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Reverse Shooting of Exchange Rates

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Abstract

Reverse shooting of the exchange rate has been put forward in this paper by scrutinizing the adjustment and evolution of the exchange rate towards its new long-run equilibrium level following a change in money supply. Joint and sequential effects of covered interest rate parity and the sticky price on the rise, from the short-term through the long-run horizon, result in a feature of reverse shooting of the exchange rate. Regardless of what the immediate response of the exchange rate to the change in money supply can be argued for, reverse shooting homogenizes the evolution path of exchange rate adjustment and movement from different views.

JEL No: F31, F37

Key words: exchange rate, reverse shooting

1. Introduction

The Dornbusch (1976) model of exchange rate determination has defined a high-water mark of theoretical simplicity and elegance in international finance (Rogoff 2002). One of the most famous features of the model is overshooting. Due to its prominence and influence, the overshooting proposition has been tested empirically over time. The first significant empirical studies of the Dornbusch model include Driskill (1981) on the issues of short-term overshooting, intermediate-term dynamics and long-run proportionality of the exchange rate. The empirical evidence appears to confirm the overshooting hypothesis, using the Swiss franc vis-à-vis the US dollar exchange rate for the period 1973–1977. Proposing a real interest rate differential model as an alternative to the flexible price monetary model and Dornbusch's sticky price monetary model, findings in Frankel (1979) reject both models based on the results of coefficient restrictions. Nonetheless, the results show some weak evidence of overshooting in depreciation following a fall in the nominal interest rate differential and the subsequent appreciation. It can be observed that the Dornbusch model has cast empirical controversies from the onset in spite of its theoretical elegance, which leads to various modifications to the model and model's assumptions and various modified hypotheses – one of them being exchange rate undershooting.

Overshooting in the Dornbusch model is observed by incorporating uncovered interest rate parity (UIRP) into the demand for money function, in conjunction with a kind of exchange rate expectations formation. Ironically, it is this exchange rate expectations formation with which agents foresee, target and adjust to the new long-run equilibrium exchange rate that results in overshooting. That is, the outcome of the action – the seemingly rational prediction and adjustment is inconsistent with the purpose of the action itself.

As the new long-run equilibrium is not attainable immediately and in the short-term, or even in the medium-run, targeting the new long-run equilibrium in exchange rate adjustment in the short-term does not seem rational. If the agents foresee a transition path towards the new long-run equilibrium, adjustments based on the transition path may be reasonable and realistic in a competitive foreign exchange market. With simplicity, the present paper proposes that, in addition to a new long-run equilibrium exchange rate, there is a new short-term exchange rate target following a change in money supply. While the exchange rate converges to the new long-run equilibrium exchange rate eventually in the long-run, it moves towards and around the new short-term exchange rate target immediately upon the change in money supply and in the short-term. The transition path thereby derived may still feature overshooting; it may feature undershooting and reverse shooting as well, with the latter being the main outcome of the proposition and analysis of this study.

The rest of the paper is organized as follows. The next section introduces the construct of this study and presents the analytical framework. Section 3 demonstrates several illustrating cases while section 4 concludes this study.

2. Exchange rate adjustments in the short-term and the long-run

This section first brings in a simplified version of the overshooting model. Departing from the basic overshooting model, the construct based on the proposition of the paper that the exchange rate, following a change in money supply, moves towards a new short-term exchange rate target in the short-term and converges to a new long-run equilibrium exchange rate in the long-run is developed.

2.1.1. The making of overshooting

UIRP states that the expected change in the foreign exchange rate is equal to the interest rate differential between the domestic country and the foreign country. In the case of a small open economy, the expected change in the foreign exchange rate is equal to the difference between the prevailing domestic interest rate and its long-run equilibrium rate – the world's interest rate:

$$E_t(\Delta e_{t+1}) = r_t - r^* \tag{1}$$

where e_t is the exchange rate in logarithms, $\Delta(e_{t+1}) = e_{t+1} - e_t$, r_t is the domestic interest rate, and r^* is the long-run equilibrium interest rate where a time subscript is not relevant. Exchange rate expectations that are formed in the following way:

$$E_t(\Delta e_{t+1}) = \theta(\bar{e} - e_t) \tag{2}$$

where $\bar{e}$ is the long-run exchange rate and $\theta > 0$ is a coefficient. Equation (2) states that the expected change in the exchange rate follows a dynamic adjustment process that the exchange rate will revert to its long-run rate, with the speed of adjustment being decided by the value of θ. The domestic currency is expected to depreciate when the exchange rate is below its long-run level and is expected to appreciate when it is above its long-run level. The adjustment is swift with θ is large and slow when θ is small. Combining equation (1) with equation (2) leads to:

$$r_t = r^* + \theta(\bar{e} - e_t) \tag{3}$$

The demand for money equation is the standard version given below:

$$m_t - p_t = \phi y_t - \lambda r_t \tag{4}$$

in which $\phi > 0$ and $\lambda > 0$. Inserting equation (3) into equation (4) yields:

$$m_t - p_t = \phi y_t - \lambda r^* - \lambda\theta(\bar{e} - e_t) \tag{5}$$

Assume that the system is in equilibrium at $t = 0$, with $p_0 = \bar{p}$, $e_0 = \bar{e}$ and $y_0 = \bar{y}$. Assume also there is an increase in money supply Δm at $t = 0$. Then, equation (6a) is for the system prior to the increase of money supply and equation (6b) represents the system upon the increase of money supply:

$$m_0 - p_0 = \phi y_0 - \lambda r^* - \lambda\theta(\bar{e} - e_0) \tag{6a}$$

$$m_0 + \Delta m - p_{0^+} = \phi y_{0^+} - \lambda r^* - \lambda\theta(\bar{e}_n - e_{0^+}) \tag{6b}$$

Since the price is fixed in the short-term and output is not supposed to be affected, subtracting equation (6a) from equation (6b) yields:

$$\begin{aligned} \Delta m &= -\lambda\theta(\bar{e}_n - e_{0^+}) + \lambda\theta(\bar{e} - e_0) \\ &= -\lambda\theta(\bar{e}_n - \bar{e}) + \lambda\theta(e_{0^+} - e_0) \end{aligned} \tag{7}$$

where $\bar{e}_n$ is the new long-run equilibrium exchange rate to be established consistent with the new quantity of money supply. Dornbusch (1976) argues rightfully that the increase in the long-run equilibrium exchange rate is equal to the increase of money supply. Equation (7) can be re-arranged as follows:

$$\Delta e_{0^+} = e_{0^+} - e_0 = \bar{e}_n - \bar{e} + \frac{\Delta m}{\lambda\theta} = \Delta m + \frac{\Delta m}{\lambda\theta} = \left(1 + \frac{1}{\lambda\theta}\right)\Delta m > \Delta m \tag{7'}$$

Equation (7') demonstrates exchange rate overshooting, i.e., depreciation in the exchange rate is greater than what is justified by the amount of increase in money supply.

2.1.2. Exchange rate adjustments in the short-term and in the long-run

The analysis below departs from that of Dornbusch (1976). Instead of targeting the new long-run equilibrium exchange rate immediately upon the increase of money supply and all along subsequently, there is a transition period in exchange rate adjustments. The exchange rate, following a change in money supply, moves towards a new short-term exchange rate target, $\bar{e}_{n1}$, in the short-term, and converges to a new long-run equilibrium exchange rate, $\bar{e}_{n2}$, in the long-run. A weighting function is applied so the weight allocated to the short-term adjustment process is gradually decreasing over time and the weight allocated to the long-run adjustment process is gradually increasing over time:

$$E_t(\Delta e_{t+1}) = e^{-\kappa t}\theta(\bar{e}_{n1} - e_t) + \left(1 - e^{-\kappa t}\right)\theta(\bar{e}_{n2} - e_t) \qquad (8)$$

It turns into the expression of equation (2) when t becomes sufficiently large; and the larger the parameter κ, the faster the exchange rate converges to its new long-run equilibrium level. A simple function reflecting the sticky feature of the price is also produced:

$$p_t = p_0 + \left(1 - e^{-\gamma t}\right)\Delta m \qquad (9)$$

The price is unchanged upon the increase of money supply, reflecting its fixed feature in the short-term. The price increases by the same amount of the increase in money supply in the long-run. The smaller the parameter γ, the stickier is the price. Finally, output is assumed to be unaffected by the change in money supply:

$$y_t \equiv y_0 \tag{10}$$

A system with the above features is then represented by the following equation:

$$\begin{aligned} & m_t - \left[p_0 + \left(1 - e^{-\gamma t}\right)\Delta m\right] \\ & = \phi y_0 - \lambda r^* - e^{-\kappa t}\lambda\theta(\bar{e}_{n1} - e_t) - \left(1 - e^{-\kappa t}\right)\lambda\theta(\bar{e}_{n2} - e_t) \end{aligned} \tag{11}$$

Rearrangements of equation (11) lead to:

$$\begin{aligned} e_t - e_0 &= \frac{1}{\lambda\theta}\left[\Delta m - \left(1 - e^{-\gamma t}\right)\Delta m\right] + e^{-\kappa t}\left(\bar{e}_{n1} - \bar{e}\right) + \left(1 - e^{-\kappa t}\right)\left(\bar{e}_{n2} - \bar{e}\right) \\ &= e^{-\gamma t}\frac{\Delta m}{\lambda\theta} + e^{-\kappa t}\left(\bar{e}_{n1} - \bar{e}\right) + \left(1 - e^{-\kappa t}\right)\left(\bar{e}_{n2} - \bar{e}\right) \end{aligned} \tag{12}$$

with the change immediately following the increase in money supply being:

$$\Delta e_{0^+} = e_{0^+} - e_0 = \frac{\Delta m}{\lambda\theta} + \left(\bar{e}_{n1} - \bar{e}\right) \tag{13}$$

Reverse shooting takes place if $\frac{\Delta m}{\lambda\theta} + \left(\bar{e}_{n1} - \bar{e}\right) < 0$. From equation (1) and equation (4), it can be inferred:

$$\bar{e}_{n1} - \bar{e} = \Delta r_t = -\frac{\Delta m}{\lambda} \tag{14}$$

Bringing equation (14) into equation (15) yields:

$$\Delta e_{0^+} = e_{0^+} - e_0 = \frac{\Delta m}{\lambda}\left(\frac{1}{\theta} - 1\right) \tag{15}$$

Therefore, $\frac{1}{\lambda}\left(\frac{1}{\theta} - 1\right) < 0$, or $1 < \theta < 2$, corresponds to reverse shooting; $\frac{1}{\lambda}\left(\frac{1}{\theta} - 1\right) > 1$ corresponds to overshooting; $0 < \frac{1}{\lambda}\left(\frac{1}{\theta} - 1\right) < 1$ corresponds to undershooting; and the exchange rate explodes if $\theta \geq 2$.

By definition:

$$\bar{e}_{n2} - \bar{e} = \Delta m \tag{16}$$

Bringing equation (14) and equation (16) into equation (12) yields:

$$
\begin{aligned}
e_t - e_0 &= e^{-\gamma t}\frac{\Delta m}{\lambda\theta} + e^{-\kappa t}\left(\bar{e}_{n1} - \bar{e}\right) + \left(1 - e^{-\kappa t}\right)\left(\bar{e}_{n2} - \bar{e}\right) \\
&= e^{-\gamma t}\frac{\Delta m}{\lambda\theta} - e^{-\kappa t}\frac{\Delta m}{\lambda} + \left(1 - e^{-\kappa t}\right)\Delta m \\
&= \frac{\Delta m}{\lambda\theta}\left[e^{-\gamma t} - \theta e^{-\kappa t} + \lambda\theta\left(1 - e^{-\kappa t}\right)\right]
\end{aligned}
\tag{17}
$$

Overshooting in terms of equation (7') is a special case of the above equation at $t = 0$ with $\kappa \to \infty$, i.e., expected instant transition of the exchange rate to the new long-run equilibrium level. Equation (17) depicts the evolution path towards the new long-run equilibrium exchange rate. With a reasonable κ, the exchange rate, after some fluctuations, increases gradually or the domestic currency depreciates gradually while the sticky price is gradually rising, eroding the purchasing power of the currency vis-à-vis the foreign currency.

2.1.3. Further discussion

There is a difference between a falling domestic interest rate as a consequence of an expansion in the monetary base and an act of cutting the interest rate to boost the economy. The former suggests that UIRP have effect initially with which the currency appreciates. With increased confidence in the economy, this may last if the economy indeed expands or is still expected to expand. However, since no growth in the economy is assumed, the currency would devalue when the increase in money supply takes effect upon the price gradually, which erodes the purchasing power of the currency gradually and the currency depreciates consequently. Accompanied by a rising domestic interest rate that is reverting to the level of the world's interest rate and is also to curb rising inflation expectations due to the increase in money supply, the currency depreciates further. Hence, reverse shooting followed by depreciation is more realistic to depict the movement and adjustment of the exchange rate following an increase in money supply. As regard to the latter, an immediate increase in the exchange rate or depreciation of the domestic currency is the convention, which can be taken as overshooting since, at zero increase in money supply, the increase in the exchange rate is always the greater mania. Since the question here is the effect of an increase in money supply rather than that of a cut in the interest rate without increasing money supply, the former case is more pertinent, so is reverse shooting in response.

3. Illustrating cases

This section provides several illustrating cases to demonstrate the evolution of the exchange rate towards its new long-run equilibrium level, following an increase in money supply. The parameters in equation (17) are set according to the discussion of equation (15) with reference to their ranges and the corresponding immediate response of the exchange rate upon the increase in money supply. Figure 1 to Figure 5 demonstrate the adjustment of the exchange rate towards its new long-run equilibrium level along their evolution path for the cases of (immediate) reverse shooting, overshooting and undershooting. Part (a) of the figure exhibits the change and adjustment in the exchange rate along the evolution path over a 12-month horizon until the exchange rate has well settled down at its new long-run equilibrium level; while part (b) of the figure exemplifies the change and adjustment over a 3-month period to enlarge the exchange rate movement upon the increase in money supply and in the following short- to medium-term. Prior to the increase in money supply, the exchange rate in logarithms is $e_0 = Ln(2) = 0.6931$ that has settled down at the long-run equilibrium rate. Money supply is increased by 10 percent, i.e., $\Delta m = 0.1$, at the time $t = 0$. Changes in money supply can be reasonably and realistically smaller than 10 percent, which though does not affect the results. The new long-run equilibrium exchange rate is therefore 0.6931+0.1=0.7931.

{Figure 1}

{Figure 2}

Figure 1 illustrates a reverse shooting case with the parameters γ=0.5, κ=0.1, λ=0.3, θ=1.2. Upon the increase in money supply, the exchange rate decreases, or reversely shoots, to 0.6376. It continues to decrease to 0.5341 and then starts to increase. In about three weeks to one month time, it comes back to its prior level before the increase in money supply. Around two months into the new regime, the exchange rate is below its new long-run equilibrium exchange rate by 5-10 percent of the increase in money supply, or the domestic currency has depreciated by 90-95 percent of the whole amount of depreciation caused by the increase in money supply. By the end of thee months, the exchange rate is below its new long-run equilibrium exchange rate by 1 percent of the increase in money supply, or the domestic currency has depreciated by 99 percent of the whole amount of depreciation caused by the increase in money supply. By the end of 12 months, it is well settled down at its new long-run equilibrium exchange rate. Figure 2 exhibits a reverse shooting case where the adjustment is slower, with γ=0.5, κ=0.02, λ=0.8, θ=1.5, which takes longer time for the exchange rate to converge to its new long-run equilibrium rate. By the end of three months, the exchange rate is below its new long-run equilibrium exchange rate by 65 percent of the increase in money supply, or the domestic currency has only depreciated by 35 percent of the whole amount of depreciation caused by the increase in money supply; and by the end of 12 months, the exchange rate has not totally settled down yet at its long-run equilibrium exchange rate.

{Figure 3}

The overshooting case is illustrated in Figure 3, with the parameters γ=0.5, κ=0.1, λ=0.3, θ=0.5. Different from the Dornbusch (1976) model where the exchange rate decreases monotonously after the initial overshooting or the domestic currency appreciates all along to its new long-run equilibrium level, Figure 3 exhibits that the exchange rate falls below its prior level before the increase in money supply, and then picks up again. That is, the domestic currency appreciates in the short-term and then depreciates gradually in the long-run. This is more plausible, taking into consideration that the price is sticky – fixed in the short-term and subject to change in the long-run. In addition, the shape of the curve or the evolution path in Figure 3 also demonstrates reverse shooting. After the initial overshooting of the exchange rate to 1.0265, the movement in the exchange rate reverts and decreases to 0.5850, and then starts to increase. Similar to the case illustrated in Figure 1, around two months into the new regime, the exchange rate is below its new long-run equilibrium exchange rate by 5-10 percent of the increase in money supply, or the domestic currency has depreciated by 90-95 percent of the whole amount of depreciation caused by the increase in money supply. By the end of thee months, the exchange rate is below its new long-run equilibrium exchange rate by 1 percent of the increase in money supply, or the domestic currency has depreciated by 99 percent of the whole amount of depreciation caused by the increase in money supply. By the end of 12 months, it is well settled down at its new long-run equilibrium exchange rate. Finally, Figure 4 and Figure 5 are for undershooting. Figure 4 shows a case of under shooting, with the parameters γ=0.1, κ=0.04, λ=1.5, θ=0.5. Upon the increase in money supply, the exchange rate increases to 0.7598 that is below its new long-run equilibrium level. After the initial increase, the exchange rate decreases and then picks up. Around three months into the new regime, the exchange rate is below its new long-run equilibrium exchange rate by 5-10 percent of the increase in money supply, or the domestic currency has depreciated by 90-95 percent of the whole amount of depreciation caused by the increase in money supply. The case in Figure 5 is the same as that in Figure 4 except κ=0.01. After the initial increase, the exchange rate reverts and decreases to 0.6741 before it starts to increase over the long-run. As follows, the shape of the curve or the evolution path for overshooting and undershooting all demonstrates reverse shooting as well, and reverse shooting is the common phenomenon that can be shared by the different views of exchange rate adjustments.

{Figure 4}

{Figure 5}

4. Concluding remarks

This paper scrutinizes the adjustment and evolution of the exchange rate towards its new long-run equilibrium level, upon a change in money supply, in the short-term through to the long-run horizon. Reverse shooting of the exchange rate has been put forward. Upon an increase in money supply, the interest rate falls with UIRP taking effect initially with which the currency appreciates, and then the sticky price rises gradually from the medium-term and over the long-run in which the currency depreciates, featuring a phenomenon of reverse shooting. As the rising sticky price erodes the purchasing power of the domestic the currency vis-à-vis the foreign currency gradually, the domestic currency depreciates gradually or the exchange rate increases gradually, following the initial appreciation of the currency in the short-term. Regardless of what the immediate response of the exchange rate to the change in money supply can be argued for, reverse shooting homogenizes the evolution path of exchange rate adjustment and movement for all of them.

A scenario different from the one in this study is that the economy is indeed boosted by the monetary policy, or the increase in money supply is a response to an expanding economy or expected growth in the economy. In this case, the domestic currency would appreciate, stay at the lowered level of the exchange rate, and would not depreciate afterwards in the medium-term to the long-run. Inspecting equation (4) tells this story. An increase in money supply is accompanied by a falling interest rate; then UIRP takes effect and the exchange rate decreases. Holding the world's output and interest rate constant, while the domestic interest rate is reverting to the world level of interest rates, a growing or expanded economy means that the price does not need to rise, leaving the exchange rate at the appreciated level without setting out on depreciation.

As economic systems and economic activity are sophisticated, there can be realities, illusions and disillusions. An increase in money supply with a real need for the growing economy leads to a permanent decrease in the exchange rate. Whereas an increase in money supply with an illusion for economic growth that turns out to be a disillusion leads to domestic currency depreciation, following certain initial appreciation of the currency to varied degrees, which also exhibits reverse shooting as in the core part of this study.

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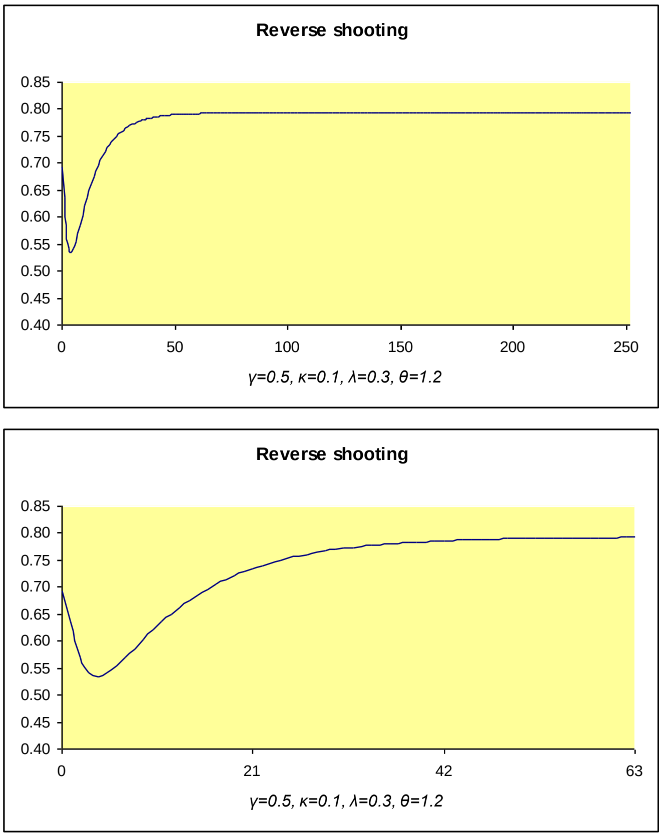
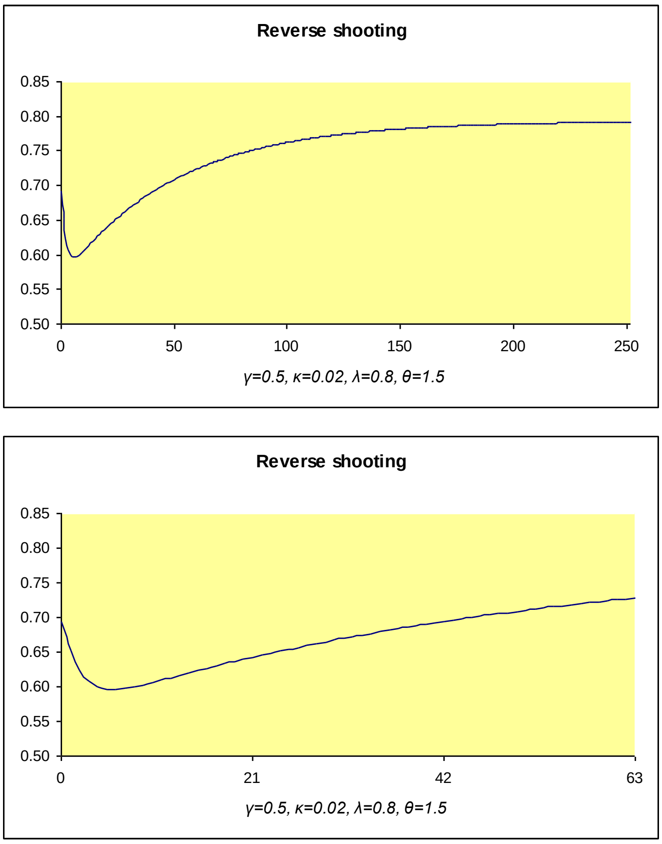


(a)

(b)

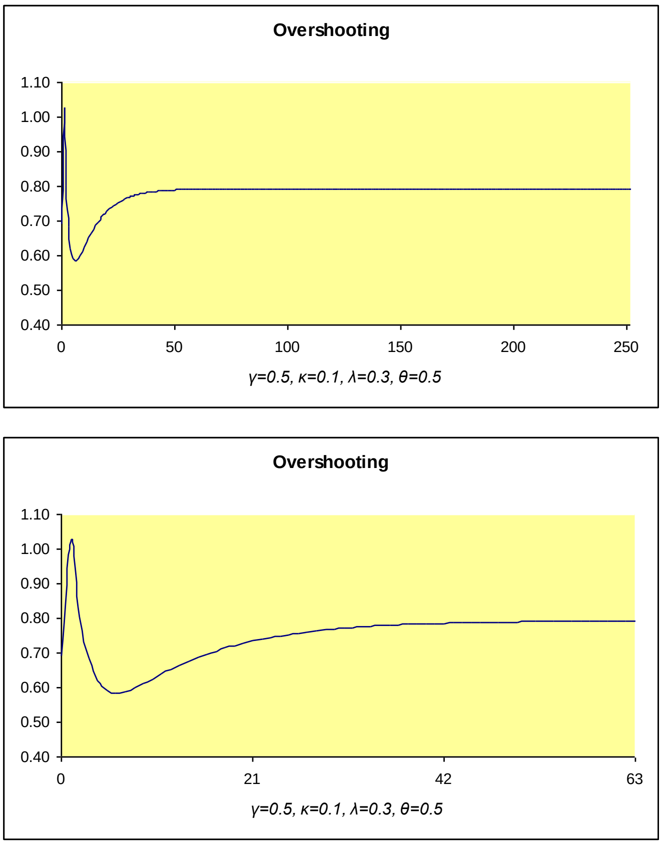
Figure 1. Reverse shooting (γ=0.5, κ=0.1, λ=0.3, θ=1.2)



(a)

(b)

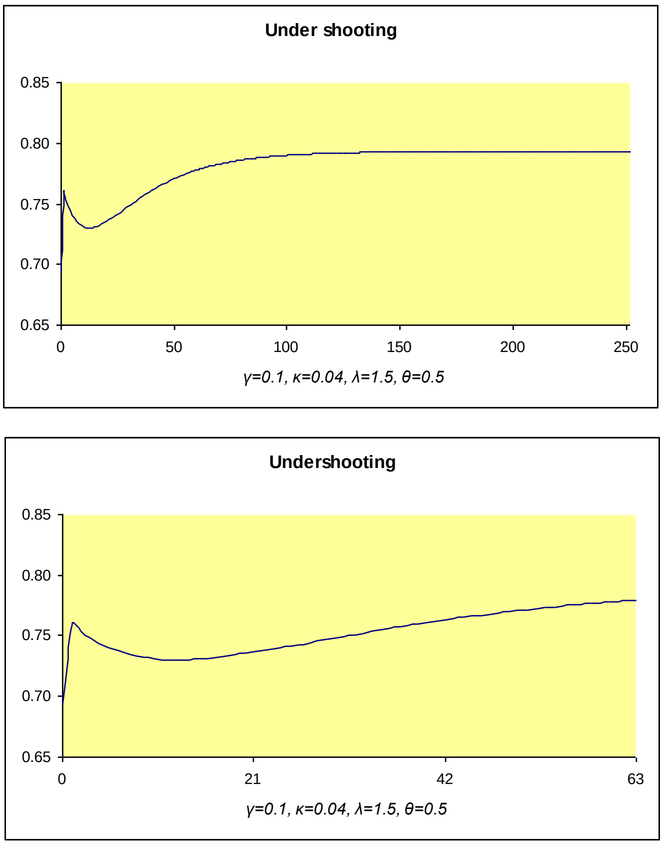
Figure 2. Reverse shooting (γ=0.5, κ=0.02, λ=0.8, θ=1.2)



(a)

(b)

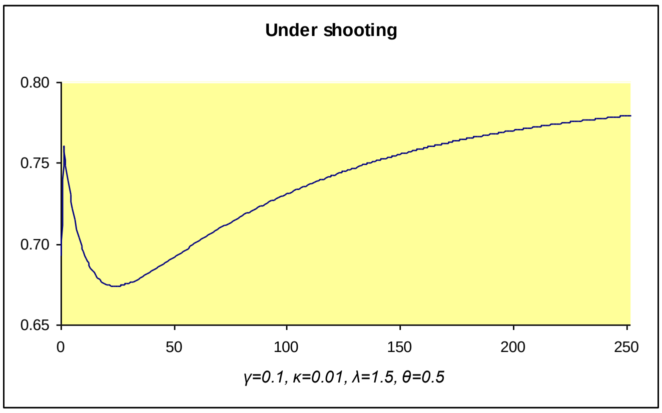
Figure 3. Overshooting (γ=0.5, κ=0.1, λ=0.3, θ=0.5)



(a)

(b)

Figure 4. Undershooting (γ=0.1, κ=0.04, λ=1.5, θ=0.5)



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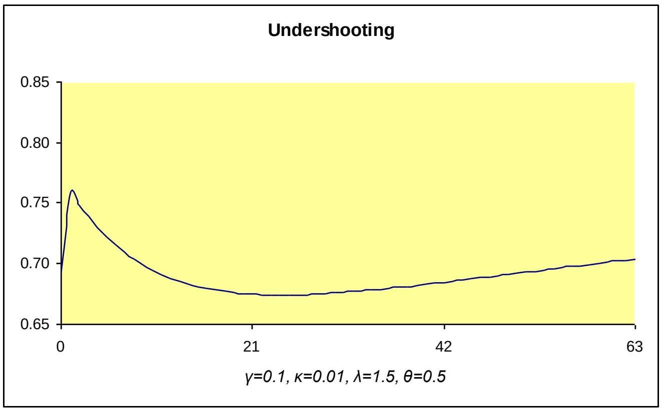

(b)

Figure 5. Undershooting (γ=0.1, κ=0.01, λ=1.5, θ=0.5)