^{0r} \in \Re_+$, for all $(l \in \mathcal{D}_2) \cap (m(l) \in \mathcal{D}_1)$ and $r = 1, 2$.*

B3 Vector v has non-negative components with distribution function G.

Proof: 2: Under Assumption B1, from the marginal distribution of $(T_1^1, \Delta_1^1, T_1^2, \Delta_1^2)$ we can identify Λ_k^r, $\forall k \in \mathcal{D}_1$ and $r = 1, 2$, by invoking Proposition 3, part (a), of Abbring and van den Berg (2003a). Identification of second-spells functions, both for $r = 1$ and $r = 2$, is now considered in sequential steps. In step **(a)**, identification of lagged duration functions is shown. Step **(b)** concerns identification of the second-spells baseline hazards. Finally, step **(c)** deals with identification of the individual heterogeneity distribution.

(a) From a large data set we can compute the subdensity

$$Q_{ac}^{1\prime\prime}(t_1^1, t_2^1, t_1^2, t_2^2) \equiv \frac{\partial^2 Q_{ac}^1(t_1^1, t_2^1, t_1^2, t_2^2)}{\partial t_1^1 \partial t_2^1} = \left[\frac{\partial^2 S}{\partial t_{oa}^1 \partial t_{ac}^1}\right]_{\substack{t_{oa}^1 = t_{ob}^1 = t_1^1 \\ t_{ac}^1 = t_{ad}^1 = t_2^1}}$$
$$= \lambda_{oa}^1(t_1^1)\lambda_{ac}^1(t_2^1)h_{ac}^1(t_1^1)D_{ac}\mathscr{L}_G(s_{oa}, s_{ob}, s_{ac}, s_{ad}, s_{be}, s_{bf}), \quad (13)$$

where $D_{ac}\mathscr{L}_G() \equiv \partial^2 \mathscr{L}_G\{s_{oa}, s_{ob}, s_{ac}, s_{ad}, s_{be}, s_{bf}\}/\partial s_{oa}\partial s_{ac}$. We can also compute the subdensity

$$Q_{ac}^{2\prime\prime}(t_1^1, t_2^1, t_1^2, t_2^2) \equiv \frac{\partial Q_{ac}^2(t_1^1, t_2^1, t_1^2, t_2^2)}{\partial t_1^2 \partial t_2^2} = \left[\frac{\partial^2 S}{\partial t_{oa}^2 \partial t_{ac}^2}\right]_{\substack{t_{oa}^2 = t_{ob}^2 = t_1^2 \\ t_{ac}^2 = t_{ad}^2 = t_2^2}}$$
$$= \lambda_{oa}^2(t_1^2)\lambda_{ac}^2(t_2^2)h_{ac}^2(t_1^2)D_{ac}\mathscr{L}_G(s_{oa}, s_{ob}, s_{ac}, s_{ad}, s_{be}, s_{bf}). \quad (14)$$

If we divide the subdensity in (13) by the subdensity in (14), the component related to the second derivative of the Laplace transform drops out. This is the advantage of having variation within individual in repeated realizations data. Indeed, fix $(t_2^1, t_1^2, t_2^2) \in \Re_+^3$ and pick $(t_1^1, t_{oa}^{01}) \in \Re_+^2$. Remind that $h_{ac}^1(t_{oa}^{01}) = 1$ and consider $Q_{ac}^{1\prime\prime}/Q_{ac}^{2\prime\prime}$:

$$\frac{\frac{Q_{ac}^{1\prime\prime}(t_1^1, t_2^1, t_1^2, t_2^2)}{Q_{ac}^{2\prime\prime}(t_1^1, t_2^1, t_1^2, t_2^2)}}{\frac{Q_{ac}^{1\prime\prime}(t_{oa}^{01}, t_2^1, t_1^2, t_2^2)}{Q_{ac}^{2\prime\prime}(t_{oa}^{01}, t_2^1, t_1^2, t_2^2)}} = \frac{\frac{\lambda_{oa}^1(t_1^1)\lambda_{ac}^1(t_2^1)h_{ac}^1(t_1^1)}{\lambda_{oa}^2(t_1^2)\lambda_{ac}^2(t_2^2)h_{ac}^2(t_1^2)}}{\frac{\lambda_{oa}^1(t_{oa}^{01})\lambda_{ac}^1(t_2^1)h_{ac}^1(t_{oa}^{01})}{\lambda_{oa}^2(t_1^2)\lambda_{ac}^2(t_2^2)h_{ac}^2(t_1^2)}} = \frac{\lambda_{oa}^1(t_1^1)h_{ac}^1(t_1^1)}{\lambda_{oa}^1(t_{oa}^{01})h_{ac}^1(t_{ac}^{01})}. \quad (15)$$

Since λ_{oa}^1 has already been identified, we get identification of h_{ac}^1 (up to a constant). Similarly, by fixing $(t_1^1, t_2^1, t_2^2) \in \Re_+^3$ and picking $(t_1^2, t_{ac}^{02}) \in \Re_+^2$ we can identify h_{ac}^2. Identification of h_{ad}^r, h_{be}^r, and h_{bf}^r, for $r = 1, 2$, is analogously yielded working on $Q_l^{1\prime\prime}/Q_l^{2\prime\prime}$ with $l \in \mathcal{D}_2 - \{ac\}$.

(b) With the normalization $\Lambda_{ac}^2(t^0) = 1$ and working on the ratio $Q_{ac}^{1\prime\prime}/Q_{ac}^{2\prime\prime}$, we get

$$\Lambda_{ac}^{2}(t_{2}^{2})=\int_{0}^{t_{2}^{2}}\Big[\int_{0}^{t^{0}}\frac{Q_{ac}^{1\prime\prime}(t_{1}^{1},\tau_{2}^{1},t_{1}^{2},\tau_{2}^{2})\lambda_{oa}^{2}(t_{1}^{2})h_{ac}^{2}(t_{1}^{2})}{Q_{ac}^{2\prime\prime}(t_{1}^{1},\tau_{2}^{1},t_{1}^{2},\tau_{2}^{2})\lambda_{oa}^{1}(t_{1}^{1})h_{ac}^{1}(t_{1}^{1})}d\tau_{2}^{1}\Big]^{-1}d\tau_{2}^{2}. \qquad (16)$$

Since λ_{oa}^1, λ_{oa}^2, h_{ac}^1, and h_{ac}^2 have already been identified, fixing $(t_1^1, t_1^2) \in \Re_+^2$ and letting t_2^2 vary over $\Re_+$ yield identification of Λ_{ac}^2. From similar computations we get

$$\frac{\Lambda_{ac}^{1}(t_{2}^{1})}{\Lambda_{ac}^{2}(t_{2}^{2})}=\int_{0}^{t_{2}^{1}}\Big[\int_{0}^{t_{2}^{2}}\frac{Q_{ac}^{2\prime\prime}(t_{1}^{1},\tau_{2}^{1},t_{1}^{2},\tau_{2}^{2})\lambda_{oa}^{1}(t_{1}^{1})h_{ac}^{1}(t_{1}^{1})}{Q_{ac}^{1\prime\prime}(t_{1}^{1},\tau_{2}^{1},t_{1}^{2},\tau_{2}^{2})\lambda_{oa}^{2}(t_{1}^{2})h_{ac}^{2}(t_{1}^{2})}d\tau_{2}^{2}\Big]^{-1}d\tau_{2}^{1}. \qquad (17)$$

Since Λ_{ac}^2, λ_{oa}^1, λ_{oa}^2, h_{ac}^1, and h_{ac}^2 have already been identified, fixing $(t_1^1, t_1^2) \in \Re_+^2$ and letting t_2^1 vary over $\Re_+$ yield identification of Λ_{ac}^1. Identification of all the other second-spell integrated baseline hazards is obtained by analogously working on $Q_l^{1\prime\prime}/Q_l^{2\prime\prime}$ with $l \in \mathcal{D}_2 - \{ac\}$.

(c) The distribution of $\cap_{r=1,2}(T_1^r, \Delta_1^r, T_2^r, \Delta_2^r)$ provides data on the survivor function $S(t_1^1, t_2^1, t_1^2, t_2^2)$ for $(t_1^1, t_2^1, t_1^2, t_2^2) \in \Re_+^4$. By Equation (10) we have

$$\begin{aligned}S(t_1^1,t_2^1,t_1^2,t_2^2)=\mathscr{L}_G\Big\{&\sum_{r=1,2}\Lambda_{oa}^r(t_1^r)\ ,\ \sum_{r=1,2}\Lambda_{ob}^r(t_1^r)\ ,\ \sum_{r=1,2}\Lambda_{ac}^r(t_2^r)h_{ac}^r(t_1^r)\ ,\\ &\sum_{r=1,2}\Lambda_{ad}^r(t_2^r)h_{ad}^r(t_1^r)\ ,\ \sum_{r=1,2}\Lambda_{be}^r(t_2^r)h_{be}^r(t_1^r)\ ,\ \sum_{r=1,2}\Lambda_{bf}^r(t_2^r)h_{bf}^r(t_1^r)\Big\}. \qquad (18)\end{aligned}$$

All the functions entering $\mathscr{L}_G$ have already been identified. Hence, $\mathscr{L}_G$ can be traced on a non-empty open set by appropriately varying $(t_1^1, t_2^1, t_1^2, t_2^2)$. As $\mathscr{L}_G$ is real analytic, it is uniquely determined on $\Re_+^6$. Uniqueness of the Laplace transform concludes identification of G. ∎

With repeated realizations of the lagged duration of interest, the multiple-spells MPH model is identified under weaker assumptions. We need neither the finite-mean of the individual heterogeneity distribution nor regressor variation. The latter implies that we can relax some of the separability assumptions, which are instead required with single realization data, and that the baseline hazards, lagged duration functions, and individual heterogeneity distribution can depend on x. Note however that the individual heterogeneity components v are not allowed to vary from the first to the second realization of the process. Whether or not this assumption is reasonable depends on the application. If it is more reasonable to assume that the unobserved heterogeneity components are realization-specific, Theorem 1 can be iteratively applied under the corresponding required assumptions.

5 Discussion

As Abbring (2008) pointed out, Abbring and van den Berg's (2003b) duration models with treatment effects can be reformulated in a competing risks framework. Consider an individual in a certain origin state o, e.g. unemployment, that at each point of time can leave this state either for e, e.g. employment, or p, e.g. participation to some kind of training program. Those individuals who ended up in p can still move to state e after some random time. The interest lies in understanding whether transition from unemployment to the training program makes people more likely to end up in employment than in the case in which the program is not provided. Such a framework is encompassed in the more general model analysed so far. Moreover, in this study the hazard rate of leaving p for e is allowed to depend on the pre-treatment duration, i.e. the duration of the preceding unemployment event.

On the basis of this competing risks reformulation, identification of Abbring and van den Berg's (2003b) models with heterogeneous treatment effects[7] can be extended in our framework to allow the treatment effect to depend also on the pre-treatment duration, provided that pre-treatment duration affects the hazard proportionally.

Under the same assumptions as in Abbring and van den Berg (2003b) and Assumption A5 (Assumption B2 with repeated realizations data), t_2+t_1 periods since the beginning of the origin state we can define and identify the treatment effect

$$\delta(t_2|t_1,x,v_{oe},v_{pe}) \equiv \frac{\lambda_{pe}(t_2)\phi_{pe}(x)h_{pe}(t_1)v_{pe}}{\lambda_{oe}(t_2+t_1)\phi_{oe}(x)v_{oe}}, \tag{19}$$

which depends on observables, unobservables, and pre-treatment duration. Equation (19) compares two conditional instantaneous probabilities of entering employment evaluated at the same time since the beginning of the unemployment spell o. The numerator of (19) is the instantaneous probability of entering employment from the treatment p conditional on surviving t_2 periods in the treatment and having spent t_1 quarters in unemployment before the treatment. The denominator is the instantaneous probability of

[7] We are referring to Abbring and van den Berg's (2003b) Models 1B and 2B, pp. 1507 and pp. 1510, respectively. Empirical studies applying such models can be found in van den Berg et al. (2002) and Zijl et al. (2004).

directly entering employment from unemployment conditional on surviving $t_1 + t_2$ periods in unemployment.

We can compare the treatment effect in (19) with the one provided in Abbring and van den Berg (2003b, pp. 1508) by adapting it to our notation. They coincide when it is imposed that $\lambda_{pe}(t_2)h_{pe}(t_1) = \lambda_{pe}(t_2+t_1)$. However, in general the two models are non-nested.[8]

In a single realization framework, identification of δ requires separability assumptions on observables, unobservables, pre-treatment duration (lagged duration), and current duration dependence. We have nonetheless seen in Section 4 that, with repeated realizations of the first and second spell, separability on observables is not needed. Theorem 2 indeed implies that the covariates can enter δ in a general way and the identification of a treatment effect in which, for instance,

$$\delta^r(t_2^r|x, t_1^r, v_{oe}, v_{pe}) = \frac{\lambda_{pe}^r(t_2^r|x)h_{pe}^r(t_1^r|x)v_{pe}}{\lambda_{oe}^r(t_2^r + t_1^r|x)v_{oe}}, \quad \text{with } r = 1, 2, \tag{20}$$

and where the unobserved heterogeneity distribution may depend on x.

6 Conclusions

This paper focuses on identifiability of the effect of a spell duration on the duration of the subsequent spell when individuals are under dependent competing risks of exit. We show that under the MPH assumption lagged duration dependence is identified without exclusion restrictions over risks and/or over spells and without parametric functional-form assumptions. In contrast to Omori (1998), we do not need the regressor effects to take value on the set of positive real numbers but just on a non-empty open set of it. This is a condition more likely to be satisfied in empirical applications, in particular if spell- and time-varying explanatory variables are available. A standard assumption in the MPH single-realization literature (e.g. Elbers and Ridder, 1982; Honoré, 1993; Abbring and van den Berg, 2003a) is the finite-mean assumption on the individual heterogeneity distribution, which is required for identification (Ridder, 1990) and also necessary here when data provide information on single realization of the lagged duration.

If data provide information for each individual on repeated realizations

[8]They are nested in the particular case in which in our model the transition intensities are log-linear in t_1 and t_2, i.e. $\lambda_{pe}(t_2) = \exp(\alpha t_2)$ and $h_{pe}(t_1) = \exp(\beta t_1)$, and in Abbring and van den Berg (2003b) the baseline hazard is log-linear in its argument, i.e. $\lambda_{pe}(t_2+t_1) = \exp[\gamma(t_2 + t_1)]$.

of the lagged durations, timings of the process, and transition, variation within individuals can be exploited to identify the model under weaker assumptions. The finite-mean assumption on the individual heterogeneity distribution can now be relaxed and, as opposed to Omori (1998), we do not require the separability of the effect of covariates, which are allowed to enter the functional forms of the model in a general way.

Finally, reformulating Abbring and van den Berg's (2003b) duration model with a dynamically assigned binary treatment as a multiple-spell competing risks model, our result suggests that the timing of events conveys information to identify a treatment effect that can be heterogeneous not only because of observed and unobserved individual characteristics but also because of different pre-treatment durations, provided that pre-treatment duration affects the hazard proportionally.

References

Abbring, J.H., "The Event-History Approach to Program Evaluation," in D. Millimet, J. Smith, and E. Vytlacil, eds., *Advances in Econometrics, Volume 21*, Oxford: Elsevier Science, 2008, chapter 2, pp. 33–55.

__ **and G.J. van den Berg**, "The Identifiability of the Mixed Proportional Hazards Competing Risks Model," *Journal of the Royal Statistical Society Series B*, 2003a, *65* (3), 701–710.

__ **and** __ , "The Nonparametric Identification of Treatment Effects in Duration Models," *Econometrica*, 2003b, *71* (5), 1491–1517.

Bonnal, L., D. Fougere, and A. Serandon, "Evaluating the Impact of French Employment Policies on Individual Labour Market Hystories," *Review of Economic Studies*, 1997, *64* (4), 683–713.

Cockx, B. and M. Picchio, "Are Short-Lived Jobs Stepping Stones to Long-Lived Jobs?," 2009. Université catholique de Louvain, mimeo.

Cox, D.R., *Renewal Theory*, London: Methuen, 1962.

Doiron, D. and T. Gørgens, "State Dependence in Youth Labor Market Experiences, and the Evaluation of Policy Interventions," *Journal of Econometrics*, 2008, *145* (1), 81–97.

Elbers, C. and G. Ridder, "True and Spurious Duration Dependence: The Identifiability of the Proportional Hazard Model," *Review of Economic Studies*, 1982, *49* (2), 403–409.

Frijters, P., "The Non-Parametric Identification of Lagged Duration Dependence," *Economics Letters*, 2002, *75* (3), 289–292.

Gagliarducci, S., "The Dynamics of Repeated Temporary Jobs," *Labour Economics*, 2005, *12* (4), 429–448.

Gaure, S., K. Røed, and L. Westlie, "The Impacts of Labour Market Policies on Job Search Behavior and Post-Unemployment Job Quality," 2008. IZA discussion paper No. 3802, Bonn.

Gritz, R.M., "The Impact of Training on the Frequency and Duration of Employment," *Journal of Econometrics*, 1993, *57* (1–3), 21–51.

Heckman, J.J. and B.O. Honoré, "The Identifiability of the Competing Risks Model," *Biometrika*, 1989, *76* (2), 325–330.

Honoré, B.R., "Identification Results for Duration Models with Multiple Spells," *Review of Economic Studies*, 1993, *60* (1), 241–246.

Lancaster, T., *The Econometric Analysis of Transition Data*, Cambridge: Cambridge University Press, 1990.

Lee, S., "Identification of a Competing Risks Model with Unknown Transformations of Latent Failure Times," *Biometrika*, 2006, *93* (4), 996–1002.

Omori, Y., "The Identifiability of Independent Competing Risks Models with Multiple Spells," *Oxford Bulletin of Economics and Statistics*, 1998, *60* (1), 107–116.

Ridder, G., "The Non-Parametric Identification of Generalized Accelerated Failure-Time Models," *Review of Economic Studies*, 1990, *57* (2), 167–181.

Tsiatis, A., "A Nonidentifiability Aspect of the Problem of Competing Risks," *Proceedings of the National Academy of Sciences of the United States of America*, 1975, *72* (1), 20–22.

van den Berg, G.J., "Duration Models: Specification, Identification, and Multiple Durations," in J.J. Heckman and E. Leamer, eds., *Handbook of Econometrics, Volume V*, Amsterdam: North-Holland, 2001, chapter 55, pp. 3381–3460.

—, **A. Holm, and J.C. van Ours**, "Do Stepping-Stone Jobs Exist? Early Career Paths in the Medical Profession," *Journal of Population Economics*, 2002, *15* (4), 647–666.

Widder, D.V., *The Laplace Transform*, Princeton: Princeton University Press, 1941.

Zijl, M., G.J. van den Berg, and A. Heyma, "Stepping Stones for the Unemployed: the Effect of Temporary Jobs on the Duration until Regular Work," 2004. IZA discussion paper No. 1241, Bonn.

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