Does Trading Volume Really Explain Stock Returns Volatility?

by

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and

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Abstract

Assuming that the variance of daily price changes and trading volume are both driven by the same latent variable measuring the number of price-relevant information arriving on the market, the Mixture of Distribution Hypothesis (MDH) represents an intuitive and appealing explanation for the empirically observed correlation between volume and volatility of speculative assets.

This paper investigates to which extent the temporal dependence of volatility and volume is compatible with a MDH model through a systematic analysis of the long memory properties of power transformations of both series.

It is found that the fractional differencing parameter of the volatility series reaches its maximum for a power transformation around 0.75 and then decreases for other order moments while the differencing parameter of the trading volume remains remarkably unchanged. The volatility process thus exhibits a high degree of intermittence whereas the volume dynamic appears much smoother. The results suggest that volatility and volume may share common short-term movements but that their long-run behavior is fundamentally different.

Keywords: Volatility Persistence, Long Memory, Trading Volume.

JEL Classification: C13, C52, G15.

1. Introduction

The relations among trading volume, stock returns and price volatility, the subject of empirical and theoretical studies over many years, have recently received renewed attention with the increased availability of high frequency data. A vast amount of the empirical research has documented what is now known as the "stylized facts" about asset returns and trading volume. In particular, speculative asset returns are found to be leptokurtic relative to the normal distribution and exhibit a high degree of volatility persistence. The same abnormality is found for the trading volume which also happens to be is positively correlated with squared or absolute returns.

A meaningful approach for rationalizing the strong contemporaneous correlation between trading volume and volatility – as measured by absolute or squared returns – is provided by the so-called Mixture of Distribution Hypothesis (MDH) introduced by Clark (1973). In this model, the variance of daily price changes and trading volume are both driven by the same latent variable measuring the number of price-relevant information arriving on the market. The arrival of unexpected "good news" results in a price increase whereas "bad news" produces a price decrease. Both events are accompanied by above-average trading activity in the market as it adjusts to a new equilibrium. The absolute return (volatility) and trading volume will thus exhibit a positive correlation due to their common dependence on the latent information flow process.

Another successful specification for characterizing the dynamic behavior of asset price volatility is based on the AutoRegressive Conditionally Heteroskedastic (ARCH) model of Engle (1982) and the Generalized ARCH (GARCH) of Bollerslev (1986). In this class of models, the conditional variance of price changes is a simple function of past information contained in previous price changes. The autoregressive structure in the variance specification allows for the persistence of volatility shocks, enabling the model to capture the frequently observed clustering of similar-sized price changes, the so-called GARCH effects.

These univariate time series models, however, are rather silent about the sources of the persistence in the volatility process. In the search of the origin of these GARCH effects, Lamoureux and Lastrapes (1990) analyze whether they can be attributed to a corresponding time series behavior of the information arrival process in Clark's mixture model. Inserting the contemporaneous trading volume in the conditional variance specification shows that this variable has significant explanatory power and that previous price changes contain negligible additional information when volume is included in the variance equation.

This inference, however, is based on the assumption that trading volume is weakly exogenous, which is not adequate if price changes and trading volume are jointly determined. As explained by Andersen (1996) it seems to be necessary to analyze the origin of GARCH effects in a setting where trading volume is treated as an endogenous variable. Tauchen and Pitts (1983) refined Clark's univariate mixture specification by including the trading volume as an endogenous variable and proposed a Bivariate Mixture Model (BMM) in which volatility and trading volume are jointly directed by the latent number of information arrivals. This implies that the dynamics of both variables are restricted to depend only on the time series behavior of the information arrival process. Hence, if the bivariate mixture models are the correct specification, the time series of trading volume provides information about the factor which generates the persistence in the volatility process.

Unfortunately, recent empirical studies reveal some shortcomings in the bivariate mixture models. Lamoureux and Lastrapes (1994) show that the estimated series of latent information arrival process does not fully account for the persistence of stock price volatility. Similar results were obtained by Andersen (1996) and Liesenfeld (1998) even in a context where an autoregressive structure is put on the latent information arrival process. In order for the BMM to be able to successfully explain the observed features of the price changes and volume series, Liesenfeld (2001) even presents a generalized mixture model where the latent process includes two components (the number of information arrivals and the traders' sensitivity to new information), both endowed with their own dynamic behavior.

Although from a market microstructure perspective, the BMM representation is intuitively appealing, the absence of strong empirical support for the model seems to suggest that volatility and trading volume have too different dynamics to be directed by the same latent process as suggested by the BMM. It also appears that the fundamental differences of behavior, making the BMM untenable, should be looked for in the structure of temporal dependencies of both series.

To this respect, an extensive empirical literature has developed over the past decade for modeling the temporal dependencies in financial markets volatility. A common finding to emerge from most of the studies concerns the extremely high degree of own serial dependencies in the series of absolute or squared returns. However, the available empirical evidence regarding the dynamic dependencies in financial market trading volume is more limited. Lobato and Velasco (2000) analyze the long memory property for the trading volume and volatility (as measured by squared or absolute returns) of 30 stocks composing the Dow Jones Industrial Average index. They conclude that return volatility (R_t^2 or $|R_t|$) and trading volume (V_t) possess the same long memory parameter, lending some support to Bollerslev and Jubinski's (1999) mixture model where a common latent process exhibiting long memory is used.

In an investigation of the long-run dependencies in stock returns, Ding Granger and Engle (1993) explain, however, that power transformations other than unity or square have to be considered to fully characterize the long-run property of a financial series. Considering the temporal properties of the functions $|R_t|^q$ for positive values of q, they show that the power transformations of returns do exhibit long memory with quite high autocorrelations for long lags and that this property is strongest for $q=1$ or near 1 compared to both smaller and larger positive values.

The main contribution of this paper is to find out to which extent the temporal dependence of volatility and volume of speculative assets is compatible with a MDH model through a systematic analysis of the long memory properties of power transformations of order q of both the return and the trading volume series (i.e.,

$|R_t|^q$ and V_t^q). To this end we follow the methodology introduced in Ding, Granger and Engle (1993) and Ding and Granger (1996): the analysis of long memory is tantamount to studying the decay rate of the autocorrelation function. The output of such an analysis yields the fractional integration parameter commonly denoted by d. In this paper, it is obtained through the semiparametric techniques developed by Robinson (1994, 1995a and 1995b). The results obtained are quite surprising: whereas the fractional differencing parameter, d, reaches its maximum for $q = 0.75$ and then decreases for higher order moments in the case of the volatility, the same differencing parameter remains remarkably unchanged in the case of the trading volume. Hence, the volatility process appears to be more complex than the volume process and exhibits a higher degree of intermittence[1].

Restating the results in the very simple and intuitive framework developed by Lamoureux and Lastrapes (1990), we observe that the inclusion of trading volume in the conditional variance equation of these stocks does not change the degree of temporal dependence. That is, it leaves the level of volatility persistence, as measured by the sum $\alpha + \beta$, virtually unchanged and the volume coefficient is not significant. Trading volume is only able to explain the volatility persistence of stocks with the lower degree of intermittence. In this situation, we recover the appealing result of Lamoureux and Lastrapes (1990), namely the fact that volume becomes highly significant and the volatility persistence measured by $\alpha + \beta$ decreases to zero. Our results suggest that volatility and volume may share common short-term movements but that their long-run behavior is fundamentally different.

In the search for improvements of the BMM framework that enable to account for the asymmetric behavior of volume and volatility on the short- and long-run, two competing models were recently presented in the literature. On the one hand, Bollerslev and Jubinski (1999) find that the long-run dependencies of volume and volatility are common but that the short-run responses to certain types of "news" are not necessarily the same across the two variables. With a different specification, Liesenfeld (2001) explains that the short-run volatility dynamics are directed by the

[1] Broadly speaking, what we mean by intermittence is brutal movements in the volatility series.

information arrival process, whereas the long-run dynamics are associated with the sensitivity to new information. On the contrary, the variation of the sensitivity to news is largely irrelevant for the behavior of trading volume which is mainly determined by the variation of the number of information arrivals. Our results obtained using semiparametric methods outside this BMM framework thus lend support to Liesenfeld's specification in the sense that it differenciates volume and volatility for their long-run behavior.

The remainder of this paper is organized as follows. Section 2 briefly reviews the methodology of the Ding, Granger and Engle test. The data, the empirical estimations and the results are presented in Section 3. An intuitive correspondence with the MDH framework of Lamoureux and Lastrapes (1990) is discussed in Section 4 while the last section concludes.

2. Long-Run Dependencies in Volatility and Volume

In agreement with the efficient market theory, empirical studies have shown that although stock market returns are uncorrelated at lags larger than a few minutes, where some microstructure effects might apply, absolute and squared returns - common measures of volatility - do exhibit long-range dependencies in their autocorrelation function. In order to better define the notion of long memory, we follow Robinson (1994) among others. A stationary process presents long memory if its autocorrelation function $\rho(j)$ has asymptotically the following rate of decay:

$$\rho(j) \approx L(j)\, j^{2d-1} \text{ as } j \to \infty, \tag{1}$$

where $L(j)$ is a slowly varying function[2] and $d \in (0,1/2)$ is the parameter governing the slow rate of decay of the autocorrelation function. This parameter d measures the degree of long-range dependence of the series. In this context, the long memory property of the absolute returns should be written as:

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[2] Such that $\lim_{j\to\infty} L(\lambda j)/L(j) = 1, \forall \lambda > 0$.

$$\rho(|R_t|, j) \approx L(j)\, j^{2d-1} \text{ as } j \to \infty . \qquad (2)$$

Studying a large variety of speculative assets, Taylor (1986) first highlighted the existence of such an empirical regularity in the autocorrelation of the absolute returns.

Applying the Granger & Newbold (1977) techniques for power transforms of Normal distributions, Andersen & Bollerslev (1997) push the analysis one step further and theoretically show that, in this context, any power transformation of the absolute returns, $|R_t|^q$, possesses this long memory property. Namely, that:

$$\rho(|R_t|^q, j) \approx j^{2d-1} \qquad (3)$$

where j is large and denotes the jth information arrival process and d the hyperbolic rate of decay or the fractional differencing parameter ($0 < d < 1/2$). From an empirical viewpoint, Ding, Granger and Engle (1993) use the S&P 500 stock index to study the decay rate of the autocorrelation function when different power transformations of the absolute returns are analyzed (i.e., $|R_t|^q$ for $q = 0.25, 0.5, ..., 2$). They indeed conclude to the existence of a long memory property regardless the value for the parameter q and also show that the slowest decay rate for the autocorrelation function is obtained for values of q close to 1.

Whatever its form, the MDH framework does not mean a causal relationship between the variance of daily price changes and trading volume. Both variables are assumed to be driven by the same latent process measuring the number of price-relevant information arriving in the market. As such, it implies a common long-range dependence in the volatility and the volume processes. If the MDH represents a correct specification of the contemporaneous behavior of volatility and volume, the autocorrelation function of the latter process should exhibit the same rate of decay as the autocorrelation function of volatility as represented by $|R_t|$. Hence one should observe the following:

$$\rho(V_t, j) \approx j^{2d-1} \text{ as } j \to \infty \text{ and } 0 < d < 1/2 , \qquad (4)$$

with V_t being the trading volume.

Moreover, under some specific distributional assumptions (see Bollerslev and Jubinski (1999)), the cross-correlations between the volatility and the trading volume may also present the same hyperbolic decay:

$$corr(|R_t|, V_{t-j}) \approx corr(V_t, |R_{t-j}|) \approx j^{2d-1}. \qquad (5)$$

One way of testing the adequacy of the MDH models is thus through an analysis of the long memory behavior of the volatility and volume processes as well as the rate of decay of their cross-correlations functions. In this direction, we apply the Ding, Granger and Engle (1993) approach and do not restrict our analysis to a single power transformation of both series. Rather, we investigate the rate of decay of the autocorrelation functions $\rho(|R_t|^q, j)$ and $\rho(V_t^q, j)$ for different values of the power term (i.e., for $q = 0.25, 0.5, ..., 4$). In addition to representing a new method for testing the simultaneous behavior of volatility and volume, our approach offers the interesting property of providing a test for the MDH that does not rely on any parametric specification of the latent process.

In this paper, we use a semiparametric framework to estimate the degree of fractional differencing d. Although this type of approach necessarily results in an efficiency loss compared to parametric methods (like MLE or GMM), it allows avoiding problems resulting from model misspecifications in the parametric case (Bollerslev and Jubinski (1999)). The approach relies on the spectrum $f(\omega)$ of a covariance stationary process X_t, at frequency ω, defined by:

$$\text{cov}(|X_t|, \tau) = \int_{-\pi}^{\pi} f(\omega)\ \exp(i\,\omega\,\tau)\ d\omega, \qquad (6)$$

with $\tau = 0, \pm 1, ...$. If the series is fractionally integrated, then, for frequencies ω close to 0,

$$f(\omega) \approx C\,\omega^{-2d} \text{ as } \omega \to 0^+, \qquad (7)$$

where C is a strictly positive constant. Nevertheless, the spectrum of a long memory process has a singular feature at frequency zero as $\lim_{\omega \to 0^+} f(\omega) = \infty$. Hence, instead of assuming the knowledge of this process at all frequencies, one only establishes some hypothesis concerning the behavior of the spectral density in the neighborhood of the origin (around the low frequencies). As there is no parametric assumption about the spectrum outside the neighborhood of the origin, the approach is called semiparametric.

Let the process for the absolute returns or the trading volume X_t be:

$$(1-L)^d \mid X_t \mid = \eta_t \ , \tag{8}$$

with L being the lag operator and η_t representing a stationary and ergodic process with a bounded spectrum, $f_\eta(\omega)$, at all frequencies ω. Then, the spectrum for the process X_t will be:

$$f(\omega) = \left| \left[1-\exp(-i\,\omega)\right]^{-d} \right|^2 f_\eta(\omega), \tag{9}$$

with $f_\eta(\omega)$ being positive, even, continuous and bounded away from zero and from infinity. In this framework, d controls for the long memory characteristics whereas $f_\eta(\omega)$ integrates the short term behavior. The only thing that we need to specify concerning the form of the function $f_\eta(\omega)$ is that in the neighborhood of the origin, i.e. $\omega \to 0$,

$$f(\omega) = \mid \omega \mid^{-2d} f_\eta(0). \tag{10}$$

We then have:

$$\ln\left[\, f(\omega) \,\right] \approx \ln f_\eta(0) - 2\,d\ \ln(\omega), \tag{11}$$

and the spectrum is approximately log-linear for the long-run frequencies.

A widely known and commonly used semiparametric estimator for d based directly on this relation is the so-called GPH log-periodogram regression estimator introduced by Geweke and Porter-Hudak (1983)) and denoted by $\hat{d}_{GPH}$. It is obtained by running the following regression:

$$\ln\left[\, I(\omega_j) \,\right] = \beta_0 - 2\,d\ln\left|1-\exp(-i\,\omega_j)\right| + e_j, \tag{12}$$

where T denotes the sample size and $I(\omega_j)$ is the series periodogram[3] at the *jth* Fourrier frequency, $\omega_j = 2\pi j / T \in (0, \pi)$. Hence, the logarithm of the sample periodogram ordinates is regressed on a constant and the (lowest) Fourrier frequencies. The GPH regression estimator $\hat{d}_{GPH}$ is then simply calculated as being $-1/2$ times the estimated slope of this regression.

As $f(\omega) = |\omega|^{-2d} f_\eta(0)$ only works for ω_j close to zero, we must restrict the regression to the Fourrier frequencies in the neighborhood of the origin. This is why the regression is run by using only the first m Fourrier frequencies close to zero (i.e. , $j = l+1, l+2, ..., m$), where l and m are the trimming and truncation parameters.

The consistency of this estimator is provided by Robinson (1995a and 1995b) under regularity conditions (namely, $m \to \infty$, $l \to \infty$ but $\frac{l}{m} \to 0$ and $\frac{m}{T} \to 0$) as well as the assumption of normality of the analyzed series. In this situation, the estimator itself is asymptotically Gaussian, having a variance equal to $(1/m) * (\pi^2 / 24)$. However, the absolute returns $|R_t|$ and the trading volume V_t, like most financial series, violate the Gaussian assumption and invalidate the asymptotic theory for the $\hat{d}_{GPH}$ estimator. In order to overcome this difficulty, we thus introduce the less restrictive estimator adopted by Andersen and Bollerslev (1997). Denoted by $\hat{d}_{AP}$, this most robust estimator is based on the average periodogram ratio for two frequencies close to zero as shown below:

$$\hat{d}_{AP} = \frac{1}{2} - \frac{\ln\left[\hat{F}(\tau\,\omega_m) / \hat{F}(\omega_m)\right]}{2\ln(\tau)}, \tag{13}$$

where $\hat{F}(\omega)$ is the average periodogram, $\hat{F}(\omega) = \frac{2\pi}{T} \sum_{j=1}^{m} I(\omega_j)$ for frequencies $j = 1, 2, ..., m$ ($m \to \infty$ but $\frac{m}{T} \to 0$) and $0 < \tau < 1$. By construction, the estimated

[3] $I(\omega_j) = (2\pi T)^{-1} \left| \sum_{t=1}^{T} |X_t| \exp(i\, t\, \omega_j) \right|^2$.

parameter[4] $\hat{d}_{AP}$ is in the stationary range since it is below $1/2$. Moreover, Lobato and Robinson (1997) prove that $\hat{d}_{AP}$ is asymptotically Gaussian for $0 < d < 1/4$ and non normally distributed for $1/4 \leq d < 1/2$.

In the following empirical analysis, we thus use the Andersen and Bollerslev estimator $\hat{d}_{AP}$ to measure the long-run dependencies in the absolute moments of order q $(q = 0.25, 0.5, ..., 4)$ of both the return and the trading volume series.

3. Breaking Out the Conventional Viewpoint

The data set used for our empirical work consists in daily prices and trading volume for 50 London Stock Exchange "blue chips" quoted between January 1990 and May 2001. All series were collected from Datastream and include 2874 observations. To save space and to ease the presentation, results are only provided for six stocks: Allied Domecq, Hilton GP, British Land, Barclays, Reuters GP and Dixons GP. They are representative, however, of what is obtained for the whole sample. Returns are calculated as differences of price logarithms and the trading volume is also used in logarithm[5].

Table 1 presents the usual descriptive statistics both for the return and volume series of each of the six stocks. The sample moments for all return series indicate empirical distributions with heavy tails relative to the normal. The return series also exhibit a positive asymmetry except for Dixons GP returns that happen to be negatively skewed. Not surprisingly, the Jarque-Bera statistic rejects normality for each of the return series at the 5% level of significance, a level that is used throughout this paper. Trading volume also appears to be non-normally distributed although the leptokurtosis and the asymmetry are less pronounced.

Insert about here Table 1

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[4] In our estimations we use $m = T^{1/2}$ and the scalar $\tau = 0.25$.

[5] The tests were also done on the trading volume expressed by the number of shares and the results are qualitatively the same.

Since the early work of Harris (1986 and 1987), several papers have presented tests of the mixture of distribution hypothesis using different speculative assets and data frequencies. However, although Harris' tests only rely on simple predictions emanating from the assumption that prices and volume evolve at uniform rates in transaction times (namely, basic tests on the correlation of volume or number of trades with prices and squared prices or else on the autocorrelation functions of these variables), the following studies rely on specific distributional assumptions or parameterizations for the directing process.

Indeed, in the univariate setting, returns are modeled by a subordinated process with the traded volume regarded as a proxy for the directing process and tests are then performed relative to specific distributional assumptions for this variable (see Clark (1973) or Richardson and Smith (1994)). In the bivariate setting, both returns and volume are assumed to be directed by a latent process and empirical tests crucially depends on the selected dynamic for this variable (see Andersen (1996) Watanabe (2000) or Liesenfeld (2001)).

In this study we try to build our tests for the MDH in a nonparametric framework to recover the generality of Harris' first investigations of the model. As explained in the previous section, the MDH framework does not imply at all a causal relationship between the variance of daily price changes and trading volume. Since both variables are assumed to be driven by the same latent process, they must exhibit the same long-range dependence. Hence, if the MDH represents a correct specification of the contemporaneous behavior of volatility and volume, the autocorrelation function of the latter process should exhibit the same rate of decay as the autocorrelation function of volatility. The same hyperbolic decay may also be found for the cross-correlations between the volatility and the trading.

Our tests for the adequacy of the MDH models will thus be carried out through an analysis of the long memory behavior of the volatility and volume processes as well as the rate of decay of their cross-correlation functions. This approach thus provides new tests for the MDH that do not rely on any parametric specification of the latent process.

Insert about here Figure 1

Figure 1 starts this analysis by a representation of the autocorrelograms obtained for the absolute returns – our measure of volatility – and the trading volume of six LSE stocks. Consistent with Ding and Granger (1996), the autocorrelations present the slow, hyperbolic decay, typically found in long memory processes. Moreover, most of these autocorrelations are positive and statistically significant, as lying outside the Gaussian confidence bandwidths.

We already observe, however, some important differences in the behavior of the autocorrelation function for the trading volume relative to that of the absolute returns. The autocorrelation of absolute returns seems to die away much faster in the case of British Land, Hilton GP, and to some extents Barclays, than it does for the trading volume series, implying the possibility of a different long-run behavior.

Given the importance of the existence or non-existence of a common long-run behavior of volatility and volume for the MDH model, a formal test of the presence of long-run dependencies in both series is required. To this end, we use the so called Lo's R/S long-term dependence test. Lo's (1991) modified R/S statistic for long-range dependence in a financial series X may be defined as follows:

$$Q(m,t) = \frac{1}{\hat{s}_{T,m}}\left[\max_{1\le k\le T}\sum_{j=1}^{k}(X_j - \overline{X}) - \min_{1\le k\le T}\sum_{j=1}^{k}(X_j - \overline{X})\right], \tag{14}$$

where m is a truncature parameter, $\overline{X}$ is the sample mean (i.e. $\overline{X} = \frac{1}{T}\sum_{j=1}^{T} X_j$) and the quantity $\hat{s}^2_{T,m}$ represents an estimator of $\sigma^2 = \sum_j \mathrm{cov}(X_j, X_0)$ defined by $\hat{s}^2_{T,m} = \frac{1}{T}\sum_{j=1}^{T}(X_j - \overline{X})^2 + 2\sum_{j=1}^{T}\omega_j(m)\hat{\gamma}_j$, with $\omega_j(m) = 1 - \frac{j}{m+1}$ and $\hat{\gamma}_j = \frac{1}{T}\sum_{i=1}^{T-j}(X_i - \overline{X})(X_{i+j} - \overline{X})$, $0 \le j < T$. Lo computed confidence intervals for the statistic $(1/\sqrt{T})Q(m,T)$, namely he uses the interval $[0.809, 1.862]$ as the 95% acceptance region for the null hypothesis of no long-range dependence.

To better understand the link between the R/S statistic and the fractional integration parameter d, let us recall that when $m = 0$, the R/S statistic amounts to estimating the limit of $\log Q(0,T)/\log T$ called Hurst coefficient. This coefficient, usually denoted by H is related to the fractional integration parameter by $H = d - 1/2$.

The results for Lo's long-term dependence test are provided in Table 2. In agreement with Lobato and Velasco (2000), for all stocks, both the absolute returns and the trading volume series exhibit long-run dependence (i.e., $0 < d < 1/2$) as the statistics remain outside the 95% confidence interval. Hence one cannot reject the MDH when tests of a similar long memory property for volatility and trading volume are carried out with a power transformation of unity (i.e., using $|R_t|^q$ and V_t^q with $q = 1$).

Insert about here Table 2

Although the existence of the same long-run dependence in the cross-correlations of volatility and volume (equation (5)) requires more stringent assumptions and thus not represent in itself a way of rejecting all possible versions of the MDH, it remains an interesting feature to study. For our sample of six stocks, Figure 2 shows these cross-correlation functions. We observe the same hyperbolic decay as for the autocorrelation function of the absolute returns or the trading volume series in the case of four stocks, lending more support to the assumption that these series may be driven by the same latent process. For Barclays and British Land, however, the cross-correlations are not significant since they stay inside the confidence interval. If this cannot be regarded as such as a sufficient feature to reject the MDH for these stocks, we may expect, if it still holds, the relation between volatility and volume to be much weaker.

Insert about here Figure 2

To fully compare the long-run behavior of volatility and volume, we follow Ding, Granger and Engle (1993) and do not restrict the analysis to a single power transformation of both series. Indeed, we now investigate the rate of decay of the autocorrelation functions $\rho(|R_t|^q, j)$ and $\rho(V_t^q, j)$ for different values of the power term (i.e., for $q = 0.25, 0.5, ..., 4$). Instead of giving all the corresponding graphs, we summarize the results through a parameter measuring the level of long memory for each series and each value of the power transformation q. This parameter is the degree of fractional differencing d. As explained in the previous section we use the Andersen and Bollerslev (1997) semiparametric estimator, $\hat{d}_{AP}$, based on the average periodogram ratio for two frequencies close to zero and defined in equation (13). Figure 3 shows the obtained parameters both for the absolute returns and the trading volume series as a function of the power transformation q while Table 3 provides the corresponding values.

Insert about here Figure 3 and Table 3

First of all, being always in the interval $(0,1/2)$, the estimated $\hat{d}_{AP}$ reveals the presence of the long memory in almost all the power transformations of both the absolute returns and the trading volume series. However, a striking feature appears: whereas the fractional differencing parameter takes almost the same values for the trading volume regardless of the power transformation q, the results are rather different for the volatility –measured by the absolute returns -. For the latter, the rate of decay of the autocorrelation function has its maximum around $q = 0.75$ and then decreases, more or less rapidly, to significantly smaller values. This result was obtained for the 50 LSE stocks used in this study. Indeed, the rate of decay of the autocorrelation function of the power transformation of the absolute returns always presents a maximum for an exponent q in the range $[0.5,1]$ and decreases significantly for larger values while the same estimator $\hat{d}_{AP}$ remains remarkably insensitive to the power transformation q for the trading volume series.

Moreover, we observe that the difference of behavior of the degree of fractional differencing is strongest in the case of stocks that exhibit no long-run dependence in the cross-correlations. Indeed, the fractional integration parameter $\hat{d}_{AP}$ of the volatility process decreases from 0.3745 to 0.1705 for Barclays and even from 0.3048 to 0 for British Land while the estimators for the volume series remain virtually at the same level. These stocks clearly present a very different long-term behavior for the volatility and volume processes. Overall, the simultaneous analysis of the fractional differencing parameter for the volatility and the trading volume series at different power transformations clearly shows that both processes present fundamentally different behaviors in the long-run.

To fully understand the difference in behavior, it seems interesting to link the analysis of the fractional differencing parameter to the so-called intermittency of a process. Indeed, the usual tool to analyze the local smoothness of a process X is by computing its structure functions, namely the sample moments of its increments (that is $E(|X(t,t+T)|^q)$ where $X(t,t+T)$ is the increment of X between time t and $t+T$ and $q \geq 0$). The resulting scaling law may either be a linear function of q, namely $E(|X(t,t+T)|^q) \approx T^{qH}$ where H is again the Hurst exponent (and $H = d - 1/2$), or a nonlinear function of q, i.e. $E(|X(t,t+T)|^q) \approx T^{\zeta(q)}$. In the former situation, X exhibits smooth trajectories while, in the latter situation, the process presents a very unsmooth local structure and is said to be intermittent.

The presence of very different values for the fractional differencing parameter d when different power transformations of the absolute returns are used is thus clearly a sign of intermittence in the volatility process. On the contrary, the trading volume seems to be a much smoother process that might be associated with a linear scaling law. This underlines once more the different time behavior between volatility and volume. From the time dependency and the time aggregation viewpoints, the volatility process appears to be much more complex than the trading volume process.

4. Volume Versus GARCH Effects Revisited

Many empirical studies have shown that the GARCH model of Bollerslev (1986) is particularly successful in its ability to capture the clustering of similar-sized price changes through a conditional variance that depends on the past squared residuals of the process. In its GARCH (1,1) form, the model corresponds to the following equations:

$$R_t = \mu_{t-1} + \varepsilon_t \,, \tag{15}$$

$$\varepsilon_t \,/\, \varepsilon_{t-1}, \varepsilon_{t-2}, \ldots \sim N(0, \sigma_t^2)\,, \tag{16}$$

$$\sigma_t^2 = \omega + \alpha\, \varepsilon_{t-1}^2 + \beta\, \sigma_{t-1}^2 \,, \tag{17}$$

where ε_t is the conditionally Gaussian residual and μ_{t-1} represents the conditional mean. Since the focus of this section is strictly on the impact of the trading volume in the variance equation, we simply assume an autoregressive process of order 1 for all stocks. The degree of persistence in the volatility is measured by the sum of the coefficients α and β.

Using the MDH framework, Lamoureux and Lastrapes (1990) argue that the observed GARCH effects in financial time series may be explained as a manifestation of time dependence in the rate of evolution of intraday equilibrium returns. They suggest that the daily number of information arrivals directing the price process may be proxied by the trading volume. Then, the focus of their analysis is to assess the degree of volatility persistence that remains in a GARCH (1,1) model conditional on the knowledge of the mixing variable (i.e., the trading volume). To do this, they use the previous GARCH (1,1) model where the conditional variance equation is replaced by the following:

$$\sigma_t^2 = \omega + \alpha\, \varepsilon_{t-1}^2 + \beta\, \sigma_{t-1}^2 + \gamma V_t \,. \tag{18}$$

Equation (18) now models the conditional variance of returns as a GARCH (1,1) process with the trading volume V_t as explanatory variable. The mixture model predicts that γ should be positive and significant. Moreover, the persistence of variance as measured by $\alpha + \beta$ should become negligible if accounting for the uneven flow of information with the trading volume explains the presence of GARCH effects in the data.

Their empirical analysis based on 20 actively traded stocks strongly supports the MDH framework. The sample period used for their study is very small, however, and does not include financial crises. Using different data and time periods, many studies (see for instance Kamath, Chatrath and Chaudhry (1993) or Sharma, Mougoue and Kamath (1996)) strongly question the informational power of trading volume in the GARCH setting. Using the 50 LSE stocks, we re-estimate the GARCH (1,1) model without and with volume as described respectively by equations (17) and (18). Results are summarized in Tables 4 and 5.

Insert about here Table 4 and Table 5

The estimated GARCH (1,1) models without volume all support the existence of a strong persistence in the volatility process with a sum of coefficients α and β always above 0.9. Results of the inclusion of volume as an explanatory variable in the variance equation, however, provide mitigated conclusions, not entirely rejecting the Lamoureux and Lastrapes (1990) findings nor giving them an unconditional support. Indeed, for two stocks, namely Allied Domecq and Hilton GP, the trading volume has an important explanatory power. When included in the conditional variance equation, the coefficient γ is significantly positive and the persistence in volatility as measured by $\alpha+\beta$ is much smaller. For two other stocks, namely Reuters GP and Dixons GP, the trading volume has a limited explanatory power: even if the volume coefficient is significantly positive, GARCH effects are almost the same and the persistence in volatility does not change significantly[6]. Finally, for the last two stocks, namely British Land and Barclays, the trading volume does not explain the volatility at all: the coefficient γ is not statistically significant and the persistence in volatility is not affected by the inclusion of volume in the equation.

[6] This is the most common result that we had on the whole sample of the 50 LSE stocks.

An interesting way of understanding why, in some situations, the volume is able to capture GARCH effects while most of the time is does not allow explaining the fluctuations of volatility is by linking the results obtained in the simple Lamoureux and Lastrapes (1990) framework with our findings on the behavior of the fractional differencing parameter. Indeed, the informational content of the trading volume happens to be relevant to understanding the dynamics of volatility only in situations where the difference of behavior in the parameter d is small for the volume and volatility series. On the contrary, when the volatility process presents a high level of intermittence (typically, the case of British Land and Barclays) while the volume process remains very smooth, their structures become too different to be driven by the same latent process. In this situation, the MDH does not hold and the Lamoureux and Lastrapes approach fails to capture the GARCH effects of the volatility through the inclusion of the trading volume in the variance equation.

5. Conclusion

The systematic presence of leptokurtosicity in asset returns as well as the documented positive correlation between trading volume and squared or absolute returns have found their most convincing theoretical explanation in the so-called Mixture of Distribution Hypothesis (MDH). The model assumes that volatility and volume are directed by the latent number of information arrivals. Far from being neutral, this framework of analysis implies a strong probabilistic relation between both variables, and in particular the existence of a common temporal dependence.

Focusing on the long memory properties of power transformations of absolute returns and trading volume, this paper investigates, in a non parametric setting, to which extent the temporal dependence of volatility and volume of speculative assets is compatible with the MDH model. We apply the methodology introduced in Ding, Granger and Engle (1993) and Ding and Granger (1996) and compute the fractional integration parameter of both series (that is, we study the rate of decay of their autocorrelation functions).

The results obtained are quite surprising: whereas the fractional differencing parameter reaches its maximum for power transformations around $q = 0.75$ and then decreases for higher order moments in the case of the volatility, the same differencing parameter remains remarkably unchanged in the case of the trading volume. The volatility process thus exhibits a high degree of intermittence whereas the volume dynamic appears much smoother.

Reformulating the results in the very intuitive framework introduced by Lamoureux and Lastrapes (1990), we obtain that stocks for which the trading volume has virtually no explanatory power relative to the GARCH effects also correspond to those for which the difference in the fractional parameters of volume and volatility is the strongest.

The results suggest that volatility and volume may share common short-term movements but that their long-run behavior is fundamentally different. This leaves an open window for researchers willing to re-discuss the volatility-volume relationship.

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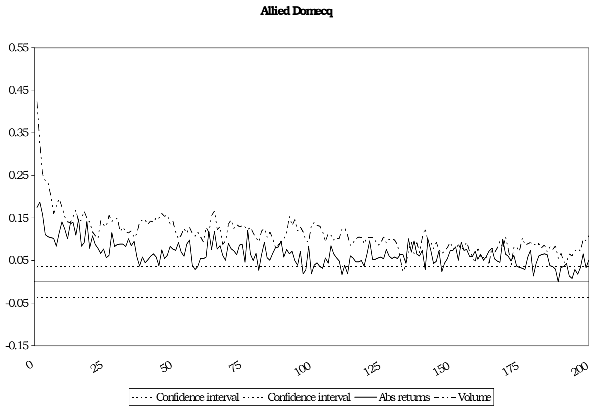
Figure 1

Autocorrelograms for the Absolute Returns - Volatility - and Traded Volume.

We plot the autocorrelations of the absolute returns – as a measure of volatility – and the trading volume of six stocks traded on the London Stock Exchange Market, for lag 1 to lag 200. For all stocks we see the slow hyperbolic decay, which characterises long memory processes, i.e. $\rho(|R_t|, j) \approx j^{2d-1}$ and $\rho(V_t, j) \approx j^{2d-1}$ as $j \to \infty$ and $0 < d < 1/2$. The autocorrelations remain above the 95% confidence interval ($\pm 1.96/\sqrt{T}$) for long lags (even as long as 200 for some stocks). However, the autocorrelation of absolute returns seems to die out much faster in the case of British Land, Hilton GP and even Barclays compared to the trading volume series.


Allied Domecq
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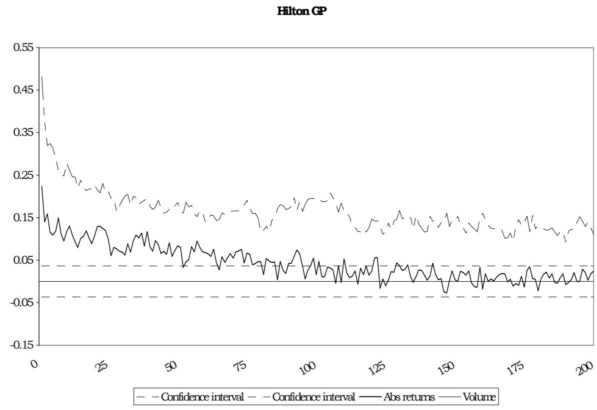
0 25 50 75 100 125 150 175 200
Confidence interval
Confidence interval
Abs returns
Volume


Hilton GP
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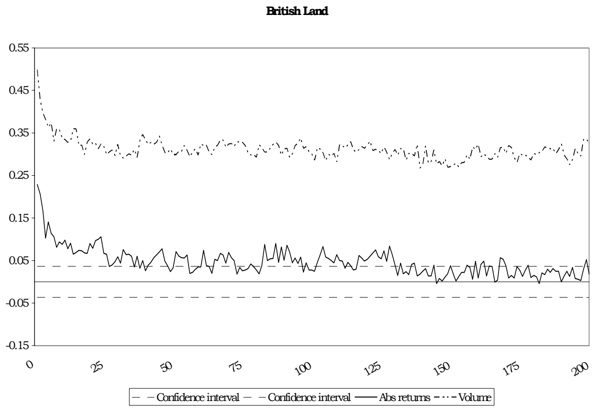
Confidence interval
Confidence interval
Abs returns
Volume


British Land
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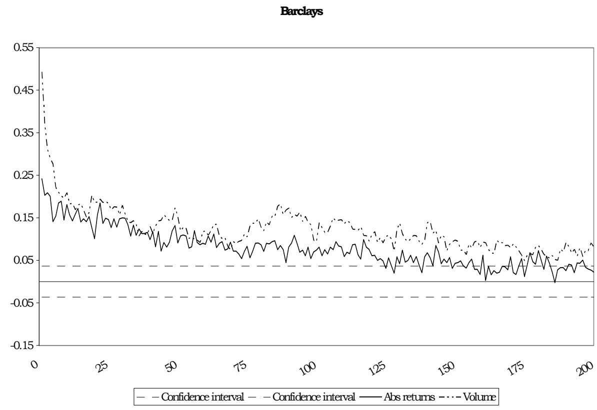
0 25 50 75 100 125 150 175 200
Confidence interval
Confidence interval
Abs returns
Volume


Barclays
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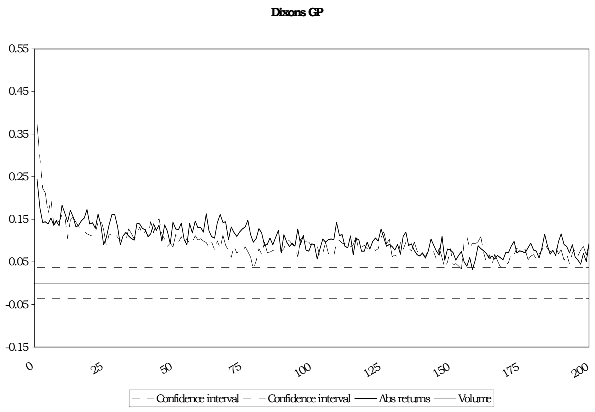
0 25 50 75 100 125 150 175 200
Confidence interval
Confidence interval
Abs returns
Volume


Dixons GP
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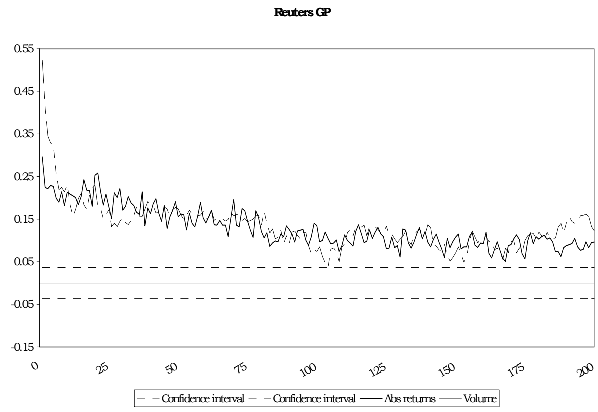
0 25 50 75 100 125 150 175 200
Confidence interval
Confidence interval
Abs returns
Volume


Reuters GP
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-0.05
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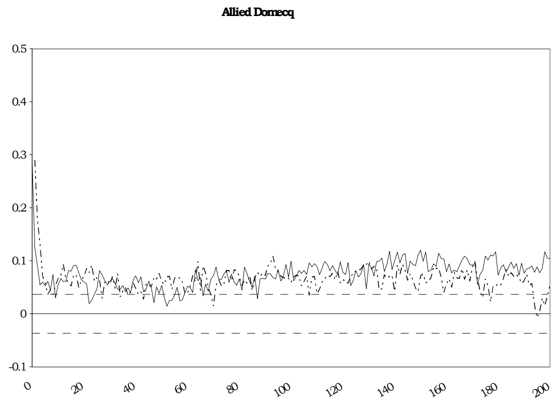
0 25 50 75 100 125 150 175 200
Confidence interval
Confidence interval
Abs returns
Volume

Figure 2

Cross-correlations of the Absolute Returns - Volatility - and Traded Volume.

We analyze the cross-correlations functions of volatility and traded volume for the six stocks. The straight line corresponds to $corr(|R_t|, V_{t-j})$ and the dotted one to $corr(V_t, |R_{t-j}|)$. The graphs show the same hyperbolic decay as for the autocorrelation functions of the absolute returns and volume, i.e. $corr(|R_t|, V_{t-j}) \approx corr(V_t, |R_{t-j}|) \approx j^{2d-1}$. Nevertheless, for Barclays and British Land the cross-correlations are not significant, as they lie inside the 95% Gaussian confidence interval.


Allied Domecq
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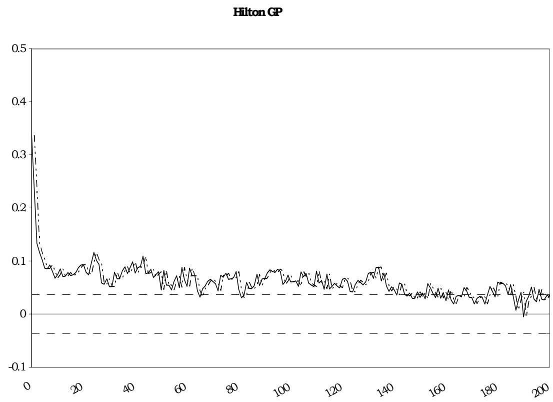
Hilton GP
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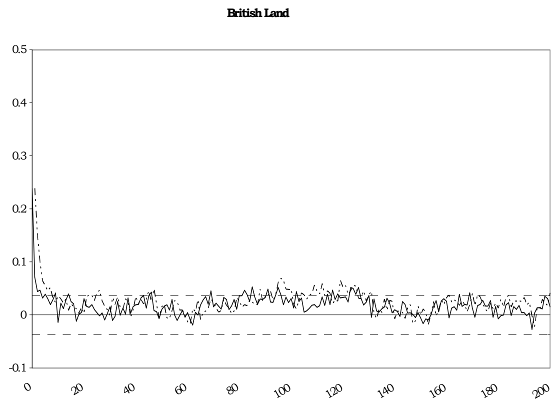
British Land
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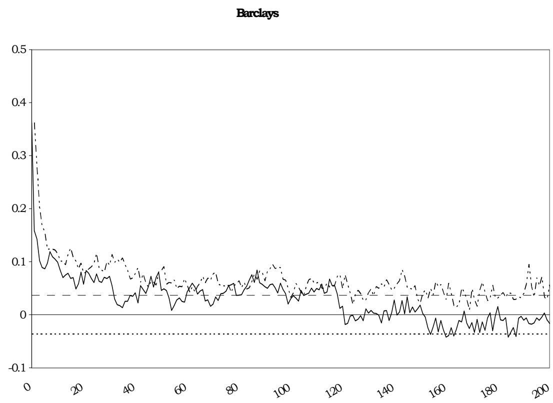
Barclays
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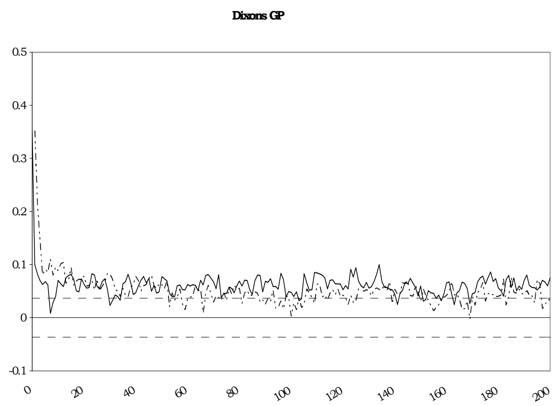
Dixons GP
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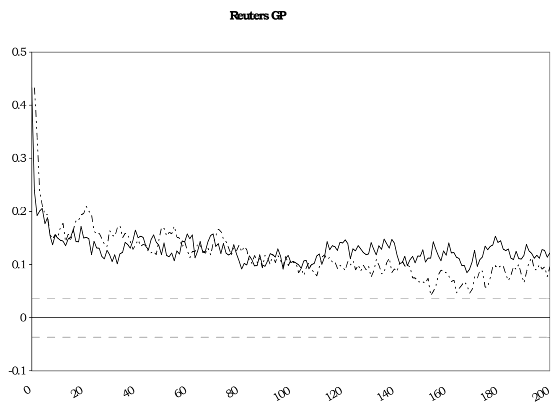
Reuters GP
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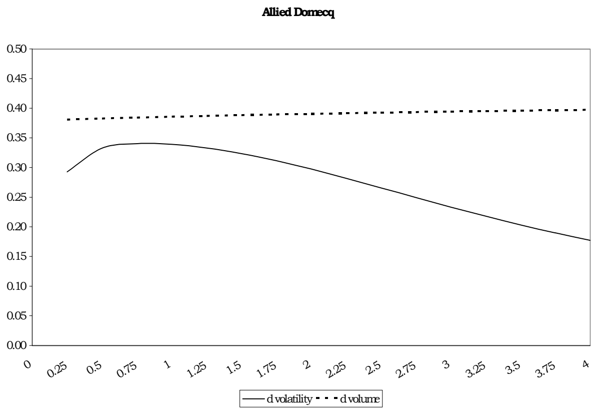
Figure 3

Fractional Differencing Parameter d and Power Transformation q.

Figure 3 investigates the rate of decay of the autocorrelation functions $\rho(|R_t|^q, j)$ and $\rho(V_t^q, j)$ for different values of the power term (i.e., for $q = 0.25, 0.5, ..., 2$). The fractional differencing parameter is calculated using the semiparameter estimator of Andersen and Bollerslev (1997) based on the average periodogram ratio for two frequencies close to zero, $\hat{d}_{AP}$. The fractional differencing parameter $\hat{d}_{AP}$ takes almost the same values for the trading volume regardless of the power transformation q for the volume (linear dotted line) whereas the graph shows a humped shape for the volatility. For the latter, after a maximum value around $q = 0.75$, the fractional differencing parameter decreases to significantly smaller values for superior values of the power transform. The decrease is very substantial in the case of British Land and Barclays.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4
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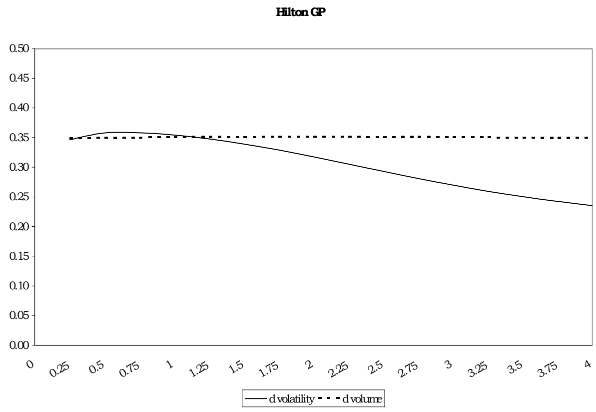
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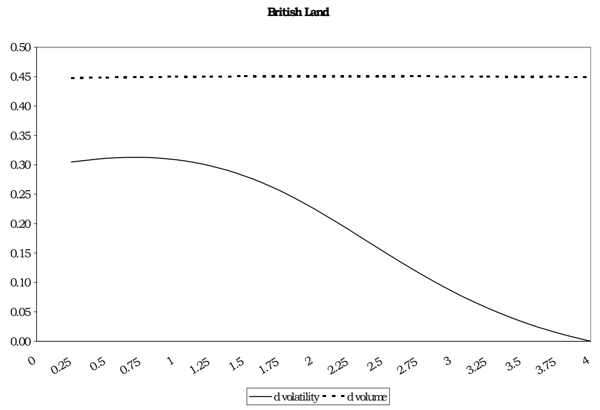
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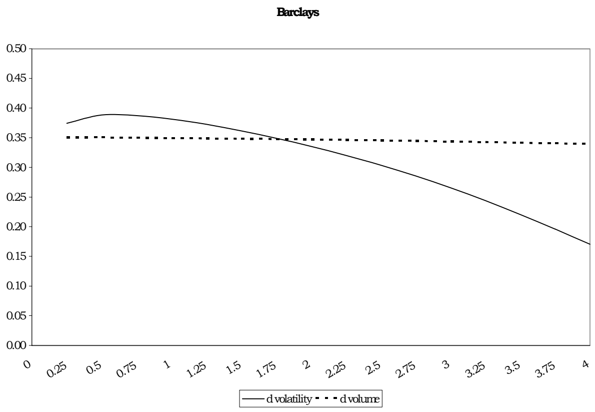
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3.75
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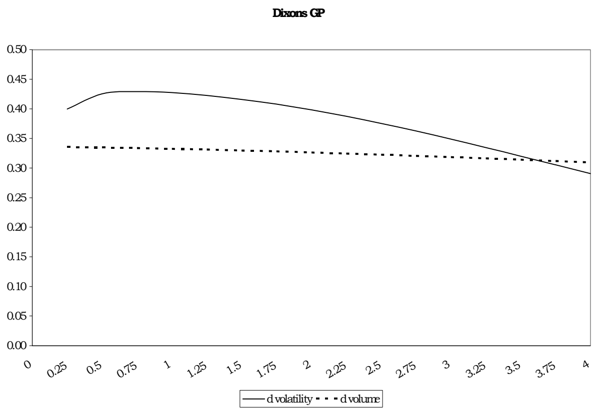
d volatility
d volume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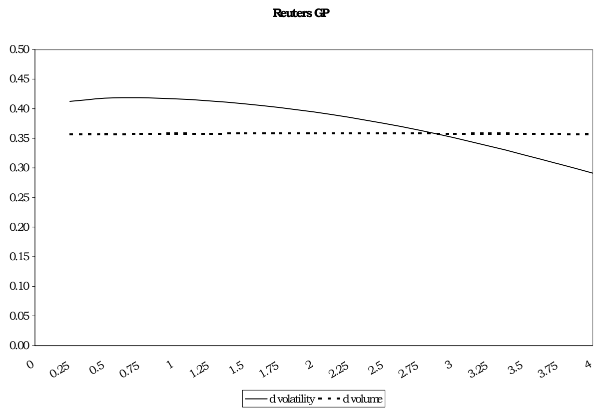
Reuters GP
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4
d volatility
d volume

Table 1

Descriptive Statistics for Daily Stock Returns and Traded Volume.

Descriptive statistics of six stocks traded on the London Stock Exchange over the period January 1990 to May 2001 are presented in Table II. For our analysis, we use both returns measured as $R_t = 100 \times \ln(P_t / P_{t-1})$ and traded volumes expressed in logarithm. The number of observations for the period of analysis is $T = 2874$. The 95% confidence interval for a test of normality is given by $\pm 1.96 * \sqrt{6/T}$ for the sample skewness and $3 \pm 1.96 * \sqrt{24/T}$ for the sample kurtosis. We also provide the Jarque-Bera test for normality. We use * to indicate significance at the 5 percent level.

Descriptive Statistics

	Barclays				British Land			
	Returns		Volume		Returns		Volume	
Mean	0.0583	Mean	9.5956	Mean	0.0101	Mean	6.6278
Variance	3.8507	Variance	0.3666	Variance	3.3366	Variance	1.3652
Skewness	0.1994*	Skewness	-0.0022	Skewness	0.5253*	Skewness	-0.9393*
Kurtosis	6.0121*	Kurtosis	4.4811*	Kurtosis	10.4855*	Kurtosis	4.5546*
Jarque-Bera	1105.50*	Jarque-Bera	262.70*	Jarque-Bera	6842.22*	Jarque-Bera	712.04*

	Dixons GP				Allied Domecq			
	Returns		Volume		Returns		Volume	
Mean	0.0683	Mean	8.7848	Mean	0.0147	Mean	8.1345
Variance	5.9235	Variance	0.7891	Variance	2.8547	Variance	0.4683
Skewness	-0.2449*	Skewness	-0.9453*	Skewness	0.3021*	Skewness	-0.1714*
Kurtosis	11.3915*	Kurtosis	5.8394*	Kurtosis	10.6184*	Kurtosis	4.5383*
Jarque-Bera	8461.31*	Jarque-Bera	1393.59*	Jarque-Bera	6994.08*	Jarque-Bera	297.47*

	Hilton GP				Reuters GP			
	Returns		Volume		Returns		Volume	
Mean	-0.0097	Mean	8.1169	Mean	0.0514	Mean	8.2072
Variance	5.3381	Variance	0.5798	Variance	5.6882	Variance	0.4961
Skewness	0.3372*	Skewness	0.0893	Skewness	0.1004*	Skewness	-0.2285*
Kurtosis	8.8465*	Kurtosis	3.7208*	Kurtosis	11.1163*	Kurtosis	3.8169*
Jarque-Bera	4147.77*	Jarque-Bera	66.05	Jarque-Bera	7893.42*	Jarque-Bera	104.94*

Table 2

Lo's Long Term Dependence Test for Absolute Returns and Traded Volume.

Lo's (1991) modified R/S statistic for long-range dependence is presented in Table 2 to formally test the presence of long-term dependence in absolute returns and trading volume. In agreement with Lobato and Velasco (2000), for all stocks, both the absolute returns and the trading volume series exhibit long-run dependence (i.e., $0 < d < 1/2$) as the statistics remain outside the 95% confidence interval.

Lo's Long Term Dependence Test

Absolute Returns

Barclays		British Land		Dixons GP		Allied Domecq		Hilton GP		Reuters GP	
m	**Q(m,T)**	**m**	**Q(m,T)**	**m**	**Q(m,T)**	**m**	**Q(m,T)**	**m**	**Q(m,T)**	**m**	**Q(m,T)**
2	5.4030*	2	4.4180*	2	5.6072*	2	5.3139*	2	4.0261*	2	7.0982*
5	4.5412*	5	3.8087*	5	4.8528*	5	4.6051*	5	3.5078*	5	5.8891*
8	**4.0459***	**8**	**3.4737***	**8**	**4.3695***	**6**	**4.4541***	**8**	**3.1843***	**10**	**4.8485***
10	3.7990*	10	3.3175*	10	4.1191*	10	4.0039*	10	3.0232*	12	4,5691*

Trading Volume

Barclays		British Land		Dixons GP		Allied Domecq		Hilton GP		Reuters GP	
m	**Q(m,T)**	**m**	**Q(m,T)**	**m**	**Q(m,T)**	**m**	**Q(m,T)**	**m**	**Q(m,T)**	**m**	**Q(m,T)**
12	3.1361*	12	5.5628*	10	4.0667*	10	4.3217*	12	3.6933*	15	4.1161*
15	2.9444*	15	5.1158*	12	3.8808*	12	4.1165*	15	3.4313*	17	3.9697*
19	2.7485*	**20**	4.5749*	**13**	3.8003*	**16**	3.8049*	**19**	3.1700*	**21**	3.7207*
22	2.6268*	22	4.4050*	15	3.6546*	18	3.6789*	22	3.0151*	24	3.5685*

Table 3

Decay Rate of the Autocorrelation Functions of Absolute Power Transformations for Returns and Volume.

To gauge the influence of the power transformation q on the decay rate of the autocorrelation functions $\rho(|R_t|^q, j)$ and $\rho(V_t^q, j)$, Table 3 presents the values of the obtained Andersen and Bollerslev (1997) fractional differencing parameters $\hat{d}_{AP}$ both for power transformations of the absolute returns (volatility) and the trading volume when $q = 0.25, 0.5, \ldots, 4$.

Rate of Decay for Absolute Power Transformations of Returns and Traded Volume

	Barclays		***British Land***		***Dixons GP***		***Allied Domencq***		***Hilton GP***		***Reute***
q	Volatility	Volume	Volatility	Volume	Volatility	Volume	Volatility	Volume	Volatility	Volume	Volatility
0.25	0.3745	0.3507	0.3048	0.4476	0.3996	0.3353	0.2927	0.3807	0.3465	0.3489	0.4122
0.5	0.3883	0.3504	0.3111	0.4485	0.4253	0.3346	0.3327	0.3824	0.3579	0.3495	0.4177
0.75	0.3872	0.3499	0.3129	0.4491	0.4292	0.3336	0.3404	0.3840	0.3581	0.3500	0.4184
1	0.3813	0.3494	0.3089	0.4496	0.4273	0.3324	0.3392	0.3855	0.3543	0.3504	0.4168
1.25	0.3725	0.3488	0.2987	0.4500	0.4226	0.3311	0.3331	0.3868	0.3478	0.3507	0.4134
1.5	0.3616	0.3482	0.2814	0.4503	0.4161	0.3296	0.3236	0.3881	0.3393	0.3509	0.4084
1.75	0.3492	0.3476	0.2568	0.4505	0.4079	0.3280	0.3116	0.3893	0.3291	0.3510	0.4021
2	0.3355	0.3469	0.2257	0.4506	0.3983	0.3263	0.2975	0.3904	0.3176	0.3510	0.3945
2.25	0.3203	0.3461	0.1903	0.4506	0.3875	0.3244	0.2820	0.3914	0.3055	0.3510	0.3857
2.5	0.3038	0.3453	0.1533	0.4505	0.3755	0.3225	0.2659	0.3924	0.2931	0.3509	0.3755
2.75	0.2857	0.3444	0.1176	0.4504	0.3626	0.3205	0.2495	0.3933	0.2812	0.3507	0.3640
3	0.2659	0.3435	0.0851	0.4502	0.3490	0.3184	0.2334	0.3942	0.2700	0.3505	0.3513
3.25	0.2442	0.3425	0.0571	0.4500	0.3347	0.3163	0.2180	0.3950	0.2597	0.3502	0.3374
3.5	0.2208	0.3414	0.0338	0.4497	0.3201	0.3141	0.2034	0.3957	0.2505	0.3499	0.3225
3.75	0.1959	0.3404	0.0149	0.4494	0.3053	0.3118	0.1897	0.3965	0.2424	0.3496	0.3070
4	0.1703	0.3392	0.0000	0.4490	0.2906	0.3095	0.1770	0.3971	0.2353	0.3493	0.2911

Table 4

GARCH (1,1) Parameter Estimates without Trading Volume.

We use the LSE "blue chips" stocks over the period January 1990 to May 2001 to estimate by maximum likelihood the parameters of a GARCH (1,1) model without volume. Results for the six stocks discussed in this paper are presented in Table 4 where heteroskedastic-consistent t-values are also provided in parenthesis. All stocks exhibit a high level of volatility persistence as measured by the sum $\alpha + \beta$.

	GARCH (1, 1) Model without Trading Volume			
	ω	α	β	$\alpha + \beta$
Barclays	0.0482*	0.0805*	0.9085*	0.9889
	(3.297)	(6.542)	(64.38)	
British Land	0.0678*	0.0986*	0.8875*	0.9860
	(3.955)	(7.64)	(61.67)	
Dixons GP	0.1109*	0.0867*	0.8988*	0.9853
	(3.949)	(6.68)	(59.97)	
Allied Domecq	0.0897*	0.0861*	0.8830*	0.9690
	(4.241)	(6.569)	(49.24)	
Hilton GP	0.0572*	0.0596*	0.9318*	0.9912
	(3.529)	(7.373)	(106.5)	
Reuters GP	0.0440*	0.0826*	0.9135*	0.9960
	(3.338)	(6.501)	(71.69)	

Table 5

GARCH (1,1) Parameter Estimates with Trading Volume.

Table 5 follows the work of Lamoureux and Lastrapes (1990) and presents the parameter estimates of a GARCH (1,1) model that includes the trading volume as explanatory variable in the variance equation. Again, the study was done on 50 LSE "blue chips" stocks even though results are only presented for six stocks. Significance of the parameters is measured by the heteroskedastic-consistent t-values. The last column gives the level of volatility persistence by the sum $\alpha + \beta$ when volume has been included in the equation.

GARCH (1, 1) Model with Trading Volume

	ω	α	β	γ	$\alpha + \beta$
Barclays	-0.5591	0.0962*	0.8825*	0.0676	0.97853
	(-1.780)	(5.952)	(40.31)	(1.903)	
British Land	0.0284	0.0999*	0.8849*	0.0066	0.98467
	(0.5027)	(7.429)	(56.55)	(0.7127)	
Dixons GP	-1.0955*	0.1237*	0.8364*	0.1557*	0.95996
	(-4.411)	(6.624)	(31.59)	(4.515)	
Allied Domecq	-2.8069*	0.1920*	0.5828*	0.4213*	0.7746
	(-10.98)	(8.710)	(16.03)	(11.13)	
Hilton GP	-7.4209*	0.2121*	0.2183*	1.2577*	0.43022
	(-26.99)	(8.326)	(9.618)	(26.92)	
Reuters GP	-1.4614*	0.1200*	0.8462*	0.2023*	0.96593
	(-4.562)	(7.591)	(37.94)	(4.519)	