Market Power, Human Capital andGrowth *

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Abstract

In this paper we study the economic determinants of the inter-sectoral distribution of skilled workers and the long-run consequences of imperfect competition on growth within an R&D-based growth model with human capital accumulation. We find that steady-state growth is driven only by incentives to accumulate human capital and is independent of scale effects. In the model imperfect competition has a positive growth effect, while influencing the allocation of human capital to the different economic activities. Contrary to general wisdom, high R&D investment is not always associated with high output growth.

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Introduction

Recent theoretical and empirical advancements in the endogenous growth literature suggest that not only Research and Development (R&D) activity, but also human capital accumulation is a primary determinant of economic development. However, while in the R&D-based growth models an important (and recent) research line has already investigated whether the presence of imperfect competition in the product market may be growth-enhancing or not,[1] this has not yet been done within an integrated economic growth model with R&D and human capital accumulation. Still, another issue that has been largely neglected by this branch of growth literature is the analysis of the economic determinants of the sectoral distribution of human capital when this factor input is allowed to grow over time.

The aim of this paper is to fill these gaps in the literature. In order to do this it combines in the simplest possible way the basic Lucas (1988) model of human capital accumulation with (a generalization of) the Grossman and Helpman (1991, Ch. 3, pp. 43/57) model of endogenous technical change without knowledge spillovers. The reason why we concentrate on this last peculiar model is simple. We are interested in studying the nexus between market power (measured by the elasticity of substitution among different varieties of capital goods) and growth and the main forces underlying the inter-sectoral allocation of skilled workers within an economy where the true lever to economic development is schooling investment and not the R&D externality.[2]

In more detail, we consider a model economy that is made up of a representative household and firms. For simplicity purposes the representative household consists of only one agent who is involved in four types of activities: consumption goods production, intermediate goods manufacturing, human capital investment and R&D effort. Population is stationary in the economy and consumption goods are produced within a perfectly competitive environment in which prices are taken as given and each input is compensated according to its own marginal product. The intermediate goods sector consists of monopolistic producers of differentiated products entering

[1] See, among others, Aghion and Howitt (1996, 1997, 1998a,b), Aghion *et al.* (1997a,b), van de Klundert and Smulders (1995, 1997), Bucci (2002a).

[2] What happens to the *market power/growth nexus* and the *inter-sectoral allocation of human capital* in a model where there is no human capital accumulation and the true engine of growth is represented by the externality in the R&D activity is deeply studied in Bucci (2002a).

the production function of the homogeneous final good as an input. The representative household invests portions of its fixed-time endowment to acquire formal education. Finally, in the model purposive R&D activity is the source of technological progress. Technical progress happens, indeed, through inventing new varieties of horizontally differentiated capital goods within a separate and competitive R&D sector. When a new blueprint is discovered in the R&D sector, an intermediate-goods producer acquires the perpetual patent over it. This allows the intermediate firm to manufacture the new variety and practice monopoly pricing forever. A peculiarity of this economic system is that all the sectors do employ skilled workers. This is done because, as already mentioned, it is one of our objectives to study in detail the economic determinants of the inter-sectoral distribution of human capital when this factor input is allowed to grow over time, thus spurring economic growth.

Our main findings are threefold. First of all, as in the basic Lucas (1988) model, human capital accumulation depends on the parameters describing preferences and the skill acquisition technology. However, unlike Lucas (1988), the presence of imperfect competition conditions in the intermediate products market both has growth effects and influences the allocation of the available human capital stock to the different sectors employing this input. Secondly, as it is common in recent endogenous growth theory,[3] our model does not display any *scale effect*, since growth does not depend on the total available human capital stock. Finally, we find that the relationship between the equilibrium output growth rate and the share of resources (human capital) invested in R&D activity is absolutely non-monotonic and crucially depends on the productivity of the *schooling technology*.

Our analysis here is related to other works, both in its scope and its methodological approach. Arnold (1998) also develops an endogenous growth model that integrates purposive R&D activity with human capital accumulation and where the true engine of growth is represented by the investment in schooling. But his work is mainly motivated by the attempt of rejecting, on theoretical grounds, two main predictions of standard growth models based on R&D (namely that the equilibrium growth rate is very much sensitive to policy changes and to the level of resources used in research). Blackburn *et alii* (2000) extend Arnold's model in the direction of a fuller micro-foundation of the R&D process and obtain the same results with no further new insights.

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[3] See Eicher and Turnovsky (1999) for a detailed discussion on *non-scale models of economic growth*.

However, both these two works do not deal at all with the determinants of the inter-sectoral allocation of skilled workers and with the long-run influences of imperfect competition on growth. Indeed, in Blackburn *et alii* (2000) intermediate firms do not employ directly human capital, since they use forgone consumption to produce. This is the main reason why we consider that framework as truly inadequate to answer the questions we would like to answer in this paper.

To the best of our knowledge, and within a similar framework, this is one of the first attempts in this direction. In this respect, we should probably mention a recent paper by Jones and Williams (2000) aimed at analysing whether a decentralised economy undertakes too little or too much R&D in the presence of some distortions to the research activity.[4] Still, in this paper there is no human capital accumulation, no evaluation of the possible long-run links between (im)perfect competition and growth and capital goods and research are produced devoting units of foregone consumption. Finally, other two works that come closer to ours are Bucci (2001, 2002b). The main difference with respect to the first of these two papers is that the present contribution endogenises the shares of human capital devoted to each economic activity (that are kept exogenous in that approach), whereas the difference with respect to the second one is that this paper does represent a simple generalization of it (encompassing it as a special case).[5]

The rest of the paper is organised as follows. Section 1 introduces the basic model and in Section 2 we study the general equilibrium of it. Section 3 examines the steady-state properties of the economy under investigation and in Section 4 we compute the equilibrium output growth rate. In Section 5, we solve for the inter-sectoral allocation of human capital, present some comparative statics results about the main economic determinants of the shares of the reproducible input

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[4] These distortions are represented respectively by the *surplus appropriability problem,* the presence of *knowledge spillovers* and the *creative destruction* and *congestion externalities*.

[5] Other works that take explicitly into account the interaction between endogenous technological change and human capital formation are Stokey (1988) and Young (1993), Grossman and Helpman (1991, Ch.5.2), and Eicher (1996), Redding (1996) and Restuccia (1997). Though, all these remain limited in many respects. In the first two (Stokey, 1988 and Young, 1993), for example, skill accumulation happens through learning-by-doing and on-the-job-training in the production activity, rather than a separate education sector. In Grossman and Helpman (1991, Ch.5.2), a separate education sector does exist but, strangely enough, it does not require any skilled worker to operate. Eicher (1996) develops a rich model in which both human capital and technological innovation are endogenous. However, this paper is solely concerned with steady-state predictions on the relationship between relative supply of skilled labour and relative wage. Restuccia (1997), on the other hand, builds a dynamic general equilibrium model with schooling and technology adoption. But the primary concern of the paper is to study how these two elements may be amplifying the effects of productivity differences on income disparity. Finally, Redding (1996) emphasises the potential interaction between investment in education and investment in research and shows under which conditions such an interaction may give rise to co-ordination problems and under-development traps.

(skilled work) devoted to each economic activity and briefly discuss the most important findings. Section 6 presents the results concerning the steady-state predictions of the model concerning the relationship between the type of production functions (employed in the downstream sector), the sectoral distribution of human capital, imperfect competition and growth in some selected cases and finally Section 7 concludes.

1. The Model Economy

Consider an economy with three different productive sectors. There exists an undifferentiated consumers good, which is produced using skilled labour and capital goods (intermediate inputs). These are available, at time *t*, in n_t different varieties. In order to produce such inputs, intermediate firms employ only human capital. Technical progress takes place as a continuous expansion, through purposive Research and Development (R&D) activity, of the set of available horizontally differentiated intermediates. R&D is skill intensive as well. Unlike the traditional *R&D-based growth models*, I assume that the supply of human capital may grow over time. In this connection, following the pathbreaking papers by Uzawa (1965) and Lucas (1988), I postulate the existence of a representative household that chooses plans for consumption (*c*), asset holdings (*a*, to be defined later) and human capital (*h*). For the sake of simplicity, I also assume that the representative household of this economy has unit measure. In the model there is no physical capital and unskilled labour. Human capital is a homogeneous input and can be employed to produce the final output, intermediates, new human capital and to invent new varieties of capital goods (research).

1.1 The Consumers Good Sector

The homogeneous, undifferentiated consumers good is produced within a *competitive industry*. Such an industry is populated by a large number of identical firms and employs the following constant returns to scale aggregate production function:

(1) $$Y_t = AH_{Yt}^{1-l}\left[\int_0^{n_t}(x_{jt})^a \, dj\right]^{\frac{l}{a}}, \qquad A>0, \quad 0 \le l \le 1, \quad 0 \le \frac{l}{1+l} < a < 1.$$

As in Bucci (2002a), we have written the production technology in use in the downstream sector in such a way to disentangle the (equilibrium) monopolistic mark-up set in the intermediate sector and the degree of returns from specialization.[6] Another reason why we employ the production function of equation (1) is that this technology allows to encompass as particular cases (and depending on the value of l) two recent models of endogenous growth[7] (one of which is not *R&D-based*) that in their original version do not include human capital accumulation. Even with respect to these models we are interested to study in this paper their potential implications (as for the monopoly power/growth relationship and the sectoral distribution of human capital) when a positive supply of skilled workers is explicitly introduced in them.

Unlike Bucci (2002a), we take here a different view by considering an economy where the true lever to economic development is human capital accumulation (and not the R&D externality). In addition, another important difference with that paper is that we want to find out here the equilibrium inter-sectoral allocation of human capital in terms of *shares* (and not *stocks*) of this (growing) factor input being devoted to each sector in the steady-state and analyse its main economic determinants.

According to equation (1), output at time t (Y_t) is obtained combining skilled work (H_{Yt}) and n different varieties of intermediate inputs, each of which is employed in the quantity x_j. a, l and A are technological parameters. The latter (total factor productivity) is strictly positive, whereas l is (not strictly) between zero and one. The restriction on a assures that in a symmetric equilibrium the istantaneous profit accruing to a generic intermediate producer at a certain point in time is inversely related to the number of varieties existing at the same date.

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6 This point is made clear by Benassy (1998, p. 63). Indeed, in a moment we will show that (under additional assumptions) the mark-up charged over the marginal cost by the monopolistic producers of intermediate inputs is $1/a$. At the same time, from equation (1), it is possible to see that in a symmetric equilibrium (in which the total production of intermediates, X, is spread evenly between the n brands) the degree of returns to specialization (the exponent of n) is equal to $l(1/a-1)$. This one is clearly different from the monopoly power measure and, more importantly, depends not only on a but also on l. In this sense, the model we present here is a simple extension of Bucci (2002b).

7 Namely the Rebelo's (1991) and Grossman and Helpman's (1991, Ch. 3, pp. 43/57) models.

Since the industry is competitive, in equilibrium each variety of intermediates receives its own marginal product (in terms of the *numeraire* good, the final output):

$$(2) \qquad p_{jt} = A\, l H_{Yt}^{1-l} \left[\int_0^{n_t} (x_{jt})^a \, dj \right]^{\frac{l}{a}-1} (x_{jt})^{a-1} .$$

In equation (2) p_{jt} represents the inverse demand function faced, at time *t*, by the generic *j-th* intermediate producer. As it is common in the first generation *innovation-based* growth literature, the elasticity of substitution between two generic intermediates coincides with the price-demand elasticity faced by each capital goods producer and is equal to $1/(1-a)$.[8]

1.2 The Intermediate Goods Sector

The capital goods industry is *monopolistically competitive* and each intermediate input is produced using the same technology:

$$(3) \qquad x_{jt} = B \cdot h_{jt} , \qquad \forall j \in (0, n_t), \qquad \text{B>0.}$$

This production function is characterised by constant returns to scale in the only input employed (human capital) and, according to it, one unit of skills is able to produce (at each time) the same constant quantity of whatever variety. *B* measures the productivity of human capital employed in this sector. Following Romer (1990) and Grossman and Helpman (1991, Chap.3), we continue to assume that each intermediate good embodies a design created in the R&D sector and that there exists a patent law which prohibits any firm from manufacturing an intermediate good without the consent of the patent holder of the design.

The generic *j-th* firm maximises (with respect to x_{jt}) its own instantaneous profit function under the (inverse) demand constraint (equation (2)).

[8] This result is obtained under the specific assumption that each firm producing intermediate inputs is so small that a marginal increase in the quantity of output it produces does not change the quantities produced by its market rivals and, then, total intermediate output.

Continuing to assume that in the intermediate sector there exists no strategic interaction among firms, the resolution of this maximisation program gives the optimal price set by the generic *j-th* intermediate producer for one unit of its own output:

(4) $$p_{jt} = \frac{1}{a}\frac{w_{jt}}{B}.$$

From equations (4) and (2), the wage rate accruing at time t to one unit of human capital employed in the capital goods sector (w_{jt}) is equal to:

(4') $$w_{jt} = ABa\,lH_{Yt}^{1-l}\left[\int_0^{n_t}(x_{jt})^a\,dj\right]^{\frac{l}{a}-1}(x_{jt})^{a-1}.$$

In a symmetric equilibrium (where $x_{jt} = x_t$, $\forall j \in (0, n_t)$), each local monopolist faces the same wage rate ($w_{jt} = w_t$, $\forall j \in (0, n_t)$) and equation (4) can be recast as:

(4'') $$p_{jt} = \frac{1}{a}\frac{w_{jt}}{B} = \frac{1}{a}\frac{w_t}{B} = p_t, \qquad \forall j \in (0, n_t).$$

The hypothesis of symmetry is dictated by the way through which each variety of intermediates enters the final output technology and by the fact that all the capital goods producers use the same production function (equation (3)).

Hence, when all the capital goods firms are identical, they produce the same quantity (x_t), face the same wage rate accruing to intermediate skilled workers (w_{jt}) and fix the same price for one unit of their own output. The price is equal to a constant *mark-up* ($1/a$) over the marginal cost (w_{jt}/B). In equilibrium the wage rate accruing to one unit of human capital employed in the intermediate sector (w_{jt}) will be the same (and equal to w_t) for all the sectors where this factor input is employed. This is due to the hypothesis that skilled workers are homogeneous in this model economy (they have the same productivity irrespective of the economic activity they perform).

Defining by $H_{jt} \equiv \int_0^{n_t} h_{jt}\,dj$ the total amount of human capital employed in the intermediate sector at time t and under the assumption of symmetry among capital goods producers ($x_{jt} = x_t$, $\forall j \in (0, n_t)$) we obtain:

$$(5) \qquad x_{jt} = \frac{B \cdot H_{jt}}{n_t} = x_t, \qquad \forall j \in (0, n_t).$$

Finally, the instantaneous profit function of a generic *j-th* intermediate firm will be:

$$(6) \qquad p_{jt} = \left(p_t - \frac{1}{B} w_t \right) \cdot x_t = A\, l(1-a) \cdot H_{Yt}^{1-l} \cdot (n_t)^{\frac{l-a}{a}} \left(\frac{B \cdot H_{jt}}{n_t} \right)^{l} = p_t, \qquad \forall j \in (0, n_t).$$

Since we are dealing with a *monopolistic competition market*, p will be decreasing in n (the number of intermediate firms existing at time *t*) if and only if $a > l/1+l$. This explains the restriction on a we have introduced in equation (1).

Equation (6) says that, just as *x* and *p*, so too the instantaneous profit is equal for each variety of intermediates in a symmetric equilibrium.

1.3 The Research Sector

Producing the generic *j-th* variety of capital goods entails the purchase of a specific blueprint (the *j-th* one) from the competitive research sector, characterised by the following aggregate technology:

$$(7) \qquad \dot{n}_t = C \cdot H_{nt}, \qquad C>0,$$

where n_t denotes the number of capital goods varieties existing at time *t*, H_n is the total amount of human capital employed in the sector and *C* is the productivity of the research skilled workers. The production function of new ideas coincides with the one employed by Grossman and Helpman (1991) in their Chapter 3 model ***without knowledge spillovers*** (pp.43-57). It implies that in the present framework knowledge is a completely *private good* (the returns to the discovery of a new blueprint are fully appropriated by the inventor[9]) and *non-cumulative* (in order

[9] Indeed, in our model it is assumed that there exists a perfect patent protection system (based on infinitely-lived patents) that allows the successfull innovator to accrue the whole gains from his/her R&D efforts and prevents someone from imitating (or even from innovating around the original innovation). See Bucci and Saglam (2000) for a model of endogenous growth where patent lifetime is not infinite.

to invent new varieties of intermediate inputs only skilled labour is employed and not also the available stock of disembodied technological capital approximated by the existing number of designs). In the original Grossman and Helpman's model without knowledge spillovers this specification of the R&D process implies the cessation of growth in the log run. In our model, instead, this does not happen since in our economy the true engine of growth is represented by human capital accumulation (and not by the R&D externality). In this sense the model we present here shares the same conclusions of many other models with purposive R&D activity and skills accumulation.[10] At the same time we think that concentrating our attention in the main text on the simple R&D technology of equation (7) helps in avoiding potential complications concerning the nature of knowledge spillovers, the way these last are to be more appropriately modelled and the problem of technology diffusion. In this respect, it is well known that the empirical (IO) literature on knowledge spillovers is almost unbounded and particularly controversial. Griliches (1992), for example, supports the idea that such spillovers are quite important in the R&D process. On the other hand, the surveys by Keely and Quah (1998) and Keely (2001) cast many doubts on the strength of R&D spillovers in real life (both at the micro and macro level).[11] On the basis of such inconclusive empirical results it is a fair conclusion that our hypothesis on the R&D capital aggregate production function seems to be as extreme as the (opposite) one of immediate and instantaneous spillovers in innovation activity and generally adopted by the pathbreaking *R&D-based growth models* (Romer, 1990; Grossman and Helpman, 1991, Chap. 3, pp. 57-65; Aghion and Howitt, 1992).

As the research sector is competitive, new firms will enter it till when all profit opportunities will be completely exhausted. The zero profit condition amounts, in this case, to set:

(8) $$\frac{1}{C} w_{nt} = V_{nt}$$

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10 Notably Arnold (1998) and Blackburn *et al.* (2000).

11 At the micro level Keely and Quah (1998) conclude that *"…knowledge spillovers do occur. However, the physical clustering of innovation suggests that spillovers do not happen automatically or completely"* (p. 24). At the macro level, instead, they notice that *"…spillovers across regions do occur. At the same time, however, these spillovers are generally incomplete"* (p. 25). According to Keely (2001): *"…Although in principle a patent's information spills over to other firms, there is a large empirical literature that suggests such spillovers are in practice neither so immediate nor widespread"* (p.5).

(9) $$V_{nt} = \int_{t}^{\infty} \exp\left[-\int_{t}^{t} r(s)ds \right] p_{jt}\, dt\,, \qquad t > t\,.$$

Symbols used in equations (8) and (9) have the following meaning: w_n is the wage rate accruing to one unit of human capital devoted to research; the term $\exp\left[-\int_{t}^{t} r(s)ds \right]$ is a present value factor which converts a unit of profit at time t into an equivalent unit of profit at time *t; r* is the real rate of return on the consumers' asset holdings; p_j is the profit accruing to the *j-th* intermediate producer (once the *j-th* infinitely-lived patent has been acquired) and V_n is the market value of one unit of research output (the generic *j-th* idea allowing to produce the *j-th* variety of capital goods). Notice that V_n is equal to the discounted present value of the profit flow a local monopolist can potentially earn from *t* to infinity and coincides with the market value of the *j-th* intermediate firm (since there is a one to one relationship between number of patents and number of capital goods producers).

1.4 Consumers

We consider a closed economy in which there exists only one representative infinitely-lived household that holds assets in the form of ownership claims on firms and chooses plans for consumption (*c*), asset holdings (*a*) and human capital (*h*). For the sake of simplicity, I assume that the only household populating this economy has unit measure and there is no population growth. This hypothesis implies that, at each *t*, the household's own stock of human capital (*h*) equals the economy aggregate stock of this factor input (*H*). Following Lucas (1988), we also assume that the household is endowed with one unit of time and optimally allocates a fraction *u* of its time endowment to productive activities (research, capital goods and consumer goods production) and the remaining fraction (*1-u*) to non-productive activities (schooling). As it will be clearer later on, given the household's choice of the optimal *u* (that we denote by *u**), the labour market clearing conditions will determine the decentralised allocation of the productive human capital between manufacturing of intermediate and consumers goods and invention of new ideas (research).

With an instantaneous utility function $u(c_t) = \log(c_t)$, the decision problem of the household can be written as:

(10) $$\underset{\{c_t,u_t,a_t,h_t\}_{t=0}^{\infty}}{Max} U_0 \equiv \int_0^{\infty} e^{-rt} \log(c_t)dt \ , \qquad r > 0$$

s.t.:

(11) $$\dot{a}_t = r_t a_t + w_t u_t h_t - c_t$$

(12) $$\dot{h}_t = d(1-u_t)h_t , \qquad d > 0$$

a_0, h_0 given.

The control variables of this problem are c_t and u_t, whereas a_t and h_t are the state variables. Equation (10) is the intertemporal utility function; equation (11) is the budget constraint and equation (12) represents the human capital supply function.[12] The symbols used have the following meaning: r is the subjective discount rate; c denotes consumption of the homogeneous final good; w is the wage rate accruing to one unit of skilled labour[13] and d is a parameter reflecting the productivity of the education technology.

With m_{1t} and m_{2t} denoting respectively the shadow price of the household's asset holdings and human capital stock, the first order conditions are:

(13) $$\frac{e^{-rt}}{c_t} = m_{1t}$$ (14) $$m_{1t} = m_{2t}\frac{d}{w_t}$$

(15) $$m_{1t} r_t = -\dot{m}_{1t}$$ (16) $$m_{1t} w_t u_t + m_{2t} d(1-u_t) = -\dot{m}_{2t} .$$

Equation (13) gives the discounted marginal utility of consumption, which satisfies the dynamic optimality condition in equation (15). Equation (14) is the static optimality condition for the allocation of time, equating the marginal benefit and the marginal cost of an additional unit of skills devoted to working. The marginal cost involves the cost associated with future reductions in human capital, as expressed by the other dynamic optimality condition in equation (16).

[12] Notice that we assume no depreciation for human capital. This hypothesis is completely harmless in the present context and serves the scope of simplifying the analysis. Also notice that we consider the variant of the basic Lucas model (1988) in which spillovers from education are internalised. This is done because we are explicitly assuming that there is only one household (of unit measure) within this economy and population is stationary.

[13] In equilibrium there exists only one wage rate accruing to skilled workers since human capital is homogeneous.

Conditions (13) through (16) must satisfy the constraints (11) and (12), together with the transversality conditions:

$$\lim_{t\to\infty} m_{1t} a_t = 0 ; \qquad \lim_{t\to\infty} m_{2t} h_t = 0$$

2. General Equilibrium

In order to find out the equilibrium of the model under the symmetry hypothesis $(x_{jt} = B \cdot H_{jt} / n_t = x_t , \ \forall j \in (0, n_t))$ first notice that, for given *u** (the optimal fraction of skills devoted by consumers to production activities), the optimal allocation of human capital among research, capital and consumers goods production is found solving simultaneously the following labour market clearing conditions:

(17) $H_Y + H_j + H_n = u * H , \qquad \forall t$

(18a) $w_j = w_n$

(18b) $w_j = w_Y$

Since human capital is perfectly homogeneous in the model, we impose that: 1) it is paid the same wage rate across all the productive sectors where this input is employed (equations (18a) and (18b)); 2) the sum of the human capital stocks allocated to each market is equal to the total stock of productive human capital available at time *t* (equation (17)).

Since the total value of the household's assets must equal the total value of firms, the following condition must be checked in a symmetric equilibrium:

(19) $a = nV_n$

where V_n is given by (9) and satisfies the following asset pricing equation:

(19a) $\dot{V}_n = rV_n - p_j$

with:

(19b) $p_j = \frac{l}{1-l}(1-a)\frac{H_Y w_Y}{n}$, and (19c) $w_Y = \frac{A(1-l)}{H_Y^l} n^{\frac{l}{a}} \cdot \left(\frac{BH_j}{n}\right)^l$.

Recall that one new idea allows a new intermediate firm to produce one new variety of capital goods. In other words, there exists a one-to-one relationship between number of ideas, number of capital goods producers and number of intermediate input varieties. This explains why, in equation (19), the total value of the household's assets (*a*) is equal to the number of profit-making intermediate firms (*n*) times the market value (V_n) of each of them (equal to the market value of the corresponding idea). Equation (19a) simply suggests that the interest on the value of the *j-th* generic intermediate firm (rV_n) should be equal, in equilibrium, to the sum of two terms:

- the instantaneous monopoly profit (p_j) coming from the production of the *j-th* capital good;
- the capital gain or loss matured on V_n during the time interval *dt* ($\dot{V}_n$).

We can now move to the steady-state equilibrium.

3. Steady State Equilibrium

In this paragraph we characterise the steady state (or balanced growth path) equilibrium of the model. We first start with a formal definition of balanced growth path equilibrium:

Definition: Balanced Growth Path (or Steady-State) Equilibrium

A balanced growth path (or steady state) equilibrium is an equilibrium where the growth rate of all the variables depending on time is constant, human (H) and knowledge (n) capital are complements ($R \equiv H_t / n_t$ is constant at each time *t*) and H_Y, H_j, H_n all grow at the same constant rate as H.[14]

[14] The hypothesis that human and technological capital are complements (the value of the ratio $R = H_t / n_t$ remains invariant along the balanced growth path) may be justified on both theoretical and empirical grounds. From the theoretical point of view, Redding (1996) clearly shows that the complementarity relationship between skilled workers and technology does represent a crucial element in explaining the existence of poverty traps in many less developed countries, due to the joint presence of low levels of skills and R&D investment in these areas. He also shows that, under particular conditions, the complementarity hypothesis between human capital and R&D is also responsible for the existence of multiple steady states. On the empirical side, instead, many contributions claim the relevance of the skill-technology connections even at the sectoral level (Goldin and Katz, 1998), whereas de la Fuente and da Rocha (1996) also find evidence of strong complementarities between human capital stock and investment in R&D for the OECD countries.

With this definition of balanced growth path, when g_H (the growth rate of H) is constant u is constant as well (see equation (12)).[15] This means that, along a balanced growth path, the household will optimally decide to devote a constant fraction of its fixed time endowment to working (u*) and education (1-u*) activities. Solving explicitly the consumers' problem, it is possible to show that the following results do hold in the long-run equilibrium (see the Appendix for mathematical details):

(20) $$r = \frac{a\, d + l(1-a)(d-r)}{a};$$

(21) $$g_{H_Y} = g_{H_j} = g_{H_n} = g_n = g_H = d - r\ ;$$

(22) $$g_c = g_a = (d-r)\left[\frac{a + l(1-a)}{a}\right];$$

(23) $$\frac{H_j}{n} = \frac{a\, d}{C(1-a)}$$

(24) $$\frac{H_Y}{n} = \frac{(1-l)\, d}{lC(1-a)}$$

(25) $$u^* = \frac{r}{d}.$$

According to result (20), the real interest rate (*r*) is constant. Equation (21) states that along a balanced growth path, the number of new ideas (*n*), the household's total human capital stock (*H*) and the human capital stocks devoted respectively to the consumers good production (H_Y), to the intermediate sector (H_j) and to research (H_n) all grow at the same constant rate, given by the difference between the schooling technology productivity parameter (d) and the subjective discount rate (r). Equation (22) gives the equilibrium growth rate of consumption and household's asset holdings. Equations (23) and (24), instead, give respectively the equilibrium values of the constant H_j / n and H_Y / n ratios, whereas equation (25) represents the optimal constant fraction of the household's time endowment that it will decide to allocate to working activities (*u**). For the growth rate of the variables in equations (21) and (22) to be positive and bounded, d should be strictly greater than r and bounded. The condition $d > r$ also assures that $0 < u^* < 1$.

[15] As already said in paragraph 1.4, given the assumptions on the size of the representative household and the population growth rate, $h \equiv H$ (which implies that we can use g_H instead of g_h).

4. Endogenous Growth

To compute the output growth rate of this economy in a symmetric, balanced growth equilibrium, first rewrite equation (1) as follows:

$$Y_t = AH_{Yt}^{1-l} n_t^{\frac{l}{a}} \left(\frac{B \cdot H_{jt}}{n_t} \right)^l = \Psi H_{Yt}^{1-l} n_t^{\frac{l}{a}} , \qquad \Psi \equiv A \left(\frac{B \cdot H_{jt}}{n_t} \right)^l .$$

Then, taking logs of both sides of this expression, totally differentiating with respect to time and recalling that in the steady-state equilibrium $g_{H_Y} = g_n = g_H = d - r$ (see equation (21) above), we obtain:

$$\text{(1a)} \qquad \frac{\dot{Y}_t}{Y_t} \equiv g_Y = g_c = g_a = \left[\frac{a + l(1-a)}{a} \right] g_H = [1 + l(b-1)] \cdot (d - r) , \qquad b \equiv 1/a .$$

Hence, economic growth depends only on a (the inverse of which can be easily interpreted as a measure of the monopoly power enjoyed by each intermediate local monopolist), l (which represents the share of total income being devoted in a symmetric equilibrium to the purchase of all the available capital goods varieties[16]) and the accumulation rate of human capital (g_H). In this last respect the model supports the main conclusion of that branch of the endogenous growth literature pioneered by Uzawa (1965) and Lucas (1988).[17] As a consequence, and in line with this literature, our analysis does not display any scale effect, since g_Y depends neither on the absolute dimension of the economy (its total human capital stock), nor on the population growth rate (that, indeed, is equal to zero in our model).[18]

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16 $l \equiv \frac{\int_0^{n_t} (p_{jt} \cdot x_{jt}) dj}{Y_t}$.

17 Benhabib and Spiegel (1994), Islam (1995) and Pritchett (1996) all suggest that, unlike Lucas (1988), international differences in per-capita growth rates depend exclusively on differences in the respective human capital stocks each country is endowed with. However, Jones (1995a,b) points out that the *scale effect hypothesis* should be rejected on empirical grounds.

18 The *no-scale-effect prediction* is indeed shared by many other models (*e.g.* Kortum, 1997; Aghion and Howitt, 1998a, Chap. 12; Dinopoulos and Thompson, 1998; Peretto and Smulders, 1998; Segerstrom, 1998; Young, 1998; Howitt, 1999; Blackburn *et al.*, 2000; Bucci, 2001, among others). See Jones (1999) and Eicher and Turnovsky (1999) for recent surveys.

As it is usual in this class of (symmetric) growth models with horizontal product differentiation, in the expression for the equilibrium growth rate of output the term $l(b-1)$ measures the *returns to specialization*. Notice that such returns depend (positively) not only on b (the monopoly power), but also on l. The intuition behind this result is simple: the higher the mark-up rate that can be charged over the marginal cost in the monopolistic sector and the share of national income spent on the intermediate inputs, the higher the returns an intermediate producer may obtain from specialising in the production of the marginal variety of capital goods. Additionally, it is probably worth pointing out that b enters the equilibrium growth rate when (and only when) l is not equal to zero (i.e. when capital goods are an input in the production of the final good). This is clear when one considers that the only product market where imperfect competition prevails is the intermediate one in the model.

Since I am particularly interested in analysing those factors potentially able to influence the *inter-sectoral competition* for the acquisition of human capital in the present context, we have first to determine an expression for the equilibrium human to technological capital ratio ($R \equiv H/n$). At this aim, we use equation (17), with $u^* = r/d$, $H_j/n = a\,d/(1-a)C$ and $H_Y/n = (1-l)\,d/\,lC(1-a)$, and obtain:

(26) $$\frac{H_n}{n} = R\frac{r}{d} - \frac{a\,d}{C(1-a)} - \frac{(1-l)\,d}{(1-a)\,lC} \quad \Rightarrow$$

(26') $$g_n = C\frac{H_n}{n} = RC\frac{r}{d} - \frac{a\,d}{(1-a)} - \frac{(1-l)\,d}{l(1-a)}.$$

Equating the last expression above to equation (21) yields:

(27) $$R \equiv \frac{H_t}{n_t} = \frac{d\left[d - lr(1-a)\right]}{lr(1-a)C}, \quad \forall t.$$

In the next section, I compute the equilibrium shares of human capital devoted to research (s_n), capital goods production (s_j), final good manufacturing (s_Y) and human capital accumulation (s_H).

5. Human Capital, R&D and Growth

Given R, the shares of human capital devoted to each sector employing this factor input in the decentralised long-run equilibrium are the following:

(28) $$s_j \equiv \frac{H_j}{H} = \frac{H_j}{n} \cdot \frac{n}{H} = \frac{H_j}{nR} = \frac{a\, lr}{d - lr(1-a)};$$

(29) $$s_Y \equiv \frac{H_Y}{H} = \frac{H_Y}{n} \cdot \frac{n}{H} = \frac{H_Y}{nR} = \frac{r(1-l)}{d - lr(1-a)};$$

(30) $$s_n \equiv \frac{H_n}{H} = \frac{H_n}{n} \cdot \frac{n}{H} = \frac{H_n}{nR} = \frac{lr(d-r)(1-a)}{d[d - lr(1-a)]}, \qquad \frac{H_n}{n} = \frac{d-r}{C};$$

(31) $$s_H \equiv \frac{H_H}{H} = 1 - u^* = \frac{d-r}{d}.$$

5.1. Some Comparative Statics Results

From equation (28) it is possible to state the following comparative statics results (in this section I'll assume $g_H > 0$, which implies $d > r > 0$, and $0 < l < 1$. Later on I will analyse what happens to the main variables of the model under the special cases $l = 0$ and $l = 1$):

(28a) $\frac{\partial s_j}{\partial a} > 0$; (28b) $\frac{\partial s_j}{\partial d} < 0$; (28c) $\frac{\partial s_j}{\partial r} > 0$ (28d) $\frac{\partial s_j}{\partial l} > 0$.

Equations (28a) through (28d) say that the equilibrium share of human capital devoted to capital goods production depends negatively on the human capital accumulation productivity parameter (d) and positively on a , the subjective discount rate (r) and the share of total income devoted to the purchase of intermediate inputs. I am particularly interested in investigating the impact that the monopoly position enjoyed by each local intermediate producer (and measured by the elasticity of substitution among capital goods) may have on the main variables of the model in the long-run equilibrium. At this aim, first notice that $1/a$ does represent, as already mentioned, the optimal mark-up rate charged over the marginal cost by the intermediate producers and accordingly it can be used as a measure of the degree of monopoly power present in the uncompetitive sector. The higher a , the higher the elasticity of substitution between two generic intermediate inputs (equal to $1/(1-a)$). This means that they become more and more alike when a grows and, as a consequence, the price elasticity of the derived demand curve faced by a local monopolist (equal, again, to $1/(1-a)$) tends to be infinitely large when a tends to one. In a word, the toughness of competition in the intermediate sector is strictly (and positively) depending on the level of a. Conversely, the inverse of a ($1/a$), can be viewed as a proxy for how uncompetitive the sector is.

Intuitively, what equation (28a) tells us is that when a increases, the degree of competition within the capital goods market increases and, then, the aggregate intermediate output and the human capital demand coming from this sector do increase as well (s_j goes up). Therefore, a re-allocation of the available human capital among all the sectors employing this input does happen.

As for the equilibrium share of human capital devoted to the consumers good sector, we conclude that:

(29a) $\frac{\partial s_Y}{\partial a} < 0$; (29b) $\frac{\partial s_Y}{\partial d} < 0$; (29c) $\frac{\partial s_Y}{\partial r} > 0$; (29d) $\frac{\partial s_Y}{\partial l} < 0$.

Hence, unlike what happens for s_j, now an increase in the mark-up rate does increase the decentralised equilibrium share of human capital devoted to the production of the final good. Again, the economic intuition behind this result is quite simple: an increase in the mark-up rate (and in this way in the price) of all the intermediate inputs, *ceteris paribus*, makes it more profitable for the final good producers to substitute human capital for capital goods. As a consequence, the demand for this factor input (H_Y) increases and, for given total human capital stock, s_Y increases as well. The effects of r and d on s_Y are exactly the same as those found on s_j. Finally, an increase in l pushes unambiguously s_Y down. This happens because when l rises, then the investment (in terms of human capital sectoral shares) in the capital goods and R&D sectors grows up, whereas it remains unchanged in the education sector (as we will show in a moment).

Coming now to the comparative statics results for the equilibrium share of human capital devoted to R&D activity (s_n), we find that:

$$\text{(30a)}\quad \frac{\partial s_n}{\partial a} < 0,$$

This means that the impact that the intermediate sector monopoly power exerts upon s_n is unambiguously positive. This result is intuitively explained as follows: a higher mark-up increases, *ceteris paribus*, the flow of profits accruing to intermediate producers, which in turn increases the market value of one unit of research output (a new design), raising the share of the available human capital devoted to R&D. Thus, this paper extends one of the main results of Jones and Williams (2000)[19] to a framework where human capital may be accumulated over time and all the sectors employ skilled workers.

On the contrary, concerning the effect that d and r do have on s_n, it is possible to see that it is not unambiguous and crucially depends on the absolute value of the human capital accumulation productivity parameter (d). Indeed:

$$\text{(30b)}\quad \frac{\partial s_n}{\partial d} > 0 \quad \text{and} \quad \text{(30c)}\quad \frac{\partial s_n}{\partial r} < 0 \quad \text{when} \quad r < d < r\left(1+\sqrt{1-l(1-a)}\right);$$

$$\text{(30b')}\quad \frac{\partial s_n}{\partial d} < 0 \quad \text{and} \quad \text{(30c')}\quad \frac{\partial s_n}{\partial r} > 0 \quad \text{when} \quad d > r\left(1+\sqrt{1-l(1-a)}\right).$$

In addition:

$$\text{(30d)}\quad \frac{\partial s_n}{\partial l} > 0.$$

This result seems particularly intuitive since it says that when the share of total income which goes to the purchase of capital goods increases, then the share of resources devoted to R&D (in order to invent new varieties of intermediate inputs) increases as well.

The comparative statics results for s_H and R are as follows:

$$\text{(31a)}\quad \frac{\partial s_H}{\partial a} = 0; \qquad \text{(31b)}\quad \frac{\partial s_H}{\partial d} > 0; \qquad \text{(31c)}\quad \frac{\partial s_H}{\partial r} < 0; \qquad \text{(31d)}\quad \frac{\partial s_H}{\partial l} = 0$$

$$\text{(27a)}\quad \frac{\partial R}{\partial a} > 0; \qquad \text{(27b)}\quad \frac{\partial R}{\partial d} > 0; \qquad \text{(27c)}\quad \frac{\partial R}{\partial r} < 0; \qquad \text{(27d)}\quad \frac{\partial R}{\partial l} < 0;$$

$$\text{(27e)}\quad \frac{\partial R}{\partial C} < 0.$$

[19] In Jones and Williams (2000) there is no human capital accumulation and the inter-sectoral competition for the same resource (foregone consumption) is restricted to the intermediate and research sectors. However, as in the present model, in Jones and Williams (2000) the mark-up is determined by the elasticity of substitution between intermediate capital goods, too.

The share of human capital devoted to education activity (s_H) is completely independent on a and l (depending only on the technological and preference parameters, respectively d and r). On the other side, the ratio of human to technological capital (R) depends negatively on C[20] and l and positively on a.[21]

Below I report a table that summarizes in a more compact way all the comparative statics results:[22]

	d	r	l	a	$R \equiv \frac{H}{n}$	s_n	s_j	s_Y	s_H	g_Y
$r < d < r\left(1+\sqrt{1-l(1-a)}\right)$	↑				+	+	-	-	+	+
$d > r\left(1+\sqrt{1-l(1-a)}\right)$	↑				+	-	-	-	+	+
$r < d < r\left(1+\sqrt{1-l(1-a)}\right)$		↑			-	-	+	+	-	-
$d > r\left(1+\sqrt{1-l(1-a)}\right)$		↑			-	+	+	+	-	-
$\forall\, l \in (0;1)$			↑		-	+	+	-	0	+
$\forall a \in (0;1)$				↑	+	-	+	-	0	-

Table 1: *Comparative Statics Results Summary*

[20] The higher the research human capital productivity, the higher the number of capital goods invented until a certain date, the lower R. A similar effect also explains the negative impact of l on R.

[21] R can also be written as $d \cdot s_H / C \cdot s_n$.We already know that when a increases s_n increases as well (whereas s_H does not change). At the same time, when a increases s_n decreases (whereas s_H does not change). From these effects the impact of l and a on R follows immediately.

[22] In the first and second row of Table 1 I see what happens to *R*, s_n, s_j, s_Y, s_H and g_Y when d increases and falls respectively in the following two intervals: 1) $r < d < r\left(1+\sqrt{1-l(1-a)}\right)$ and 2) $d > r\left(1+\sqrt{1-l(1-a)}\right)$. In rows number 3 and 4 I do the same with r. Finally, in the last two rows I analyse respectively what happens in the long run equilibrium to the main variables of the model when l and a do increase.

5.2. The inter-sectoral allocation of human capital: a discussion

Looking at Table 1 (first row), one can see that when $r < d < r\left(1+\sqrt{1-l(1-a)}\right)$, then *R*, s_n, s_H and g_Y are positively correlated to each other[23] and that there exists a negative relationship between s_n and s_H, on the one side, and s_j and s_Y, on the other. Clearly, this means that, within the above mentioned interval, when the productivity of human capital in the education sector increases, then the economy will allocate more resources to education and research and less resources to intemediates and final output production. This, in turn, has the effect to boost economic growth (which in the model depends on the investment in schooling) and to make human capital relatively abundant with respect to technological capital (R increases).[24]

On the other hand, when d is sufficiently high ($d > r(1+\sqrt{1-l(1-a)})$), there exists a positive relationship between s_n, s_j and s_Y and a negative relationship between these variables and s_H, *R* and g_Y. Hence, the hypothesis one can infer is that when human capital is particularly productive in the education sector, further increases in d push the investment in formation up and reduce the investment (in terms of human capital) in the other three sectors competing for the same input. All this leads to an increase in both the steady state growth rate of the economy and the ratio *R*.

The effect that an increase in r has on the main variables of the model is perfectly consistent with what we have just said: in the interval $r < d < r(1+\sqrt{1-l(1-a)})$ there exists a positive relationship between *R*, s_n, s_H and g_Y and a negative relationship between these variables and s_j and s_Y. Instead, in the interval $d > r(1+\sqrt{1-l(1-a)}$, the relationship linking *R*, s_H and g_Y is positive, whereas the relationship linking these variables with s_n, s_j and s_Y is negative.

Overall, the result comes out that, contrary to Jones and Williams (2000) where the steady state share of R&D is monotonically increasing in the steady state output growth rate, the relationship

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23 In this section by positively (negatively) "correlated" we mean that a certain set of endogenous variables move in the same (different) direction of the exogenous shock that hits an exogenous variable.

24 Evidently *R* can also be written as: $R = \frac{a\,d}{s_j(1-a)C}$. An increase in d determines an increase in the numerator, a reduction in the denominator (through the effect on s_j) and an unambiguous increase in *R*.

between s_n and g_Y is non-monotonic in a context where human capital is allowed to grow over time through optimizing behaviour of rational agents and skills and technological capital are complements to each other in the steady-state equilibrium. The result stated by Jones and Williams remains true in the present context only when the productivity parameter of human capital accumulation is sufficiently low. The intuition behind this result is the following: when human capital is particularly productive in the education sector, a decentralised economy will invest more resources in this last sector rather than the other ones employing the same factor input. Since in this model aggregate growth is driven solely by human capital accumulation, this shift of resources towards education is at the root of the negative relationship between g_Y and s_n when d is sufficiently high.

From Table 1 we also notice that the share of total income devoted to the purchase of technologically advanced goods (l) is positively correlated with s_j and negatively correlated with s_Y. As in Blackburn et al. (2000), instead, the consumers' decision about how much time to invest in education and training (s_H) is exclusively driven by r and d (and independent on l and a). Via *returns to specialization* ($l(1-a)/a$), both the growth rate of the economy (g_Y) and s_n depend positively on l. Because of this last effect, *R* goes down when l increases.[25]

Finally, it is important pointing out that variations in a do play an important role in shaping the equilibrium allocation of human capital between sectors ($1/a$ is positively correlated with s_n and s_Y and negatively correlated with s_j). More importantly, the present analysis shows that introducing in the simplest possible way human capital accumulation in a generalization of the basic Grossman and Helpman model (1991, Chap. 3, pp. 43/57) *without knowledge spillovers* delivers an unambiguously positive correlation between imperfect competition ($1/a$) and growth (g_Y) when $0 < l < 1$. The intuition behind this result goes as follows. Looking at the production function of the homogeneous final good, one realises that in the steady-state equilibrium (with *x* constant), an increase in the level of output may be determined either by the growth of H_Y, or the growth of *n*, or the growth of both. Since the growth of *n* is induced (for the complementarity hypothesis between human and technological capital) by the growth of *H* (independent of the

[25] Indeed, *R* is also equal to $(d-r)/C \cdot s_n$.

mark-up rate), the only way for the market power to influence output growth is through varying the level of H_Y. In turn, H_Y can be decomposed in two parts:

$$H_Y \equiv s_Y \cdot H \,, \quad s_Y = H_Y / H \,.$$

While a variation in $1/a$ has no effect on the schooling decision of the household, an increase in the market power variable exerts an unambiguously positive effect on s_Y. In other words, it is through allocating a higher share of human capital towards the final output sector that monopoly power positively affects growth in the model.

In principle this result may seem counterintuitive since previous papers (notably Aghion-Dewatripont-Rey, 1997; Aghion-Harris-Vickers, 1997) and Aghion and Howitt (1996, 1998b)) clearly maintain that product market competition is unambiguously good for growth. Still, this conclusion hinges on the assumption that the engine of growth is the continuous improvement of the *quality level* of already existing goods. In comparison with these approaches what the present paper shows is that results concerning the long run relationship between competition and growth might well change within a horizontal differentiation model of endogenous growth featuring the following two characteristics: the true lever to economic growth is human capital accumulation and the choice of utility-maximasing agents to invest in formal education complements the one of profit-seeking firms to invent new varieties of intermediate goods.

6. Technology, the sectoral distribution of human capital and the interplay between market power and growth under some special cases.

All the results stated up to now have been obtained under the assumptions that d is strictly greater than r [26] and l is strictly between zero and one. In this section, while keeping the assumption that $d > r$, we study how the sectoral distribution of human capital and the relationship between imperfect competition and growth do change when l is assumed to be respectively equal to zero, one and a (i.e., when the production function in the downstream sector varies).

[26] This hypothesis assures that the human capital accumulation rate is positive in the long run equilibrium.

Case (a): $l = 0$.

In this case the technologies adopted in each economic sector (in the symmetric long-run equilibrium) read as:

$Y_t = aH_t$, $\quad a \equiv \frac{Ar}{d}$ (for the final goods production);

$x_{jt} = B \cdot h_{jt}$, $\quad \forall j(0, n_t)$ (for the capital goods production);

$\dot{n}_t = C \cdot H_{nt}$ (for research);

$\dot{h}_t = (d - r) \cdot h_t$ (for human capital supply),

and the model we are dealing with is the Rebelo (1991)-Lucas (1988) one or "Ah-model". The main variables of the model assume the following values:

$$s_j = 0; \qquad s_Y = \frac{r}{d}; \qquad s_n = 0; \qquad s_H = \frac{d - r}{d}; \qquad r = d;$$

(32) $g_{H_Y} = g_{H_j} = g_{H_n} = g_n = g_H = g_c = g_a = g_Y = d - r$.

As is well known, both in Rebelo (1991) and Lucas (1988), technical progress happens through devoting resources to physical (human) capital accumulation rather than a deliberate R&D activity aimed at expanding the set of available (horizontally differentiated capital goods). In case (a) this is reflected in the fact that the intermediate inputs do not enter the final goods production technology and $s_j = s_n = 0$. Thus, all the human capital is distributed between the final output (s_Y) and education (s_H) sectors.

Since capital goods are not productive inputs, market power ($1/a$), which in the model outlined in the previous sections emerges from the intermediate sector, does not play any role on the growth rate of output (g_Y). As in Lucas (1988), this last coincides with the growth rate of human capital and is equal to the difference between the productivity of the schooling technology ($d = r$)[27] and the subjective discount rate (r). Finally, it is worth noticing that, in a long run equilibrium where each sector gets a constant fraction of the available stock of human capital, s_Y

[27] See again Lucas (1988).

affects only the level of output ($Y_t = As_Y H_t$), whereas its growth rate is solely driven by s_H ($g_Y = d \cdot s_H$).

Case (b): $l = 1$.

In this case the technologies adopted in each economic sector (in the symmetric long-run equilibrium) are the following:

$$Y_t = A\left[\int_0^{n_t} (x_{jt})^a \, dj\right]^{\frac{1}{a}}, \qquad \text{(for the final goods production);}$$

$$x_{jt} = B \cdot h_{jt}, \qquad \forall j(0, n_t) \qquad \text{(for the capital goods production);}$$

$$\dot{n}_t = C \cdot H_{nt} \qquad \text{(for research);}$$

$$\dot{h}_t = (d - r) \cdot h_t \qquad \text{(for human capital supply),}$$

and the model we are dealing with is the Lucas (1988)-Grossman and Helpman (1991, Chap. 3, pp. 43/57) one. The main variables of the model assume the following values:

$$s_j = \frac{ar}{d - r(1-a)}; \qquad s_Y = 0; \qquad s_n = \frac{r(1-a)(d-r)}{d[d - r(1-a)]}; \qquad s_H = \frac{d-r}{d};$$

$$r = \frac{a\,d + (1-a)(d-r)}{a}; \qquad g_{H_Y} = g_{H_j} = g_{H_n} = g_n = g_H = d - r;$$

(33) $$g_c = g_a = g_Y = \frac{d-r}{a}$$

In this case human capital enters only indirectly (through the capital goods) the final output technology, whereas it continues to be employed in all the remaining sectors ($s_Y = 0$ and s_j, s_n and s_H are all positive). As in the previous case, the accumulation rate of human capital is equal in equilibrium to $d - r$, but now the growth rate of output (g_Y) depends positively and unambiguously on the mark-up rate ($1/a$). The reason is that in the present case g_Y is a function not only of s_H, but also of s_n:

$$g_Y = \frac{(1-a)\,r\,d \cdot s_n}{r(1-a) - d \cdot s_n} + d \cdot s_H$$

and it is easy to show that $\frac{\partial s_n}{\partial(1/a)} > 0$ and $\frac{\partial g_Y}{\partial s_n} > 0$. In other words, in this particular case, unlike what happens when $0 < l < 1$, it is through allocating a higher share of human capital from the intermediate sector ($\partial s_j / \partial(1/a) < 0$) towards the research sector that monopoly power positively affects growth.

Case (c): $l = a$.

The last special case we want to deal with in this section is the case where $l = a$.[28] Under this assumption the technologies adopted in each economic sector (in the symmetric long-run equilibrium) are the following:

$Y_t = AH_{Yt}^{1-a} \cdot \int_0^{n_t} (x_{jt})^a \, dj$, (for the final goods production);

$x_{jt} = B \cdot h_{jt}$, $\forall j(0, n_t)$ (for the capital goods production);

$\dot{n}_t = C \cdot H_{nt}$ (for research);

$\dot{h}_t = (d - r) \cdot h_t$ (for human capital supply).

The main variables of the model assume the following values:

$$s_j = \frac{a^2 r}{d - ar(1-a)}; \quad s_Y = \frac{r(1-a)}{d - ar(1-a)}; \quad s_n = \frac{ar(1-a)(d-r)}{d[d - ar(1-a)]}; \quad s_H = \frac{d-r}{d};$$

$$r = d(2-a) - r(1-a); \quad g_{H_Y} = g_{H_j} = g_{H_n} = g_n = g_H = d - r;$$

(34) $g_c = g_a = g_Y = (2-a)(d-r)$

In the present case human capital is employed in each economic sector. Thus, we can identify this case (unlike the two previous ones) as that in which the inter-sectoral competition for the same input (skilled workers) is tougher (s_j, s_Y, s_n and s_H are all positive). As in the previous cases, the accumulation rate of human capital is equal in equilibrium to $d - r$, but now (unlike case (b)) the

[28] This is the only case considered in Bucci (2002b), where the market power and the returns to specialization are not disentengled (they both depend exclusively on a in a symmetric long-run equilibrium).

relationship between s_n and $1/a$ is non-monotonic and the growth rate of output (g_Y) is a positive and non-linear (concave) function of the mark-up rate ($1/a$).

The main results concerning the relationship between the production technology in use in the downstream sector, the inter-sectoral distribution of human capital, the monopoly power and the aggregate growth rate within an integrated model of R&D and human capital accumulation is summarised by the following propositions:

Result 1

In a generalised, integrated growth model of deterministic R&D activity and human capital accumulation where the true engine of growth is represented by the supply of skilled workers à la Lucas (1988), as the one described by the equilibrium equations (20) through (27) and (1a), there always exists (except when $l = 0$*) a positive relationship between monopoly power (*$1/a$*) and aggregate growth (*g_Y*).*

Proof:
See equations (1a), (32), (33) and (34).

The reason why there exists no relationship between market power and growth when $l = 0$ is that in this case there is neither an intermediate sector, nor a research one (accordingly, the output growth rate is completely independent on the mark-up that arises in the model from the capital goods sector). Throughout the main text we have already given the intuition for the result of a positive relationship between $1/a$ and g_Y in the other cases. What such a result seems to suggest is the following: as for the long-run relationship between monopoly power (measured in terms of elasticity of substitution among capital goods) and growth, we may substantially replicate the same result obtained in the basic Schumpeterian model of growth[29] using an horizontal product differentiation model where the true engine of growth is human capital accumulation and skilled workers and technological capital are complements to each other in the very long run.

[29] Notably, Aghion and Howitt (1992).

Result 2

In a generalised, integrated growth model of deterministic R&D activity and human capital accumulation where the true engine of growth is represented by the supply of skilled workers à la Lucas (1988), as the one described by the equilibrium equations (20) through (27) and (1a), both the type of technology being used in the final output sector and the inter-sectoral competition for the (growing) human capital do affect the relationship between aggregate growth (g_Y *) and monopoly power (* $1/a$ *).*

Indeed, such a relationship is linear in case (b) – where human capital is not directly employed in the final output sector, whose technology is of the CES type – and concave in case (c) – where human capital is used everywhere and the final output technology is (an extension of) Cobb/Douglas. Similar results are obtained in a model where the growth engine is the R&D externality and there is no human capital accumulation (see Results 1 and 2 in Bucci (2002a)).

Result 3

In a generalised, integrated growth model of deterministic R&D activity and human capital accumulation where the true engine of growth is represented by the supply of skilled workers à la Lucas (1988), as the one described by the equilibrium equations (20) through (27) and (1a), both the type of technology being used in the final output sector and the inter-sectoral competition for the (growing) human capital do affect the level of the equilibrium growth rate. This last is higher whenever the final output technology is CES and does not employ human capital.

Proof:

From equations (32), (33) and (34) one easily concludes that: g_Y *(case b) >* g_Y *(case c) >* g_Y *(case a).*

This result parallels Results 3 and 4 of Bucci (2002a). Therefore, also when human capital is allowed to grow over time the highest possible growth rate is obtained within a *Grossman and Helpman-type economy*. On the contrary, in the present context, the lowest growth rate does prevail in a *Rebelo-Lucas-type-economy*, where the final output technology is linear in the human capital input and all the existing markets (final output and education) are perfectly competitive.

7. Concluding Remarks

In this paper we have analysed the steady-state predictions of an endogenous growth model with both purposive R&D activity and human capital accumulation where the engine of growth is represented by the supply of skills *à la Lucas* (1988). In the economy, human and technological capital are complements to each other and there exists no pecuniary externality in their accumulation process. In addition, it has been assumed that human capital enters as an input in all the sectors in order to analyse the economic forces underlying the inter-sectoral distribution of skilled workers. Using a theoretical framework where technological progress shows up in the form of the creation of new horizontally differentiated capital goods, the long-run relationship between imperfect competition and growth has also been addressed. The results of the model can be summarised as follows. First of all, the steady-state human capital accumulation depends solely on the parameters describing preferences and the productivity of the schooling technology. As a consequence, since the true engine of growth is represented by the supply of skills, the model does not display any scale effect. Many other endogenous growth models nowadays share this property. Secondly, we founded that the share of human capital devoted to research is not monotonically increasing in the steady-state growth rate of output and depends positively on the market power enjoyed by intermediate producers. Finally, as for the impact of monopoly power on the other main variables of the model, the presence of imperfect competition conditions among the capital goods producers turns out to have always positive growth effects (except when the share of national income spent on the purchase of capital goods is exactly equal to zero) and may dramatically influence the allocation of the reproducible factor input (human capital) to the economic sectors employing it. We think this is as an important as an alternative result in comparison with other papers that, unlike the approach taken here, consider the technological progress as basically stemming from a continuous vertical differentiation process.

In the light of these results, two important questions still remain open in the future research agenda. First, a through empirical test on the economic determinants of the long-run allocation of human capital among alternative uses (with particular emphasis to the R&D activity) deserves surely further attention. Secondly, it would be worth studying how the results obtained here would change if one assumed that economic growth is simultaneously induced by two engines of growth (schooling investment and R&D activity).

APPENDIX

In this Appendix, I derive the set of results (20) through (25) in the main text.

From equation (12), when g_h is constant u_t turns out to be constant as well. This means that in equilibrium the household devotes a constant fraction of its own time endowment to working (*u*) and education (*1-u*) activities. Consequently, the optimal *u* (*u**) will be constant and endogenously determined through the solution to the household decision problem. Consider now this problem (equations (10) through (12) in the main text), whose first order conditions (equations (13) through (16)) are reported below for convenience, together with the consumer's constraints and the transversality conditions:

(11) $\dot{a}_t = r_t a_t + w_t u_t h_t - c_t$ (12) $\dot{h}_t = d(1-u_t)h_t$, $d > 0$

(13) $\dfrac{e^{-rt}}{c_t} = m_{1t}$ (14) $m_{1t} = m_{2t}\dfrac{d}{w_t}$

(15) $m_{1t} r_t = -\dot{m}_{1t}$ (16) $m_{1t} w_t u_t + m_{2t} d(1-u_t) = -\dot{m}_{2t}$

$\lim_{t\to\infty} m_{1t} a_t = 0$ $\lim_{t\to\infty} m_{2t} h_t = 0$.

From now on I will omit in this appendix the index *t* near the time dependant variables. Combining equations (14) and (16) we get:

(1) $\dfrac{\dot{m}_2}{m_2} = -d$,

whereas, from (15):

(2) $\dfrac{\dot{m}_1}{m_1} = -r$.

In a symmetric steady state equilibrium H_Y, H_j, H_n and *n* all grow at the same constant rate as H (denoted by g_H). This in turn implies that *x* (the output produced in this symmetric equilibrium by each local monopolist, and equal to BH_j/n) is constant over time and (from equations (4') and (19c) in the main text) the wage rate accruing to one unit of skilled labour ($w_j = w_Y = w$) grows at a rate equal to $\left[\frac{l(1-a)}{a} g_H\right]$. Then, using equation (14) in this appendix, we get:

(3) $\frac{\dot{m}_1}{m_1} = \frac{\dot{m}_2}{m_2} - \frac{l(1-a)}{a} g_H \quad \Rightarrow$

(3') $r = d + \frac{l(1-a)}{a} g_H$.

This means that in equilibrium (when g_H is constant), the real interest rate (r) is constant as well. From equation (6) in the main text it follows that the instantaneous profit accruing to each capital goods' producer does grow at the rate $\frac{l(1-a)}{a} g_H$ as well. Hence, through simple computations, equation (9) in the main text may be re-written as:

(9a) $$V_{nt} = \int_t^{\infty} A\, l(1-a) H_{Yt}^{1-l} n_t^{\frac{l-a}{a}} \left(\frac{BH_{jt}}{n_t}\right)^l e^{-r(t-t)} dt = A\, l(1-a) \left(\frac{BH_{jt}}{n_t}\right)^l \frac{H_{Yt}^{1-l} \cdot n_t^{\frac{l-a}{a}}}{d}.$$

According to the equation above, the market value (the discounted flow of future profits) of a generic *j-th* intermediate firm (equal to the market value of the corresponding *j-th* idea) grows in the long run equilibrium at the rate $\frac{l(1-a)}{a} g_H$. Using equations (8) in the main text and (9a) above, it is possible to conclude that:

(9b) $$w_{nt} = AC\, l(1-a) \left(\frac{BH_{jt}}{n_t}\right)^l \frac{H_{Yt}^{1-l} \cdot n_t^{\frac{l-a}{a}}}{d}.$$

Employing equations (18a) and (4') in the main text and equation (9b) above, we get:

(4) $\frac{H_j}{n} = \frac{a\, d}{C(1-a)}$,

whereas using equations (18b), (4') and (19c) in the main text, the result comes out that:

(5) $\frac{H_Y}{n} = \frac{1-l}{a\, l} \cdot \frac{H_j}{n} = \frac{(1-l)\, d}{lC(1-a)}$.

Combining equations (13) and (15), we find the usual Euler equation, giving the optimal household's consumption path:

(6) $\frac{\dot{c}}{c} \equiv g_c = r - r = d - r + \frac{l(1-a)}{a} g_H$.

Dividing both sides of equation (11) by *a*, we get:

(7) $\frac{c}{a} = r + wu\frac{h}{a} - g_a$.

We already know that in the steady-state equilibrium *r*, *u* and g_a are constant. Therefore, for the ratio *c/a* to be constant it should be the case that wh/a is constant. Indeed, *h* grows at the rate g_H , w $(= w_Y = w_j = w_n)$ grows at the rate $\frac{l(1-a)}{a} g_H$ and $g_a = \frac{l(1-a)+a}{a} g_H$.[30] Hence, we can conclude that in equilibrium the growth rate of wh/a is equal to zero and the ratio *c/a* is constant. In other words, consumption (*c*) and asset holdings (*a*) do grow at the same constant rate along a balanced growth path. This implies that:

(8) $g_c = g_a = r - r = d - r + \frac{l(1-a)}{a} g_H$.

Finally, to find out the optimal *u** one first equates equation (6) in this appendix with the value of g_a and obtains:

(9) $g_{H_Y} = g_{H_j} = g_{H_n} = g_n = g_H = d - r$.

Then, plugging equation (9) into (12):

(10) $\frac{\dot{h}}{h} \equiv g_H = d(1-u) = d - r \Rightarrow u^* = \frac{r}{d}$.

For g_H to be strictly positive, d should be strictly greater than r, which in turn implies $0 < u^* < 1$. When $g_H = d - r > 0$, the real interest rate and the growth rate of consumption and asset holdings become respectively:

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[30] This follows immediately from the fact that $a = nV_n$ (equation (19) in the main text), $g_n = g_H$ (for the complementarity hypothesis between human and technological capital) and $g_{V_n} = \frac{l(1-a)}{a} g_H$ (see equation (9a) in this appendix).

(3’’) $r = \dfrac{a\, d + l(1-a)(d-r)}{a} > 0$;

(8’) $g_c = g_a = r - r = (d-r)\left[\dfrac{a + l(1-a)}{a}\right]$>0.

Also notice that when r and g_a assume the values written above and $g_H = d - r$, then the two transversality conditions are trivially checked since:

$\lim_{t\to+\infty} m_{1t}\cdot a_t = \lim_{t\to+\infty} m_{10}\cdot a_0\cdot e^{-rt} = 0$, and

$\lim_{t\to+\infty} m_{2t}\cdot h_t = \lim_{t\to+\infty} m_{20}\cdot h_0\cdot e^{-rt} = 0$.

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