Identifying and Forecasting the Turns of the Japanese Business Cycle

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Abstract

In this paper we identify and try to predict the turning points of the Japanese business cycle. As a measure of the business cycle we use a composite economic indicator (CEI). This indicator is endowed with nonlinear dynamics to capture the asymmetries between different cyclical phases. Two types of nonlinear dynamics are considered: Markov switching and smooth transition autoregression (STAR). The performance of these models in terms of forecasting the business cycle turns is compared. Both types of models produce statistically equivalent in-sample forecasting results, whilst the CEI with exponential STAR tends to outperform the CEI with Markov-switching and logistic STAR in the out-of-sample prediction.

Keywords: composite economic indicator, Markov switching, smooth transition autoregression, turning points, reference cycle, forecasting
JEL classification: C4, C5.

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1 Introduction

The crucial question many economic agents face is how to detect and predict timely the shifts between the cyclical phases. Possessing such a knowledge they would be able to adjust their activities in advance to the ups and downs of the business cycle. In order to address this question we suggest a composite economic indicator (CEI) which is constructed using the nonlinear dynamic factor analysis.

The CEI provided with the nonlinear dynamics permits detecting the business cycle turns and, moreover, predicting the turning points, which is impossible when the *ad hoc* techniques like a very popular Bry-Boschan method — see Bry & Boschan (1971) are used.

The CEI with Markov switching (CF-MS) became already quite a standard tool of forecasting the business cycle dynamics. It was applied to the US data by Chauvet (1998) and Kim & Nelson (1999), to the data of several European economies by Kaufmann (2000), and to the Brazilian data by Chauvet (2001). In this paper we propose another nonlinear model: CEI with smooth transition autoregressive dynamics (CF-STAR). It can be useful in the situations where CF-MS is not supported by the data, while the hypothesis of the STAR dynamics cannot be rejected.

In section 2 we set up the linear dynamic factor model and three nonlinear models. In section 3 these models are estimated, whereas in section 4 their forecasting performance is evaluated with respect to the Japanese reference cycle dates. Section 5 concludes the paper. All the tables are in Appendix.

2 Models

The models of CEI introduced in this section embody two key characteristics of the business cycle: comovement of different time series across the cycle and the asymmetric behavior of the economy at different cyclical phases. While the linear model captures only first of these features, the nonlinear models incorporate both characteristics.

The nonlinear models rely on the idea that the economic conditions evolve through a limited number of alternating and recurrent regimes, or states. In the simplest case we distinguish between two regimes: expansion and recession.

The economy behaves asymmetrically under the different regimes. This translates into the CEI model's parameters (for example, the growth rates and the volatility) depending on the state.

The nonlinear models below, though coincide in assuming the existence of qualitatively different regimes, differ in the way of modelling the transition between these regimes. In the Markov switching model the shifts between the regimes depend on unobserved state variable, whilst in the STAR model these shifts are conditioned on the past history of some observed variable.

2.1 Linear dynamic common factor

The linear dynamic factor model as a tool of constructing the composite economic indicator was adopted by Stock & Watson (1989). We will denote it simply as CF. It is expressed as follows:

$$\Delta y_t = \gamma_i(L)\Delta C_t + u_t \tag{1}$$

$$\Delta C_t = \mu + \sum_{i=1}^{p} \phi_i \Delta C_{t-i} + \sigma_\varepsilon \varepsilon_t \tag{2}$$

$$\Psi(L)u_t = \Sigma_\eta \eta_t \tag{3}$$

where Δ is the first-order difference operator; y_t is the $n \times 1$ vector of the observable time series; C_t is the dynamic common factor in levels; u_t is the $n \times 1$ vector of the idiosyncratic components; ε_t are the normally distributed residuals with unit variance and zero mean; μ and ϕ_i $(i = 1, 2, ..., p)$ are the common factor's state-dependent intercepts and autoregressive coefficients, respectively.

The lag polynomial matrices of the specific factors, Ψ_j $(j = 1, ..., q)$, are diagonal.

Σ_η is a diagonal variance-covariance matrix. The shocks to the specific factors are assumed to be serially and mutually uncorrelated and normally distributed with unit variances and zero means: $\eta_t \sim NIID\,(O_n, I_n)$, where O_n is the $n \times 1$ vector of zeros and I_n is the $n \times n$ identity matrix.

Here we estimate the linear model without intercept. This is admissible because the data are previously demeaned and standardized.

2.2 Dynamic common factor with regime switching

The linear dynamic factor model of Stock & Watson was extended to the Markov switching (CF-MS) case by Kim (1994) and Kim & Nelson (1999). Equations (1) and (3) are kept unchanged, whereas the state-dependent terms are introduced into equation (2) which as result looks like:

$$\Delta C_t = \mu_1^{MS} s_t + \mu_2^{MS}(1 - s_t) + \sum_{i=1}^{p} \left[\phi_{1i}^{MS} s_t + \phi_{2i}^{MS}(1 - s_t) \right] \Delta C_{t-i} + \\ + \left[\sigma_{\varepsilon 1} s_t + \sigma_{\varepsilon 2}(1 - s_t) \right] \varepsilon_t \qquad (4)$$

where y_t is the $n \times 1$ vector of the observable time series; C_t is the dynamic common factor in levels; ε_t are the normally distributed residuals with unit variance and zero mean; s_t is the regime variable taking m values, where m is the number of the regimes; μ_j^{MS} and ϕ_{ji}^{MS} ($j = 1, 2$ and $i = 1, 2, ..., p$) are the common factor's state-dependent intercepts and autoregressive coefficients, respectively. Thus, for $m = 2$, $s_t = 1, 0$. Given that $\mu_1^{MS} > \mu_2^{MS}$, regimes 1 and 2 may be interpreted as an ascending trend and a descending trend states, respectively. In this model the intercept term, μ_i^{MS}, and the residual variance of the common factor, $\sigma_{\varepsilon j}^2$ ($j = 1, 2$), are state-dependent, i.e., they are different for the different regimes.

The transition probabilities, $p_{ij} = \Pr(s_t = j | s_{t-1} = i)$, sum up to one when added across all the possible states for the given regime in the previous period: $\sum_{j=1}^{m} p_{ij} = 1 \ \forall i$ for m states.

2.3 Dynamic common factor with smooth transition autoregression

The novelty of this paper is the application of STAR to the unobserved common factor model. The technique itself as applied to the observed univariate time series was developed by Chan & Tong (1986) as well as by Teräsvirta and his coauthors (e.g. Granger & Teräsvirta (1993)).

The common factor model with smooth transition autoregression (CF-STAR) is apparently very similar to its counterpart with regime switching. However, there is a crucial difference between the two approaches: while in CF-MS the state variable determining shifts from one regime to another is unobserved, in CF-STAR the switches between regimes are conditioned upon the past values of the composite indicator itself or upon those of some other *observed* variable. In the present case the situation is complicated by the fact that we do not observe the CEI itself. Hence we should condition the changes in regimes on its past *estimated* values.

The only difference between the two models is the equation describing evolution of the common dynamic factor:

$$\Delta C_t = \mu_1^{STAR} F_t + \mu_2^{STAR}(1 - F_t) + \sum_{i=1}^{p} \left[\phi_{1i}^{STAR} F_t + \phi_{2i}^{STAR}(1 - F_t) \right] \Delta C_{t-i} + \\ + \left[\sigma_{\varepsilon 1} F_t + \sigma_{\varepsilon 2}(1 - F_t) \right] \varepsilon_t \quad (5)$$

where $\mu_1^{STAR} > \mu_2^{STAR}$ are the state-dependent intercepts; ϕ_{ji}^{STAR} ($j = 1, 2$ and $i = 1, 2, ..., p$) are the state-dependent autoregressive coefficients; $F_t \equiv F_t(\Delta C_{t-d}; \lambda, r)$ is some smooth transition function. In the present study we are using two specifications of the transition function. First, the logistic specification (denoted as CF-LSTAR) which allows capturing the asymmetries between the business cycle phases:

$$F_t(\Delta C_{t-d}; \lambda, r) = \frac{1}{1 + \exp(-\lambda(\Delta C_{t-d} - r))} \quad (6)$$

where $\lambda > 0$ is the parameter determining the abruptness of transition (the greater is its value the sharper are the switches between the regimes); ΔC_{t-d} is playing the role of the so-called transition variable; $d > 0$ is called the transition delay; r is the transition threshold. The shifts between the two different regimes (say, high growth and low growth) depend on deviation between the past CEI's growth rate and some threshold, r. If, for instance, the past common factor's growth rate exceeded the threshold, the high growth regime becomes more probable.

Second, the exponential specification (denoted as CF-ESTAR) of the transition function:

$$F_t(\Delta C_{t-d}; \lambda, r) = 1 - \exp(-\lambda(\Delta C_{t-d} - r)^2) \quad (7)$$

Again as in the CF-MS case, the residual variances of the specific factors are mutually and serially uncorrelated and normally distributed with unit variance and zero mean.

3 Estimation

The Japanese composite economic indicators were constructed using five monthly macroeconomic time series starting in January 1973 and ending in January 2003. In fact, these are the series corresponding to the "type 1 dataset" of Fukuda and Onodera (2001) who have estimated a Japanese CEI using the linear dynamic factor model. Table 1 lists the series and gives their description.

As a benchmark the linear CF model was used. We started with determining the optimal lag structure of this benchmark model, that is, the order of the autoregressive polynomials of the common and specific factors. The Akaike (AIC) and Schwartz (SBIC) information criteria were applied. The log-likelihood values of the linear CF with different orders of autoregressive polynomials of the common and specific factors together with the corresponding Akaike and Schwartz quantities are presented in Table 2. The AIC and SBIC come up with optimal combinations (1,3) and (1,2), respectively. We chose the combination (1,2) as more parsimonious. It corresponds to the common factor following AR(1) and the specific factors following AR(2).

Next, we have tested the common factor dynamics for linearity. The alternative was the STAR-type nonlinearity. The LM-type tests based on the third- and fourth-order Taylor expansion of the STAR transition function around $\lambda = 0$ were conducted as in van Dijk et al. (2000). To conduct these tests the estimated values of the common factor, obtained from the linear CF(1,2) model, were used.

The third-order Taylor expansion of the logistic transition function results in:

$$\begin{aligned}\Delta\hat{C}_t = \mu_1 + \sum_{i=1}^{p}\phi_{1i}\Delta\hat{C}_{t-i} + \sum_{i=1}^{p}\phi_{2i}\Delta\hat{C}_{t-i}\Delta\hat{C}_{t-d} + \\ + \sum_{i=1}^{p}\phi_{3i}\Delta\hat{C}_{t-i}\Delta\hat{C}^2_{t-d} + \sum_{i=1}^{p}\phi_{4i}\Delta\hat{C}_{t-i}\Delta\hat{C}^3_{t-d}\end{aligned} \tag{8}$$

where $\Delta\hat{C}_t$ is the linear estimate of the growth rate of the common factor.

Under this condition the null hypothesis (linear CF) is as follows: $\phi_{j1} = \phi_{j2} = ... = \phi_{jp} = 0$ $(j = 2, 3, 4)$. This hypothesis can be tested with F-statistic denoted as LM_3.

In order to select between the logistic and exponential STAR specifications we use the test proposed by Escribano & Jordá (2001). This requires estimating the fourth-order Taylor approximation:

$$\Delta\hat{C}_t = \mu_1 + \sum_{i=1}^{p}\phi_{1i}\Delta\hat{C}_{t-i} + \sum_{i=1}^{p}\phi_{2i}\Delta\hat{C}_{t-i}\Delta\hat{C}_{t-d} + \sum_{i=1}^{p}\phi_{3i}\Delta\hat{C}_{t-i}\Delta\hat{C}^2_{t-d} + \\ + \sum_{i=1}^{p}\phi_{4i}\Delta\hat{C}_{t-i}\Delta\hat{C}^3_{t-d} + \sum_{i=1}^{p}\phi_{5i}\Delta\hat{C}_{t-i}\Delta\hat{C}^4_{t-d} \quad (9)$$

The null of linearity is $\phi_{j1} = \phi_{j2} = ... = \phi_{jp} = 0$ $(j = 2, 3, 4, 5)$. The corresponding test statistic has an F-distribution and is labelled as LM_4.

The p-values of the tests are reported in Table 3. The null of linearity is rejected at 5% significance level for the delay $d = 4$. In other words, the STAR nonlinearity can be accepted when the transition variable is $\Delta\hat{C}_{t-4}$. This circumstance was used to specify the CF-STAR model. The choice between LSTAR and ESTAR depends on two statistics introduced by Escribano & Jordá (2001) and denoted by LML and LME. We will not reproduce the corresponding formulae here referring the reader instead to Escribano & Jordá (2001). The decision rule is: if the minimum p-value corresponds to LME, select LSTAR, otherwise select ESTAR. The Japanese data support the hypothesis of the exponential STAR dynamics. Nevertheless, we estimate both models for the sake of comparison of their turning points forecasting performance.

Before estimation the data were normalized by subtracting the mean and dividing the resulting series by the raw series' standard deviation.

In all the nonlinear models the common factor was specified as AR(1), whereas the specific factors as AR(2). In CF-MS only common factor's intercept is supposed to be state-dependent. In CF-LSTAR and CF-ESTAR both the common factor's intercept and its residual variance are assumed to be state-dependent. The reason why we do not make the CF-MS residual variance state-dependent is because in this case the model is detecting not the recession – expansion regimes but the low volatility – high volatility regimes capturing the long-term volatility decline which started in 1970s. This result we obtained when we tried to estimate the CF-MS model with both intercept and residual variance being state dependent.

In both CF-STAR models the exponent of the transition function, $F_t(\Delta C_{t-d}; \lambda, r)$, was standardized by division by the common factor's residual variance, σ^2_ε, to make the abruptness parameter λ scale-free and easier to interpret, as suggested by Skalin & Teräsvirta (1999).

Standardized logistic transition function:

$$F_t(\Delta C_{t-d}; \lambda, r) = \frac{1}{1 + \exp(-\lambda(\Delta C_{t-d} - r)/\sigma_\varepsilon)} \tag{10}$$

Standardized exponential transition function:

$$F_t(\Delta C_{t-d}; \lambda, r) = 1 - \exp(-\lambda(\Delta C_{t-d} - r)^2/\sigma_\varepsilon^2) \tag{11}$$

Both models were estimated using the method of maximum likelihood. For more details on the maximum likelihood estimation of the CF-MS model see Kim and Nelson (1999). The procedure is easily extended to the CF-STAR case. The parameter estimates of the linear CF, both CF-STAR models, and CF-MS model (along with their standard errors in parentheses) are reported in Tables 4-7.

4 Evaluation

The forecasting ability of each of the models in question cannot be examined directly, since CEI is unobserved and hence we cannot test which of the models replicates it better. Therefore the performance of the two nonlinear models is evaluated from the viewpoint of capturing and forecasting the turning points of the business cycle. These turns are detected by the conditional low growth regime probabilities derived from each nonlinear model. In the CF-STAR models these probabilities are computed as $1 - F_t(\Delta C_{t-d}; \lambda, r)$, while in the CF-MS model these are the conditional filtered and smoothed probabilities.

In the Japanese case the issue of the reference cycle chronology is rather ambiguous. There is one "official" chronology published by the Economic and Social Research Institute (ESRI). The dating decisions made by ESRI are based on the aggregation (computing a percentage of the series in expansion) of the Bry-Boschan chronologies of 11 macroeconomic variables complemented by the expert judgements[1]. The variables used to construct the ESRI's reference chronology are listed in Table 8.

[1]The description of the ESRI's dating methodology was kindly provided by the member of the Institute Yasuko Ikemoto.

There are also two chronologies (for the classical and growth cycles) provided by the Economic Cycle Research Institute (ECRI). The ECRI claims that its classical cycle chronology is constructed in a similar way to the NBER dating of the US business cycle. The primary indicators used by ECRI to establish the Japanese classical cycle chronology (in line with the NBER approach) are listed in Table 8. For each of these series a set of turning points is defined, and the "clusters" of turns for all the series serve as the base for establishing the overall chronology. This dating is double-checked using the quarterly real GDP and the unemployment rate, which tends to lag at troughs. The ECRI's Japanese Coincident Index is also used as a guide in case of any ambiguity[2].

We are going to use both the ECRI's classical cycle dates for Japan and the ESRI's turning points dates as the alternative Japanese reference cycle chronologies. While coinciding in some instances with the ECRI's classical cycle chronology, the latter discovers additional cycles. The dating approaches of the ECRI's classical cycle and of the ESRI's cycle seem to be pretty similar and their differences are caused mainly by the differences in their composition.

Both chronologies are shown in Table 9. In addition, we have simulated the ESRI's chronology up to January 2003 (columns 5-6 of Table 9), since the official ESRI's dates are only available until December 2001. In order to simulate the reference dates ten out of eleven component series were used (except for the operating income which is a quarterly series). To each of these series the Bry-Boschan algorithm (implemented as a GAUSS code by M.Watson) was applied. A diffusion index was constructed as a percentage of the series in expansion. Finally, the simulated ESRI's reference chronology was computed using the following rule: expansion when the diffusion index exceeds 0.5, recession otherwise. The short contraction of May-September 1995 was detected when simulating the ESRI's procedure. However, we do not accept it, because it does not satisfy the minimum phase duration criterion (6 months) imposed in the Bry-Boschan algorithm.

The ECRI's chronology uncovers four cycles, where the last one starting in August 2000 is incomplete. Both the official and simulated ESRI datings report seven cycles where the last commences in October 2000 and is incomplete (official chronology) or starts in August 2000 and ends in January 2002 (simulated chronology). The ESRI produces much more signals than the

[2]This information has been provided to us by the ECRI member Anirvan Banerji.

ECRI classical cycle chronology. The four ECRI cycles coincide, although not perfectly — the average absolute difference for the peaks being 4.25 months and for troughs being 3.33 months — with four of the seven ESRI cycles. The differences between the official and simulated ESRI's chronologies are much less important: on average 2.14 months for peaks and 0.5 months for troughs.

Figure 1 compares the three nonlinear models, on the one hand, with the linear CF model, on the other hand, in terms of the behavior of the common factor in levels. It is constructed as a partial sum of the common factor's growth rates, $\Delta\hat{C}_t$.

The profiles of the composite economic indicators constructed using the CF-MS and CF-STAR are quite similar to that of the CEI estimated using the linear model. Apart from the differences in the levels which are easily explained by the nonstationary nature of $\hat{C}_t$, given the way it is constructed, the upward and downward movements of the linear indicator are readily replicated by those of the nonlinear models.

The ECRI recessions depicted on Figure 1 by the shadowed areas correspond to the periods of the prolonged declines of the CEIs. There are four episodes of short downswings (1977, 1980, 1982, and 1986) of the CEIs which are not coupled with the ECRI's contractions. Another episode where the CEIs and the ECRI's chronology diverge is the recession of 1991-1993 where the CEIs fell down well in advance of the outbreak of the ECRI's contraction.

If we inspect the ESRI's chronology, however, we will see that it recognizes the recessions of 1977, 1980-1982 (a single longer contraction instead of two shorter ones as suggested by the CEIs), and 1986. It also signals the start of the 1991-1993 recessions more than one year earlier than it does ECRI.

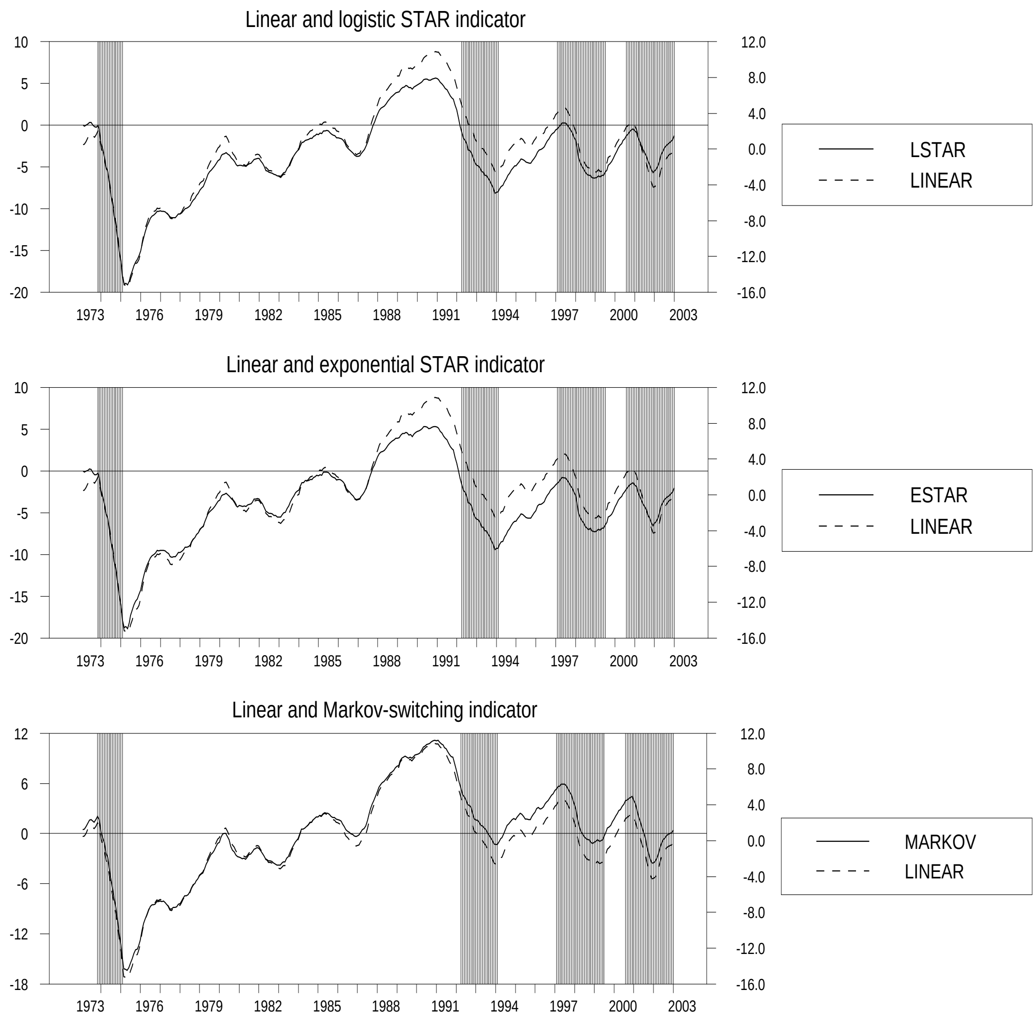
In sum, the CEIs' profiles seem to conform better to the ESRI's dating than to that of the ECRI.

The informal judgement about the "goodness-of-fit" of these models can be made from the visual inspection of Figures 2-3 displaying the positive growth regime probabilities derived from the CF-STAR and CF-MS, on the one hand, and the ECRI's business cycle dating, on the other hand. The shaded areas correspond to the ECRI's recessions. In the case of CF-MS model we dispose of the filtered and smoothed regime probabilities. The CF-STAR regime probabilities and the CF-MS filtered regime probabilities are the most volatile. Anyway, all the regime probabilities seem to sufficiently accurately recognize the ECRI dates.

Figure 2 displays the negative growth regime probabilities derived from

Figure 1: The profiles of Japanese nonlinear composite economic indicators

1973:1-2003:1



the CF-LSTAR and CF-ESTAR and plotted against the ECRI's dates represented by the shadowed areas.

The CF-ESTAR model appears to produce less false alarms than CF-LSTAR. The CF-ESTAR derived low regime probabilities are less volatile than those of the logistic model. CF-ESTAR correctly detects four true recessions and does not signal any false recession, while CF-LSTAR comes up with four true and two false contractions. Interestingly, these two "false" contractions are false from the point of view of ECRI but are true from the standpoint of ESRI.

Both models announce the beginning of the 1991-1993 recession a few months earlier than the ECRI dating but at about the same moment as the ESRI's dating. We have noticed this already when examining the profiles of the linear and nonlinear CEIs. The last two completed recessions detected by both models (1992-1993 and 1997-1998) are significantly shorter than the reference cycle contractions.

The low growth regime (filtered and smoothed) probabilities corresponding to the CF-MS are graphed on Figure 3.

Again the shadowed areas correspond to the ECRI's chronology. The filtered conditional probabilities of CF-MS model detect four true signals and give one false signal in 1980, while the smoothed probabilities capture all the true signals without any false alarms.

The CF-MS and CF-STAR conditional probabilities are relatively close to each other, the only important difference being the recession of 1991-1993 which is much shorter according to the CF-MS model.

The formal analysis of both in-sample and out-of-sample performance of CF-STAR and CF-MS was undertaken using the quadratic probability score (QPS) suggested by Diebold & Rudebusch (1989). This method compares the recession probabilities derived from some model to a generally accepted business cycle dating.

The QPS is defined as in Layton & Katsuura (2001):

$$QPS = \frac{1}{T}\sum_{t=1}^{T}(P_t - D_t)^2 \qquad (12)$$

where T is the number of observations; P_t is the model-derived probability; D_t is the binary variable taking value of 1 during the reference cycle recessions and 0 during the reference cycle expansions. QPS is limited within the interval [0,1]. The smaller is QPS the better is the correspondence between

Figure 2: Japanese CF-STAR low growth regime probabilities vs. ECRI dates

1973:1-2003:1

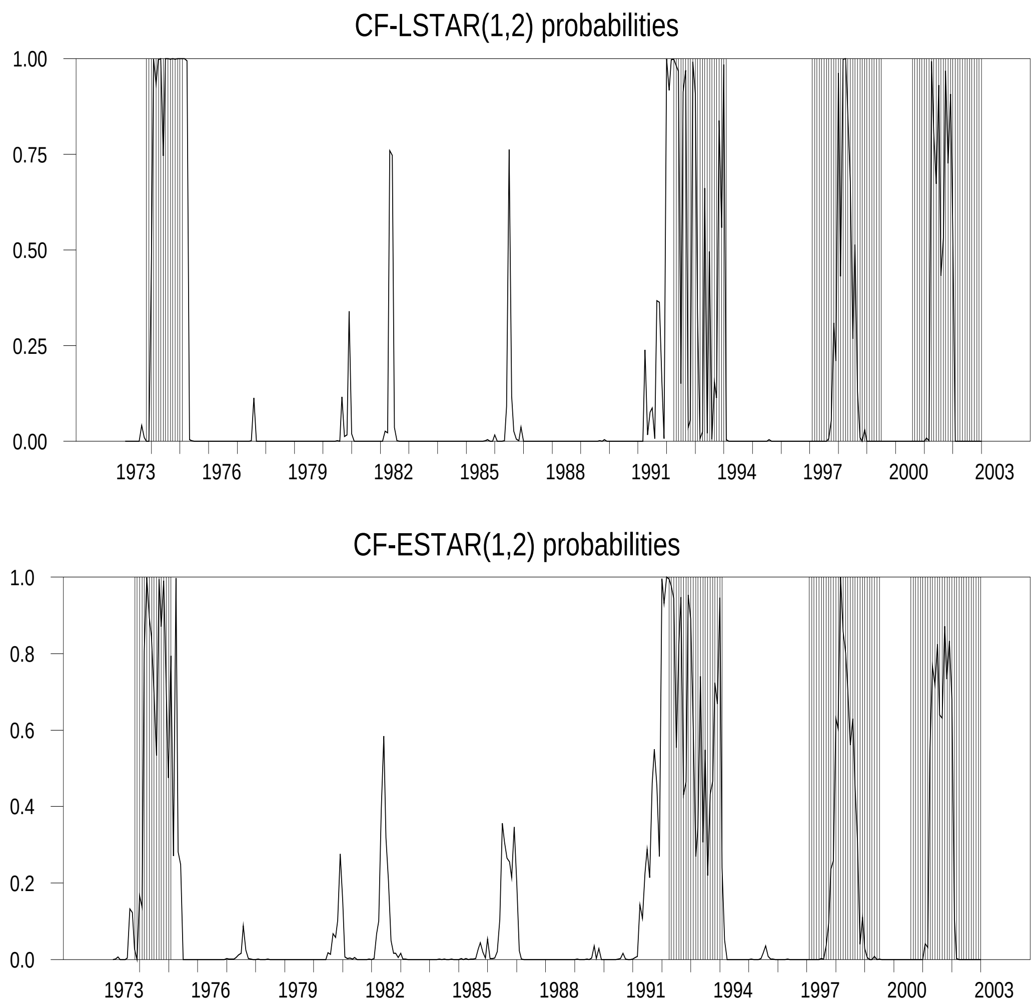
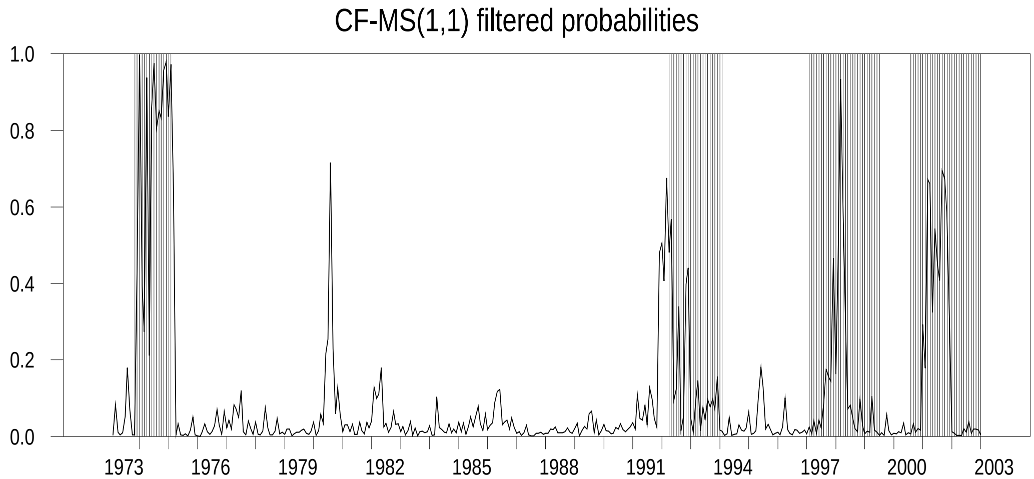
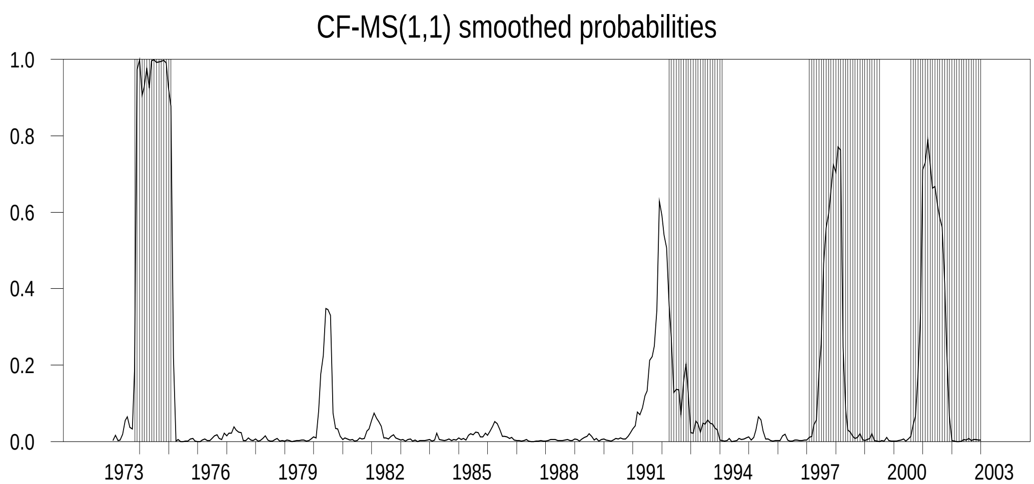


Figure 3: Japanese CF-MS low growth regime probabilities vs. ECRI dates

1973:1-2002:7





the model-derived probabilities and the "official" business cycle chronology.

To test whether the differences in the QPS of different models are statistically significant we use the Diebold-Mariano statistic proposed by Diebold & Mariano (1994).

For the in-sample evaluation we used the conditional recession probabilities — filtered probabilities Pr(low growth regime in period $t|I_t$) and smoothed probabilities Pr(low growth regime in period $t|I_T$) in CF-MS, or Pr(low growth regime in period $t|\Delta C_{t-1}$) in CF-STAR — estimated using the whole sample. As the reference cycle dates we use both the ECRI and ESRI chronologies. Here $I_t = \{\Delta C_t, \Delta C_{t-1}, ..., \Delta C_1\}$ is the information set consisting of the whole history of the CEI up to the period t. However, given that the official ESRI dates were not updated since the December 2001, to compute the QPS we used the probabilities over 1973:1-2001:12.

The results of the comparison of in-sample forecasting performance of the three nonlinear models are presented in Table 10. The columns 2 and 3 contain the point estimates of the QPS with respect to the ECRI and ESRI dates, correspondingly. In the columns 4-6 the upper diagonal submatrix reports the DM-statistic for ECRI chronology, while the lower diagonal matrix reports DM-statistic for ESRI dating. The asterisks denote the statistically significant differences in predicting accuracy. The DM-statistic is computed by comparing the loss differentials (with respect to a binary coded reference dating) of the regime probabilities of one nonlinear model to the loss differentials of another nonlinear model.

The ranking of different forecasting models, according to their point estimates of QPS, regardless of the reference cycle chronology, is as follows: the CF-ESTAR probabilities, smoothed conditional probabilities of CF-MS and CF-LSTAR probabilities, and finally the filtered probabilities of CF-MS. However, when the confidence intervals are taken into account, the in-sample performance of the filtered low growth regime probabilities derived from all the models proves to be statistically equal. CF-ESTAR in-sample prediction is better than that of CF-LSTAR at 10% significance level with respect to the ESRI dating.

To compare the out-of-sample forecasting accuracy of the three nonlinear models, the predictions with forecasting horizons ranging from 1 month to 6 months were made. The period for out-of-sample forecasting exercise was: 1998:1-2001:12.

The algorithm of the out-of-sample forecasting was the following. First, each model was estimated for the subsample 1973:1-1997:7 and the 1-, 2-,

..., 6-month ahead forecasts were made. Next, the estimation subsample was augmented by one month and the forecasts were computed. The forecasting procedure was repeated until 2001:11 was reached.

The regime probabilities of the CF-MS model were predicted using the forecasting formula from Hamilton (1994, p. 694). The CF-STAR regime probabilities were computed using the following two-step procedure:

$$\hat{F}_{T+1} \equiv F_{T+1}(\Delta\hat{C}_T; \hat{\lambda}, \hat{r}) = \frac{1}{1+\exp(-\hat{\lambda}(\Delta\hat{C}_T - \hat{r}))} \tag{13}$$

$$\begin{aligned} \Delta\hat{C}_{T+1} = {} & \hat{\mu}_1^{STAR}\hat{F}_{T+1} + \hat{\mu}_2^{STAR}(1-\hat{F}_{T+1}) + \\ & + \sum_{i=1}^{p}\left[\hat{\phi}_{1i}^{STAR}\hat{F}_t + \hat{\phi}_{2i}^{STAR}(1-\hat{F}_t)\right]\Delta\hat{C}_T \end{aligned} \tag{14}$$

where the parameters and variables with hats are those estimated for the period from *1* to *T*. Based on these data the forecasts are made for the period covering h following months, that is, $[T+1, T+h]$.

In addition to the "standard" DM-test of the differences in forecasting accuracy, the modified DM-test suggested by Harvey et al. (1997) was applied. This test is especially designed to compare the out-of-sample prediction records. As its authors claim, it is less over-sized than the standard DM-test which tends to over-reject the null hypothesis of no difference in forecasting accuracy of two models being compared. The modified DM-test (DM^*) is related to the standard one (DM) in a following way:

$$DM^* = DM\left(\frac{T+1-2h+T^{-1}h(h-1)}{T}\right)^{1/2} \tag{15}$$

where T is the sample size; h is the forecasting horizon. Harvey et al. (1997) report that the best results are obtained when the critical values of the Student's t rather than standard normal distribution are employed. Here we follow their recommendation when computing the p-values of modified DM-test.

The results of testing the out-of-sample forecasting accuracy are reported in Table 11a for the ECRI's reference chronology and in Table 11b for the ESRI's reference dating. The second column contains the point estimates of QPS. In the columns 3-5 two submatrices are displayed: the upper diagonal submatrix reports the DM-statistic, while the lower diagonal matrix reports the modified DM-statistic. The asterisks denote the statistically significant differences in predicting accuracy.

Arithmetically under both reference cycle chronologies CF-ESTAR dominates both CF-MS and CF-LSTAR over all forecasting horizons. Moreover, this dominance is statistically significant. Under ECRI's dating CF-LSTAR outperforms CF-MS up to 5-month horizon after which they give statistically equal results, whereas under the ESRI's dating CF-LSTAR is always outperforming the CF-MS.

It should be stressed that at the moment we did these estimations the ESRI had not decided yet on the trough date of the latest contraction. On the one hand, it means that our comparisons of the three methods over the second subsample were subject to changes depending on the ESRI decision, although we tried to minimize this effect by choosing for the out-of-sample forecasting the period before December 2001. On the other hand, it is worth noting that all the three models showed a sharp decrease in the forecast recession probabilities about the end of 2001, thus implying that the economy is getting out of recession. In June 6, 2003 the ESRI finally released the new reference cycle chronology, according to which the preliminary date of the latest trough is January 2002. This coincides exactly with our simulated ESRI's chronology which signals a trough in January 2002 and confirms our prediction results.

5 Concluding remarks

In this paper we have considered three alternative nonlinear single-factor models of the composite economic indicator: a model with Markov switching and its two counterparts with smooth transition autoregression: CF-LSTAR and CF-ESTAR.

The empirical analysis of these three models was conducted based on the Japanese monthly coincident series. Both in-sample and out-of-sample turning points forecasting abilities of the models were compared using the quadratic probability score test: the model-derived datings were contrasted to the ECRI's and ESRI's reference cycle chronologies. In the in-sample forecasting all the models appear to give statistically equivalent results, while the leader of the out-of-sample forecasting is undoubtedly the CF-ESTAR model, which is immediately followed by the CF-LSTAR.

We came to more or less similar conclusions in Kholodilin (2002) where the US composite economic indicator is constructed using the same three nonlinear models. In that case CF-ESTAR dominated other two models up to 3-month out-of-sample forecasting horizon.

In addition, both CF-ESTAR and CF-LSTAR for the moment appear to be computationally less expensive than the common factor model with Markov switching. Hence it can be concluded that CF with smooth transition autoregressive dynamics, especially CF-ESTAR, is a reasonable alternative to the CF-MS model.

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6 Appendix

Table 1. The component series of Japanese composite economic indicator
Monthly data, 1973:1–2003:1

Series	Short-hand	Description
Index of industrial production (mining & manufacturing)	IP	1995=100, SA
Index of wholesale trade sales	WTS	% change
Effective job offer rate (job offers / applicants)	EJO	times
Index of non-scheduled worked hours (manufacturing)	NWN	2000=100
Large industrial power consumption	LIP	10^6 kWh

Source: Economic and Social Research Institute, Cabinet Office, Government of Japan (http://www.esri.cao.go.jp/en/stat/menu.html).

Table 2. Optimal lag structure of the linear common factor model
Japanese data 1973:1-2003:1

Comb	LogLik	AIC	SBIC
(0,0)	-2420.30	-4840.60	-4840.60
(0,1)	-2260.41	-4530.82	-4550.26
(0,2)	-2200.21	-4420.42	-4459.31
(0,3)	-2185.77	-4401.54	-4459.87
(1,0)	-2275.15	-4552.30	-4556.19
(1,1)	-2187.20	-4386.40	-4409.73
(1,2)	-2146.79	-4315.58	**-4358.36**
(1,3)	-2137.63	**-4307.26**	-4369.48
(2,0)	-2274.99	-4553.98	-4561.76
(2,1)	-2183.80	-4381.60	-4408.82
(2,2)	-2146.11	-4316.22	-4362.89
(2,3)	-2137.31	-4308.62	-4374.73
(3,0)	-2268.57	-4543.14	-4554.81
(3,1)	-2183.60	-4383.20	-4414.31
(3,2)	-2145.78	-4317.56	-4368.12
(3,3)	-2136.90	-4309.80	-4379.80

Note 1: Comb = lag combination; LogLik = value of the log-likelihood function; AIC = Akaike information criterion; SBIC = Schwartz Bayesian information criterion.
Note 2: Bold entries stand for the minima of the corresponding information criterion: (1,2) is the optimal lag combination according to SBIC, while (1,3) is the optimal lag combination according to AIC.

Table 3. Testing linearity against STAR dynamics
Japanese data 1973:1-2003:1

Delay	p-value			
	LM3	LM4	LML	LME
1	0.289	0.291	0.450	0.111
2	0.126	0.042	0.123	0.116
3	0.029	0.029	0.265	0.174
4	0.0	0.0	0.011	0.387
5	0.390	0.432	0.222	0.880
6	0.633	0.784	0.978	0.736

Note: Linearity tests: 3rd (LM3) and 4th order (LM4) Taylor approximation. Esribano-Jordá selection tests: the null of LSTAR dynamics (LML) and the null of ESTAR dynamics (LME)

Table 4. Estimated parameters of linear CF model
(Japanese macroeconomic monthly data, 1973:1-2003:1)
Log-likelihood: -2146.79

$$\Delta y_t = \begin{pmatrix} 1 \\ 0.564_{(0.08)} \\ 1.36_{(0.20)} \\ 1.67_{(0.13)} \\ 0.726_{(0.08)} \end{pmatrix} \Delta C_t + u_t$$

$$\Delta C_t = 0.854_{(0.04)} \Delta C_{t-1} + 0.056_{(0.01)} \varepsilon_t$$

$$u_t^{IP} = -0.658_{(0.06)} u_{t-1}^{IP} - 0.265_{(0.06)} u_{t-2}^{IP} + 0.532_{(0.05)} \eta_t^{IP}$$

$$u_t^{WTS} = -0.395_{(0.05)} u_{t-1}^{WTS} - 0.259_{(0.05)} u_{t-2}^{WTS} + 0.787_{(0.06)} \eta_t^{EJO}$$

$$u_t^{EJO} = 0.335_{(0.06)} u_{t-1}^{EJO} + 0.314_{(0.06)} u_{t-2}^{EJO} + 0.333_{(0.03)} \eta_t^{IP}$$

$$u_t^{NWH} = -0.007_{(0.03)} u_{t-1}^{NWH} + 0.135_{(0.06)} u_{t-2}^{NWH} + 0.411_{(0.04)} \eta_t^{NWH}$$

$$u_t^{LIP} = -0.397_{(0.05)} u_{t-1}^{LIP} - 0.201_{(0.05)} u_{t-2}^{LIP} + 0.747_{(0.06)} \eta_t^{LIP}$$

Table 5. Estimated parameters of CF-LSTAR model with delay d=4
(Japanese macroeconomic monthly data, 1973:1-2003:1)
Log-likelihood: -2101.97

$$\Delta y_t = \begin{pmatrix} 1 \\ 0.507_{(0.08)} \\ 1.15_{(0.19)} \\ 1.97_{(0.13)} \\ 0.710_{(0.09)} \end{pmatrix} \Delta C_t + u_t$$

$$\Delta C_t = 0.010_{(0.011)} F_t - 0.051_{(0.09)}(1 - F_t) + 0.823_{(0.04)} \Delta C_{t-1} + \left[0.026_{(0.01)} F_t + 0.238_{(0.08)}(1 - F_t)\right] \varepsilon_t$$

$$F_t = \frac{1}{1 + \exp(-5.2(\Delta C_{t-4} + 0.38))}$$

$$u_t^{IP} = -0.572_{(0.06)} u_{t-1}^{IP} - 0.179_{(0.06)} u_{t-2}^{IP} + 0.608_{(0.05)} \eta_t^{IP}$$

$$u_t^{WTS} = -0.360_{(0.05)} u_{t-1}^{WTS} - 0.227_{(0.05)} u_{t-2}^{WTS} + 0.832_{(0.06)} \eta_t^{EJO}$$

$$u_t^{EJO} = 0.380_{(0.05)} u_{t-1}^{EJO} + 0.325_{(0.05)} u_{t-2}^{EJO} + 0.347_{(0.03)} \eta_t^{IP}$$

$$u_t^{NWH} = -0.384_{(0.11)} u_{t-1}^{NWH} - 0.116_{(0.09)} u_{t-2}^{NWH} + 0.244_{(0.04)} \eta_t^{NWH}$$

$$u_t^{LIP} = -0.360_{(0.05)} u_{t-1}^{LIP} - 0.174_{(0.05)} u_{t-2}^{LIP} + 0.787_{(0.06)} \eta_t^{LIP}$$

Table 6. Estimated parameters of CF-ESTAR model with delay d=4
(Japanese macroeconomic monthly data, 1973:1-2003:1)
Log-likelihood: -2110.08

$$\Delta y_t = \begin{pmatrix} 1 \\ 0.486_{(0.08)} \\ 1.05_{(0.19)} \\ 2.02_{(0.14)} \\ 0.710_{(0.09)} \end{pmatrix} \Delta C_t + u_t$$

$$\Delta C_t = 0.013_{(0.01)} F_t - 0.066_{(0.10)} (1 - F_t) + 0.782_{(0.05)} \Delta C_{t-1} + \left[0.034_{(0.01)} F_t + 0.238_{(0.10)} (1 - F_t) \right] \varepsilon_t$$

$$F_t(\Delta C_{t-d}; \lambda, r) = 1 - \exp(-0.28(\Delta C_{t-d} + 0.90)^2)$$

$$u_t^{IP} = -0.557_{(0.06)} u_{t-1}^{IP} - 0.161_{(0.06)} u_{t-2}^{IP} + 0.620_{(0.05)} \eta_t^{IP}$$

$$u_t^{WTS} = -0.352_{(0.05)} u_{t-1}^{WTS} - 0.221_{(0.05)} u_{t-2}^{WTS} + 0.842_{(0.06)} \eta_t^{EJO}$$

$$u_t^{EJO} = 0.396_{(0.05)} u_{t-1}^{EJO} + 0.327_{(0.05)} u_{t-2}^{EJO} + 0.352_{(0.03)} \eta_t^{IP}$$

$$u_t^{NWH} = -0.498_{(0.13)} u_{t-1}^{NWH} - 0.196_{(0.09)} u_{t-2}^{NWH} + 0.192_{(0.04)} \eta_t^{NWH}$$

$$u_t^{LIP} = -0.356_{(0.05)} u_{t-1}^{LIP} - 0.164_{(0.05)} u_{t-2}^{LIP} + 0.792_{(0.06)} \eta_t^{LIP}$$

Table 7. Estimated parameters of CF-MS model
(Japanese macroeconomic monthly data, 1973:1-2003:1)
Log-likelihood: -2141.76

$$\Delta y_t = \begin{pmatrix} 1 \\ 0.559_{(0.07)} \\ 1.31_{(0.21)} \\ 1.64_{(0.13)} \\ 0.713_{(0.08)} \end{pmatrix} \Delta C_t + u_t$$

$$\Delta C_t = 0.039_{(0.02)} s_t - 0.321_{(0.08)}(1 - s_t) + 0.726_{(0.08)} \Delta C_{t-1} + 0.051_{(0.05)} \varepsilon_t$$

$$\pi = \begin{pmatrix} 0.983_{(0.01)} & 0.15_{(0.08)} \\ 0.017 & 0.85 \end{pmatrix}$$

$$u_t^{IP} = -0.678_{(0.06)} u_{t-1}^{IP} - 0.283_{(0.06)} u_{t-2}^{IP} + 0.516_{(0.05)} \eta_t^{IP}$$

$$u_t^{WTS} = -0.396_{(0.05)} u_{t-1}^{WTS} - 0.260_{(0.05)} u_{t-2}^{WTS} + 0.786_{(0.06)} \eta_t^{EJO}$$

$$u_t^{EJO} = 0.337_{(0.06)} u_{t-1}^{EJO} + 0.317_{(0.06)} u_{t-2}^{EJO} + 0.334_{(0.03)} \eta_t^{IP}$$

$$u_t^{NWH} = 0.012_{(0.03)} u_{t-1}^{NWH} + 0.158_{(0.07)} u_{t-2}^{NWH} + 0.414_{(0.04)} \eta_t^{NWH}$$

$$u_t^{LIP} = -0.392_{(0.05)} u_{t-1}^{LIP} - 0.196_{(0.05)} u_{t-2}^{LIP} + 0.751_{(0.06)} \eta_t^{LIP}$$

Table 8. Time series used to establish the Japanese reference cycle chronology

ECRI	ESRI
Industrial production	Production index (mining and manufacturing)
Employment	Industrial goods shipment index (mining and manufacturing)
Real earnings	Electric power consumption of large users
Real retail sales	Capacity utilization index (manufacturing industry)
	Overtime working hours index (manufacturing industry)
	Producer index level of investment good shipments (excluding transport machinery)
	Department store sales (year-on-year)
	Commercial sales index (wholesale business) (year-on-year)
	Operating income (all industries)
	Small business sales (manufacturing industry)
	Effective job offer ratio (excluding new university graduates)

Table 9. Alternative Japanese reference cycle chronologies

ECRI classical cycle		ESRI chronology		Simulated ESRI chronology	
Peak	Trough	Peak	Trough	Peak	Trough
Nov-73	Feb-75	Nov-73	Mar-75	Nov-73	Mar-75
		Jan-77	Oct-77	Jan-77	Oct-77
		Feb-80	Feb-83	Feb-80	Feb-83
		Jun-85	Nov-86	Nov-84	Nov-86
Apr-92	Feb-94	Feb-91	Oct-93	Oct-90	Jan-94
Mar-97	Jul-99	May-97	Jan-99	Mar-97	Jan-99
Aug-00		Oct-00		Aug-00	Jan-02

Table 10. In-sample performance of the nonlinear models for Japanese data in 1973:2-2001:12 with respect to ECRI's classical cycle and ESRI's reference cycle dates

Model	QPS		DM-statistic			
	ECRI	ESRI	LSTAR	ESTAR	MSf	MSs
LSTAR	0.144	0.296		1.28	0.812	0.0137
ESTAR	0.127	0.270	1.74*		1.08	0.451
MSf	0.158	0.305	0.483	1.22		0.957
MSs	0.144	0.293	0.123	0.648	0.736	

Note 1: QPS = quadratic probability score; DM = Diebold-Mariano statistic
Note 2: MSf = filtered probabilities of CF-MS; MSs = smoothed probabilities of CF-MS
Note 3: * forecasting accuracy difference is significant at 5% level

Table 11a. Out-of-sample forecasting performance of CF-MS and CF-STAR models for Japanese data
ECRI's reference dating. Forecasting sample 1998:1-2001:12

Model	QPS	DM / modified DM		
		MS	LSTAR	ESTAR
Forecasting horizon: 1 month				
MS	0.581		2.13**	2.19**
LSTAR	0.422	2.1**		0.488
ESTAR	0.406	2.16**	0.482	
Forecasting horizon: 2 months				
MS	0.603		1.99**	2.19**
LSTAR	0.463	1.93**		1.25
ESTAR	0.422	2.12**	1.21	
Forecasting horizon: 3 months				
MS	0.623		1.84**	2.09**
LSTAR	0.502	1.74**		1.71**
ESTAR	0.444	1.98**	1.62*	
Forecasting horizon: 4 months				
MS	0.641		1.60*	1.93**
LSTAR	0.541	1.48*		1.90**
ESTAR	0.471	1.79**	1.76**	
Forecasting horizon: 5 months				
MS	0.647		1.39*	1.75
LSTAR	0.562	1.26		1.88**
ESTAR	0.492	1.59*	1.70**	
Forecasting horizon: 6 months				
MS	0.652		1.30*	1.62*
LSTAR	0.571	1.16		1.71**
ESTAR	0.509	1.43*	1.51*	

QPS = quadratic probability score; DM = Diebold-Mariano statistic; modified DM = modified Diebold-Mariano statistic

* forecasting accuracy difference is significant at 10% level, ** significant at 5% level

Table 11b. Out-of-sample forecasting performance of CF-MS and CF-STAR models for Japanese data
ESRI's reference dating. Forecasting sample 1998:1-2001:12

Model	QPS	DM / modified DM		
		MS	LSTAR	ESTAR
Forecasting horizon: 1 month				
MS	0.419		2.21**	2.27**
LSTAR	0.255	2.19**		0.481
ESTAR	0.240	2.24**	0.476	
Forecasting horizon: 2 months				
MS	0.442		2.12**	2.30**
LSTAR	0.296	2.05**		1.25
ESTAR	0.255	2.23**	1.21	
Forecasting horizon: 3 months				
MS	0.465		2.03**	2.20**
LSTAR	0.335	1.92**		1.66**
ESTAR	0.279	2.09**	1.57*	
Forecasting horizon: 4 months				
MS	0.483		1.80**	2.06**
LSTAR	0.375	1.67*		1.88**
ESTAR	0.305	1.91**	1.74**	
Forecasting horizon: 5 months				
MS	0.490		1.60*	1.89**
LSTAR	0.395	1.45*		1.87**
ESTAR	0.325	1.72**	1.70**	
Forecasting horizon: 6 months				
MS	0.496		1.52*	1.71**
LSTAR	0.405	1.35*		1.79**
ESTAR	0.350	1.52*	1.59*	

QPS = quadratic probability score; DM = Diebold-Mariano statistic; modified DM = modified Diebold-Mariano statistic

* forecasting accuracy difference is significant at 10% level, ** significant at 5% level