Vintage Capital, Optimal investment and Technology Adoption *

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Abstract. In this paper, we study a vintage capital model under a general equilibrium setting. In this model firms can invest not only on new vintage capital goods, but also on existing ones. We show that the capital accumulation is a single hum-shape function, featuring slow technology diffusion.

Keywords: Embodiment, Technology adoption, Vintage capital

JEL Classification: E22, E32, O40.

1 Introduction

Vintage capital models, which were launched in the early 60s', have become increasingly popular in the economic literature. These models provide an approach for the analysis of investment volatility. The main difference between vintage capital model and the standard neoclassical growth model lies in the fact that in the former, new technological progress is embodied in new equipment, which gives rise to an endogenous process of creative destruction. Furthermore, as mentioned by Boucekkine et al (1997) and Benhabib et al (1991), optimal investment paths are no longer monotonic in contrast to the standard neoclassical growth model. In vintage capital models, the

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replacement of the old equipment is an economic decision, allowing for a formalization in the Shumpeterean vein. This replacement decision induces non-monotonic optimal paths for investment according to an echo principle(Boucekkine et al (1997) and Benhabib et al (1991)).

In the framework of Boucekkine et al (1996, 1997), the approach is purely theoretical. A common assumption to these models is that investment is only allowed for the new vintage capital, a traditional assumption in the literature (see Malcomson (1975)).As a result, there is immediate new technology diffusion.

However this assumption is not realistic. Firms use to invest in older capital goods. The main reason behind this is technology adoption. Technology adoption is costly in that it requires some specific vintage capital goods and involves learning or installation costs. Learning effects entail the concept of costly technology adoption, which motivates the investment in dominated technologies. Parente (1994) argued that the firms may be prevented from adopting new technologies as this supposes a loss in human capital (vintage specificity of human capital). However Parente did not work out this ideas in a vintage capital model. He only conducted a steady state analysis. Greenwood and Yorukoglu (1997) also combined vintage and learning effects in a computable dynamic general equilibrium model but they did so in a mostly ad-hoc way. Jovanovic and Nyarko (1996) explored the same idea in a stochastic framework, but in partial equilibrium setting.

Is it possible to build a general equilibrium vintage capital model, in which investment is allowed in both new and existing vintage capital goods? This idea was already considered by Barucci and Gozzi (1996, 2001) and Feichtinger, Hartl, Kort and Veliov (2001). In a flexible mathematical framework (McKendrick systems), they studied the replacement-adoption problem within a partial equilibrium setting, where capital accumulation is captured by a first order partial differential equation.

We will extend the above work to a general equilibrium setting. Precisely, we will consider an optimal growth model. In fact, the role of central planner maybe is crucial in technology adoption. As emphasized by Williamson(1971), "in the early 19th century, United State encouraged a faster scrapping of capital in favor of technologically superior equipment in the textile industry and then obtained rapid productivity growth in that industry "(See also Bardhan and Priale (1996)). Very recently, in the textile industry in China, though with low productivity technology and old technique, "in the early 1980s, due to large demand on domestic market, the textile industry produced unusually good profit[1]". From the long run point of view, at the end of 1990s, the textile industry of China still decided to adopt new techniques and replace the old ones step by step by offering some subsidies.

In order to establish the optimal investment rules in such situation, central planner has to consider two things: (i) how much to invest in existing vintage capital goods (*improvement in the quality of investment*), and how much to invest in new vintage capital goods (*expanding variety of investment*). Our model considers such a trade-off.

Among our results, we obtain that the stock of capital is a *single hum-shape* with respect to the vintage age for any fixed time t. In another words, there is slow technology diffusion.

This paper is arranged as follows. In Section 2, a vintage capital model is introduced under a general equilibrium setting, while Section 3 shows Pontryagin's condition and its sufficiency. Section 4 provides the analysis of equilibria, technology diffusion and investment path.

2 The Model

Suppose population is constant, labor market is competitive and there is only one final good, which can be assigned to consumption or investment and plays the role of numeraire.

Moreover, assume that the central planner solves a standard maximization problem with a linear instantaneous utility function,

$$\max_{c(t)} \int_0^\infty c(t)e^{-\rho t}dt, \tag{1}$$

subject to the budget constraint

$$y(t) = \int_0^{\overline{v}} a(t,v)k(t,v)dv, \tag{2}$$

$$\begin{aligned} c(t) &= y(t) - \int_0^{\overline{v}} \left(l(t,v)i(t,v) + \alpha(t,v)i^2(t,v)\right) dv \\ &\quad - \left(l_0(t)i_0(t) + \alpha_0(t)i_0^2(t)\right), \end{aligned} \tag{3}$$

[1]See China textile industry: Past, Present and Future, by China State Textile Industry Bureau

with $t \geq 0$, $c(t)$ and $y(t)$ present per-capita consumption and output respectively, $\rho(>0)$ is the time preference parameter, $\overline{v}$ is the oldest vintage capital still in use, that is, the lifetime of the vintage capital goods, and $a(t,v)$ is a known function of productivity of capital and satisfies the following conditions,

Assumption A

1° $a(t,v) \geq 0$, *for any* $t \geq 0$, $v \geq 0$*;*

2° $\dfrac{\partial a(t,v)}{\partial v} < 0$, *for any* $t \geq 0$*. That is, "young" vintage capital goods are more efficient than old ones.*

Moreover, $k(t,v)$ is the stock of per-capita capital goods of a given vintage age $v(\geq 0)$ at time t, which satisfies the following first order partial differential equation[2]

$$\begin{cases} \dfrac{\partial k(t,v)}{\partial t} + \dfrac{\partial k(t,v)}{\partial v} = i(t,v) - \delta k(t,v), \quad t \in (0,\infty), \ v \in (0,\overline{v}], \\ k(t,0) = i_0(t), t \in (0,\infty), \\ k(0,v) = k_0(v), v \in (0,\overline{v}], \end{cases} \tag{4}$$

where $i(t,v)$ is investment in the existing vintage capital v at time t, that is, the process of improving the quality of investment, $\delta(\geq 0)$ is the natural depreciation rate of capital, $i_0(t)$ is investment in new vintage capital goods at time t, that is, expanding variety of investment, and $k_0(v)$ is stock of capital at time 0, with vintage v. Here, we assume that

$$\begin{cases} k_0(v) \geq 0, \quad \text{given}\ , \forall v \geq 0, \\ \displaystyle\int_0^{\overline{v}} k_0(v)dv > 0, \end{cases} \tag{4.a}$$

in another words, initially in this economy there are some equipments that can be used for some periods. Moreover, we assume investments are nonnegative ($i(t,v) \geq 0$,$i_0(t) \geq 0$).

[2]The partial differential equation expresses the transition equation generalized to the classical dynamical system $\dot{k}(t) = i(t) - \delta k(t)$. From the vintage capital point of view, during time period $\triangle\, t$ at given time t, for given vintage v, the stock of capital will change from $k(t,v)$ to $k(t+ \triangle\, t, v + \lambda \triangle\, t)$, where constant $\lambda \geq 0$ describes the relation between time t and vintage capital v. Simple manipulations allows us to impose $\lambda = 1$.

The coefficients $l(t,v), l_0(t,)$, $\alpha(t,v)$, $\alpha_0(t)$ are known functions, where $l(t,v), l_0(t)$ are operating (or learning, or unit) costs, $\alpha(t,v)$ and $\alpha_0(t)$ are adoption costs with respect to existing and new vintage capital goods. Following the tradition in the investment literature, adoption costs are specified as quadratic functions of gross investment. Here, we assume that,

Assumption C *For every $v \geq 0$ and for a fixed $\epsilon > 0$, there are*

$$l(t,v) \geq 0, \quad \text{and} \quad \alpha(t,v) \geq \epsilon, \tag{5}$$

$$\frac{\partial \alpha(t,v)}{\partial v} \leq 0, \quad \frac{\partial l(t,v)}{\partial v} \leq 0. \tag{6}$$

Furthermore,

$$l_0(t) \geq \lim_{v \to 0} l(t,v), \quad \alpha_0 \geq \lim_{v \to 0} \alpha(t,v) \geq \epsilon. \tag{7}$$

Assumption C means that there are at least positive adoption costs for new and existing vintage capital goods (inequalities (5)). Moreover, ‘young’ vintage capital goods are more expensive than ‘old’ ones (inequalities (6)), and naturally, new vintage capital goods are the most expensive (see (7)).

Remark Obviously, from (4), $i(t,v)$ does not continue up to the boundary condition $i_0(t)$ at $v = 0$. As a result, the solution of (4) for $v > 0$ does not continue up to the boundary. Hence, this process has no accumulation of new capital goods.

Let $k(\cdot, v) : [0,\infty) \to L^2((0,\overline{v}]; \mathbb{R})$, where $L^2((0,\overline{v}]; \mathbb{R})$ means space of square integral functions, that is, for any fixed $v \in (0,\overline{v}]$,

$$k(t) = k(t,v) \in L^2(\mathbb{R}), \quad \int_0^\infty |k(t,v)|^2 dt < \infty.$$

Formally, (4) can be written as

$$\begin{cases} k'(t) = \mathcal{A}k(t) + i(t) + \delta_0 i_0(t), \quad t \in (0,\infty) \\ k(0) = k_0, \end{cases} \tag{8}$$

where $\mathcal{A}$ is a suitable operator in L^2,

$$-\mathcal{A}k(t) = (-\mathcal{A}k(t))(v) = \frac{\partial k(t,v)}{\partial v} + \delta k(t,v),$$

and δ_0 is Dirac's delta at the point 0,

$$\delta_0 = \begin{cases} 1, \text{ at } 0, \\ 0, \text{ otherwise.} \end{cases}$$

Substituting (2) into (3), we have

$$\begin{aligned} c(t) &= \int_0^{\overline{v}} \left[a(t,v)k(t,v) - \big(l(t,v)i(t,v) + \alpha(t,v)i^2(t,v) \big) \right] dv \\ &- \big(l_0(t)i_0(t) + \alpha_0(t)i_0^2(t) \big) . \end{aligned} \tag{9}$$

Substituting (9) into (1), the central planner's problem is equivalent to the following one

$$\begin{aligned} \max_{i(t,v),i_0(t)} \quad & \int_0^{\infty} \left[\int_0^{\overline{v}} \big(a(t,v)k(t,v) - l(t,v)i(t,v) - \alpha(t,v)i^2(t,v) \big) \, dv \right. \\ & \left. - \, l_0(t)i_0(t) - \alpha_0(t)i_0^2(t) \right] e^{-\rho t} dt, \end{aligned} \tag{10}$$

subject to (8).

3 Investment Strategy and Pontryagin Condition

In this section, we study the investment strategy depending on cost functions. Before introducing the value function, let us denote that $I_{\{v \geq t\}}$ is an indicator function of interval $v \geq t$, and $\mathcal{A}^*$ is an adjoint operator of $\mathcal{A}$, that is,

$$\mathcal{A}^* k(\tau)(v) = \frac{\partial k(\tau, v)}{\partial v} - \delta k(\tau, v).$$

Define the current value function of the maximization problem (10), (8) as

$$\begin{aligned} V(t;k) &= \int_t^{\infty} \left[\int_0^{\overline{v}} (ak(\tau,v) - li(\tau,v) - \alpha i^2) dv - l_0 i_0(\tau) - \alpha_0 i_0^2(\tau) \right] e^{-\rho\tau} d\tau \\ &- \int_t^{\infty} < q(\tau,v), k_t - (\mathcal{A}k(\tau) + i(\tau) + \delta_0 i_0(\tau)) >_v e^{-\rho\tau} d\tau, \end{aligned}$$

where $< \cdot, \cdot >$ is the inter product. The first order conditions of the above value function with respect to investments $i(\tau, v)$, $i_0(\tau)$, and state variable $k(\tau, v)$, give

$$i(t,v) = \frac{q(t,v) - l(t,v)}{2\alpha(t,v)}, \tag{11}$$

$$i_0(t) = \frac{q(t,0) - l_0(t)}{2\alpha_0(t)}, \tag{12}$$

and

$$\frac{\partial q(t,v)}{\partial v} = (\rho - \mathcal{A}^*)q(t,v) - a(t,v),$$

with transversality condition

$$\lim_{t\to\infty} q(t,v)e^{-\rho t} = 0.$$

From the above analysis, we have

Proposition 1 (Pontryagin Condition and Optimal Investment Strategy) *For any given investment strategy $i(t,v), i_0(t)$, the costate equation (shadow price of capital) is given by*

$$\begin{cases} \dfrac{\partial q(t,v)}{\partial v} = (\rho - \mathcal{A}^*)q(t,v) - a(t,v), \\ \lim_{t\to\infty} q(t,v)e^{-\rho t} = 0, \end{cases} \tag{13}$$

and the solution of the above equation is

$$\begin{aligned} q(t,v) &= \int_t^\infty e^{-\rho(\tau-t)}e^{-\delta(\tau-t)}a(\tau, v+(\tau-t))I_{[0,\overline{v}-(\tau-t)]}d\tau \\ &= \int_v^{\overline{v}} e^{-(\rho+\delta)(\tau-v)}a(\tau-v+t,\tau)d\tau. \end{aligned} \tag{14}$$

Under assumption A and C, we obtain that, at time t, there exists a unique optimal investment strategy $(i^*(t,v), i_0^*(t))$ for the optimal control problem. This strategy is given by (11) and (12). Furthermore, the optimal investment strategy does not depend on the initial value of capital, but on all 'prices', including shadow price of capital, unit and adoption costs of gross investment.

It is straightforward that the shadow price of capital, $q(t,v)$ depends on the time preference of consumers, depreciation rate of capital, and on the productivity of capital, which is important for the investment strategy.

Solving partial differential state equations (4) or (8), and assuming that $i(t,v), i_0(t)$ are locally bounded functions, we obtain,

$$k(t,v) = e^{-\delta t}k_0(v-t)I_{\{v\geq t\}} + e^{-\delta v}i_0(t-v)I_{\{v<t\}} + \int_0^{t\wedge v} e^{-\delta\tau}i(t-\tau, v-\tau)d\tau, \tag{15}$$

with $t\wedge v = \min\{t,v\}$.

From an economic point of view, if the operating time of some equipment is not as old as the age of that equipment(or operating the equipment before it is obsolete), then before the initial store of capital is scrapped out of the market, there is always some store of capital left in this economy.

However for $t > v$, it could be possible that investment in any vintage is too costly, that is, from (11), (12), for any $v \in (0, \overline{v}]$,

$$i(t, v) = 0, \quad i_0(t) = 0.$$

Then, as a result, for any $v \in [0, \overline{v}]$,

$$k(t, v) = 0, \text{ and } \int_0^{\overline{v}} a(t, v)k(t, v)dv = 0.$$

In other words, too costly vintage investment will not allow us to implement the interior solution. Even if the economy is at the interior solution, it still can end up in a corner solution–poverty trap. So we need to impose some conditions to avoid the corner solution. In particular, we assume that the 'prices ' satisfy the following conditions.

Assumption I *In the following,* $1°$ *always holds and at least* $2°$ *or* $3°$ *hold as well, where*

1°

$$0 < \alpha(t, v), \alpha_0(v) \leq \overline{\alpha} < \infty,$$

$$q(t, v) = \int_v^{\overline{v}} e^{-(\rho+\delta)(\tau-v)} a(\tau - v + t, \tau)d\tau \geq l(t, v), \quad \forall v \in (0, \overline{v}],$$

$$q(t, 0) = \int_0^{\overline{v}} e^{-(\rho+\delta)\tau} a(\tau + t, \tau)d\tau \geq l_0(t);$$

2° *There exists an interval* $I_0 \subset (0, \overline{v}]$*, such that,*

$$q(t, v) = \int_v^{\overline{v}} e^{-(\rho+\delta)(\tau-v)} a(\tau - v + t, \tau)d\tau > l(t, v), \quad \forall v \in I_0;$$

3°

$$q(t, 0) = \int_0^{\overline{v}} e^{-(\rho+\delta)\tau} a(\tau + t, \tau)d\tau > l_0(t).$$

Assumption I says that if for all the existing vintage capital goods, the remaining lifetime productivity discounted by the time preference of consumers and depreciation rate of capital is not higher than the operating cost $l(t, v)$, $v \in (0, \overline{v}]$, then we must have new vintage capital goods, with much higher productivity and the earning from this new equipment is more than the operating cost (2° and 3°). Moreover any kind of equipment adjustment has finite adoption costs and any equipment's earning is not less than the operating cost(1°) .

The above assumption implies that at least there exists $v_0 \in [0, \overline{v}]$, such that,

$$k(t, v_0) > 0, \quad \int_0^{\overline{v}} k(t, v) dv > 0.$$

Actually, it could be the case that, for any $v \in (0, \overline{v}]$,

$$q(t, v) = \int_v^{\overline{v}} e^{-(\rho+\delta)(\tau-v)} a(\tau - v + t, \tau) d\tau = l(t, v),$$

because of our exogenous constant lifetime assumption, that is, positive constant $\overline{v}$. When the equipments are no longer young, and if the operating costs are the same, after discounting the productivity, the earning can only compensate the operating cost and no more left [3] . If this is the case, either we have to invest in the new capital goods with higher productivity to replace the old ones, or adjust the assumption of lifetime $\overline{v}$, or both. In this present work, we consider the first case.

Note that Assumption I is similar to the 'piecewise continuous' assumption in Boucekkine et al (1997), and our assumption on the initial condition (4.a) is similar to the 'no hole' assumption in Boucekkine et al (1997). Actually this productivity assumption is a key assumption that drives the economy, and provides the incentive to invest in new and more productivity capital goods. Hence, in developed economies, investing in new and scrapping old vintage capital goods are necessary to keep the economy developing. Investing in the new and young equipments, though they are more costly than old ones, is a kind of saving for the total economy in long run perspective.

Proposition 2 (Necessary and sufficient condition) *Suppose that Assumptions A and C hold, moreover if Assumption I also holds, then the above Pontryagin conditions are not only necessary, but also sufficient for the original optimal control problem.*

The proof of sufficiency is given in Appendix 1.

Proposition 3 *Let Assumption I hold. For any given investment strategy $i(t, v), i_0(t)$, there exists a unique solution of the capital accumulation equation (4), given by (15).*

[3] That is the case in the previous footnote, $\lambda > 1$, which means that this vintage capital goods quickly go out of date.

Corollary 1 *Productivity of capital has a positive effect on investment strategy, while the unit and adoption costs have negative effects.*

In fact, the above two different directions effects drive the central planner to consider both sides(productivity and costs) of the investment. If the productivity $a(t,v)$ $(v \in [0, \overline{v}))$, is homogenous for all types of vintage capital goods, then the planner would like to choose the old vintage capital goods, since with the same productivity, they cost less. But then this economy loses its engine of development. On the other hand, if we assume that $l(t,v) = 0, (v > 0)$, and keep $\alpha(t,v)$ the same as before, that is, for existing vintage capital goods, there is no learning cost, the planner would like to take the youngest vintage capital, which is more efficient, and scrape the old ones at once. But as we know in reality, this is not the case. The effect of $l(t,v)$ can be compensated by increasing the adoption cost (that is the reason, we can assume that $l(t,v)$ is nonnegative, rather than strictly positive).

For $l_0(t)$ and $\alpha_0(t)$, similar interpretations can be made.

Note If in one economy, there is only one market for new capital goods, we can assume $\alpha(t,v) = 0$, $\alpha_0(t) = 0$, and $l(t,v) \geq \epsilon > 0$, $l_0(t) \geq \epsilon > 0$, for any $t > 0$, and keep all the other conditions in Assumption C, then we can rebuild the similar results of Malcomson (1975) with exogenous constant scrapping rule, in the sense of general equilibrium setting.

Furthermore choosing the optimal investment strategies above, there is an optimal capital accumulation.

Corollary 3 *Suppose that Assumption A, C and I hold, for a given productivity of capital $a(t,v)$, with optimal investment strategy (i^*, i_0^*), given by (11), (12), there is an optimal accumulation of capital given by,*

$$k^*(t,v) = e^{-\delta t}k_0(v-t)I_{\{v\geq t\}} + e^{-\delta v}i_0^*(t-v)I_{\{v<t\}} + \int_0^{t\wedge v} e^{-\delta\tau}i^*(t-\tau, v-\tau)d\tau. \tag{15'}$$

4 Steady State

In this section, we study the properties around the equilibria. For the exogenous cost functions and productivity of capital, there could be two cases, that is, time independent and time dependent.

Yorukoglu (1998, P552) noted in his paper that, IBM introduced its pentium PCs in the early 1990s at the same price as it introduced its 286 PC in the 1980s. Therefore it took less than a decade for the computing technology to improve on the order of 20 years in terms of both speed and memory capacities, without increasing the cost. In this case, we may think that the exogenous cost functions are time independent, except the productivity, which is an increasing function with respect to time t (see also Feichtinger, Hartl, Kort and Veliov (2001)).

In this present paper, we will follow Solow's belief that the technology is exponentially increasing with time t, that is, $a(t,v) = e^{a_1 t} a_2(v)$, where function $a_2(v)$ satisfies Assumption A and $0 < a_1 < \rho$(to assure the convergence of the integral). But all the cost functions only depend on the type of vintage v, rather than time t. Hence, we assume that $l(t,v) = l(v), \alpha(t,v) = \alpha(v)$ for any $t \geq 0$ and $v \geq 0$, and l_0, α_0 are constants. Moreover the other conditions in Assumption C and I hold. Simple calculation from (11), (12), (13) and (15), lead to the shadow price of capital, optimal investments as follows,

$$\begin{cases} q^*(t,v) &= \displaystyle\int_v^{\overline{v}} e^{-(\rho+\delta)(\tau - v)} a(\tau - v + t, \tau) d\tau \\ &= e^{(\rho+\delta-a_1)v} e^{a_1 t} \displaystyle\int_v^{\overline{v}} e^{-(\rho+\delta-a_1)\tau} a_2(\tau) d\tau, \\ q^*(t,0) &= e^{a_1 t} \displaystyle\int_0^{\overline{v}} e^{-(\rho+\delta-a_1)\tau} a_2(\tau) d\tau. \end{cases} \tag{16}$$

$$i^*(t,v) = \frac{q^*(t,v) - l(v)}{2\alpha(v)}, \quad i_0^*(t) = \frac{q^*(t,0) - l_0}{2\alpha_0}. \tag{17}$$

and the capital accumulation is

$$\begin{cases} k^*(t,v) = e^{-\delta t} k_0(v-t) + \displaystyle\int_0^t e^{-\delta\tau} i^*(t-\tau, v-\tau) d\tau, \ t \leq v, \\ k^*(t,v) = e^{-\delta v} i_0^*(t-v) + \displaystyle\int_0^v e^{-\delta\tau} i^*(t-\tau, v-\tau) d\tau, \ t > v. \end{cases} \tag{18}$$

By the assumption of time independence of cost functions, it is easy to see that in (16) and (17), both the shadow prices of capital and investments in the new and existing vintage capital goods are increasing functions with respect to time t for any fixed vintage capital. But the increasing investment in vintage capital goods does not match the classical results in the literature (see Boucekkine et al (1997), and Benhabib et al (1991)), they are rather consistent with the neoclassical growth model. The key reason is that we assume the time independence of unit and adoption costs, which in fact have

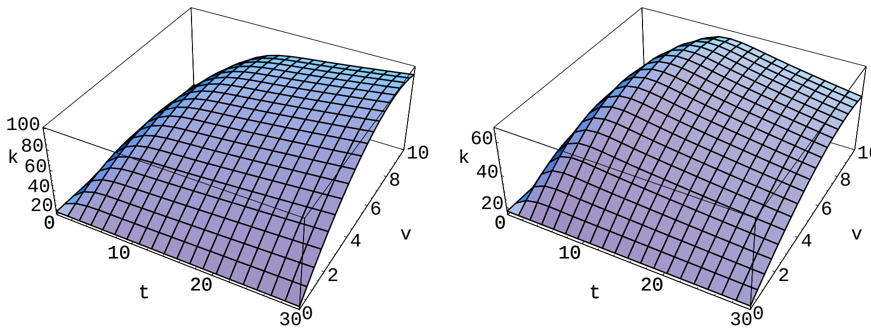

Figure 1: Left: Monotonic with time. Right: Non-monotonic with time

negative effects compared to the positive effects of the productivity of capital. It is not difficult to see (from (17)) that, if we also assume that the costs are increasing functions of time t (as Solow (1959), Barucci and Gozzi (2001)), then we lose the monotonicity of investment, see Example 1.

Example 1 Taking $\delta = 0.06$, $\rho = 0.04$, lifetime $\overline{v} = 10$, initial capital $k_0(v) = 1-0.1v$, new equipment costs are constants with respect to time, that is, $\alpha_0 = 0.5$, $l_0 = 3.98$. Moreover in the left figure, we take existing equipment costs are time independent, $\alpha(t,v) = 0.5e^{-0.1v}$ and $l(t,v) = 0.398(10-v)$, but productivity function depends on time $a(t,v) = 4e^{0.01t}e^{-0.1v}$, we got monotonic investment path and monotonic capital accumulation. In the right Figure, we take $\alpha(t,v) = 0.125e^{0.03t-0.1v}$, $l(t,v) = 0.398(10-v)$ and $a(t,v) = 4e^{0.01t}e^{-0.1v}$, then it is easy to see that there is no monotonic investment path and also the capital accumulation is not monotonic.

Actually, under the assumption of time dependent productivity and time independent cost functions, for the capital accumulation, our results are richer than in the literature so far. For existing vintage capital goods, if $t \leq v$, we have that

$$\begin{aligned}\frac{\partial k^*(t,v)}{\partial t} &= a_1 \int_0^t e^{-\delta\tau}\left[\frac{e^{-\delta\tau}q^*(t-\tau,v-\tau)}{2\alpha(v-\tau)}\right]d\tau \\ &\quad -e^{-\delta t}[\delta k_0(v-t)+k_0'(v-t)]+e^{-\delta t}\frac{q^*(0,v-t)-l(v-t)}{2\alpha(v-t)}.\end{aligned} \tag{19}$$

One can prove that for any fixed $v > 0$, and $t \leq v$, the second term in (19) is non-negative (see Appendix A2) , and the first and the last terms are positive. As a result, the investment in the existing vintage capital has positive effect on the capital accumulation, but with time passing, the old equipments storing slows down the capital accumulation process.

As for $v = 0$,we have

$$\frac{\partial k^*(t,0)}{\partial t} = e^{-\delta v}(i^*)'_0(t-v) > 0,$$

and in the case for fixed v, satisfies, $0 < v < t$, we have that

$$\frac{\partial k^*(t,v)}{\partial t} = e^{-\delta v}(i_0^*)'(t-v) + \int_0^v e^{-\delta\tau}\frac{\partial i^*(t-\tau, v-\tau)}{\partial t}d\tau \geq 0, \qquad (20)$$

That is, investment in the new or young existing vintage capital goods will increase capital capacity.

We conclude the above analysis with Proposition 4.

Proposition 4 *For a given fixed vintage capital goods and for given exponential productivity of capital and time independent cost functions, there is a unique long-run optimal investment strategy, stationary steady state of capital accumulation and shadow price of capital, given by (17), (18) and (16).*

Furthermore, there is a monotonic investment path for new and existing vintage capital goods. Investing on existing vintage capital will always increase the capital accumulation, but the storing of old vintage capital goods will slow down this accumulation procedure.

Let us now study the long-run steady state, that is $t > v$. We assume that people do not invest too much on the old equipments, that is,

$$\frac{di^*(t,v)}{dv} \leq 0, \quad \text{for any} \quad v \in (0, \overline{v}]. \qquad (21)$$

Proposition 5 *With time dependent productivity and time independent cost functions, consider a vintage capital good, which initially does not exist in the market (that is, $t > v$). Assume that there are vintage capital goods markets and (21) holds, then with the depreciation rate of capital satisfying $0 < \delta < 1$, we have that there exists a benchmark age $v^* \in (0, \overline{v})$, such that, at v^*, the investment in this vintage capital good compensates the depreciated capital ($i^*(t, v^*) = \delta k^*(t, v^*)$). Moreover, younger than this age, the optimal capital accumulation is an increasing function with respect to v, but older than this age, the capital accumulation is a decreasing function of age. In other words, there is a single hum-shape of capital accumulation with respect to age. Hence there is new technology diffusion (see Figure 2).*

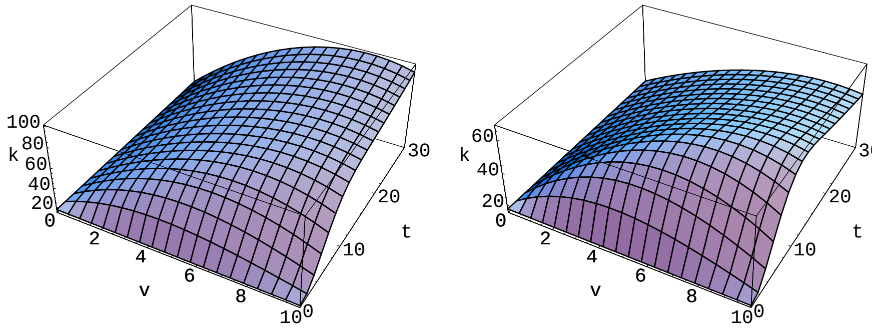

Figure 2: Single hum-shape of capital accumulation with age

Furthermore, the optimal accumulation of vintage capital goods is increasing in terms of technology level a_1*, and decreasing with respect to* $\delta, \rho, \alpha(v), l(v), l_0$ *and* α_0*.*

Proof From (18), it is easily to check that the optimal capital accumulation $k^*(t,v)$ is a decreasing function with respect to all cost functions l_0, $l(v)$, α_0, and $\alpha(v)$. As a result, any kind of increase in the cost functions will hurt the capital accumulation. Also, it is not difficult to check that $\frac{\partial k^*}{\partial a_1} > 0$, hence technology improvement will increase the capital capacity.

See Appendix 3 for the other proof. ■.

Remark As mentioned by Malcomson (1975), *a firm buys only most recent vintage of equipment*, (that is, only $v \to 0^+$ can happen), then there is immediate technology diffusion. In our model, there is a market, where vintage equipments are sold, therefore, there is not necessary immediate technology diffusion, because of cost or information delay.

Example 2 We take the same functions as in Example 1, with the only change is the direction of axels.

From Figure 2, it is easy to see that there is single hum-shape of capital accumulation with respect to age in the two cases. Moreover at steady state, there are less old equipments and new vintage capital goods than young ones. Old equipments are no longer efficient enough, so they are scrapped out of the market. But the new ones are too expensive or need some time to be accepted by the market. Initially, there are some old equipments on the market, but with time passing, they are outdated and being scrapped off.

By the way, comparing Figure 2 and Figure 3, it would be interesting to

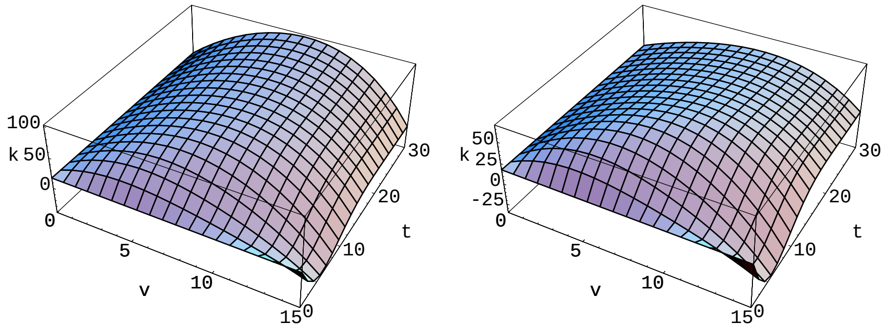

Figure 3: Prolong lifetime of vintage capital goods

study the effect of constant exogenous lifetime of vintage capital goods on capital accumulation and consumption.

It is easy to see that both $q(t,v)$ and $k(t,v)$ are also functions of lifetime $\overline{v}$. Then we can write $k(t,v), q(t,v)$ as $k(t,v;\overline{v}), q(t,v;\overline{v})$. Derivative $q(t,v;\overline{v})$ and $q(t,0;\overline{v})$ with respect to lifetime $\overline{v}$, and substituting them into the partial derivative, $\frac{\partial k(t,v;\overline{v})}{\partial \overline{v}}$, we have that

$$\frac{\partial k(t,v;\overline{v})}{\partial \overline{v}} = \frac{e^{-\delta v}e^{-(\delta+\rho)\overline{v}}e^{a_1(t+\overline{v}-v)}a_2(\overline{v})}{2}\left[\int_0^v \frac{e^{(\rho+2\delta)}\tau}{\alpha(\tau)}d\tau + \frac{1}{\alpha_0}\right] > 0.$$

Hence increasing the lifetime of equipment will lead to increase the capital accumulation– less scrapping will save some capital. However, this procedure cannot work all the time, since at the same time, we also have that

$$\begin{aligned}\frac{\partial^2 k(t,v;\overline{v})}{\partial \overline{v}^2} &= \frac{e^{-\delta v}e^{-(\delta+\rho-a_1)\overline{v}}e^{a_1(t-v)}}{2}\left[\int_0^v \frac{e^{(\rho+2\delta)}\tau}{\alpha(\tau)}d\tau + \frac{1}{\alpha_0}\right] \\ &\times[-(\delta+\rho-a_1)a_2(\overline{v}) + a_2'(\overline{v})] < 0,\end{aligned}$$

due to the fact that, $a_1 < \rho$ and Assumption A of a_2.

As a conclusion, we have the following result.

Proposition 6 *Slowing down the exogenous scrapping rule will not reduce capital capacity, but keeping on using old vintage equipment will hinder the development of the economy. Hence, replacing the old equipment by the new and more efficient one is a process of creative destruction.*

Actually, this fact was already noticed by Salter(1960), where he mentioned

"In fact there is some evidence to suggest that one of the chief reasons for Anglo-American productivity differences lies in standards of obsolescence. It is a common theme in Productivity Mission Reports that the productivity of the best plants in the United Kingdom is comparable with that of the best plants in the United States, and that the difference lies in a much higher proportion of plants employing outmoded methods in the United Kingdom– a much greater 'tail' of low-productivity plants. Such a situation is consistent with a higher standard of obsolescence in the United State which follows from a higher level of real wages (page 72–73)."

Proposition 7 *For given time dependent productivity and time independent cost functions, there is unique long-run stationary steady state of optimal consumption, which is given by*

$$\begin{aligned} c^*(t) &= \int_0^{\overline{v}} \int_0^v a(t,v) e^{-\delta(v-\tau)} \frac{q^*(t-v+\tau,\tau) - l(\tau)}{2\alpha(\tau)} d\tau dv \\ &+ \int_0^{\overline{v}} a(t,v) e^{-\delta v} \frac{q^*(t-v,0) - l_0}{2\alpha_0} dv \\ &- \int_0^{\overline{v}} \frac{(q^*)^2(t,v) - l^2(v)}{4\alpha(v)} dv - \frac{(q^*)^2(t,0) - l_0^2}{4\alpha_0}. \end{aligned} \tag{22}$$

Moreover Assumption I and $a_1 < \rho$ *are sufficient conditions to achieve the interior solution of the above optimal consumption.*

Furthermore, increasing the operating (or learning) costs, will harm the optimal consumption, while the adoption costs of new and existing vintage capital goods have ambiguous effects on consumption.

Remark The last statement of the above proposition explains the reason why sometime the new technologies are more expensive, but people still would like to invest in them. Also it shows the hint to the central planner that some time subsidies are necessary in the long run.

Proof By substituting optimal investment and optimal capital accumulation (17) and (18) into the consumption function (3),we can get (22).

The proof of admissible consumption $c(t) > 0$ is given in Appendix A4.

In that proof, we can see that Assumption I and $a_1 < \rho$ are sufficient conditions of interior solution of consumption (in fact not only for optimal consumption).

In the following we will study how the costs affect the optimal consumption.

Differentiating (22) with respect to the adoption cost of new equipment, we have

$$\begin{aligned}\frac{\partial c^*(t)}{\partial \alpha_0} &= \frac{1}{4\alpha_0^2}\left[(q^*)^2(t,0) - 2\int_0^{\overline{v}} a(t,v)e^{-\delta v} q^*(t-v,0)\right] dv \\ &+ \frac{1}{4\alpha_0^2}\left[2l_0 \int_0^{\overline{v}} a(t,v)e^{-\delta v} dv - l_0^2\right],\end{aligned}$$

from which we can see that the sign of the above expression is not clear. As a result there are ambiguous effects of the adoption cost on the consumption. But it is easy to check that if $a_1 = 0$, that is, the technology does not improve with time t, then increasing the adoption cost will hurt consumption. The reason of this is obvious: since the new technology does not improve the productivity, there is no incentive to introduce the new ones, which are costly. But if the technological process increasing with time $0 < a_1 < \rho$, it could be the case that even increasing the adoption cost, the investment in this vintage capital good still can benefit the consumers, because the productivity is high enough to compensate the extra cost. But on the other hand, because of the embodied technology, as we see in Proposition 5, technology adoption will lead to have more and more capital accumulation and also increase the shadow price of capital. As a result, firms would like to invest more in the new and more efficient equipments, which leads to the conclusion that consumers could afford less consumption goods. Hence, strictly increasing the technology level maybe not beneficial to the consumers.

Derivative (22) with respect to the operating cost of new equipment, we obtain

$$\frac{\partial c^*(t)}{\partial l_0} = \frac{1}{2\alpha_0}\left(l_0 - \int_0^{\overline{v}} a(v)e^{-\delta v} dv\right).$$

By 1° in Assumption I, it is easy to see that the difference on the right hand side of the above equation leads to $\frac{\partial c}{\partial l_0} \leq 0$. Hence, the operating cost of new vintage capital always has negative effects on the consumption.

In order to study the effect of costs of existing equipment on the consumption, we rewrite (22) as,

$$\begin{aligned}c^*(t) &= \int_0^{\overline{v}}\int_v^{\overline{v}} a(t,\tau)e^{-\delta(\tau-v)} \frac{q^*(t-\tau+v,v) - l(v)}{2\alpha(v)} d\tau dv \\ &+ \int_0^{\overline{v}} a(t,v)e^{-\delta v}\frac{q^*(t-v,0) - l_0}{2\alpha_0} dv \\ &- \int_0^{\overline{v}} \frac{(q^*)^2(t,v) - l^2(v)}{4\alpha(v)} dv - \frac{(q^*)^2(t,0) - l_0^2}{4\alpha_0}.\end{aligned} \tag{22'}$$

Differentiating (22') with respect to the operating cost of existing vintage capital, we have

$$\frac{\partial c^*(t)}{\partial l(v)} = \int_0^{\overline{v}} \frac{1}{2\alpha(v)} \left[l(v) - \int_v^{\overline{v}} a(t,\tau) e^{-\delta(\tau - v)} d\tau \right] \leq 0.$$

Thus, increasing the operating cost of existing vintage will diminish consumption.

For the effect of the adoption cost of the existing vintage capital goods, differentiating (22') with respect to $\alpha(v)$, and rearrange the terms, we get,

$$\begin{aligned} \frac{\partial c^*(t)}{\partial \alpha(v)} &= \int_0^{\overline{v}} \frac{1}{4\alpha^2(v)} \Big[(q^*(t,v))^2 - l^2(v) \\ &\quad - 2 \int_v^{\overline{v}} a(t,\tau) e^{-\delta(\tau - v)} (q^*(t - \tau + v, v) - l(v)) d\tau \Big] dv, \end{aligned}$$

which has a similar effect on the adoption cost of new equipment. ■

5 Conclusion

In this paper, under a general equilibrium setting of a vintage capital goods model, with a simple linear utility function and nearly linear output function, we obtain that there is slow technology diffusion rather than immediate diffusion, if firms can invest not only in new vintage capital goods, but also in existing ones. Moreover, keeping using old vintage capital goods will not reduce capital capacity, but it could hinder the development of the economy. Furthermore, increasing adoption costs will undermine capital accumulation, but it may be welfare improving.

Appendix A1 (Proof of sufficient condition in Proposition 2)

Denote

$$\begin{aligned}
F_1(\tau, v; k) &= e^{-\rho\tau}\left[a(\tau, v) + (q_t - \rho q + \mathcal{A}^* q)\right] k(\tau, v), \\
F_2(t, v; k) &= e^{-\rho t} q(t, v) k(t, v), \\
F_3(\tau, v; i) &= e^{-\rho\tau}\left[q(\tau, v) i(\tau, v) - l(\tau, v) i(\tau, v) - \alpha(\tau, v) i^2(\tau, v)\right], \\
F_4(\tau, v; i_0) &= e^{-\rho\tau}\left[q(\tau, v)\delta_0 i_0(\tau) dv - l_0 i_0(\tau) - \alpha_0 i_0^2(\tau)\right].
\end{aligned}$$

Obviously, we have

$$\begin{aligned}
V \;&\le \int_t^\infty \int_0^{\overline{v}} F_1(\tau, v; k) dv d\tau + \int_0^{\overline{v}} F_2(t, v; k) dv \\
&\quad + \int_t^\infty \int_0^{\overline{v}} \max_{i, i_0}(F_3(\tau, v; i) + F_4(\tau, v; i_0)) dv d\tau.
\end{aligned}$$

If for any $(\tau, v) \in [0, \infty) \times [0, \overline{v}]$, there is $(i^*(\tau, v), i_0^*(\tau))$, such that,

$$F_3(\tau, v; i^*(\tau, v)) = \max_i F_3(\tau, v; i), \quad F_4(\tau, v; i_0^*(\tau)) = \max_{i_0} F_4(\tau, v; i_0),$$

then the above $(i^*(\tau, v), i_0^*(\tau))$ is optimal for the original optimal control problem. In fact, we have

$$\begin{aligned}
\frac{\partial F_3}{\partial i} &= q(\tau, v) - l(\tau, v) - 2\alpha(\tau, v) i(\tau, v), \\
\frac{\partial F_4}{\partial i_0} &= q(\tau, v) - l_0(\tau) - 2\alpha(\tau, v) i_0(\tau),
\end{aligned}$$

and

$$\begin{aligned}
\frac{\partial^2 F_3}{\partial i^2} &= -2\alpha(\tau, v) < 0, \\
\frac{\partial^2 F_4}{\partial i_0^2} &= -2\alpha(\tau, v) < 0,
\end{aligned}$$

by Assumption C. Due to Assumption I,

$$\begin{aligned}
&\frac{\partial F_3(\tau, v; 0)}{\partial i} \ge 0, \quad \frac{\partial F_4(t, v; 0)}{\partial i_0} \ge 0 \\
&\frac{\partial F_3(\tau, v; \infty)}{\partial i} < 0, \quad \frac{\partial F_4(t, v; \infty)}{\partial i} < 0,
\end{aligned}$$

and in the first formula at least one of the inequality strictly holds, hence $F_3(\tau, v; i) + F_4(\tau, v; i_0)$ is strictly concave with respect to (i, i_0). As a result there is unique maximum point, $(i^*(\tau, v), i_0^*(\tau))$, such that,

$$\frac{\partial F_3}{\partial i} = 0, \quad \frac{\partial F_4}{\partial i_0} = 0,$$

which is equivalent to (11) and (12).

Appendix 2 (Proof)

In the following we prove that for any fixed $v > 0$, and for any $t \leq v$, we have that

$$\delta k_0(v - t) + k_0'(v - t) \geq 0. \tag{A2.1}$$

In fact, if $k_0'(t) \geq 0$, then the above always true. First, we assume that $k_0'(t) \leq 0$, $t \in (0, v)$, and $\delta k_0(v - t) + k_0'(v - t) \leq 0$, then integrating for t in $(0, v)$, we have that

$$0 \leq \delta \int_0^v k_0(v - t)dt < -\int_0^v k_0'(v - t)dt = k_0(0) - k_0(v) < 0,$$

which is a contradiction. Second, let consider that there exits an interval $[\alpha, \beta] \subset (0, v)$, such that, when $t \in [\alpha, \beta]$, there is $k_0'(t) \leq 0$ and $\delta k_0(v - t) + k_0'(v - t) \leq 0$. In fact using the same argument as above, we can obtain the contradiction by the fact that k_0 is a decreasing function in $[\alpha, \beta]$.

As a result, even if $k_0(t)$ is not monotone, we still have that (A2.1) holds.

Appendix 3 (Proof of Proposition 5)

In the case $t > v$, for any fixed t, derivative the second equation in (18) with respect to v, we have

$$\begin{aligned}
\frac{\partial k^*(t, v)}{\partial v} &= i^*(t, v) - \delta \left[e^{-\delta v} i_0^*(t - v) + \int_0^v e^{-\delta(v-\tau)} i^*(t - v + \tau, \tau)d\tau \right] \\
&\quad -a_1 \left[\frac{e^{-\delta v}}{2\alpha_0} q(t - v, 0) + \int_0^v \frac{e^{-\delta(v-\tau)}}{2\alpha(\tau)} q(t - v + \tau, \tau)d\tau \right] \\
&= i^*(t, v) - \delta k^*(t, v) \\
&\quad -a_1 \left[\frac{e^{-\delta v}}{2\alpha_0} q(t - v, 0) + \int_0^v \frac{e^{-\delta(v-\tau)}}{2\alpha(\tau)} q(t - v + \tau, \tau)d\tau \right].
\end{aligned} \tag{A3.1}$$

Taking the limit for v going to zero, we obtain

$$\lim_{v\to 0^+}\frac{\partial k^*(t,v)}{\partial v} = i^*(t,0^+) - \delta i_0^*(t) - \frac{a_1}{2\alpha_0}q(t,0).$$

We claim that, for any $0<\delta<1$,

$$i^*(t,0^+) \geq \delta i_0^*(t). \tag{A3.2}$$

If so, then we have that

$$\lim_{v\to 0^+}\frac{\partial k^*(t,v)}{\partial v} + \frac{a_1}{2\alpha_0}q(t,0) \geq 0. \tag{A3.3}$$

On the other hand, defining

$$F(v) = a_1\left[\frac{e^{-\delta v}}{2\alpha_0}q(t-v,0) + \int_0^v \frac{e^{-\delta(v-\tau)}}{2\alpha(\tau)}q(t-v+\tau,\tau)d\tau\right],$$

we can easily get the following ordinary differential equation,

$$\begin{cases} F'(v) = -(\delta+a_1)F(v) + \frac{q(t,v)}{2\alpha(v)}, \\ F(0) = \frac{a_1}{2\alpha_0}q(t,0). \end{cases}$$

Solving the above equation, we get

$$\begin{aligned} F(v) &= F(0) + e^{-(\delta+a_1)v}\left(\int_0^v \frac{q(t,\tau)}{2\alpha(\tau)}e^{(\delta+a_1)\tau}d\tau\right) \\ &\geq F(0) = \frac{a_1}{2\alpha_0}q(t,0). \end{aligned}$$

Combing with (A3.1), it follows

$$\frac{\partial k^*(t,v)}{\partial v} = i^*(t,v) - \delta k^*(t,v) - F(v) \leq i^*(t,v) - \delta k^*(t,v) - F(0),$$

and then

$$\frac{\partial k^*(t,v)}{\partial v} + F(0) \leq i^*(t,v) - \delta k^*(t,v). \tag{A3.4}$$

Due to the definition of $q(t,v)$, there is $\lim_{v\to\overline{v}} q(t,v) = 0$. As a result,

$$\begin{aligned} &\lim_{v\to\overline{v}} \left(i(t,v) - \delta k^*(t,v)\right) \\ &\quad = -\frac{l(\overline{v})}{2\alpha(\overline{v})} - \delta\left[e^{-\delta\overline{v}}i_0(t-\overline{v}) + \int_0^{\overline{v}} e^{-\delta\tau}i(t-\tau,\overline{v}-\tau)d\tau\right] < 0, \end{aligned} \tag{A3.5}$$

because of Assumption I, and at least one of $i_0(t-\overline{v})$ and $i(t-\tau, \overline{v}-\tau)$ are strictly positive.

Combining (A3.3),(A3.4) and (A3.5), we have that there exists a benchmark $v^* \in (0, \overline{v})$, such that

$$i^*(t, v^*) - \delta k^*(t, v^*) = 0,$$

and

$$\begin{cases} \dfrac{\partial k^*(t,v)}{\partial v} + \dfrac{a_1}{2\alpha_0} q(t,0) \geq 0, & 0 < v < v^*, \\ \dfrac{\partial k^*(t,v)}{\partial v} + \dfrac{a_1}{2\alpha_0} q(t,0) < 0, & v^* < v < \overline{v}. \end{cases}$$

Since $\frac{a_1}{2\alpha_0} q(t,0)$ is independent of v for any time $t > 0$, therefore, $k^*(t,v)$ is single hum-shape function of v.

Now we proof the claim (A3.2). Define

$$g(\delta) = i^*(t, 0^+) - \delta i_0^*(t) = \frac{q(t,0) - l(t, 0^+)}{\alpha(0^+)} - \delta \frac{q(t,0) - l_0}{\alpha_0}.$$

It is easy to check that

$$g(0) \geq 0, \quad g(1) \geq 0,$$

due to $\alpha_0 \geq \alpha(0^+)$ and $l_0 \geq l(0^+)$. Moreover combining with

$$g'(\delta) = -\frac{q(t,0) - l_0}{2\alpha_0} - \frac{(1-\delta)}{2\alpha_0} \int_0^{\overline{v}} a(t+s, s) e^{-(\rho+\delta)s} s ds < 0,$$

we prove that

$$g(\delta) \geq 0.$$

Appendix 4 (Proof of Proposition 7)

We prove that under conditions of the Proposition, we have that $c(t) > 0, \forall t > 0$.

From direct calculation, we obtain that,

$$\begin{aligned}
I_1 &= \int_0^{\overline{v}} a(t,v)e^{-\delta v}\frac{q^*(t-v,0)-l_0}{2\alpha_0}dv \\
&= \frac{1}{2\alpha_0}\int_0^{\overline{v}}\int_0^{\overline{v}} e^{a_1 t}a_2(v)e^{-\delta v}e^{-(\rho+\delta)s}e^{a_1(s+t-v)}a_2(s)dsdv \\
&\quad -\frac{l_0}{2\alpha_0}\int_0^{\overline{v}} e^{a_1 t}a_2(v)e^{-\delta v}dv \\
&= \frac{q(t,0)}{2\alpha_0}\int_0^{\overline{v}} e^{a_1(t-v)}a_2(v)e^{-\delta v}dv - \frac{l_0}{2\alpha_0}\int_0^{\overline{v}} e^{a_1 t}a_2(v)e^{-\delta v}dv,
\end{aligned}$$

then

$$\begin{aligned}
I &= I_1 - \frac{q^2(t,0)-l_0^2}{4\alpha_0} \\
&= \frac{q(t,0)}{2\alpha_0}\left(\int_0^{\overline{v}} e^{a_1(t-v)}a_2(v)e^{-\delta v}dv - \frac{q(t,0)}{2}\right) \\
&\quad +\frac{l_0}{2}\left(\frac{l_0}{2} - \int_0^{\overline{v}} e^{a_1 t}a_2(v)e^{-\delta v}dv\right) \\
&\geq \frac{q^2(t,0)}{2\alpha_0}\left(e^{-a_1\overline{v}} - \frac{1}{2}\right) + \frac{l_0}{2\alpha_0}\left(\frac{l_0}{2\alpha_0} - e^{(\rho-a_1)\overline{v}}q(t,0)\right) \\
&\geq \frac{l_0 q(t,0)}{2\alpha_0}\left[e^{-a_1\overline{v}} + e^{(\rho-a_1)\overline{v}} - 1\right] \\
&\geq 0,
\end{aligned}$$

by the fact that $\rho > a_1$ and $q(t,0) \geq l_0$, due to Assumption I.

Using the same argument but changing the order of the integration, we can get that

$$\begin{aligned}
II &= \int_0^{\overline{v}}\left[\int_0^{v} a(t,v)e^{-\delta\tau}i(t-\tau,v-\tau)d\tau - l(v)i(t,v) - \alpha(v)i^2(t,v)\right]dv \\
&\geq \int_0^{\overline{v}} \frac{l(v)}{2\alpha(v)}q(t,v)\left(e^{-a_1(\overline{v}-v)} + e^{(\rho-a_1)(\overline{v}-v)} - 1\right)dv \\
&\geq 0.
\end{aligned}$$

Consider Assumption I, at least one of the above two inequalities is a strict inequality. Hence, we have that with condition $\rho > a_1$ and Assumption I,

$$c(t) > 0, \forall t > 0.$$

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