Unobserved Leading and Coincident Common Factors in the Post-War U.S. Business Cycle

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Abstract

The paper introduces a two-factor model of the common leading and coincident economic indicators. Both factors are unobserved and each of them captures the dynamics of a corresponding group of the observed time series. The common leading factor is assumed to Granger-cause the common coincident factor. This property is used to estimate these two factors simultaneously and hence more efficiently. Two models of the latent leading and coincident factors are studied: a model with linear dynamics and a model with Markov-switching dynamics introduced through the leading factor intercept term. Moreover, a possibility of the individual leading variables having different leads over the common coincident indicator is considered. These models - both with linear and with regime-switching dynamics - were applied to the US monthly macroeconomic time series. The business cycle dating resulting from the nonlinear model closely corresponds to the NBER chronology and leads its turning points by 3-5 months.

JEL Classification: C5, E3

Keywords: dynamic factor analysis, Markov switching, leading indicator, coincident indicator, Granger causality

1 Introduction

In the modern macroeconomic literature many efforts are devoted to identification of a hypothetical coincident economic indicator which represents a general economic activity and allows to trace the evolution of the business cycle. It is designed to serve as a reference time series to judge about the state of the affairs in the economy. The most prominent examples of the one-factor models with the linear dynamics is Stock and Watson (1988), while those with the Markov-switching dynamics are the models proposed by Chauvet (1998), Kim and Yoo (1995).

With respect to this common coincident indicator one can then define the leading and lagging macroeconomic variables. The former of these time series are especially important since they permit to predict the changes in the state of the economy before they have occurred.

Normally, however, the leading series are not aggregated into a common leading factor. The evolution of the common coincident factor is conditioned on each of them individually, either directly through a vector autoregression (VAR) system of the common coincident factor and individual leading observed time series as in Stock and Watson (1988), Chauvet and Potter (2000) or via the time-varying transition probabilities which depend on the individual leading variables as in Kim and Yoo (1995).

This paper sets up a two-factor model where one of the latent factors is postulated as a common leading indicator, while the second factor is taken to be the common coincident indicator. A one-way Granger causality is assumed to exist coming from the former common factor to the latter one. The common leading and coincident factors are estimated from a set of observed time series which is split into a subset of leading and a subset of coincident variables.

First, we consider a linear model with leading and coincident factor following an AR process. Next, we add a regime-switching dynamics to take care of the possible asymmetries between the recession and expansion phases of the business cycle captured by both common latent factors.

The linear specification of the two-factor model is presented in section two, while section three contains a description of the model with nonlinear dynamics. In section four we apply our models to the artificial data in order to see how well these models reflect the true data-generating process. In section five the linear and Markov-switching models with leading and coincident common factors are estimated for the US monthly macroeconomic data. Section six concludes the paper. All the tables and graphs are put into the Appendix following the list of references.

2 Linear model

We consider a set of the observed time series, some of which may be defined as leading while the rest of them are treated as the coincident series. The common dynamics of the time series belonging to each of these groups are underlined by a common factor: leading corresponding to the first group and coincident corresponding to the second group. Moreover, the individual leading time series are allowed to lead the coincident factor at different lead times. For example, some series may lead the coincident factor by one period, other for two, and yet the other for three or more periods. The only characteristic which is common to all of them is that they *lead*, although for different periods of time. The idiosyncratic dynamics of each time series in particular are captured by one specific factor per each observed time series. Therefore the model can be written as follows:

$$\Delta y_t = \Gamma \Delta f_t + u_t \tag{1}$$

where $\Delta y_t = (\Delta y_{Lt} \mid \Delta y_{Ct})'$ is the $n \times 1$ vector of the observed time series in the first differences; $\Delta f_t = (\Delta f_{Lt} \mid \Delta f_{Ct})'$ is the 2×1 vector of the latent common factors in the first differences; $u_t = (u_{Lt} \mid u_{Ct})'$ is the $n \times 1$ vector of the latent specific factors; Γ is the $n \times 2$ factor loadings matrix linking the observed series with the common factors.

The dynamics of the latent common factors can be described in terms of a VAR model:

$$\Delta f_t = \mu + \Phi(L)\Delta f_{t-1} + \varepsilon_t \tag{2}$$

where μ is the 2×1 vector of the constant intercepts; $\Phi(L)$ is the sequence of p ($p = \max\{p_L, p_C\}$, where p_L is the order of the AR polynomial of the leading factor, and p_C is the order of the AR polynomial of the coincident factor) 2×2 lag polynomial matrices; ε_t is the 2×1 vector of the serially and mutually uncorrelated common factor disturbances:

$$\varepsilon_t \sim NID\left(\left(\begin{array}{c} 0 \\ 0 \end{array} \right), \left(\begin{array}{cc} \sigma_L^2 & 0 \\ 0 & \sigma_C^2 \end{array} \right) \right)$$

We assume that the leading factor Granger-causes the coincident factor but not vice versa. This assumption means that the matrices Φ_i $(i = 1, ..., p)$ are diagonal or lower diagonal for all i. For simplicity we suppose that the causality from the leading to the coincident factor is transmitted only at one lag, say τ. Thus, if $i \neq \tau$,

$$\Phi_i = \left(\begin{array}{cc} \phi_{L,i} & 0 \\ 0 & \phi_{C,i} \end{array} \right)$$

and if $i = \tau$,

$$\Phi_i = \left(\begin{array}{cc} \phi_{L,i} & 0 \\ \phi_{CL,i} & \phi_{C,i} \end{array} \right)$$

The idiosyncratic factors are by definition mutually independent and are modelled as the AR processes:

$$u_t = \Psi(L)u_{t-1} + \eta_t \tag{3}$$

where $\Psi(L)$ is the sequence of q ($q = \max\{q_{1,...}, q_n\}$, where q_i is the order of the AR polynomial of the $i-th$ idiosyncratic factor) $n \times n$ diagonal lag polynomial matrices and η_t is the $n \times 1$ vector of the mutually and serially uncorrelated Gaussian shocks:

$$\eta_t \sim \left(\begin{pmatrix} 0 \\ \vdots \\ 0 \end{pmatrix}, \begin{pmatrix} \sigma_1^2 & & 0 \\ & \ddots & \\ 0 & & \sigma_n^2 \end{pmatrix} \right)$$

To estimate this model we express it in a state-space form:

Measurement equation:

$$\Delta y_t = A\beta_t \tag{4}$$

Transition equation:

$$\beta_t = \alpha + C\beta_{t-1} + v_t \tag{5}$$

where $\beta_t = (f_t|u_t)'$ is the state vector containing stacked on top of each other vector of common factors and the vector of specific factors; v_t is the vector of the common and idiosyncratic factors' disturbances with mean zero and variance-covariance matrix Q; α is the vector of intercepts.

$$A = \begin{pmatrix} \Gamma_L & o_{n_L} & i_{q_1} & ... & 0 \\ O_{n_L \times r} & \gamma_C & 0 & ... & i_{q_n} \end{pmatrix}$$

where Γ_L is the $n_L \times (r-1)$ matrix of the leading factor loadings:

$$\Gamma_L = \begin{pmatrix} \gamma_{L,1} & 0 & & 0 \\ \gamma_{L,2} & 0 & & 0 \\ \vdots & & \ddots & \\ 0 & 0 & \cdots & \gamma_{L,n_L} \end{pmatrix}$$

in which the position of each leading factor loading depends on the lead time of a corresponding observed time series.

$O_{n \times m}$ is $n \times m$ matrix of zeros; o_m is the $m \times 1$ vector of zeros; i_m is the first row of the $m \times m$ identity matrix, and $r = \max\{p_L, \tau\}$.

$$C = \begin{pmatrix} \Phi^L & & & & 0 \\ \Phi^{CL} & \Phi^C & & & \\ & & \Psi^1 & & \\ & & & \ddots & \\ 0 & & & & \Psi^n \end{pmatrix}$$

where Φ^L is the $r \times r$ matrix:

$$\Phi^L = \begin{pmatrix} \phi_L & o'_{r-p_L} \\ I_{r-1} & O_{(r-1)\times(r-p_L)} \end{pmatrix}$$

where ϕ_L is the $1 \times p_L$ row vector of the AR coefficients of the leading factor, I_n is the $n \times n$ identity matrix, and o_m is the $m \times 1$ vector of zeros.

$$\Phi^C = \begin{pmatrix} \phi_C & 0 \\ I_{p_C-1} & o_{p_C-1} \end{pmatrix}$$

The matrices $\Psi^1, ..., \Psi^n$ have the same structure as Φ^C.

$$\Phi^{CL} = \begin{pmatrix} o'_r \\ \phi_{CL} \end{pmatrix}$$

where ϕ_{CL} is the $1 \times r$ vector of zeros with ϕ_{CL}, τ at the $\tau - th$ position.

The unknown parameters and the latent factors may be estimated using Kalman filter recursions. To save space we will not present them here, referring the reader, for instance, to Hamilton (1994) who gives very clear and systematic explanation of the Kalman filter methodology.

3 Nonlinear model

It was observed by many authors, among them by Diebold and Rudebusch (1996) that the model of the business cycle would be incomplete if it would not take into account both the comovement of various macroeconomic variables and the asymmetries between the phases of the cycle. The linear model presented in the previous section incorporates the phenomenon of the simultaneous changes in the levels of different individual time series. However, it lacks a mechanism which would reflect the qualitatively different behavior of these series during recessions and expansions. One of the ways to introduce this mechanism in our model is to add to it the regime-switching dynamics.

The Markov-switching dynamics is introduced through the leading factor intercept:

$$\Delta f_t = \mu(s_t) + \Phi(L)\Delta f_{t-1} + \varepsilon_t \tag{6}$$

where $\mu(s_t) = (\mu_L(s_t), ..., 0)'$.

s_t is the unobserved regime variable. In the two-regime (expansion-recession) case it takes two values: 0 or 1. Depending on the regime, the leading factor intercept assumes different values: low in recessions and high in expansions. Thus, the common factors grow faster during the upswings and slower (or even have negative growth rate) during the downswings of the economy.

The changes in the regimes are governed by the first-order Markov chain process, which is summarized by the transition probabilities matrix:

	$s_{t-1} = 0$	$s_{t-1} = 1$
$s_t = 0$	p_{11}	$1 - p_{22}$
$s_t = 1$	$1 - p_{11}$	p_{22}

where $p_{ij} = prob(s_t = j|s_{t-1} = i)$.

The rest of the equations of the model remains unchanged. The state-space representation of the nonlinear two-factor model may be written as:

Measurement equation:

$$\Delta y_t = A\beta_t \tag{7}$$

Transition equation:

$$\beta_t = \alpha(s_t) + C\beta_{t-1} + v_t \tag{8}$$

where $\alpha(s_t) = (\mu_L(s_t), ..., 0)'$.

It is worthwhile to notice that, since it is the dynamics of the common leading factor which include the state-dependent intercept in the current period, the conditional regime probabilities predicting the occurrence of recessions or expansions of the coincident factor are simply the conditional regime probabilities computed for the leading factor shifted forward for τ periods. Thus, the conditional regime probabilities estimated using the above model provide us with the τ-periods ahead forecast of the coincident factor regimes.

All the other system matrices are as in the linear model. Thus, we have a model expressed in the state-space form and having Markov-switching dynamics. Again, we will not reproduce here all the relevant recursions which are necessary to estimate the parameters and the unobserved state vector. On the estimation of the common factor models with Markov switching one can read in Kim (1994) or Kim and Nelson (1999).

4 Simulated examples

To see how well our models replicate the true data-generating processes where both common factors are present and the described above causal relationship is introduced, we have generated four artificial data sets and have estimated the corresponding models using as inputs the time series which may be observed. In the first two cases the dynamics are linear, while in the last two cases the common factors follow Markov-switching process. In the case one all the leading variables have the same lead time, whereas in the case two one of the leading variables leads the coincident factor at a smaller lead than the other observed leading time series. The same distinction is maintained for the cases three and four where the regime switching is added.

For the linear case one we have generated two common latent factors and five individual observable series. The first two observed time series are leading, while the three remaining are the coincident. Both the common factors (in fact, their first differences, not levels) and the idiosyncratic components are modelled as the stationary AR(1) processes. The coincident factor is positively affected by the leading factor at the lag $\tau = 3$. The true parameters of the data-generating process (DGP) are presented in the column two of the Table 1 of the Appendix. The length of all these series is 540 observations, which is comparable to the length of an ordinary Post-World War II monthly time series for the US economy. In the case two six observed series were simulated: three leading and three coincident. The first two leading series lead the common coincident indicator by three periods, while the third leading time series has a lead of only two periods. The true parameters are printed out in the column two of the Table 2 of Appendix.

To identify the model (in both cases), we set the factor loadings of the first observable variable in each subset - leading and coincident - equal to unity. Thus, we estimate only three of five factor loadings: one for the leading factor and two for the coincident factor. The model is estimated by the maximum likelihood. The estimated parameters together with the standard errors and the p-values for case 1 are reproduced in the Table 1, for case two - in the Table 2. The mere observation of the true and estimated parameters' values shows that the latter are sufficiently close to the former suggesting that the proposed model estimates the parameters generated process accurately enough.

The visual comparison of the common factors profiles suggests a very high degree of similarity of the simulated and estimated common factors, especially in the case of the latent leading factor. We do not display the graphs of the simulated data here in order to save space.

In the two cases with the Markov-switching dynamics the length of the series is also 540. In the case three the first two observable time series are leading, meanwhile the last three series are coincident. The coincident factor is again correlated to the leading factor with a lag of three periods. The same identifying normalization - by setting the factor loadings of the first observed time series in each group of the variables - is used. In the case four six observed series were generated with the same leading structure as in the case two. The parameters of the true DGP for the case three are presented in the second column of Table 3 and those for the case four - in the second column of Table 4 of the Appendix. The estimates replicate the true parameters with a sufficiently high degree of precision. Again, as in the case of the linear model, the estimated common factors series are very similar to the simulated common factors.

The probabilities obtained from the nonlinear model are used to build the business cycle chronology. If the probability of being currently in recession exceeds some margin (say, 0.5) we say that the economy may be qualified to be in a recession. The estimated model captures the recession dates pretty well. However, the smoothed recession probabilities sometimes miss the recessions when those have a very short duration. In contrast, the filtered probabilities give sometimes false alarms by announcing the arrival of recessions which did not take place. Thus, the smoothed probabilities turn out to be a more conservative dating tool than the filtered probabilities.

5 Real example

The linear two-factor model was estimated using the US monthly data from January 1959 to December 1998. To estimate the leading common factor the data from Watson (2000) were used, namely three financial time series: spread between the US Treasury bills 3-month interest rate and federal funds effective annualized rate (SFYGM3), spread between the US Treasury bills with constant maturity 1-year interest rate and federal funds effective annualized rate (SFYGT1), and NYSE common stock price index (FSNCOM). The common coincident factor was estimated based on the four real time series borrowed from Mariano and Murasawa (2000): employees on nonagricultural payrolls; personal income less transfer payments; index of industrial production; and manufacturing and trade series.

The leading time series were selected by comparing them individually to a coincident factor computed as if it were not dependent of a hypothetical leading common factor. Figure 1 shows that the correlation between these series (SFYGM3, SFYGT1 in levels and the first differences of the log of FSNCOM), on one hand, and the growth rate of the common coincident indicator, on the other hand, is relatively high at leads 4-5. It is also very important that the series are sufficiently highly correlated among each other, thus permitting to postulate existence of a latent common factor standing behind their common evolution.

Three model combinations were estimated: (0,0), (1,1), and (2,2), that is, the cases when common and idiosyncratic factors follow AR(0), AR(1), and AR(2) processes, respectively. For each of these combinations different leads between the common leading and coincident factor were tried, starting from the "zero lead" (no Granger causality) and ending with a lead of six months. The results of these experiments are displayed in Table 5. The first conclusion is that introducing a Granger causality between the leading and coincident factor seems to be a meaningful exercise - there is a significant increase in the likelihood function value when a cross-regressive term is included. Secondly, even larger positive effect is achieved when the AR(2) dynamics are allowed compared to the AR(1). Finally, in the first case (autoregression of the zero order, that is, static factor model) the "optimal lead", i.e. the lead which delivers the maximum likelihood function value, is five months, while in the other two cases (autoregression of first and second orders) the "optimal lead" is three months. The estimates of the linear two-factor model with (1,1) specification with lead equal 3 months are presented in Table 6.

The common leading and coincident factors estimated with a linear model are depicted on the two left panels of Figure 2. On the upper left panel two common factors - each estimated separately in a single-factor model - are displayed. The common factors are constructed by summing up their first differences. Thus, they are represented as random walks without drift. This is done to render the cyclical movements more visible. If we were to introduce a nonzero drift as it is done normally (e.g., by Stock and Watson (1988)), it would mask the cyclical fluctuations. The common factors estimated independently have the following specifications: leading factor is (1,1), that is, both common and specific factors are AR(1), and the coincident factor is (1,1). In the case of simultaneous estimation of the two common factors, when the coincident factor depends on the leading one, the specification is also (1,1). We can observe that in terms of the turning points the two models (with and without Granger causality between the factors) are similar, differing mainly in their "vertical profile". The latter is not surprising given that the common factors were reconstructed as random walks.

The next exercise was to incorporate the Markovian dynamics into the multifactor model. This was done through the regime-dependent itercept of the leading common factor. Since in the two-factor model the coincident factor depends on the leading one, the Markov-switching dynamics of the latter is transferred to the former. The parameter estimates of the Markov-switching two-factors model are contained in Table 7. The common leading factor is specified as AR(0), the common coincident factor follows AR(2). The lead is set equal to 3 months.

The common factors estimated assuming the regime-switching dynamics are displayed on the two right panels of Figure 2. The specification of common leading and coincident factors computed in a single-factor model are (0,0) and (1,1), respectively. In the two-factor case the leading common factor and corresponding idiosyncratic components were modeled as AR(0), while the common coincident factor and corresponding specific factors were supposed to follow AR(2) processes. Visual inspection of all four graphs depicted on Figure 2 shows that their turning points are basically the same. One important difference is that the linear models treat the recession of the early 1990s as deeper than that of the beginning of 1980s, while the nonlinear models reverse the order.

Figure 3 compares the recession probabilities (filtered and smoothed) of the leading and coincident common factors. In the first case the probabilities are computed from the two-factor model, while in the second case the recession probabilities from the single (coincident) factor model are used. One can easily see that the recession probabilities calculated for the leading factor signal the arrival of the recession phase several periods later than the coincident factor recession probabilities do. The coincident factor model suggests that there were six recessions during the January 1959 - December 1998 period, while the two-factors model uncovers five recessions. The only recession which is missing is the one in the very beginning of the sample. However, given the leading nature of the recession probabilities obtained from the 2-factors model, one can assume that this recession simply "does not fit the sample". In other words, it would be found, had we the had data starting a little bit earlier.

Next, we compare the leading and coincident recession probabilities to the National Bureau of Economic Research (NBER) chronology. This is done in Figure 4. It is evident that there is very close correspondence between the NBER dating and the coincident factor recessions. The leading factor recessions, as it is to be expected, anticipate the NBER turning points.

Finally, we calculate the cross-correlations between the leading and coincident common factors at different lags and leads. These cross-correlations are displayed on Figure 5. The data used to plot the picture were the same as those which are displayed on Figure 2. The cross-correlations were computed for the first differences of the common factors, not their levels. The reason is that the common factors in levels are not stationary, while their growth rates are. We can see that the maximum correlation approaches 0.5 and that it is achieved at lead 4-5 months, although being pretty high in the neighborhood of this point.

6 Summary

In the paper we have introduced a dynamic factor model with two common factors: leading and coincident. Each of them represents the common dynamics of a corresponding subset of the observed time series which are classified as being leading or coincident with respect to some hypothetical "state of the economy". The common leading factor Granger-causes the common coincident factor, thus allowing to use the former in the predictions of the future values of the latter. This permits to improve the forecasting of the coincident factor because of the additional information coming from the leading variables. In addition, different leads with respect to the common coincident indicator for the individual leading time series are allowed, which makes the model more flexible and realistic, since in the real life the leading time series rarely lead the coincident factor for the same periods of time.

We consider two models: a model with the linear dynamics and a model with the regime switching. The first model captures the cyclical comovements of different macroeconomic time series. The second model allows also to take care of the asymmetries which may characterize different phases of the business cycle and therefore is more complete from the standpoint of the Burns and Mitchell's definition of the business cycle as interpreted by Diebold and Rudebusch (1996).

Both models are illustrated using four artificial examples (two with the identical lead time for all the observed leading series and two with the different lead times), which show a high enough fitting ability of these models, provided that they correspond to the true data-generating process.

Quite interesting results were obtained when the model was applied to the US monthly macroeconomic data stretching from January 1959 to December 1998. A linear and a Markov-switching 2-factor models were estimated. The common coincident factor is sufficiently closely related to the common leading factor, the lead time being 3-5 months. This lead is also apparent when the recession probabilities are considered: the peaks of the low state probabilities calculated for the leading factor precede those computed for the coincident factor. Moreover, there is a tight correspondence between our estimated recession dates and those provided by the NBER. The conclusion is that we can use the two-factor model to predict the evolution of the US Post-War coincident economic indicator and the business cycle turning points in the near (up to five months) future.

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7 Appendix

Table 1. True and estimated parameters of the linear two-factor model (case 1: identical lead time)

Parameter	True	Estimated	St. error	p-value
γ_1	1	-	-	-
γ_2	0.9	0.91	0.03	0.0
γ_3	1	-	-	-
γ_4	2	2.06	0.03	0.0
γ_5	1.7	1.71	0.02	0.0
ϕ_L	0.8	0.79	0.03	0.0
ϕ_C	0.7	0.70	0.03	0.0
$\phi_{CL,3}$	0.5	0.48	0.05	0.0
ψ_1	-0.3	-0.36	0.05	0.0
ψ_2	-0.7	-0.67	0.04	0.0
ψ_3	-0.5	-0.47	0.05	0.0
ψ_4	-0.2	-0.22	0.07	0.0
ψ_5	-0.8	-0.79	0.03	0.0
σ_1^2	0.25	0.26	0.03	0.0
σ_2^2	0.36	0.36	0.03	0.0
σ_3^2	0.16	0.16	0.01	0.0
σ_4^2	0.49	0.48	0.05	0.0
σ_5^2	0.81	0.81	0.06	0.0
σ_L^2	0.25	0.24	0.03	0.0
σ_C^2	0.36	0.36	0.03	0.0

Table 2. True and estimated parameters of the linear two-factor model
(case 2: different lead time)

Parameter	True	Estimated	St. error	p-value
γ_1	1	-	-	-
γ_2	0.9	0.86	0.03	0.0
γ_3	1.5	1.43	0.04	0.0
γ_4	1	-	-	-
γ_5	2	2.03	0.05	0.0
γ_6	1.7	1.72	0.04	0.0
ϕ_L	0.8	0.79	0.03	0.0
ϕ_C	0.7	0.67	0.02	0.0
$\phi_{CL,3}$	0.5	0.58	0.04	0.0
ψ_1	-0.3	-0.35	0.05	0.0
ψ_2	-0.7	-0.72	0.03	0.0
ψ_3	-0.5	-0.51	0.06	0.0
ψ_4	-0.2	-0.16	0.05	0.0
ψ_5	-0.8	-0.82	0.03	0.0
ψ_6	-0.3	-0.36	0.05	0.0
σ_1^2	0.25	0.25	0.02	0.0
σ_2^2	0.36	0.36	0.02	0.0
σ_3^2	0.16	0.16	0.02	0.0
σ_4^2	0.49	0.50	0.03	0.0
σ_5^2	0.81	0.79	0.08	0.0
σ_6^2	0.64	0.64	0.05	0.0
σ_L^2	0.25	0.23	0.02	0.0
σ_C^2	0.36	0.33	0.03	0.0

Table 3. True and estimated parameters of the nonlinear two-factor model (case 3: identical lead time)

Parameter	True	Estimated	St. error	p-value
p_{11}	0.95	0.97	0.01	0.0
p_{22}	0.84	0.84	0.05	0.0
μ_{L1}	0.4	0.40	0.04	0.0
μ_{L2}	-0.6	-0.70	0.06	0.0
γ_1	1	-	-	-
γ_2	0.9	0.90	0.01	0.0
γ_3	1	-	-	-
γ_4	2	1.99	0.01	0.0
γ_5	1.7	1.69	0.01	0.0
ϕ_L	0.8	0.78	0.02	0.0
ϕ_C	0.7	0.70	0.02	0.0
$\phi_{CL,3}$	0.5	0.51	0.03	0.0
ψ_1	-0.3	-0.29	0.05	0.0
ψ_2	-0.7	-0.69	0.03	0.0
ψ_3	-0.5	-0.50	0.05	0.0
ψ_4	-0.2	-0.09	0.07	0.0
ψ_5	-0.8	-0.82	0.03	0.0
σ_1^2	0.25	0.26	0.02	0.0
σ_2^2	0.36	0.35	0.03	0.0
σ_3^2	0.16	0.16	0.01	0.0
σ_4^2	0.49	0.47	0.05	0.0
σ_5^2	0.81	0.81	0.06	0.0
σ_L^2	0.16	0.18	0.03	0.0
σ_C^2	0.36	0.37	0.03	0.0

Table 4. True and estimated parameters of the nonlinear two-factor model (case 4: different lead time)

Parameter	True	Estimated	St. error	p-value
p_{11}	0.95	0.95	0.01	0.0
p_{22}	0.84	0.86	0.03	0.0
μ_{L1}	0.4	0.39	0.03	0.0
μ_{L2}	-0.6	-0.58	0.05	0.0
γ_1	1	-	-	-
γ_2	0.9	0.91	0.01	0.0
γ_3	1.5	1.51	0.02	0.0
γ_4	1	-	-	-
γ_5	2	2.00	0.02	0.0
γ_6	1.7	1.70	0.02	0.0
ϕ_L	0.8	0.79	0.02	0.0
ϕ_C	0.7	0.71	0.02	0.0
$\phi_{CL,3}$	0.5	0.49	0.03	0.0
ψ_1	-0.3	-0.27	0.05	0.0
ψ_2	-0.7	-0.73	0.03	0.0
ψ_3	-0.5	-0.46	0.06	0.0
ψ_4	-0.2	-0.22	0.05	0.0
ψ_5	-0.8	-0.83	0.03	0.0
ψ_6	-0.5	-0.49	0.05	0.0
σ_1^2	0.25	0.25	0.02	0.0
σ_2^2	0.36	0.36	0.02	0.0
σ_3^2	0.16	0.17	0.02	0.0
σ_4^2	0.49	0.49	0.03	0.0
σ_5^2	0.81	0.64	0.07	0.0
σ_6^2	0.36	0.42	0.04	0.0
σ_L^2	0.16	0.17	0.02	0.0
σ_C^2	0.36	0.36	0.03	0.0

Table 5. Optimal lead determination. Likelihood function values corresponding to different AR order combinations and different leads

Lead	Combinations		
	(0,0)	(1,1)	(2,2)
0	-4236.80	-3268.76	-3199.57
1	-4205.03	-3252.69	-3181.53
2	-4199.68	-3251.89	-3181.87
3	-4194.62	-3249.97	-3181.26
4	-4191.24	-3250.69	-3183.28
5	-4190.94	-3253.93	-3186.92
6	-4196.21	-3257.28	-3190.40

Table 6. Estimated parameters of the linear two-factor model
(US macroeconomic monthly data, 1959:1-1998:12)

Parameter	Estimated	St. error	p-value
γ_{12}	5.96	2.08	0.0
γ_{13}	4.70	1.70	0.0
γ_{25}	0.86	0.06	0.0
γ_{26}	0.99	0.06	0.0
γ_{27}	0.67	0.05	0.0
ϕ_L	0.92	0.02	0.0
ϕ_C	0.49	0.05	0.0
$\phi_{CL,3}$	1.30	0.50	0.01
ψ_1	0.24	0.04	0.0
ψ_2	0.65	0.08	0.0
ψ_3	0.97	0.01	0.0
ψ_4	0.09	0.08	0.13
ψ_5	-0.06	0.06	0.19
ψ_6	-0.002	0.03	0.47
ψ_7	-0.33	0.05	0.0
σ_1^2	0.91	0.06	0.0
σ_2^2	0.08	0.01	0.0
σ_3^2	0.02	0.01	0.0
σ_4^2	0.35	0.03	0.0
σ_5^2	0.53	0.04	0.0
σ_6^2	0.37	0.04	0.0
σ_7^2	0.61	0.04	0.0
σ_L^2	0.004	0.002	0.08
σ_C^2	0.37	0.04	0.0

Table 7. Estimated parameters of the regime-switching two-factor model
(US macroeconomic monthly data, 1959:1-1998:12)

Parameter	Estimated	St. error	p-value
p_{11}	0.99	0.01	0.0
p_{22}	0.93	0.03	0.0
μ_{L1}	0.06	0.02	0.0
μ_{L2}	-0.39	0.10	0.0
γ_{12}	4.73	1.22	0.0
γ_{13}	4.57	1.18	0.0
γ_{25}	0.92	0.08	0.0
γ_{26}	1.15	0.08	0.0
γ_{27}	0.78	0.06	0.0
$\phi_{C,1}$	0.43	0.08	0.0
$\phi_{C,2}$	0.02	0.10	0.41
$\phi_{CL,3}$	1.05	0.33	0.0
ψ_{41}	0.10	0.05	0.01
ψ_{42}	0.45	0.05	0.0
ψ_{51}	-0.02	0.09	0.42
ψ_{52}	0.04	0.05	0.24
ψ_{61}	-0.06	0.10	0.29
ψ_{62}	-0.09	0.07	0.11
ψ_{71}	-0.42	0.05	0.0
ψ_{72}	-0.21	0.05	0.0
σ_1^2	0.96	0.06	0.0
σ_2^2	0.22	0.03	0.0
σ_3^2	0.27	0.03	0.0
σ_4^2	0.31	0.03	0.0
σ_5^2	0.56	0.04	0.0
σ_6^2	0.32	0.03	0.0
σ_7^2	0.55	0.04	0.0
σ_L^2	0.01	0.01	0.03
σ_C^2	0.32	0.04	0.0

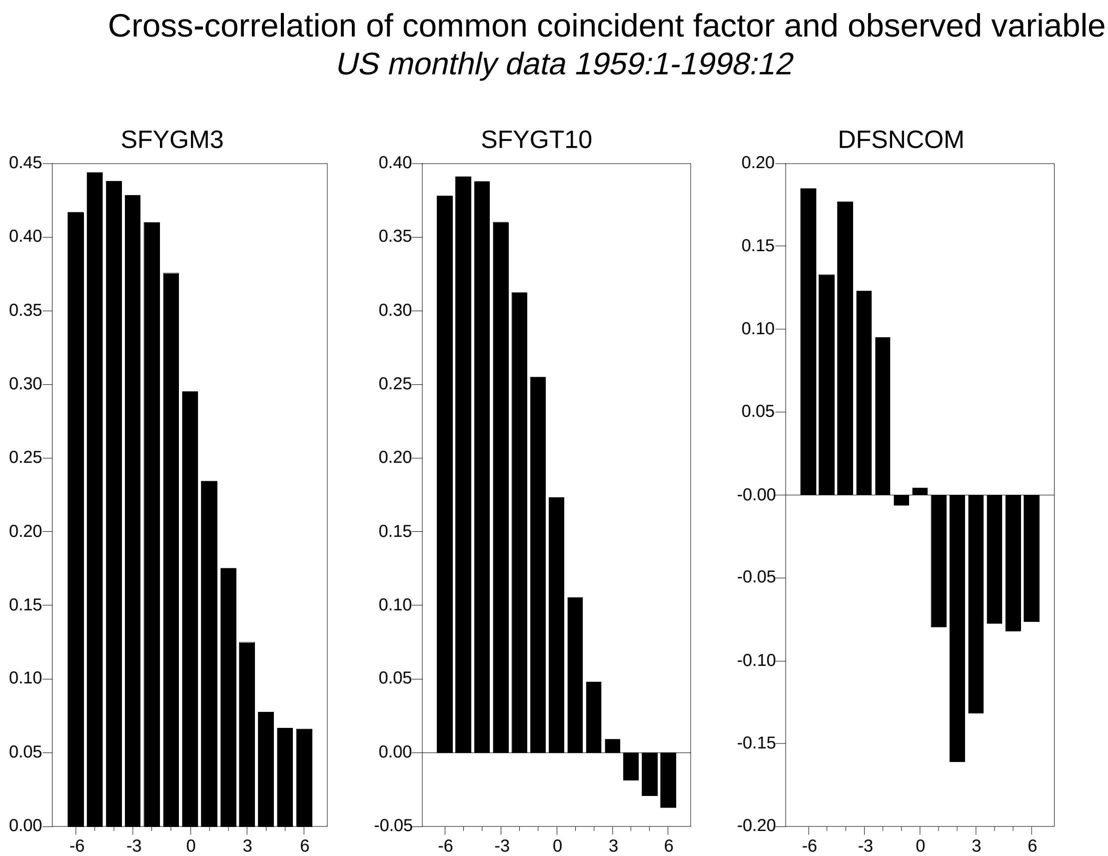

Cross-correlation of common coincident factor and observed variable
US monthly data 1959:1-1998:12
SFYGM3
SFYGT10
DFSNCOM
0.45
0.40
0.35
0.30
0.25
0.20
0.15
0.10
0.05
0.00
-0.05
-0.10
-0.15
-0.20
-6
-3
0
3
6

Figure 1:

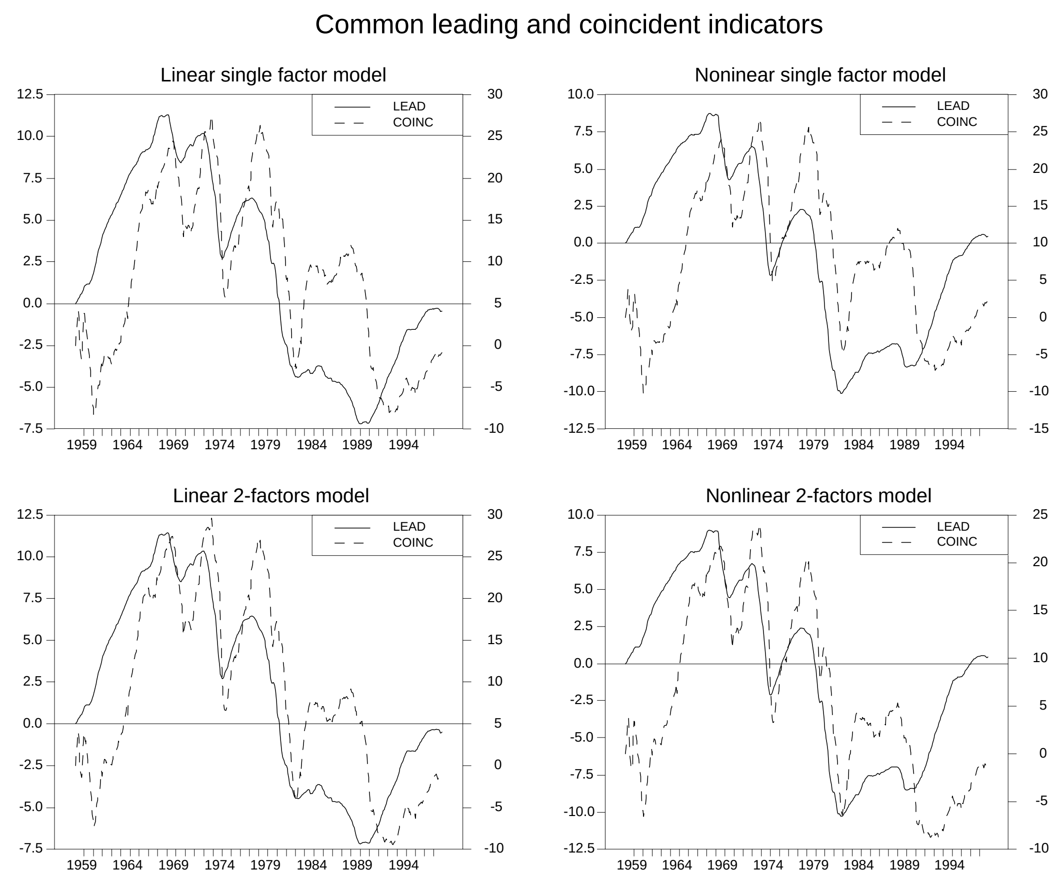

Common leading and coincident indicators
Linear single factor model
Noninear single factor model
Linear 2-factors model
Nonlinear 2-factors model
LEAD
COINC
1959 1964 1969 1974 1979 1984 1989 1994

Figure 2:

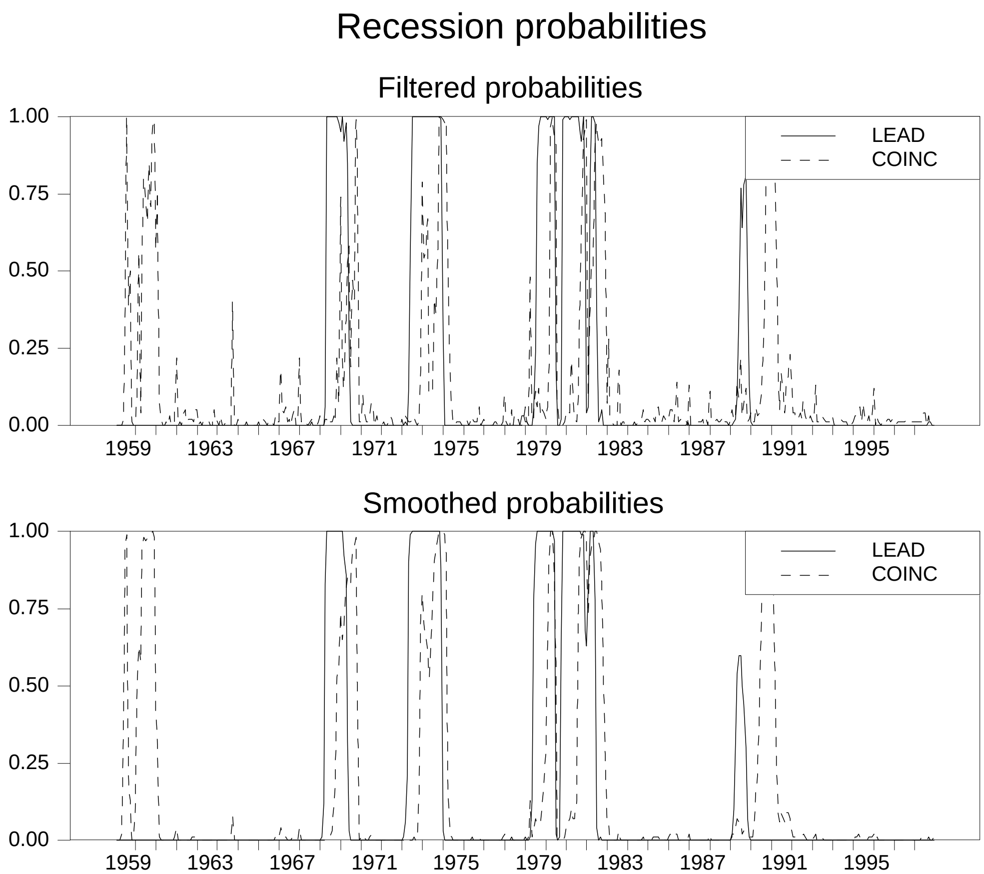
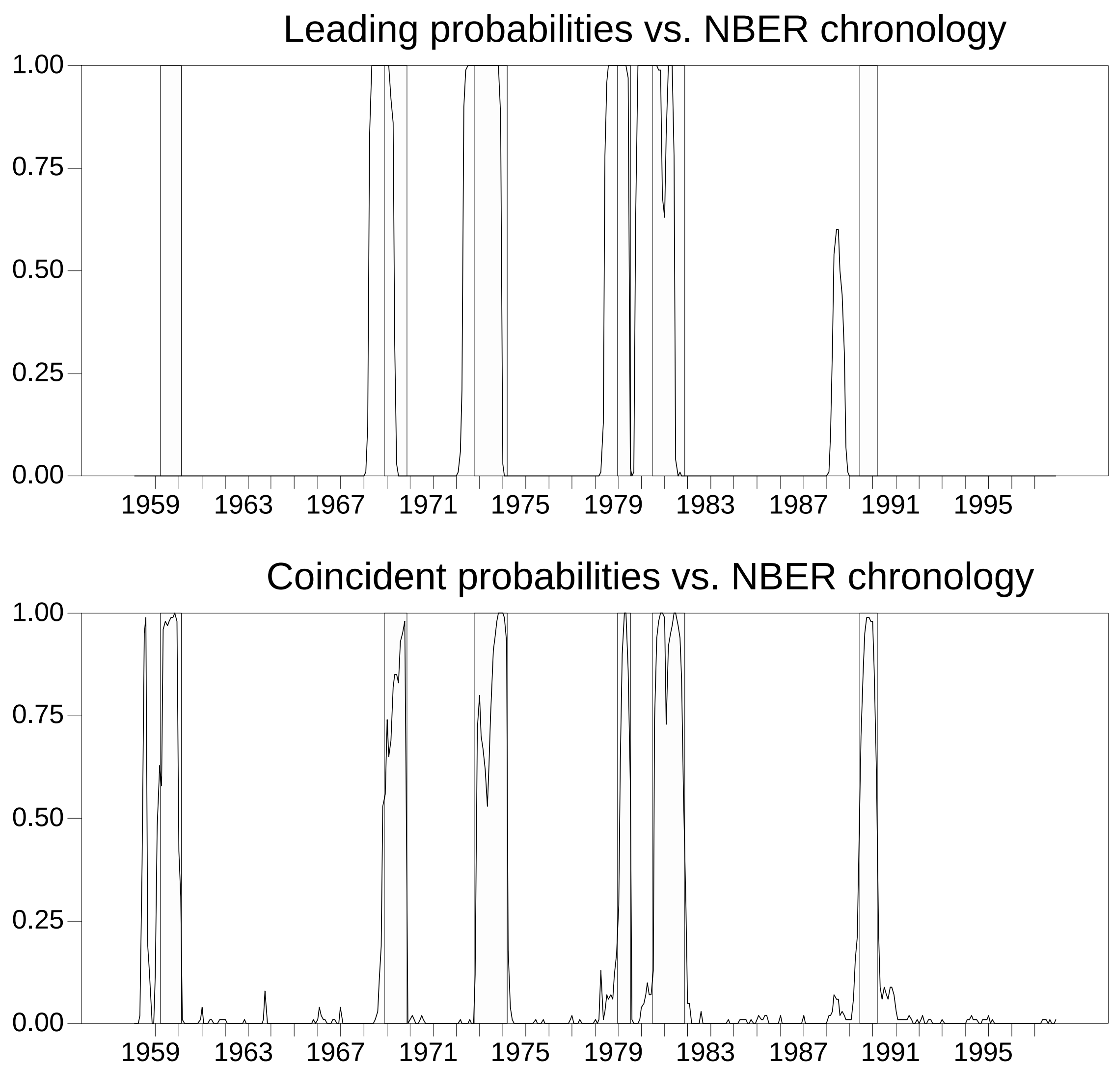

Recession probabilities
Filtered probabilities
1.00
0.75
0.50
0.25
0.00
LEAD
COINC
1959
1963
1967
1971
1975
1979
1983
1987
1991
1995
Smoothed probabilities
1.00
0.75
0.50
0.25
0.00
LEAD
COINC
1959
1963
1967
1971
1975
1979
1983
1987
1991
1995

Figure 3:



Leading probabilities vs. NBER chronology
1.00
0.75
0.50
0.25
0.00
1959
1963
1967
1971
1975
1979
1983
1987
1991
1995
Coincident probabilities vs. NBER chronology
1.00
0.75
0.50
0.25
0.00
1959
1963
1967
1971
1975
1979
1983
1987
1991
1995

Figure 4:

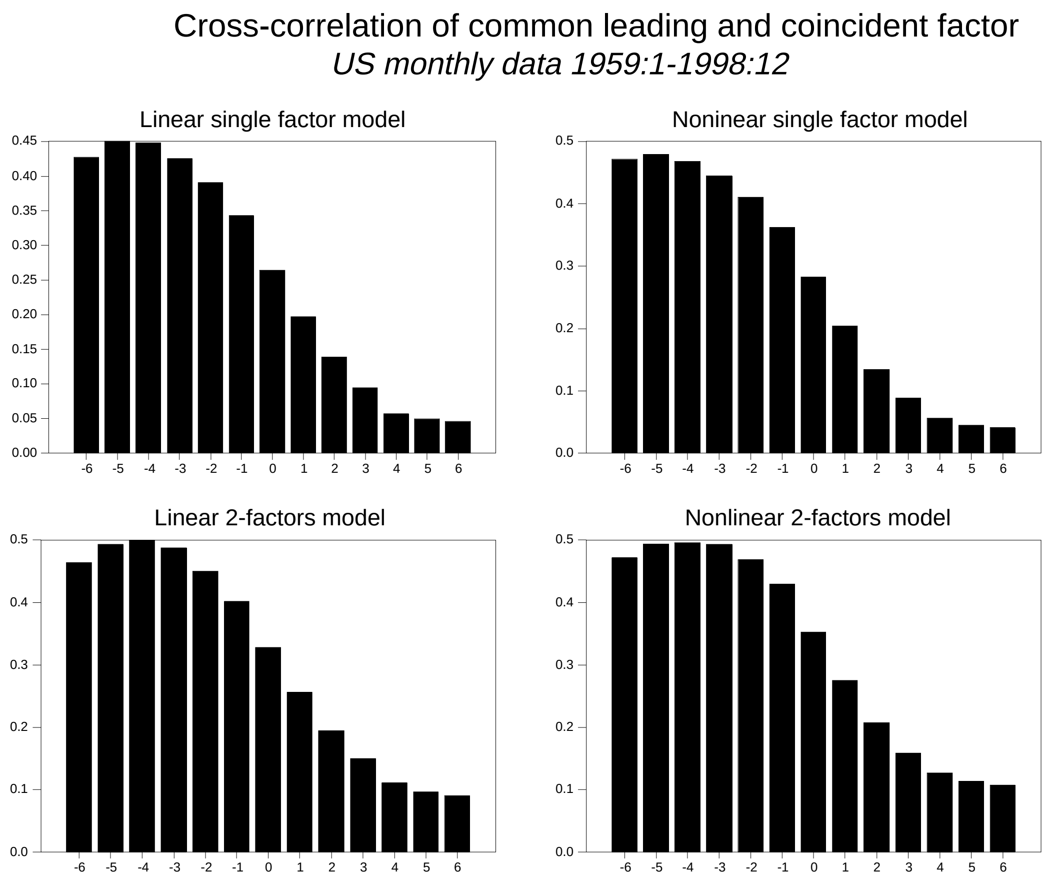

Cross-correlation of common leading and coincident factor
US monthly data 1959:1-1998:12
Linear single factor model
Noninear single factor model
Linear 2-factors model
Nonlinear 2-factors model

Figure 5: