

Education funding and regional convergence*

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March 11, 1999

Abstract

The aim of this paper is to discuss the process of regional convergence within the framework of an overlapping generations model in which the engine of growth is the accumulation of human capital. In particular, we consider different education funding systems and compare their performance in terms of growth rates and pace of convergence between two heterogeneous regions. The analysis suggests that the choice of a particular education system incorporates a possible trade-off between long run growth rate and short run convergence. In such choice, the initial capital stock and the extent of regional human capital discrepancy appear as central variables.

JEL Classification numbers: O41, F22.

Keywords: Altruism, education, growth, convergence, capital mobility.

*The financial support of the PAI P4/01 programme of the Belgian Federal government is gratefully acknowledged. We also thank Géraldine Mahieu and Jean-Pierre Vidal for their comments on an earlier draft.

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1 Introduction

Perfect capital mobility is a powerful engine to enforce convergence across countries or regions (Buiter and Kletzer, 1993). However, when there is an immobile region-specific state variable, like land (Mountford, 1995) or human capital (Buiter and Kletzer, 1995), there is room for (at least) temporary discrepancies between regions. In the case of human capital, these discrepancies should obviously be affected by the way education is financed. The purpose of this paper is to study the impact of different education funding systems on growth and on the convergence process of two regions characterized by initial disparities in the levels human capital.

The issue of convergence between different and possibly interdependent economies is central in growth theory (see, for instance Durlauf and Quah (1998)). More and more attention is nevertheless devoted to regional frameworks (see, for example, Sala-I-Martin (1996)) which might be distinguished from international ones by the fact that the economies under consideration share some common institutions. These can of course correspond to an integrated capital or labor market (see, for instance, Crettez, Michel and Vidal (1998)) but it could also refer to federal levels of decision or legislation. In such a case, we can expect the distribution of competence between geographically differentiated levels of public jurisdiction to affect the characteristics of the regional convergence process.

In this paper, we examine these issues within the framework of an overlapping generations model featuring endogenous growth engined by the accumulation of human capital. We consider two regions sharing a common capital market and a federal government. Three different types of education funding are then examined. The first one is a system under which a local authority finances the education of its region by means of locally collected taxes. In the second one, a federal government levies nation-wide taxes to finance education in both regions which possibly organizes cross-regional transfers. Finally, in the market funding system, individuals borrow to finance their education. In both public sector models, the tax rate is endogenized by implicitly assuming this choice to emerge from a voting process. In addition, these three settings are compared to the solution that would obtain under the assumption of a social planner taking into account the different externalities contained in the models.

Our question is very close to that examined in several papers related to the interplay between growth and agent's heterogeneity which, very often, illustrate the case of education finance (see, for instance Benabou (1996) or Glomm and Ravikumar (1992)). Our analysis first differs from theirs' as we investigate the properties of a market funding system. Then, the way we treat the spill-over that affects the human capital accumulation process departs from the standard specification. We indeed explicitly incorporate the possibility that the extent of such spill-over might be affected by the distance between regions (see Chua (1993)). From this point of view, distance should be interpreted as any obstacle, whether physical or institutional, that prevent the non-market interactions between agents (contacts, exchange of information, face-to-face communication). Economic geography indeed considers these interactions as an important factor in the process of technology or knowledge diffusion (see, for instance, the extensive survey by Fujita and Thisse (1997)). In our framework, the extent by which one region benefits from the others' human capital is affected by a transaction cost which reflects the role of distance in

the process of human capital spill-over.

As we show in the paper, this assumption leaves an explicit role to the "technological externality" and to the characteristics of the human capital transmission mechanisms which, together with the classical "pecuniary externality" related to the fiscal spill-over, enriches the description of the regional convergence process. In particular, even under the assumption of constant return to scale, the different education systems lead regions to converge in levels. Absent this technological externality, and in coherence with previous analysis (Benabou (1996) or Glomm and Ravikumar (1992)), only the federal funding system ensures regional convergence.

The results of this analysis can be summarized as follows: (i) nation-wide financing of education proves to foster regional convergence. In terms of the speed at which regional disparities are reduced in time, the models rank according to the order private, federal and regional system, (ii) the growth rate obtained in the long run, i.e. when convergence is achieved, is the highest (lowest) under the market funding system provided that the degree of altruism is low (high) enough and (iii) when compared to the social planner solution, the performance of a particular system is critically influenced both by the initial capital stock and by the extent of regional human capital discrepancy.

The paper is organized as follows. Section 2 formalizes the different systems. Section 3 solves the models for the growth rates and speed of convergence and ranks the different systems according to their performance in these two dimensions. Section 4 is devoted to the comparison of the solution obtained under the systems with that of the social planner. Finally, section 5 concludes.

2 The model

The model is an extension of the overlapping generations model of Allais (1947) and Diamond (1965) in which endogenous growth is made possible through human capital accumulation at the regional level. Time is discrete and goes from 0 to $+\infty$. The economy is composed of two regions, A and B . In each period, each region is populated by three generations, living for three periods. The growth rate of the population is zero and the size of the population in each region is normalized to one. When young, the representative agent benefits from education spending and builds his human capital stock; his consumption is included in his parents' consumption. When adult, he works, consumes and invests a part of his income in capital which is rented and used by the firms in the next period. When old, he therefore receives the return from his savings, consumes and dies. At each date a single physical good is produced. This good can either be consumed by the middle-aged and old generations during the period or accumulated as capital for future production. Production occurs through a constant returns to scale technology using capital and labor. Capital is perfectly mobile across regions while labor is immobile.

The production function of the representative firm in region i is given by

$$Y_{i,t} = K_{i,t}^\alpha L_{i,t}^{1-\alpha} \quad 0 < \alpha < 1$$

where $K_{i,t}$ and $L_{i,t}$ are capital and labor inputs respectively. Note the $L_{i,t}$ corresponds to effective labor as it accounts for the quantity of human capital used by the firm rather than a number of workers.

Except in the central planner case, the representative firm of each region chooses capital and labor input $K_{i,t}, L_{i,t}$ according to

$$\{K_{i,t}, L_{i,t}\} = \arg \max \{K_{i,t}^\alpha L_{i,t}^{1-\alpha} - w_{i,t}L_{i,t} - R_t K_{i,t}\} \quad (1)$$

where R_t is the interest factor and $w_{i,t}$ the wage per unit of effective labor.

The production function for human capital in region i is defined as

$$h_{i,t+1} = \psi e_{i,t}^\theta [h_{i,t} + \zeta h_{j,t}]^{1-\theta} \quad 0 < \theta < 1, 0 \leq \zeta \leq 1, \psi > 0 \quad (2)$$

with technological parameters ψ and θ . The stock of human capital is assumed to depend on some education spending ($e_{i,t}$) which must be financed in one way or the other. Moreover, it also depends on two types of spill-over which are similar to those described in Lucas (1988) models. On the one hand, the young generation inherits part of the human capital ($h_{i,t}$) of the region's adults. We therefore have a framework in which there are externalities of the local type (see Marshall (1890) or Bradley and Taylor (1996) for empirical support of this idea). On the other hand, the regional capital accumulation also hinges on the other region's stock, $h_{j,t}$, therefore allowing for cross-regional spill-over. This last effect introduces convergence forces in the model. The importance of this spill-over in the human capital accumulation process is parameterized by ζ which captures the fact that the transmission of knowledge from one region to the other is affected by distance.¹

As already mentioned, we implicitly assume that knowledge spill-over hinges in part on non-market interactions and social contacts between agents and that its quality in turn depends on the proximity of agents. Note that distance is to be interpreted in a broad sense as it may reflect some physical (geographic distance, level of communication technologies, etc...) as well as institutional (like those introducing some kind of segregation) barriers to contacts and therefore to the transmission of knowledge. The restriction $\zeta \leq 1$ models an iceberg type transport cost reflecting the idea that a fraction only of one region's human capital reaches (or affects) the other region. The function (2) is homogeneous of degree 1 and is thus at the root of endogenous growth in the model.

Finally, the clearing condition on the labor market implies in each region

$$L_{i,t} = h_{i,t} \quad (3)$$

2.1 Definition of the equilibrium

We now define the three different equilibria corresponding to the three types of education funding. We assume that, when young agents can borrow on financial markets to finance their education, the parents do not intervene in this process. On the contrary, when human capital cannot be a collateral for borrowing, there is an ad-hoc altruism factor (in opposition to rational altruism as in Barro (1974)) and parents vote for public funding. The different regimes are thus clearly exclusive as hybrid funding is not allowed.

¹Such spill-overs are in general absent in the literature, see e.g. Buiter and Kletzer (1995). Note that in the framework in which the externality hinges on an aggregate human capital index, as in Benabou (1996), the extent of cross-regional spill-over does not depend on distance. From this point of view our framework is more related to international growth models where cross-border spill-overs are affected by integration as in Rivera-Batiz and Romer (1991a) and Rivera-Batiz and Romer (1991b).

2.1.1 Regional funding

The utility function of the representative household is logarithmic and depends on consumption when adult $c_{i,t}$, consumption when old $d_{i,t+1}$ and on the amount spent on children education $e_{i,t}$. This last element reflects the ad-hoc altruism factor which is referred in the literature as “joy-of-giving” (or warm glow), because parents have a taste for giving (see e.g. Andreoni (1989)). The adults supply inelastically one unit of labor and earn $w_t h_{i,t}$, where w_t is the wage per unit of human capital and $h_{i,t}$ is the level of human capital. This income is allocated to consumption, taxes and savings $s_{i,t}$ for future consumption. When old, agents spend all their saving and accrued interest on consumption. Notice that households take their own human capital as given; we thus implicitly assume that children are not allowed to borrow on capital markets to complete the amount given by the government. As a consequence, regional education funding only rests on the regions resources.

A local government collects taxes on a regional tax base in order to finance local education spending. Education spending thus differ across regions. Taxes results from a voting process in each region.

The clearing condition on the federal capital market implies that capital of the next period is built from the savings of the adults. In each region the clearing condition on the labor market implies that the labor demand is equal to the labor supply, i.e. the existing stock of human capital.

Notice that the regional funding system is completely equivalent to a system under which parents devote part of their resources to finance the education of their own children. Indeed, if parents can choose the regional tax rate, they manage to devote the same amount of resource to education delivered by a local government compared to the case in which they directly provide education. This result obtains because, in our framework, agents are homogenous in each region and the labor supply is absent, so that taxes are non-distortionary (see, for instance, Vidal (1998) who elaborates on this possibility).

Definition 1 (regional funding) *Given the set of initial conditions $\{K_{i,0}, h_{i,0}\}$, an equilibrium with regional funding is a set of sequences*

$$\{K_{i,t+1}, L_{i,t}, w_{i,t}, h_{i,t+1}, s_{i,t}, c_{i,t}, d_{i,t+1}, R_t, e_{i,t}, \tau_{i,t}\}_{i=A,B. t \geq 0}$$

such that

- In each region, the representative household chooses his savings $s_{i,t}$ according to

$$s_{i,t} = \arg \max \{ \ln c_{i,t} + \beta \ln d_{i,t+1} + \gamma \ln e_{i,t} \} \quad (4)$$

$$s.t. \quad c_{i,t} + s_{i,t} = w_{i,t} h_{i,t} (1 - \tau_{i,t}) \quad (5)$$

$$R_{t+1} s_{i,t} = d_{i,t+1} \quad (6)$$

- The preferred regional tax rate $\tau_{i,t}$ maximizes the regional indirect utility given the local government budget constraint

$$\tau_{i,t} = \arg \max \{ \ln(w_{i,t} h_{i,t} (1 - \tau_{i,t})) + \gamma \ln(e_{i,t}) \} \quad (7)$$

$$s.t. \quad e_{i,t} = \tau_{i,t} w_{i,t} h_{i,t}$$

- In each region, the representative firm chooses capital and labor inputs $K_{i,t}$, $L_{i,t}$ according to (1). The human capital accumulates according to equation (2). The equilibrium condition on the labor markets (3) holds.
- The clearing condition on the federal capital market implies equation:

$$\sum_i K_{i,t+1} = \sum_i s_{i,t}. \quad (8)$$

2.1.2 Federal funding

Under the federal funding system, a federal government levies taxes on a nation-wide basis and uses taxes revenues to finance education spending in both regions. The common tax rate is determined by means of majority voting² and tax revenues are equally shared between the two regions to finance education. Taxes are levied on the adults income, disposable income being spent on consumption and savings which, together with the accruing interests, finance consumption when old. As in the regional system, preferences of agents are defined on adult and old consumption as well as on the level of education delivered by the government to their offspring.

Definition 2 (federal funding) *Given the set of initial conditions $\{K_{i,0}, h_{i,0}\}$, an equilibrium with federal funding is a set of sequences*

$$\{K_{i,t+1}, L_{i,t}, w_{i,t}, h_{i,t+1}, s_{i,t}, c_{i,t}, d_{i,t+1}, R_t, e_t, \tau_t\}_{i=A,B, t \geq 0}$$

such that

- In each region, the representative household chooses his savings $s_{i,t}$ according to

$$s_{i,t} = \arg \max \{ \ln c_{i,t} + \beta \ln d_{i,t+1} + \gamma \ln e_t \} \quad (9)$$

$$s.t. \quad c_{i,t} + s_{i,t} = w_{i,t} h_{i,t} (1 - \tau_t) \quad (10)$$

$$R_{t+1} s_{i,t} = d_{i,t+1} \quad (11)$$

- The preferred tax rate τ_t maximizes the average indirect utility given the federal government budget constraint

$$\tau_t = \arg \max \left\{ \sum_i [\ln(w_{i,t} h_{i,t} (1 - \tau_t)) + \gamma \ln e_t] \right\} \quad (12)$$

$$s.t. \quad 2e_t = \tau_t \sum_i w_{i,t} h_{i,t}$$

- In each region, the representative firm chooses capital and labor inputs $K_{i,t}$, $L_{i,t}$ according to (1). The human capital accumulates according to equation (2). The equilibrium condition on the labor markets (3) holds.
- The clearing condition on the federal capital market implies equation (8).

²This decision process is actually implicitly assumed here since it is shown that, given the preferences and technologies, both regions select the same tax rate. The achievement of a consensus is therefore trivial.

2.1.3 Market funding

This version of the model is an extension of Michel (1993) and de la Croix (1996) to an economy with two regions. Under the market funding system, agents finance their education by borrowing on the federal capital market and do not rely on public resources. We therefore assume that individuals have a perfect access to capital market and can use their human capital as collateral to finance their education spending. This implies that the source of education funding is nation-wide and that human capital directly competes with the physical capital formation in the process of resources allocation. One important difference with the preceding system is that education spending no longer rests on gift motives but on the return that is expected from education under the form of the wage rate associated to a particular level of human capital.

We assume that agents borrow and spend on education while young, use the accumulated human capital and retrieve the benefits from this investment while adults. At this stage, they either consume or save their income. Savings and accrued interests are completely consumed while old. The representative agent's utility is therefore defined on his adult and old consumption (respectively, $c_{i,t}$ and $d_{i,t+1}$)³. The rest is identical to the preceding system.

Definition 3 (market funding) *Given the set of initial conditions $\{K_{i,0}, h_{i,-1}\}$, an equilibrium with market funding is a set of sequences*

$$\{K_{i,t+1}, L_{i,t}, w_{i,t}, h_{i,t}, s_{i,t}, e_{i,t-1}, c_{i,t}, d_{i,t+1}, R_t\}_{i=A,B. t \geq 0}$$

such that

- In each region, the representative household chooses his own education spending $e_{i,t-1}$ and his savings $s_{i,t}$ according to

$$\{s_{i,t}, e_{i,t-1}\} = \arg \max \{ \ln c_{i,t} + \beta \ln d_{i,t+1} \} \quad (13)$$

$$s.t. \quad c_{i,t} + s_{i,t} + R_t e_{i,t-1} = w_{i,t} h_{i,t} \quad (14)$$

$$R_{t+1} s_{i,t} = d_{i,t+1} \quad (15)$$

- In each region, the representative firm chooses capital and labor inputs $K_{i,t}$, $L_{i,t}$ according to (1). The equilibrium condition on the labor markets (3) holds.
- The clearing condition on the federal capital market implies

$$\sum_i K_{i,t+1} + \sum_i e_{i,t} = \sum_i s_{i,t} \quad (16)$$

³We do not exclude the possibility for parents to care about the education level of their offspring but this does not play any role in the model as gifts or bequests are excluded. A richer specification would include both parents and children spending. However, such a model would be much complicated as this would leave the possibility for negative gifts.

2.2 Characteristics of the equilibrium

We now characterize the equilibrium trajectories for the different cases. Let us first define the regional capital-labor ratio as:

$$k_{i,t} \equiv \frac{K_{i,t}}{L_{i,t}} = \frac{K_{i,t}}{h_{i,t}} \quad (17)$$

The first order conditions of the firm's program (1) are standard and can be written as

$$\alpha k_{i,t}^{\alpha-1} = R_t \quad (18)$$

$$(1 - \alpha)k_{i,t}^\alpha = w_{i,t} \quad (19)$$

where R_t is the factor of interest prevailing on the federal capital market. The assumption of perfect capital mobility thus implies that the regional capital-labor ratios are equal. Hence, the wages per unit of human capital also equalize across regions.

$$k_{i,t} = k_t \quad \forall i \quad \text{and} \quad w_{i,t} = w_t \quad \forall i \quad (20)$$

The dynamics arising under the different funding systems will now be analyzed in terms of three stationary variables: the capital-labor ratio k_t , the ratio of workers' consumption in region B to workers' consumption in region A ,

$$z_t = \frac{c_{B,t}}{c_{A,t}},$$

and the growth factor in one region, say A , $g_{A,t} = h_{a,t}/h_{a,t-1}$. The interest of using $z_t = c_{B,t}/c_{A,t}$ is that, for our different decentralized regimes, z_t will also measure the ratio of regional human capital $h_{B,t}/h_{A,t}$.

The objective is to compare the steady growth rates of each system and the speed of regional convergence. An approximation of the speed of regional convergence near the steady state, denoted v , is computed as $1/\lambda - 1$, where λ is the eigenvalue associated to the dynamics of z

$$\hat{z}_{t+1} = \lambda^t \hat{z}_0 = \left(\frac{1}{1+v} \right)^t \hat{z}_0$$

where hatted variables denote deviations from steady state.

2.2.1 Regional funding

Under the regional funding system, the optimal regional tax rate from (7) is

$$\tau_{i,t} = \frac{\gamma}{1 + \beta + \gamma}$$

The first order necessary conditions of the household program with regional funding (4) are:

$$s_{i,t} = \frac{\beta}{1 + \beta + \gamma} w_t h_{i,t} \quad (21)$$

$$e_{i,t} = \frac{\gamma}{1 + \beta + \gamma} w_t h_{i,t} \quad (22)$$

The equilibrium is described by equations (2), (3), (5), (6), (8), (18), (19), (21) and (22). This set of equations can be reduced to a system of two non-linear difference equations of the first order, describing the dynamics of the capital-labor ratio k_t and of the ratio z_t .

As shown in the appendix, this system admits a unique steady state (balanced growth path) which is stable. If there is knowledge spill-over ($\zeta > 0$), the ratio z_t converges to 1; the speed of convergence in the regional funding system is

$$v_{\text{RF}} = \frac{2(1-\theta)\zeta}{1-\zeta+2\theta\zeta}. \quad (23)$$

and the steady state growth factor of both regions is

$$g_{\text{RF}} = (\psi\gamma^\theta)^{\frac{1-\alpha}{1-\alpha(1-\theta)}} \beta^{\frac{\alpha\theta}{1-\alpha(1-\theta)}} \left(\frac{1-\alpha}{1+\beta+\gamma} \right)^{\frac{\theta}{1-\alpha(1-\theta)}} (1+\zeta)^{\frac{(1-\theta)(1-\alpha)}{1-\alpha(1-\theta)}}$$

2.2.2 Federal funding

The first order necessary conditions of the household program with federal funding (9) and (12) are:

$$s_{i,t} = \frac{\beta}{1+\beta}(1-\tau_t)w_t h_{i,t} \quad (24)$$

$$e_{i,t} = \frac{1}{2}\tau_t w_t (h_{i,t} + h_{j,t}) \quad (25)$$

$$\tau_t = \frac{\gamma}{1+\beta+\gamma} \quad (26)$$

In this setting, saving is proportional to disposable income. In coherence with a federal education system, public resources devoted to education are equally distributed among the two regions. As a consequence, regional education level hinges on the federal stock of human capital which is the classical fiscal externality fostering convergence among regions (see, for instance, Benabou (1996)). Under the logarithmic utility function, the optimal tax rate is common to all agents (which makes the voting process on this decision trivial) and reflects their preferences. The equilibrium is then described by equations (2), (3), (10), (11), (16), (18), (19), (24), (25) and (26). Reducing these equations to a system of two non-linear difference equations of the first order, we can describe the dynamics of the capital-labor ratio k_t and of the ratio z_t .

This system admits a unique steady state (balanced growth path) which is stable, the ratio z_t converges to 1 and the speed of convergence in the federal funding system is

$$v_{\text{FF}} = \frac{2\zeta + \theta(1-\zeta)}{(1-\theta)(1-\zeta)}. \quad (27)$$

and the steady state growth factor of both regions is

$$g_{\text{FF}} = (\psi\gamma^\theta\theta^\theta)^{\frac{1-\alpha}{1-\alpha(1-\theta)}} \beta^{\frac{\alpha\theta}{1-\alpha(1-\theta)}} \left(\frac{1-\alpha}{1+\beta+\gamma} \right)^{\frac{\theta}{1-\alpha(1-\theta)}}$$

2.2.3 Market funding

The first order necessary conditions of the household program with market funding (13) are:

$$s_{i,t} = \frac{\beta}{1 + \beta} (w_t h_{i,t} - R_t e_{i,t-1}) \quad (28)$$

$$e_{i,t-1} = \left(\psi \theta \frac{w_t}{R_t} \right)^{\frac{1}{1-\theta}} [h_{i,t-1} + \zeta h_{j,t-1}] \quad (29)$$

Equation (28) states that the propensity to save out of income net of total education cost is constant. Equation (29) shows that education spending increases in the return that is expected from education under the form of the wage rate per unit of human capital. The interest factor has a negative effect on education spending as it is part of the cost of education. Using equations (2) and (29), we obtain

$$e_{i,t-1} = \left(\theta \frac{w_t}{R_t} \right)^{2(1-\theta)} h_{i,t}$$

and $c_{i,t}$ is proportional to $h_{i,t}$. As a consequence, $z_t = h_{B,t}/h_{A,t}$. The equilibrium can now be described by equations (2), (3), (14), (15), (16), (18), (19), (28) and (29). As in the previous case, this set of equations can be reduced to a system of two non-linear difference equations of the first order, describing the dynamics of the capital-labor ratio k_t and of the ratio z_t .

Again, this system admits a unique steady state (balanced growth path) which is stable while, for ζ strictly positive, the ratio z_t converges to 1; the speed of convergence in the market funding system is

$$v_{\text{MF}} = \frac{2\zeta}{1 - \zeta}. \quad (30)$$

and the steady state growth factor of both regions is

$$g_{\text{MF}} = \psi^{\frac{(1-\theta)(1-\alpha)}{1-\alpha(1-\theta)}} \left(\theta \psi \frac{1-\alpha}{\alpha} \right)^{\frac{\theta(1-\alpha)}{1-\alpha(1-\theta)}} \left(\frac{\beta(1-\alpha)(1-\theta)}{(1+\beta)(1+\zeta)(1+\theta\frac{1-\alpha}{\alpha})} \right)^{\frac{\theta}{1-\alpha(1-\theta)}} (1+\zeta)$$

3 Results

We now compare the properties of the different regimes and present the key elements in a series of propositions.

Proposition 1 (absolute convergence) *Assume that the initial levels of human capital differ across regions. Under the regional and market funding systems, there is absolute convergence of these levels if and only if ζ is strictly positive, i.e. if there are inter-regional knowledge spill-over. Moreover, in all three systems, the speed of convergence increases with the extent of knowledge spill-over.*

Proof. If $\zeta = 0$, then $v_{\text{PF}} = v_{\text{MF}} = 0$ while $v_{\text{FF}} = \frac{\theta}{1-\theta}$. Moreover,

- $\frac{\partial v_{\text{RF}}}{\partial \zeta} = \frac{2(1-\zeta)}{(1-\zeta+2\theta\zeta)^2} \geq 0.$
- $\frac{\partial v_{\text{MF}}}{\partial \zeta} = \frac{2}{(1-\zeta)^2} \geq 0.$
- $\frac{\partial v_{\text{FF}}}{\partial \zeta} = \frac{2}{(1-\theta)(1-\zeta)^2} \geq 0.$

This proposition emphasizes the role played by the knowledge spill-over in the regional convergence process. The federal funding system is indeed the only regime to display convergence if this type of externality is inoperative as it also features the classical fiscal inter-regional spill-over. This means that a market funding system, even though it allows one region to access global resources, still does not per se implies convergence between regions.⁴ Importantly, note that this result would hold in a standard framework in which the human capital accumulation process of a region also hinges on a global human capital index (see, for instance, Benabou (1996))⁵. While this also intend to capture the possibility of non-market spill-over, their introduction in such form leaves the regional convergence process unchanged.

Finally, the extent of knowledge spill-over has the expected effect of fostering regional convergence. Under the interpretation that we gave above in which it depends on the proximity of communities, this framework underlines the central role that any integration mechanism facilitating contacts and knowledge transmission between communities can play in the relative regional developments.

Proposition 2 (speed of convergence) *Assume that ζ is strictly positive. The equilibrium with federal funding displays a higher convergence speed than the equilibrium with market funding which itself has a higher convergence speed than the equilibrium with regional funding.*

Proof. By inspection of the expressions displayed above for the different systems' speed of convergence, the ranking $v_{\text{FF}} \geq v_{\text{MF}} \geq v_{\text{RF}}$ always holds for $\theta \leq 1$.

The federal funding system ensures the most rapid regional convergence since it features both the classical fiscal and knowledge spill-over. Intuitively, the market funding system implies faster convergence than the regional funding system because under, the former, regional education can be financed on the global capital market and both regions have access to the nation's resources. From this point of view, we can state that this funding system incorporates some kind of financial spill-over which, let alone (i.e. without knowledge spill-over) is nevertheless unable to produce regional convergence. These results are illustrated in figure 1. Notice that when $\zeta = 1$, the human capital spill-over is the same in both regions, and the convergence speed in the market funding and federal funding regime is infinite.

Proposition 3 (long-run growth rate) *For ζ strictly positive, the equilibrium with federal funding has the same long-run growth rate than the equilibrium with regional funding.*

⁴This result is similar to the one of Fabre (1998) who extends Michel (1993) framework to heterogeneous agents.

⁵In our case, this would amount to specify the human capital accumulation process as $h_{i,t+1} = \psi e_{i,t}^\theta [h_{i,t} + \zeta h_{j,t}]^{1-\theta} H_t$ where H_t is the global human capital index.

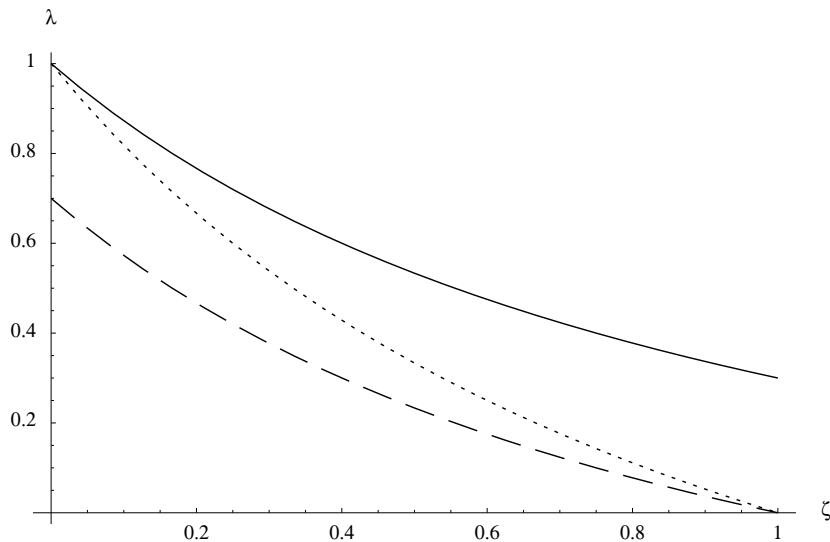


Figure 1: Convergence speed. The eigenvalue associated to the dynamics of z is plotted as a function of ζ in the three different regimes: regional (solid line), market (dotted line) and federal (dash line).

Proof. By inspection of the expressions displayed above $g_{RF} = g_{FF}$.

In the long run, provided there is knowledge spill-over, both regions converge (proposition 1). This implies that the fiscal spill-over incorporated in the federal funding system do no longer transfer resources from one region to the other so that it becomes equivalent to the regional funding system.

Lemma 1 (Altruism and growth) *When altruism is zero, the growth factor under regional funding or federal funding is zero. This growth factor is a positive function of regional altruism for low levels of altruism. There is a threshold, γ^* , above which an increase in altruism has a negative effect on growth.*

Proof.

$$\lim_{\gamma \rightarrow 0} g_{FF} = \lim_{\gamma \rightarrow 0} g_{RF} = 0.$$

It can be checked that

$$\text{Sign} \left[\frac{dg_{FF}}{d\gamma} \right] = \text{Sign} [1 + \beta - \alpha(1 + \beta + \gamma)]$$

and hence,

$$\frac{dg_{FF}}{d\gamma} \iff \gamma < \frac{(1 + \beta)(1 - \alpha)}{\alpha}$$

Growth depends on the combination of both physical and human capital accumulation. Also, as education absorbs part of the existing resources, there is a trade-off between human and physical capital. Under the regional and federal funding systems, the parents preferences for their offspring education drives the human capital accumulation process and we therefore have a critical degree of altruism that yields the growth maximizing combination of physical and human capital (see also Michel and Vidal (1998) for a similar result in a framework where regions differ in the extent of parental altruism).

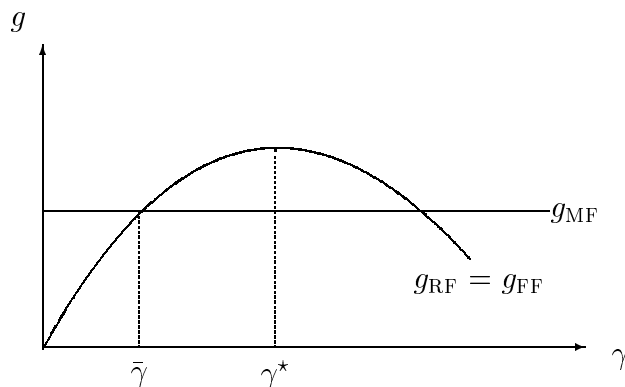


Figure 2: Altruism and growth

Proposition 4 (Growth maximising altruism) *If the degree of altruism maximizes the long run growth rate (i.e. if $\gamma = \gamma^*$), the equilibrium with regional or federal funding has a higher long run growth rate than the equilibrium with market funding.*

Proof. Using the expression obtained for g_{MF} and Lemma 1, one easily checks that, for $\gamma = \gamma^* = \frac{(1+\beta)(1-\alpha)}{\alpha}$, $g_{RF} \geq g_{MF}$ if and only if $\frac{1+\beta}{\beta\theta} \geq \left(\frac{1-\theta}{\alpha+(1-\alpha)\theta}\right) \geq 1$ which is always true for the ranges of values taken by these parameters.

Proposition 5 (Low altruism) *If the degree of altruism is low enough, the equilibrium with market funding has a higher long-run growth rate.*

Proof. By Lemma 1 we have, when $\gamma = 0$, $g_{MF} > g_{PF} = g_{FF} = 0$. By proposition 4, we have, when $\gamma = \gamma^*$, $g_{RF} = g_{FF} > g_{MF}$. As the growth rates are continuous functions of γ , there exists a $\bar{\gamma} \in]0, \gamma^*[$ such that $g_{RF} = g_{FF} = g_{MF}$. If $\gamma \leq \bar{\gamma}$ then $g_{RF} = g_{FF} \leq g_{MF}$.

As an illustration, figure 2 plots the long run growth rates obtained under the respective regimes for different values of γ .

In the regional and federal funding systems, it is the extent of parental altruism towards their offspring which determines the level of education in each period while, in the market funding system, this decision completely relies on market forces. Consequently, if the growth rate obtained under the latter can be reached under the regional or federal funding system, it is for sufficiently high education levels or equivalently sufficiently high degree of altruism.

4 Choosing the right funding system

All the equilibria described above are necessarily sub-optimal as the knowledge spill-over are not internalized by the individual agents. The question of the choice of the best regime is a typical question of finding the second-best policy. We shall address this question by explicitly distinguishing between long term issues and short-run effects linked to the initial conditions.

4.1 Long run issues

As far as long run growth rate is concerned it is useful to compare these equilibria with a first-best benchmark case which is given by the solution to the planning problem. We are particularly interested in comparing the long-run growth rate of the different systems with the optimal growth rate.

The planner maximizes the discounted sum of the utility of all future generations and allocates output between four types of activities: adult consumption, old consumption, investment in physical capital and spending on education. In the planner's problem, education do not depend on parents altruism since the planner takes into account all generations' welfare.

Definition 4 (planner's solution) *Given the set of initial conditions $\{K_{i,0}, h_{i,0}\}$, the planner's solution is a set of sequences*

$$\{K_{i,t+1}, L_{i,t}, h_{i,t+1}, c_{i,t}, d_{i,t}, e_{i,t}\}_{i=A,B; t \geq 0}$$

such that the planner allocates resources according to

$$\begin{aligned} \{c_{i,t}, d_{i,t}, e_{i,t}, L_{i,t}\} = \arg \max & \left\{ \sum_i \left[\beta \ln d_{i0} + \sum_{t=0}^{\infty} \delta^t [\ln c_{i,t} + \beta \ln d_{i,t+1}] \right] \right\} \\ \text{s.t.} & \sum_i (K_{i,t+1} + c_{i,t} + d_{i,t} + e_{i,t}) = \sum_i K_{i,t}^\alpha L_{i,t}^{1-\alpha} \\ & h_{i,t+1} = \psi e_{i,t}^\theta [h_{i,t} + \zeta h_{j,t}]^{1-\theta} \\ & L_{i,t} \leq h_{i,t} \end{aligned}$$

For this equilibrium to be defined there is an additional restriction which imposes the planner's psychological discount factor to be smaller than the inverse of the steady state growth rate. This ensures that the planner's objective function is finite. It is trivial to show that the optimal ratio z_t is equalized to one at every period:

$$z_t = \frac{c_{B,t}}{c_{A,t}} = 1 \neq \frac{h_{B,t}}{h_{A,t}} \quad \forall t \geq 0$$

Hence, the optimal convergence speed of consumptions is infinite:

$$v^* = +\infty$$

Moreover, the optimal economy converges to a balanced growth path with the growth rate given by (the computations are available from the authors upon request)

$$g^* = \delta \alpha \left(2(1 - \delta \alpha) - \frac{\delta \theta (1 - \alpha)}{1 - \delta(1 - \theta)} \right)^{\frac{\theta(1-\alpha)}{1-\alpha(1-\theta)}} \left(\frac{\psi(1 + \zeta)^{1-\theta}}{\delta \alpha} \right)^{\frac{1-\alpha}{1-\alpha(1-\theta)}}$$

Figure 3 compares g^* with g_{MF} and g_{FF} for the range of relevant values of γ and δ . The other parameters have been fixed to $\alpha = .3$, $\beta = .7$, $\psi = 2.3$, $\theta = .3$ and $\zeta = .1$.

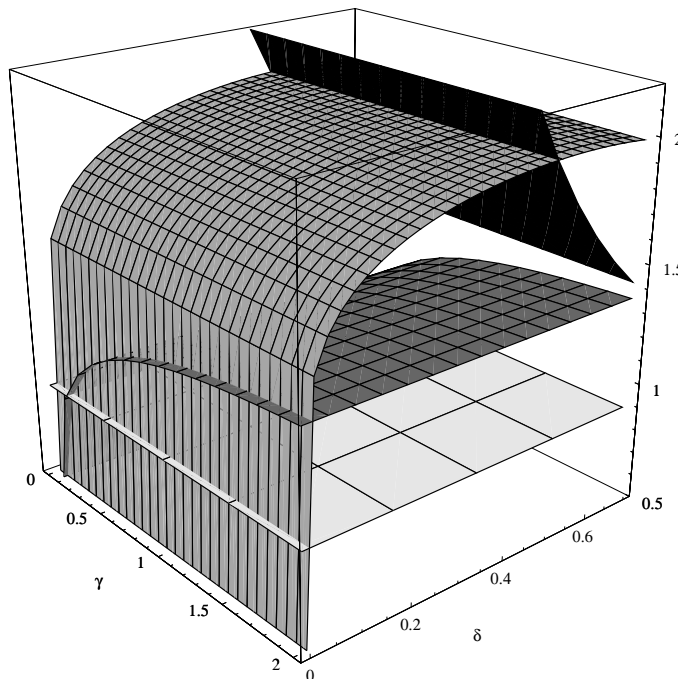


Figure 3: Long-run growth rates. Market funding (light grey), Federal and Regional funding (dark grey), planner (grey). The constraint $g^* < 1/\delta$ is represented in black.

Extensive simulation exercises show that the form of the surfaces is robust to alternative parameter values.

Let us first consider the effect of the degree of altruism (γ). We retrieve the results of proposition 5: if this degree is low (resp. high), parents will vote for low (resp. high) taxes in the regional and federal funding cases, while market funding will be characterized by a higher (resp. lower) growth rate. Comparing the outcome of the regional or federal funding systems with the planner preferred growth rate, we observe that, except for very small value of δ and high value of γ , the planner solution displays higher growth rates. Indeed, as joy-of-giving altruism is ad-hoc, there is no reason for it to take care of the externality and to lead to the planner's outcome.

Turning to the role of the social planner time preference, we first note that if δ is very low, the planner mainly cares about current generations in which case g^* is close to zero, implying that the economy disappears after one period (remember that g is a growth factor). For higher values of δ the time horizon of the social planner is far enough so that the planner's solution yields higher rates of human capital accumulation to guarantee high welfare levels to future generations. For δ high enough, the planner's growth rate exceeds that of the decentralized systems. In particular, the too low growth rate of market funding results from the presence of the positive human capital externalities that are not taken into account by agents in their decision. From a long-run perspective, this would justify the adoption by a benevolent planner of the system yielding the highest growth rate.

4.2 Effects linked to the initial conditions

The initial conditions are important in choosing the best system. Initial conditions include the economy-wide capital stock and the dispersion of human capital. We analyze their effect in turn.

The initial capital stock: When we consider the expressions for the growth rates in the different regimes, we observe that the capital stock has quite different effects in the market funding case than in the two other cases. Indeed, the regional growth rate in the market funding case are proportional to $\theta/(1 - \theta) \ln k_t$ while they are proportional to $\alpha\theta \ln k_t$ in the two other funding regimes. As a consequence, the growth rates in the first periods are more sensitive to the initial capital stock when market funding prevails. Intuitively, if the initial stock of capital is very low, interest rates are very high, and it is very expensive to borrow in order to finance the education spending. Hence, even if market funding was preferred for long-run reasons, this system can be very costly in the adjustment period if the initial capital stock is low. In the opposite, if the stock of capital is very high and interest rates very low, market funding is advantageous, at least in the short-run.

The initial dispersion of human capital: Given the concavity of agents' preferences, we have seen that the utilitarian social planner promotes homogeneous distribution of consumption across regions. From this point of view alone, one would therefore systematically select the funding system which ensures the highest speed of convergence. On the other hand, the social planner also cares about the long run growth rates reached under the different systems.

Accordingly, since from Proposition 3, the federal and regional funding systems lead to the same long run growth rate, the former should always be preferred to the latter because it ensures more rapid convergence. On the contrary, a trade-off between growth in the long run and convergence in the short run might emerge between the federal and the market funding systems provided that the long run growth rate proves higher under the latter (remember that, according to Proposition 2, the federal funding system features a higher convergence speed than the market funding system). It is therefore for the choice between these two regimes that the initial conditions concerning the distribution of human capital will play a role. One easily checks that the larger $|z_0 - 1|$, the more the federal systems dominates the market system from the planner's point of view as its preference for a fast convergence system increases with the extent of regional disparities.

4.3 Regimes specific technological externalities

Until now, the technological externality was considered as given and common to all regimes. It would nevertheless be reasonable to think that it is not systematically the case. Since the diffusion of human capital across regions is assumed to be based on social contacts, we should consider the possibility that their quality or frequency in fact depend on the institutional environment. In particular, the conjecture that the extent of inter-communities contacts is higher under a federal than under a regional funding system seems plausible (part of the technological spill-over is due to the fact that communities

share some common institutions like, for instance, a federal ministry of education), which amounts to assume $\zeta_{FF} \geq \zeta_{RF}$.

The interest of this case lies in that it also embodies a potential trade-off between long run growth and short run convergence. Suppose that instead of having a social planner who decides on the adoption of a particular regime, this choice is in fact the result of a voting process in which both regions express their preferences. In the case where $\zeta_{FF} = \zeta_{RF}$, no consensus can be reached since the region with higher (resp. lower) human capital endowment would always prefer the regional (resp. federal) system as the one that ensures the highest level of human capital in subsequent periods for that region.

On the contrary, if $\zeta_{FF} > \zeta_{RF}$, from the expressions of the regimes' growth rates, then $g_{FF} > g_{RF}$. Consequently, the initially high human capital region might face a trade off between higher human capital levels in the short run (if it selects the regional funding system) and higher long run growth rate (if it selects the federal funding system). A consensus could be obtained if the richer region prefers the federal funding system which is more likely if the initial human capital discrepancy ($|z_0 - 1|$) is low and if regional decision makers care about future generations.

5 Conclusion

In this paper, we addressed the issue of regional convergence within the framework of an overlapping generations model where growth is related to the accumulation of human capital. We considered the possibility of an initial discrepancy in terms of the regional stock of human capital. Moreover, regions share a common and integrated capital market implying a perfect mobility of physical capital which is known as an important convergence force.

One important assumption of the model was to suppose that the extent of the human capital spill-over, by which one region benefits from the other region human capital, depends on the distance between regions. Accordingly, the impact on the regional accumulation process of the other region's human capital stock was formalized as being tempered by something similar to an iceberg transport costs.

We then considered different education funding systems and compared their performance in terms of the long run growth rates and pace of convergence between regions. One system, namely the one referred to as the market funding in which individuals borrow on the whole capital market, is rarely examined in the existing literature (see, for instance, Benabou (1996) or Glomm and Ravikumar (1992)).

The analysis first suggests that a nation wide source of funding, either under the form of a market funding system or of a federal government which redistributes tax levies between regions (federal funding system), enhances the regional convergence process compared to the case in which regions rely on their own resources to finance education (regional funding system). From a long run growth rate perspective, the federal and regional systems proved to yield the same outcome while the market system was shown to generate different growth rates.

Second, from a social planner point of view and abstracting from political economy considerations, the regional funding system case will never be chosen (it features the same

growth rate as the federal system but also slower convergence).

It is optimal to chose market funding system for ever if the following three conditions are met: (i) altruism is low, (ii) the initial dispersion of capital is low and (iii) the initial capital stock is high.

If the initial capital is high but altruism is strong enough, one might chose market funding system temporarily, then switch to the federal funding system.⁶ If both altruism and initial capital are low, one might choose the federal system temporarily, then switch to market when interest rates become low enough.

As a consequence the choice of a particular education system was shown to incorporate a possible trade-off between long run growth rate and short run convergence. In such case, it was shown that the choice of a particular system was influenced by the initial capital stock for a given initial regional discrepancy.

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⁶As the dynamics are only backward-looking, the current outcome does not depend on expectations about the future and such a switch does not introduce specific difficulties.

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Appendix

Regional funding: In the regional funding system, the equilibrium is described by the following system

$$z_{t+1} = z_t \left(\frac{1 + \zeta z_t^{-1}}{1 + \zeta z_t} \right)^{1-\theta} \quad (31)$$

$$k_{t+1} = \frac{\beta}{\psi \gamma^\theta} \left(\frac{1 - \alpha}{1 + \beta + \gamma} \right)^{1-\theta} k_t^{\alpha(1-\theta)} \frac{1 + z_t}{(1 + \zeta z_t)^{1-\theta} + z_t(1 + \zeta z_t^{-1})^{1-\theta}} \quad (32)$$

The growth factor in one region, say A , can be computed following

$$g_{A,t+1} = \psi \left(\frac{\gamma(1-\alpha)}{1+\beta+\gamma} \right)^\theta k_t^{\alpha\theta} (1+\zeta z_t)^{1-\theta}$$

The first equation (31) gives z_{t+1} as a function of z_t alone, say $z_{t+1} = \phi(z_t)$. This function admits a unique non-trivial fixed point at $z = 1$. It is clear that, for values of ζ and θ between 0 and 1, the function ϕ is monotonous. Moreover, we notice that $\lim_{z_t \rightarrow 0} \phi(z_t) = +\infty$ and $\lim_{z_t \rightarrow +\infty} \phi(z_t) = 0$. These are sufficient to establish that the steady state

$$z = 1$$

is globally stable.

Given that z_t converges monotonically to 1, the equation (32) can be solved as

$$\ln k_{t+1} = (\alpha(1-\theta))^{t+1} \ln k_0 + \sum_{i=0}^t (\alpha(1-\theta))^i \nabla_{t-i} + \text{constant}$$

where

$$\nabla_t = \ln \left(\frac{1+z_t}{(1+\zeta z_t)^{1-\theta} + z_t(1+\zeta z_t^{-1})^{1-\theta}} \right)$$

converges to a constant. Hence, the dynamics in k_t converges too. The steady state

$$k = \left(\frac{\beta}{\psi\gamma^\theta} \left(\frac{1-\alpha}{1+\beta+\gamma} \right)^{1-\theta} \frac{1}{(1+\zeta)^{1-\theta}} \right)^{\frac{1}{1-\alpha(1-\theta)}}$$

is thus globally stable.

Finally, the growth rate converges to

$$g = (\psi\gamma^\theta)^{\frac{1-\alpha}{1-\alpha(1-\theta)}} \beta^{\frac{\alpha\theta}{1-\alpha(1-\theta)}} \left(\frac{1-\alpha}{1+\beta+\gamma} \right)^{\frac{\theta}{1-\alpha(1-\theta)}} (1+\zeta)^{\frac{(1-\theta)(1-\alpha)}{1-\alpha(1-\theta)}}$$

By linearizing (31) around $z = 1$, we have

$$\hat{z}_{t+1} = \left(\frac{1-\zeta+2\theta\zeta}{1+\zeta} \right) \hat{z}_t$$

where hatted variables denote deviations from steady state. The speed of convergence is the approximated by equation (23) of the main text.

Federal funding: In the federal funding regime, the equilibrium is described by the following system

$$z_{t+1} = \left(\frac{z_t + \zeta}{1 + \zeta z_t} \right)^{1-\theta} \tag{33}$$

$$k_{t+1} = \frac{\beta}{\psi\gamma^\theta} \left(\frac{1-\alpha}{1+\beta+\gamma} \right)^{1-\theta} k_t^{\alpha(1-\theta)} \frac{2^\theta (1+z_t)^{1-\theta}}{(1+\zeta z_t)^{1-\theta} + z_t(1+\zeta z_t^{-1})^{1-\theta}} \tag{34}$$

The regional growth rate is given by

$$g_{A,t+1} = \psi \left(\frac{\gamma(1-\alpha)}{1+\beta+\gamma} \right)^\theta k_t^{\alpha\theta} \left(\frac{1+z_t}{2} \right)^\theta (1+\zeta z_t^{-1})^{1-\theta} \quad (35)$$

The global stability of this system is established in the same way as that of the preceding funding systems. The steady-state is then characterized by

$$z = 1$$

$$k = \left(\frac{\beta}{\psi\gamma^\theta} \left(\frac{1-\alpha}{1+\beta+\gamma} \right)^{1-\theta} \frac{1}{(1+\zeta)^{1-\theta}} \right)^{\frac{1}{1-\alpha(1-\theta)}}$$

$$g = (\psi\gamma^\theta)^{\frac{1-\alpha}{1-\alpha(1-\theta)}} \beta^{\frac{\alpha\theta}{1-\alpha(1-\theta)}} \left(\frac{1-\alpha}{1+\beta+\gamma} \right)^{\frac{\theta}{1-\alpha(1-\theta)}} (1+\zeta)^{\frac{(1-\theta)(1-\alpha)}{1-\alpha(1-\theta)}}$$

The linearization of (33) around $z = 1$ yields

$$\hat{z}_{t+1} = (1-\theta) \frac{1-\zeta}{1+\zeta} \hat{z}_t$$

and the speed of convergence is given by equation (27) of the main text.

Market funding : In the market funding system, the equilibrium is described by the following system

$$z_{t+1} = \frac{z_t + \zeta}{1 + \zeta z_t} \quad (36)$$

$$k_{t+1} = \left(\frac{\beta(1-\alpha)(1-\theta)}{(1+\beta)(1+\zeta) (\psi\theta^{\frac{1-\alpha}{\alpha}})^{\theta/(1-\theta)} \psi (1 + \psi\theta^{\frac{1-\alpha}{\alpha}})} \right)^{1-\theta} k_t^{\alpha(1-\theta)} \quad (37)$$

And the regional growth rate is given by

$$g_{A,t} = \psi \left(\theta\psi \frac{1-\alpha}{\alpha} \right)^{\frac{\theta}{1-\theta}} k_t^{\frac{\theta}{1-\theta}} (1 + \zeta z_{t-1}) \quad (38)$$

The global stability of this system is established in the same way as that of the preceding funding systems. The steady-state is then characterized by

$$z = 1$$

$$k = \left(\frac{\beta(1-\alpha)(1-\theta)}{(1+\beta)(1+\zeta) (\psi\theta^{\frac{1-\alpha}{\alpha}})^{\theta/(1-\theta)} \psi (1 + \theta^{\frac{1-\alpha}{\alpha}})} \right)^{\frac{1-\theta}{1-\alpha(1-\theta)}}$$

$$g = \psi^{\frac{(1-\theta)(1-\alpha)}{1-\alpha(1-\theta)}} \left(\theta\psi \frac{1-\alpha}{\alpha} \right)^{\frac{\theta(1-\alpha)}{1-\alpha(1-\theta)}} \left(\frac{\beta(1-\alpha)(1-\theta)}{(1+\beta)(1+\zeta) (1 + \theta^{\frac{1-\alpha}{\alpha}})} \right)^{\frac{\theta}{1-\alpha(1-\theta)}} (1 + \zeta)$$

By linearizing (36) around $z = 1$, we obtain

$$\hat{z}_{t+1} = \frac{1-\zeta}{1+\zeta} \hat{z}_t$$

and the speed of convergence is given by equation (30) of the main text.