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The Hicks-Moorsteen Productivity Index Satisfies the Determinateness Axiom

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Abstract

There are two total factor productivity indices available in the literature based on a primal notion of the technology. In a ratio tradition, these are the Malmquist and the Hicks-Moorsteen productivity indices. In a difference perspective, the Luenberger and Luenberger-Hicks-Moorsteen productivity indicators are based upon a sightly different concept. The purpose of this note is to establish that -in contrast to the Malmquist index- the Hicks-Moorsteen type of productivity index (as well as its difference-based counterpart) is well-defined and satisfies the determinateness property, since the underlying distance functions are always feasible.

Keywords: Malmquist productivity index, Hicks-Moorsteen productivity index, determinateness.

JEL: C43, D21, D24

1 Introduction

Discrete time Malmquist input, output and productivity indexes based upon distance functions as general technology representations (Caves, Christensen and Diewert (1982)) have been made empirically tractable by Färe et al. (1995). Exploiting the relation between distance functions and radial efficiency measures, they propose computing the distance functions composing the Malmquist index using deterministic, nonparametric technologies and, following Nishimizu and Page (1981), they distinguish between technological

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change and technical efficiency change. This ratio-based primal productivity index has been employed in at least hundreds of empirical applications.[1] An important problem that was already present in the original Färe et al. (1995) contribution is that some of the distance functions constituting the Malmquist productivity index may well be infeasible when estimated upon general technologies. For instance, an output-oriented Malmquist index is indeterminate when the adjacent period distance function comparing an observation in one period to the technology in the other period cannot be computed since for that particular observation the corresponding linear programming problem is infeasible (e.g., if there is one input in the evaluated observation that is smaller than the smallest similar input dimension in the technology). Unfortunately, few empirical studies explicitly report statistics on the occurrence of this infeasibility problem in the Malmquist productivity index (e.g., Glass and McKillop (2000)), thereby masking the prevalence of this problem and contributing to its neglect in the literature. Chambers (2002) introduced a more general primal productivity indicator -known as the Luenberger productivity indicator- in terms of differences between directional distance functions, the latter functions generalising the Shephardian distance functions.[2] Inevitably, infeasibilities can also occur for these directional distance functions and thus the more general Luenberger productivity indicator does not satisfy the determinateness property in index theory as well. The latter property -one of Fisher's (1922) original axioms- can be phrased as requiring that an index remains well-defined even when one or more of its arguments become zero or infinity (see, e.g., Eichhorn (1976) and Samuelson and Swamy (1974) for conflicting views on this determinateness axiom).

Bjurek (1996) proposes an alternative Hicks-Moorsteen (or Malmquist Total Factor Productivity (TFP)) index, as a ratio of Malmquist output and input indices, partly to avoid this indeterminateness problem of the Malmquist index (see his page 310).[3] Empirical applications of this Hicks-Moorsteen index have been relatively rare (e.g., Bjurek, Førsund and Hjalmarsson (1998), Grifell-Tatjé and Lovell (1999) or Nemoto and Goto (2005)). Bjurek (1996) states that this Hicks-Moorsteen productivity index has a TFP interpretation and Grifell-Tatjé and Lovell (1999) illustrate this numerically.

[1] While most studies employ deterministic, nonparametric technologies to compute distance functions (probably because of their advantages: (i) no problem handling multiple outputs, (ii) no functional form imposed on technology, and (iii) no restrictive assumptions regarding input remuneration), this same index can be computed using distance function estimates based on parametric technology specifications (e.g., Fuentes, Grifell-Tatjé and Perelman (2001)).

[2] Diewert (2005) distinguishes between "indicators" and "indexes" to denote productivity measures based on differences respectively ratios.

[3] Bjurek (1996) reasons in terms of infeasibilities in linear programming based estimates, thereby neglecting the general nature of the problem. Notice that Diewert (1992) already mentioned this index and attributed its origin to Hicks (1961) and Moorsteen (1961), but it was almost completely ignored in his analysis.

The Luenberger-Hicks-Moorsteen indicator, defined by Briec and Kerstens (2004), is a difference based version of this ratio-based Hicks-Moorsteen (or Malmquist TFP) index that inherits its determinateness. However, the claim by Bjurek (1996) that the Hicks-Moorsteen (or Malmquist TFP) index satisfies the determinateness axiom has never been proven.

While this issue has received relatively little attention in the productivity index literature, determinateness is potentially important when use is made of productivity indices to formulate public and private policies. For instance, the implementation of incentive regulatory mechanisms in a variety of network industries (in the context of price cap regulation) would be seriously hampered when productivity change cannot be measured for some of the regulated firms (see, e.g., Estache, Perelman and Trujillo (2007) for a study employing a Malmquist index in a regulatory context).

The sole purpose of this note is exactly to prove the claim that the Hicks-Moorsteen (or Malmquist TFP) productivity index satisfies the determinateness property, thereby elucidating the mechanism behind. To develop this result, this contribution is structured in the following way. Section 2 provides the basic definitions of the various distance functions and the Hicks-Moorsteen (or Malmquist TFP) productivity index. The next section shows that the Hicks-Moorsteen productivity index is determinate by focusing on defining a short-run version of this index.

2 Definitions of Technology and Hicks-Moorsteen Productivity Index

We first introduce the assumptions on technology and the definitions of the distance functions. The latter provide the components for computing productivity indices.

2.1 Technology and Distance Functions

Production technology transforms inputs $x = (x_1, ..., x_n) \in \mathbb{R}^n_+$ into outputs $y = (y_1, ..., y_p) \in \mathbb{R}^p_+$. For each time period t, the production possibility set $T(t)$ summarises the set of all feasible input and output vectors and is defined as follows:

$$T^t = \left\{(x^t, y^t) \in \mathbb{R}^{n+p}_+ : x^t \text{ can produce } y^t\right\}. \tag{2.1}$$

Throughout the paper technology satisfies the following conventional assumptions: (T.1) $(0, 0) \in T^t$, $(0, y^t) \in T^t \Rightarrow y^t = 0$, i.e., no free lunch; (T.2) the set $A(x^t) = \{(u^t, y^t) \in T^t : u^t \leq x^t\}$ of dominating observations is bounded $\forall x^t \in \mathbb{R}^n_+$, i.e., infinite outputs are not allowed with a finite input vector; (T.3) T^t is closed; and (T.4) $\forall (x^t, y^t) \in T^t$, $(u^t, v^t) \geq 0$ and

$(x^t, -y^t) \leq (u^t, -v^t) \Rightarrow (u^t, v^t) \in T^t$, i.e., fewer outputs can always be produced with more inputs, and inversely (strong disposal of inputs and outputs). Remark that we do not need the traditional convexity assumption.

Efficiency is estimated relative to production frontiers using distance or gauge functions. Distance functions are related to the efficiency measures of Farrell (1957). The Farrell efficiency measure $E_t(x^t, y^t)$ is the inverse of the Shephard distance function. In the input-orientation, this measure $E_t^i(x^t, y^t)$ indicates the minimum contraction of an input vector by a scalar λ still remaining in the technology:

$$E_t^i(x^t, y^t) = \inf_{\lambda} \left\{ \lambda : (\lambda x^t, y^t) \in T^t, \lambda \geq 0 \right\} . \tag{2.2}$$

An output efficiency measure $E_t^o(x^t, y^t)$ searches for the maximum expansion of an output vector by a scalar θ to the production frontier, i.e., $E_t^o(x^t, y^t) = \sup_{\theta} \{ \theta : (x^t, \theta y^t) \in T^t, \theta \geq 1 \}$.

Under constant returns to scale, input and output efficiency measures are linked: $E_t^o(x^t, y^t) = [E_t^i(x^t, y^t)]^{-1}$ (Färe and Lovell (1978)). For all $(a, b) \in \{t, t+1\}^2$, the time-related versions of the Farrell input efficiency measure is given by

$$E_a^i(x^b, y^b) = \inf_{\lambda} \left\{ \lambda : (\lambda x^b, y^b) \in T^a \right\} \tag{2.3}$$

if there is some λ such that $(\lambda x^b, y^b) \in T^a$ and $E_a^i(x^b, y^b) = +\infty$ otherwise. Similarly, in the output case, $E_a^o(x^b, y^b) = \sup_{\theta} \left\{ \theta : (x^b, \theta y^b) \in T^a \right\}$ if there is some θ such that $(x^b, \theta y^b) \in T^a$ and $E_a^o(x^b, y^b) = -\infty$ otherwise.

2.2 The Hicks-Moorsteen (or Malmquist TFP) Index

Following Bjurek (1996), a Hicks-Moorsteen productivity (or Malmquist TFP) index with base period t is defined as the ratio of a Malmquist output quantity index at base period t and a Malmquist input quantity index at base period t:

$$HM_t(x^t, y^t, x^{t+1}, y^{t+1}) = \frac{MO_t(x^t, y^t, y^{t+1})}{MI_t(x^t, x^{t+1}, y^t)} \tag{2.4}$$

where

$$MO_t(x^t, y^t, y^{t+1}) = E_t^o(x^t, y^t) / E_t^o(x^t, y^{t+1})$$

and

$$MI_t(x^t, x^{t+1}, y^t) = E_t^i(x^t, y^t) / E_t^i(x^{t+1}, y^t).$$

When the Hicks-Moorsteen productivity index is larger (smaller) than unity, it indicates productivity gain (loss).

A base period $t+1$ Hicks-Moorsteen productivity index is defined as follows:

$$HM_{t+1}(x^t, y^t, x^{t+1}, y^{t+1}) = \frac{MO_{t+1}(x^{t+1}, y^{t+1}, y^t)}{MI_{t+1}(x^t, x^{t+1}, y^{t+1})} \tag{2.5}$$

where

$$MO_{t+1}(x^{t+1}, y^{t+1}, y^{t}) = E^{o}_{t+1}(x^{t+1}, y^{t})/E^{o}_{t+1}(x^{t+1}, y^{t+1})$$

and

$$MI_{t+1}(x^{t}, x^{t+1}, y^{t+1}) = E^{i}_{t+1}(x^{t}, y^{t+1})/E^{i}_{t+1}(x^{t+1}, y^{t+1}).$$

A geometric mean of these two Hicks-Moorsteen productivity indexes is (Bjurek (1996: 310)):

$$HM_{t,t+1} = [HM_t.HM_{t+1}]^{1/2}, \tag{2.6}$$

where the arguments of the functions have been suppressed for reasons of space.

Notice that the denominator (numerator) of both the Malmquist output and input quantity index in base period t ($t+1$) compares a "hypothetical" or pseudo-observation consisting of inputs and outputs observed from different periods to a technology in period t ($t+1$). In Bjurek's (1996: 310) words, the feasibility of this index is due to the fact that "all input efficiency measures included meet the condition that the period of the technology is equal to the period of the observed output quantities" and "all output efficiency measures included meet the condition that the period of the technology is equal to the period of the observed input quantities".

Also note that the same index can be defined in a static context to measure relative productivity between production units (see Caves, Christensen and Diewert (1982)). This simply requires substituting the above time superscripts with a unit superscript.

The relations between this ratio-based Hicks-Moorsteen and the more popular Malmquist productivity indexes have been established in Färe, Grosskopf and Roos (1996): both indices coincide under (i) inverse homotheticity of technology; and (ii) constant returns to scale. Similar conditions have been proven to relate the Luenberger-Hicks-Moorsteen and the Luenberger indicators in an additive setting (Briec and Kerstens (2004)).

3 The Hicks-Moorsteen Index is Determinate: Proof and Illustration by Means of a Short-Run Hicks-Moorsteen Index

3.1 Construction of Feasible Farrell Technical Efficiency Measures with Fixed Input and Output Subvectors

Ouellette and Vierstraete (2004) define a short-run input-oriented Malmquist productivity index and are among the few studies reporting infeasibilities. By focusing on the definition of a short-run (or sub-vector) Hicks-Moorsteen productivity index, it is possible to show the mechanism guaranteeing the well-definedness of the underlying efficiency measures.

Inspired from the construction of the Hicks-Moorsteen productivity index, we provide a general method for defining a feasible adjacent-time period Farrell measure of technical efficiency when some inputs or outputs are fixed at their current levels in the short run.

Introducing notation, we denote $x^t = (x^{f,t}, x^{v,t})$ so that $x_i^t = x_i^{f,t}$ for $i = 1...n_f$ and $x_i^t = x_i^{v,t}$ for $i = n_f + 1...n$, where $n_f \in \{0, 1, ..., n-1\}$. Similarly, we denote $y^t = (y^{f,t}, y^{v,t})$ so that $y_j^t = y_j^{f,t}$ for $j = 1...p_f$ and $y_j^t = y_j^{v,t}$ for $j = p_f + 1, ..., p$, where $p_f \in \{0, 1, ..., p-1\}$. This notation implies that there is always at least one variable input and one variable output dimension. We define the time-related input subvector Farrell measure of technical efficiency by:

$$E_a^{i,f}(x^b, y^b) = \inf_{\lambda}\left\{\lambda : (x^{f,b}, \lambda x^{v,b}, y^b) \in T^a, \lambda \geq 0\right\}, \tag{3.1}$$

if there is some λ such that $(x^{f,b}, \lambda x^{v,b}, y^b) \in T^a$ and $E_a^{i,f}(x^b, y^b) = +\infty$ otherwise. The subvector Farrell output measure by:

$$E_a^{o,f}(x^b, y^b) = \sup_{\theta}\left\{\theta : (x^b, y^{f,b}, \theta y^{v,b}) \in T^a, \theta \geq 1\right\}, \tag{3.2}$$

if there is some θ such that $(x^b, y^{f,b}, \theta y^{v,b}) \in T^a$ and $E_a^{o,f}(x^b, y^b) = -\infty$ otherwise.

However, the above mentioned measures are sometimes undefined, i.e., they may not obtain a finite value. In such a case, one cannot compute a productivity index involving adjacent period comparisons. Similar to the Malmquist index, the resulting Hicks-Moorsteen productivity index is infeasible. The next example illustrates this problem.

Example 3.1 *Assume that*

$$T^t = \left\{(x^t, y^t) \in \mathbb{R}_+^2 \times \mathbb{R}_+ : \min\{x_1^t, x_2^t\} \geq (1+t)y^t\right\}$$

Assume that $t = 0, 1$, $(x^0, y^0) = (1, 1, 1)$ *and* $(x^1, y^1) = (2, 2, 1)$. *Suppose moreover that* $n^f = 1$. *In such a case, we have* $E_1^{i,f}(x^0, y^0) = +\infty$, *while* $E_1^i(x^0, y^0) = 1/2$.

Inspired by Bjurek's approach, one can overcome this problem by constructing two other input and output time related versions of these short run measures. In the input oriented case, we have:

$$E_a^{i,f}(x^{f,a}, x^{v,b}, y^a) = \inf_{\lambda}\left\{\lambda : (x^{f,a}, \lambda x^{v,b}, y^a) \in T^a\right\} \tag{3.3}$$

if there is some λ such that $(x^{f,a}, \lambda x^{v,b}, y^a) \in T^a$ and $E_a^{i,f}(x^{f,a}, x^{v,b}, y^a) = +\infty$ otherwise. On the output side, we have:

$$E_a^{o,f}(x^a, y^{f,a}, y^{v,b}) = \sup_{\theta}\left\{\theta : (x^a, y^{f,a}, \theta y^{v,b}) \in T^a\right\} \tag{3.4}$$

if there is some θ such that $(x^a, y^{f,a}, \theta y^{v,b}) \in T^a$ and $E_a^{o,f}(x^a, y^{f,a}, y^{v,b}) = -\infty$ otherwise.

In the following, we say that input factors are essential if $(x^t, y^t) \in T^t$ and $y^t \neq 0$ implies that $x_i^t > 0$ for $i = 1...n$.

Lemma 3.2 *Assume that the production technology satisfies T.1-T.4 and that the inputs are essential. For $(a,b) \in \{t, t+1\}^2$, if $x^{v,b} \neq 0$, then*

$$0 < E_a^{i,f}(x^{f,a}, x^{v,b}, y^a) < +\infty$$

Proof. Let us consider the vector $(x^{f,a}, \lambda^* x^{v,b}, y^a)$, where

$$\lambda^* = \max_{\substack{i=n_f+1...n \\ x_i^{v,b}>0}} \left\{ \frac{x_i^{v,a}}{x_i^{v,b}} \right\}.$$

Elementary calculus indicates that $(x^{f,a}, \lambda^* x^{v,b}) \geq (x^{f,a}, x^{v,a}) = x^a$. From the strong disposability assumption, we deduce that $(x^{f,a}, \lambda^* x^{v,b}) \in T^a$ and consequently $E_a^{i,f}(x^{f,a}, x^{v,b}, y^a) < +\infty$. Moreover, since the factors are essential and $x^{v,b} \neq 0$, the second inequality follows. Q.E.D.

Lemma 3.3 *Assume that the production technology satisfies T.1-T.4. For $(a,b) \in \{t, t+1\}^2$, if $y^{v,a} \neq 0$ and $y^{v,b} \neq 0$, then $0 < E_a^{o,f}(x^a, y^{f,a}, y^{v,b}) < +\infty$.*

Proof. Let us consider the vector $(x^a, y^{f,a}, \theta^* y^{v,b})$, where

$$\theta^* = \min_{\substack{j=p_f+1...p \\ y_j^{v,b}>0}} \left\{ \frac{y_j^{v,a}}{y_j^{v,b}} \right\}.$$

Thus, $(y^{f,a}, \theta^* y^{v,b}) \leq (y^{f,a}, y^{v,a}) = y^a$ and from the strong disposability assumption we deduce that $(y^{f,a}, \theta^* y^{v,b}) \in T^a$ and $E_a^{o,f}(x^a, y^{f,a}, y^{v,b}) < +\infty$. Moreover, since $y^{v,a} \neq 0$ and $y^{v,b} \neq 0$, we deduce that $E_a^{o,f}(x^a, y^{f,a}, y^{v,b}) > 0$. Q.E.D.

These results immediately translate into our main result with respect to the Hicks-Moorsteen productivity index.

Proposition 3.4 *If for all $(a,b) \in \{t, t+1\}^2$, we have $y^a \neq 0$ and $y^b \neq 0$, then the Hicks-Moorsteen productivity index (2.6) is well-defined.*

Proof. Since there is no free lunch, the result follows directly from taking $n_f = p_f = 0$ in Lemmas 3.2 and 3.3. Q.E.D.

Figure 1 shows an input isoquant from technology in period t and two observations $(x^{f,t}, x^{v,t})$ and $(x^{f,t+1}, x^{v,t+1})$. It is clearly impossible to achieve the distance from $(x^{f,t+1}, x^{v,t+1})$ to the input isoquant in period t in the direction of the variable input dimension. By contrast, when creating the pseudo-observation $(x^{f,t}, x^{v,t+1})$ a distance can be measured relative to this isoquant.

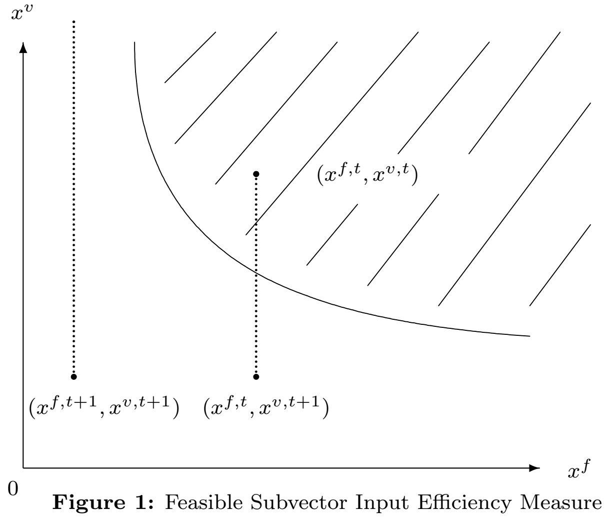

Figure 1: Feasible Subvector Input Efficiency Measure

3.2 A Determinate Hicks-Moorsteen Productivity Index with Subvectors

A base period t short-run Hicks-Moorsteen feasible productivity index is defined as follows:

$$HM_t^f(x^t, y^t, x^{t+1}, y^{t+1}) = \frac{MO_t^f(x^t, y^t, y^{t+1})}{MI_t^f(x^t, x^{t+1}, y^t)} \tag{3.5}$$

where

$$MO_t^f(x^t, y^t, y^{t+1}) = E_t^{o,f}(x^t, y^{f,t}, y^{v,t}) \Big/ E_t^{o,f}(x^t, y^{f,t}, y^{v,t+1})$$

and

$$MI_t^f(x^t, x^{t+1}, y^t) = E_t^{i,f}(x^{f,t}, x^{v,t}, y^t) \Big/ E_t^{i,f}(x^{f,t}, x^{v,t+1}, y^t).$$

A base period $t+1$ short-run Hicks-Moorsteen feasible productivity index is defined as follows:

$$HM_{t+1}^f(x^t, y^t, x^{t+1}, y^{t+1}) = \frac{MO_{t+1}^f(x^{t+1}, y^{t+1}, y^t)}{MI_{t+1}^f(x^t, x^{t+1}, y^{t+1})} \tag{3.6}$$

where

$$MO_{t+1}^f(x^{t+1}, y^{t+1}, y^t) = E_{t+1}^{o,f}(x^{t+1}, y^{f,t+1}, y^{v,t}) \Big/ E_{t+1}^{o,f}(x^{t+1}, y^{f,t+1}, y^{v,t+1}),$$

and

$$MI^f_{t+1}(x^t, x^{t+1}, y^{t+1}) = E^{i,f}_{t+1}(x^{f,t+1}, x^{v,t}, y^{t+1}) \Big/ E^{i,f}_{t+1}(x^{f,t+1}, x^{v,t+1}, y^{t+1}).$$

A geometric mean of these two feasible short-run Hicks-Moorsteen productivity indexes is:

$$HM^f_{t,t+1} = [HM^f_t . HM^f_{t+1}]^{1/2}, \tag{3.7}$$

where the arguments of the functions have again been suppressed to save space and the index remains determinate because it employs feasible efficiency measures of the type (3.3) and (3.4).

By contrast, the following variation on this short-run Hicks-Moorsteen productivity index is not well-defined. For reasons of space, we limit ourselves to only developing the base period t case. A base period t short-run Hicks-Moorsteen productivity index that is infeasible is defined as follows:

$$HM^{f\prime}_t(x^t, y^t, x^{t+1}, y^{t+1}) = \frac{MO^{f\prime}_t(x^t, y^t, y^{t+1})}{MI^{f\prime}_t(x^t, x^{t+1}, y^t)} \tag{3.8}$$

where

$$MO^{f\prime}_t(x^t, y^t, y^{t+1}) = E^{o,f}_t(x^t, y^{f,t}, y^{v,t}) \Big/ E^{o,f}_t(x^t, y^{f,t+1}, y^{v,t+1})$$

and

$$MI^{f\prime}_t(x^t, x^{t+1}, y^t) = E^{i,f}_t(x^{f,t}, x^{v,t}, y^t) \Big/ E^{i,f}_t(x^{f,t+1}, x^{v,t+1}, y^t).$$

The infeasibility results from employing efficiency measures of the type (3.1) and (3.2).

A comparison with the previous version index shows that the feasibility of the Malmquist output quantity index is achieved by comparing period t inputs and outputs with period t inputs and fixed outputs and period $t+1$ variable outputs. By contrast, the latter infeasible case compares period t inputs and outputs with period t inputs and period $t+1$ fixed and variable outputs. Thus, by simply keeping the fixed outputs firmly in the previous period, the output efficiency measures can be evaluated with respect to the resulting pseudo-observation. This logic is clearly in line with the basic intuitions cited above from Bjurek (1996).

4 Conclusions

This contribution has demonstrated that the Hicks-Moorsteen productivity index satisfies the determinateness property. By contrast, the far more popular Malmquist productivity index does not meet this demand. The same result obviously transposes to the difference based counterparts of both these indices (i.e., the Luenberger-Hicks-Moorsteen productivity indicator compared to the Luenberger productivity indicator). These two types of productivity indices are thus clearly structurally different, even though empirical differences have sometimes been found to be minor (e.g., Bjurek, Førsund and Hjalmarsson (1998)).

One plausible consequence is that one may wonder whether it is meaningful to mix up these two structurally different type of productivity indices, as it has been done in certain methodological developments. For instance, some decompositions of the Hicks-Moorsteen productivity index (e.g., Nemoto and Goto (2005)) include components that are based on a Malmquist type of index and hence these could be infeasible, despite the fact that the overall index is well-defined. This situation is potentially confusing. In a similar vein, some decompositions of the Malmquist productivity index (e.g., the input and output bias components in Färe et al (1997): see their fn. 5 on p. 123) include components that are based on a Hicks-Moorsteen type of index. This situation probably requires some further reflection.

Because of this determinateness property, we expect the Hicks-Moorsteen productivity index to gain in popularity in future empirical work, especially when infeasible solutions are simply unacceptable from a policy point of view (e.g., in incentive-based regulatory mechanisms where the efficiency requirements of price caps must be determined under all circumstances to avoid gaming the regulator).

References

Bjurek, H. (1996) The Malmquist Total Factor Productivity Index, *Scandinavian Journal of Economics*, 98(2), 303-313.

Bjurek, H., F.R. Førsund, L. Hjalmarsson (1998) Malmquist Productivity Indices: An Empirical Investigation, in: R. Färe, S. Grosskopf, R. Russell (eds) *Index Numbers: Essays in Honour of Sten Malmquist*, Boston, Kluwer.

Briec, W., K. Kerstens (2004) A Luenberger-Hicks-Moorsteen Productivity Indicator: Its Relation to the Hicks-Moorsteen Productivity Index and the Luenberger Productivity Indicator, *Economic Theory*, 23(4), 925-939.

Caves, D.W., L.R. Christensen, W.E. Diewert (1982) The Economic Theory of Index Numbers and the Measurement of Inputs, Outputs and Productivity, *Econometrica*, 50(6), 1393-1414.

Chambers, R.G. (2002) Exact Nonradial Input, Output, and Productivity Measurement, *Economic Theory*, 20(4), 751-765.

Diewert, W.E. (1992) Fisher Ideal Output, Input and Productivity Indexes Revisited, *Journal of Productivity Analysis*, 3(3), 211-248.

Diewert, W.E. (2005) Index Number Theory Using Differences Rather than Ratios, *American Journal of Economics and Sociology*, 64(1), 347-395.

Eichhorn, W. (1976) Fisher's Tests Revisited, *Econometrica*, 44(2), 247-256.

Estache, A., S. Perelman, L. Trujillo (2007) Measuring Quantity-Quality Trade-Offs in Regulation: The Brazilian Freight Railways Case, *Annals of Public and Cooperative Economics*, 78(1), 1-20.

Färe, R., E. Grifell-Tatjé, S. Grosskopf, C.A.K. Lovell (1997) Biased Technical Change and the Malmquist Productivity Index, *Scandinavian Journal of Economics*, 99(1), 119-127.

Färe, R., S. Grosskopf, B. Lindgren, P. Roos (1995) Productivity Developments in Swedish Hospitals: A Malmquist Output Index Approach, in: A. Charnes, W.W. Cooper, A.Y. Lewin, and L. M. Seiford (eds) *Data Envelopment Analysis: Theory, Methodology and Applications*, Boston, Kluwer.

Färe, R., S. Grosskopf, P. Roos (1996) On Two Definitions of Productivity, *Economics Letters*, 53(3), 269-274.

Färe, R., C.A.K. Lovell (1978) Measuring the Technical Efficiency of Production, *Journal of Economic Theory*, 19(1), 150-162.

Farrell, M. (1957) The Measurement of Productive Efficiency, *Journal of the Royal Statistical Society*, 120A(3), 253-281.

Fischer, I. (1922) *The Making of Index Numbers*, Boston, Houghton-Mifflin.

Fuentes, H.J., E. Grifell-Tatjé, S. Perelman (2001) A Parametric Distance Function Approach for Malmquist Productivity Index Estimation, *Journal of Productivity Analysis*, 15(1), 79-94.

Glass, J.C., D.G. McKillop (2000) A Post Deregulation Analysis of the Sources of Productivity Growth in UK Building Societies, *Manchester School*, 68(3), 360-385.

Grifell-Tatjé, E., C.A.K. Lovell (1999) A Generalized Malmquist Productivity Index, *TOP*, 7(1), 81-101.

Hicks, J.R. (1961) Measurement of Capital in Relation to the Measurement of Other Economic Aggregates, in: E.A. Lutz, D.C. Hague (eds) *The Theory* *of Capital*, London, Macmillan.

Moorsteen, R.H. (1961) On Measuring Productive Potential and Relative Efficiency, *Quarterly Journal of Economics*, 75(3). 451-467.

Nemoto, J., M. Goto (2005) Productivity, Efficiency, Scale Economies and Technical Change: A New Decomposition Analysis of TFP Applied to the Japanese Prefectures, *Journal of the Japanese and International Economies*, 19(4), 617-634.

Nishimizu, M., J. Page (1982) Total Factor Productivity Growth, Technological Progress and Technical Efficiency Change: Dimensions of Productivity Change in Yugoslavia, 196578, *Economic Journal*, 92(368), 920-936.

Ouellette, P., V. Vierstraete (2004) Technological Change and Efficiency in the Presence of Quasi-Fixed Inputs: A DEA Application to the Hospital Sector, *European Journal of Operational Research*, 154(3), 755-763.

Samuelson, P., S. Swamy (1974) Invariant Economic Index Numbers and Canonical Duality: Survey and Synthesis, *American Economic Review*, 64(4), 566-593.