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International Business Cycle Coherence and Phases

- A spectral analysis of output fluctuations of G7 economies

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INTERNATIONAL BUSINESS CYCLE COHERENCE AND PHASES

- A spectral analysis of output fluctuations of G7 economies

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Abstract - This paper examines international linkages of co-movements in output fluctuations amongst G7 economies in the frequency domain. The paper has identified patterns in international business cycle co-movements among the G7, offering a general outlook of international business cycle co-movements and detailing the lower frequency, higher frequency and middle range characteristics of international linkages of output fluctuations. The main findings of the study are that co-movements among G7 economies are considerably stronger at lower frequencies, with clearer patterns of linkages of international output fluctuations, than those at higher frequencies and in middle ranges. The results and findings show support for real business cycle theory being extended to an international arena, with long effect real shocks impacting economies across borders.

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Key words: business cycles, frequency domain, coherence, phase

1. Introduction

This paper examines international business cycle amongst major industrialized economies in terms of coherence and phases in the frequency domain. It analyses the cross spectra of output of these of economies in different cycle components or at different frequencies, and focuses on the patterns of co-movement in terms of coherence, coincidence and phase leads/lags, which contrasts lead/lag relations and correlations in the time domain and offers another means of looking at international business cycle issues. Intuitively, the approach is to inspect the degree to which one output variable differs from another in time series behavior in the frequency domain by analyzing their spectra and cross spectra. It examines the similarities and synchronous relations in the spectra of time series; the former is measured in coherence and latter measured by phases in the cross spectrum. Characteristically, as spectrum and cross spectrum components are depicted against frequencies of time series, spectral analysis is particularly helpful in the study of cyclical co-movements, such as international business cycles. Therefore, the approach in the frequency domain may present a fuller picture of international business cycle fluctuations with the same (amount of) information available to us in the time domain, which is utilized in a way more appropriately and effectively for this type of investigation.

The term "business cycle" is itself controversial in its definitions and measurement, arising from the differences in research methodologies, investigating techniques, application purposes, and policy considerations. Conventional definition states that business cycles are periodic but irregular up and down movements in economic activity, measured by fluctuations in real GDP and other economic variables. A full business cycle is identified as a sequence of four phases: contraction, trough, expansion, and peak, whereas the time span between, for example, two peaks, varies from time to time, so do the magnitude of peaks or troughs. Further analysis involves more details of business cycles such as large peaks/troughs and small peaks/troughs occurring at different time intervals, indicating business cycle components. For example, Schumpeter's (1939) long waves and the accompanied notions of long cycles, medium cycles and short cycles are alternations of states of economic activity or business conditions in different lengths of time period, which amounts to a decomposition of business cycle components in accordance with their frequencies of occurrences.

It is not exaggerated to claim that the first studies of business cycles adopted the time domain approach as well as the frequency domain approach almost simultaneously, in as early as the first half of the 20th century, when the notion of business cycles started to attract attention from economists and governments alike in their search for an understanding of the patterns in economic activity and a possible therapy for mitigating the damage caused by severe economic downturn. Although most empirical studies since then have been in the time domain, the business cycle is more an issue in the frequency domain arising from its two features: cyclical fluctuations and cycle components. With regard to the second feature of components of business cycles, other types of transformation can be effective as well, such as analysis in the state space. In an extreme case of (the decomposition of) cycle components, the longest "cycle" is the trend and the rest is the cycle, as in Beveridge and Nelson (1981), Watson (1986) and Clark (1987). While the Beveridge-Nelson decomposition (Beveridge and Nelson 1981) is performed by the Box-Jenkins method as an ARIMA model in the time domain, Watson (1986) and Clark (1987) resort to the state space to decompose output into two unobserved components of the trend and the cycle, being executed with the help of Kalman filters. Cochrane's (1988) persistence measure for output is concerned with the relative importance or contributions of trends and cycles in output, which appears to be in the time domain but is indeed in the frequency domain, since the measure is a special case of spectral analysis at the zero frequency point. Most recently, A'Hearn and Woitek (2001) study business cycles in the frequency domain, using annual historical industrial output (industrial production) data of 13 countries from around 1865 to 1913 for empirical univariate and bivariate analysis. When studies are centered on international business cycles, i.e., when the analysis is multivariate, other dimensions of investigation are introduced to assess the closeness or the degree of co-movement of two or more output time series. While such closeness or co-movement can be evaluated in both time domain and frequency domain, the features of business cycles, as pointed above, suggest that analysis in the frequency domain will be more advantageous for multivariate cases compared with univariate cases.

In the last quarter century, the world economy has become increasingly integrated. As such, fluctuations in output in individual economies are increasingly influenced by fluctuations in the world economy in international business cycles. This has added an additional dimension to the study of business cycles, namely, interactions and co-movements between national economies. Such interactions and co-movements, though existed well before the emergence of the interest in business cycles and business cycle theory in the early 20th century, have become non-negligible in their roles and, consequently, in their research, only fairly recently. Backus *et al.* (1992), extending Kydland and Prescott (1982), are among the first to study real business cycles in a two-country setting. In their model, they allow residuals in the shocks to be correlated across countries, and there is diffusion of technology shocks between countries. They perform empirical work on diffusion and correlation for the US and an aggregate of European countries, based on estimates of Solow residuals. One of their findings that is particularly relevant is that openness substantially alters the nature of some of the closed economy co-movements. In a similar framework, Ambler *et al.* (2002) propose a theoretical model for international transmission of business cycles that is simulated to study and predict the cross-country correlation of economic activity. Backus and Kehoe (1992) document business cycle evidence in ten countries with more than 100 years' annual data from around the 1860s to the 1980s. Fluctuations in real output, expenditure, price levels and monetary aggregates in the individual countries are analyzed, and correlations in output between the countries are presented. Following Backus and Kehoe (1992), Basu and Taylor (1999) provide an international historical perspective on business cycles, employing annual time series data running from approximately 1870 to the 1990s. They present volatility and first order autocorrelation for output, consumption, investment, the current account, and prices for a pool of 15 countries, and correlation between the US and the pool of the rest 14 countries. Further, exchange rate volatility is examined for a pool of 20 countries and real wage cyclicality is inspected for a pool of 13 countries.

It is evident that research on international business cycles is not as extensive as that on business cycles in closed economies. Many empirical studies are more about international comparisons of business cycle features in individual countries than international business cycle linkages and co-movements between national economies; and the co-movement examined is overwhelmingly the correlation of output fluctuations between countries. The present study goes beyond of documenting a relationship between national economies in terms of correlation. It attempts to identify patterns in the interaction between individual economies, covering the whole spectrum of short, medium and long cycles, (and trends at extremity) and the phase relations. The empirical investigation is further empowered by the frequency domain method to achieve the set objectives effectively. This study contributes to the literature in three ways. Firstly, it opens up a new channel of research to gain knowledge in such important aspects of international business cycle coherence and phases that are either overlooked or unable to be quantified previously. Secondly, unlike most empirical studies in the area that use long, historical, and annual data, our data set covers the last quarter century in the quarterly frequency. Consequently, the present study is able to render empirical implications that are more relevant to contemporary welfare and has a more dynamic feature too. Finally, as the co-movement between individual economies is investigated from all the perspectives of short, medium, and long runs, instead of contemporary and led/lagged correlations, the present study is able to explain and encompass some of the rival views, such as whether the British economy is more close to the US economy or the Continental European economy in business cycles.

Prior to presenting and discussing the frequency domain approach to studying business cycles in Section 3, it is worthwhile having some basic sense of this approach and the rationale for its adoption in this empirical study. Let us conceive two time series of economic activities that consist of the components of the quarterly cycle, the annual cycle, and the bi-annual cycle. If the two time series have the annual cycle in the same phase, i.e., without lead or lag, but the first time series has a one-quarter lead over the second in the quarterly cycle and has a three-quarter lag over the second in the bi-annual cycle, what results would be expected in a traditional regression analysis in the time domain? Probably none of the regression coefficients at lag zero, one, or three is significant. Even if one or several coefficients have been estimated, traditional time domain regression only tells, for example, that a change in the first time series is caused by the change in the second time series three periods earlier. It, however, does not tell the characteristics of the association between the two time series. Consequently, the coincident link in the annual cycle, and the lead/lag relations in the quarterly cycle and the bi-annual cycle can be overlooked, leading to a possibly wrong conclusion. The frequency domain approach in this study attempts to identify such relations, which is especially effective for research in cyclical fluctuations featured by business cycles.

The rest of the paper is organized as follows. Section 2 discusses the methodological aspects of the approach, presenting the frequency domain representations of cycles, spectra and cross spectra. Section 3 reports empirical results and discusses the findings and their implications. Finally, section 4 concludes.

2. The frequency domain approach

Spectral analysis, or studies in the frequency domain, is one of the unconventional subjects in time series econometrics. Analysis in the frequency domain does not bring in new or additional information, it is simply an alternative method with which information is observed, processed and abstracted. This is sometimes helpful. Depending on the characteristics of the issues, analysis in one domain may be more powerful than in the other. For example, cycles are better and more explicitly observed and represented in the frequency domain. Correlations in the time domain and cross spectra in the frequency domain deal with the relationship between two time series from different perspectives and have defined links. In the following, we briefly introduce the ideas of the Fourier transform and spectra, cross spectra, coherence, and phases.

2.1. The spectrum, phase and coherence

The spectrum of a time series is the frequency domain representation of the time series, which reveals the characteristics of the time series from its frequency domain, rather than its time domain, perspectives. The spectral density function of a discrete random process $\Delta X_t = X_t - X_{t-1}$ (t=1,...N) is:

$$h(k) = \sum_{\tau=-(N-1)}^{N-1} R(\tau) e^{-j\tau\frac{2\pi k}{N}} \qquad (1)$$

where $R(\tau)$ is the autocovariance function of ΔX_t, i.e., $R(\tau) = E\{(\Delta X_t - \mu)(\Delta X_{t-\tau} - \mu)\}$ and $\mu = E\{\Delta X_t\}$. The inverse Fourier transform of equation (1) is:

$$R(\tau) = \frac{1}{N} \sum_{k=-(N-1)}^{N-1} h(k) e^{jk\frac{2\pi\tau}{N}} \qquad (2)$$

Setting τ=0 in equation (2), we have:

$$R(0) = E\{(\Delta X_t)^2\} = \frac{1}{N} \sum_{k=-(N-1)}^{N-1} h(k) e^{jk\frac{2\pi\tau}{N}} \qquad (3)$$

It is the mean squared value of the process and has the meaning of power of the process, so equation (1) is called the power spectrum. $R(\tau)$ usually takes real values and is an even function, i.e., $R(-\tau) = R(\tau)$. Accordingly, the spectral density function can be written as:

$$h(k) = \sigma_X^2 + 2\sum_{\tau=1}^{N-1} R(\tau)\cos\left(\frac{2\pi\tau k}{N}\right) \qquad (4)$$

If we replace $R(\tau)$, the autocovariance function of ΔX_t, by the covariance between two time series, i.e., $Cov_{X,Y}(\tau) = E\{(\Delta X_t - \mu_X)(\Delta Y_{t-\tau} - \mu_Y)\}$, $\mu_X = E\{\Delta X_t\}$ and $\mu_Y = E\{\Delta Y_t\}$, then we get the cross spectrum of the two time series in the form of:

$$h_{X,Y}(k) = \sum_{\tau=-(N-1)}^{N-1} Cov_{X,Y}(\tau) e^{-j\tau\frac{2\pi k}{N}} \qquad (5)$$

$Cov(\tau)$ is in general not an even function, so equation (5) cannot take the form of equation (4), and $h_{X,Y}(k)$ is in general a complex number:

$$h_{X,Y}(k) = c(k)\cos(\frac{2\pi k}{N}\tau) + jq(k)\sin(\frac{2\pi k}{N}\tau) \tag{6}$$

Unlike the univariate Fourier transform where the imaginary part is zero, the cross spectrum has both magnitude and phase as follows:

$$m(k) = \sqrt{c^2(k) + q^2(k)} \tag{7}$$

and

$$p(k) = \tan^{-1}\frac{q(k)}{c(k)} \tag{8}$$

Equations (7) and (8) are called magnitude spectrum and phase spectrum respectively. It can be seen, from the above analysis, that if $Cov_{X,Y}(\tau)$ is an even function, then the phase spectrum is zero, i.e., there is no overall lead of series X_t over series Y_t, and vice versa. With equations (7) and (8), the cross spectrum can also be expressed as:

$$h_{X,Y}(k) = m(k)e^{jp(k)} \tag{9}$$

so that both magnitude and phase are shown explicitly.

Another measure of the closeness of two time series is coherence, defined, in a very similar way to the correlation coefficient, as:

$$Coh_{X,Y}(k) = \frac{h_{X,Y}(k)}{h^{1/2}_{X,X}(k)h^{1/2}_{Y,Y}(k)} \tag{10}$$

If we make comparison of the measures in the frequency domain with those in the time domain, then the cross spectrum of equation (5) is corresponding to covariance in the time domain, which is not standardized; the coherence as with equation (10) is corresponding to correlation in the time domain, which are standardized by the square roots of the two time series′ spectra and the two time series′ standard deviations respectively; and the phase of equation (8) addresses leads and lags. The closeness of two time series is straightforwardly observed with the standardized measures of coherence, together with the phase measure, which we adopt in this study.

While the row periodogram illustrated above produces an unbiased estimate of the spectrum, it is inconsistent, as the variance of the estimators does not go to zero as the number of data points grows. Therefore, it is usually to let the time series pass through a spectral window, which is a procedure called smoothing, to get a consistent estimate of the spectral density or cross-spectral density. There are two requirements for this to produce consistency. First, the window width must go to infinity as the number of data points increases to ensure that the variance goes to zero. That is, a window should not be too narrow to produce imprecise estimates. Second, the window width must increase at a rate slower than that in the number of data points to ensure that the bias goes to zero. A too wide window will flatten the peaks and troughs too much. To express mathematically, it is: $N \to \infty$, $M \to \infty$, $N >> M$; where N is the number of data points and M is the window size. The design and choice of window types is also important. Although a window is essential for consistency, it can produce some spurious frequencies or ripples, especially when the window edges are sharp, e.g., a rectangular window. A window with curved edges mitigates this problem by scaling the ends of the data so they merge smoothly with the zeros on either side. Bartlett′s window and the tent window are examples.

According to Priestley (1981, 1996), $h_{x,y}(k)$ and $Coh_{x,y}(k)$ follow normal distributions. With Bartlett's window, the standard error of $h_{x,y}(k)$ is:

$$Std\left[h_{x,y}(k)\right]=h_{x,y}(k)\sqrt{\frac{2M}{3N}}, \qquad for\ k\neq 0 \tag{11}$$

and

$$Std\left[h_{x,y}(k)\right]=h_{x,y}(k)\sqrt{\frac{4M}{3N}}, \qquad for\ k=0 \tag{12}$$

The standard error of $Coh_{x,y}(k)$ is:

$$Std\left[Coh_{x,y}(k)\right]=Coh_{x,y}(k)\sqrt{\frac{2M}{3N}}, \qquad for\ k\neq 0 \tag{13}$$

and

$$Std\left[Coh_{x,y}(k)\right]=Coh_{x,y}(k)\sqrt{\frac{4M}{3N}}, \qquad for\ k=0 \tag{14}$$

The window width should be appropriately chosen in relation to the sample size, so that the standard error of the coherence estimate is reasonably small and that the coherence estimate is statistically significant, in addition to the unbiasedness and consistency requirements.

{Figure 1 about here}

We demonstrate phase relations as adopted by RATS graphically with explanations. Figure 1 demonstrates several special cases of the relationship between time series. Coherence is plotted against the left hand side axis (in blue) with minimum being zero and maximum being one; and phases are plotted against the right hand side axis (in red) with minimum being -π (a half cycle lag) and maximum being π (a half cycle lead). Figure 1(a) shows perfect coherence but one time series has one phase lag vis-à-vis the other at all frequencies, and Figures 1(b), 1(c) and 1(d) show perfect coherence while there exist two phase lags, three phase lags and four phase lags, respectively, between them at all frequencies. Any lead between a half cycle and a full cycle ($\pi<p(k)<2\pi$) is regarded as a lag between a half cycle and zero lag (i.e.,$-\pi< p(k)-2\pi<0$). Figure 2 exhibit phase corresponding lags and leads in the time domain. Figure 2(a) shows that a one-quarter lead/lag in quarterly changing time series data is a half cycle lead/lag, and it is point (1, π) in Figure 1(a) with 1 being the highest frequency and π being half cycle (2π is a full cycle). Similarly, Figure 2(b) shows that a one-quarter lead/lag in semi-annually changing data is 1/4 of the cycle, and it's corresponding point is (0.5, π/2) in Figure 1(a) with 0.5 being half of the highest frequency and π/2 being 1/4 cycle; and Figure 2(c) shows that a one-quarter lead/lag in annually changing data is 1/8 of the cycle, and it is point (0.25, π/4) in Figure 1(a) with 0.25 being 1/4 of the highest frequency and π/4 being 1/8 cycle. A two-quarter lead/lag in quarterly changing dada of Figure 2(a) is equivalent to zero lead/lag, which is point (1, 0) in Figure 1(b). A two-quarter lead/lag in semi-annually changing dada of Figure 2(b) is equivalent to a half cycle lead/lag, which is point (0.5, π) or (0.5, -π) in Figure 1(b) (notice that a half cycle lead and a half cycle lag have the same meaning with regard to phases). A two-quarter lead/lag in annually changing dada of Figure 2(c) is equivalent to 1/4 cycle lead/lag, which is point (0.25, π/2) in Figure 1(b). A three-quarter lag is equivalent to a one-quarter lead in quarterly changing data of Figure 2(d), which is point (1, π) in Figure 1(c).

{Figure 2 about here}

2.2. Overall lead/lag and coincidence statistics

In general, we can regard a phase point above 0 as phase lead and a phase point below 0 as phase lag, the larger the absolute value of the phase, the larger the lead/lag. As observing a cross spectrum requires technicalities in the frequency domain, we provide summary statistics for phase leads/lags. Statistic φ is the average of the phase values on the whole spectrum, measuring the overall lead/lag between two time series. Statistic θ is the average of the absolute phase values on the whole spectrum, measuring the degree of departure from a coincident relation between two time series. φ is zero when there are equal positive and negative phase values in a cross spectrum, but the two time series may not be coincident. A small θ suggests that there are substantial coincident elements in the two time series. A small φ with a large θ implies that there are no overall leads/lags in the time series but the time series are not coincident either. A large positive (negative) φ means large phase leads (lags) in the time series. The maximum value φ that can take is π^{-}, and the minimum value is $-\pi^{+}$. The maximum value θ that can take is π^{-} and the minimum value is 0. We will analyze the empirical results with the above method of interpretations and the two summary statistics in the next Section.

2.3. *Frequencies ranges*

For the purpose of inspecting business cycle properties in the short, medium and long terms, we divide the whole spectrum into four sections. For quarterly data, point 1.00 on the spectrum refers to the quarterly frequency that completes a full cycle in two quarters (see Figure 2(a)); point 0.50 represents the semi-annual frequency that completes a full cycle in a year (see Figure 2(b)); and point 0.25 is the annul frequency that completes a full cycle in two years (see Figure 2(c)). Point 0.75 can be regarded a "four-monthly frequency" on a quasi-continuous base. The range from 0.75 up to 1.00 on the spectrum is taken for the higher frequencies. The choice is meant to be close to the quarterly frequency and, at the same time, sufficient spectrum components, or energy, are covered. Then the scope between 0.20 and 0.75 is for the medium frequencies. With 0.20 corresponding to a 1.25-year cycle, this, roughly speaking, represents cycles around the annual frequencies. The range from 0.05 to 0.20, the lower frequencies, is kept for the traditional business cycle ranges, or longer cycles over one year[1], covering both the traditional short cycles of 3-5 years and long cycles of up to 10 years. Finally, the range of the spectrum from 0 to 0.05 is for the long-run trend in business cycles. It includes long cycles over 10 years, so while preserving the long-run features of business cycles, there are sufficient spectrum components in this spectrum

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[1] Our method is substantially different from those using annual data. According to the Nyquist sampling theorem, if a time series is sampled at the frequency of 2f, then all frequency components lower than f would be reserved and can be recovered. In other words, any frequency components higher than f would be lost and unrecoverable. If annual data are ever to achieve the same results as quarterly data, one should assume there are no fluctuations that have a frequency higher than two quarters in GDP. This assumption, however, is highly unlikely to hold. So the spectrum based on annual data involves distortions. Refer to one of the books on signals and systems, e.g., Ziemer, R.E., Tranter, W.H. and Fannin, D.R. (1993), Signals and Systems: Continuous and Discrete, 3rd ed, MacMillan Publishing Company, NY.

range. Using quarterly data and the RATS procedure, a half-cycle lead is one quarter at point 1.00, two quarters at point 0.50, one year at point 0.25, and two years at point 0.125. Therefore, a same phase lead/lag value at different frequency points represents different time lengths, though it means the same fraction of a cycle.

3. Empirical results and discussions

The data used in this study are quarterly GDP of the US, Japan, Germany, France, the UK, Italy and Canada, starting in the first quarter, 1973 and ending in the fourth quarter, 1999, at constant prices, i.e., they are GDP or output in real terms. The choice of the period is influenced by the following considerations. The events around 1973 marked a number of strategic changes of the world economically as well as politically. The US President Nixon's announcement on August 15, 1971 of the end to the US commitment to convert US dollars to gold at a fixed price, the signing and withdrawal of the Smithsonian agreement in December 1971 and March 1973 eventually completed the transition to the floating exchange rate regime in 1973. A year ago in 1972, Nixon made a historic visit to the People's Republic (the PR), effectively ending an era featured by two competing camps built on rival ideologies. The strategic alliance of the US and the PR has shifted ever since, which changed the landscape of the world. The divide between the capitalist economy and the socialist planning economy started to thaw, and non-state owned rural and township enterprises thrived in the eastern coast of the PR in the last years of the Mao Zedong era. It can be concluded that these events and strategic changes dominated the world for approximately a whole quarter in the final episode of the 20th century, with their impact eventually appearing to diminish only fairly recently. This period witnessed globalization, liked or disliked, reaching every corner of the world; and this period observed the coming and (nearly) going of the G7 and the prologue to the Gn,[2] with the diminishing role of the recent annual summits that were remembered more for the clash than their economic significance.

3.1. *Time domain statistics*

Prior to progressing to the examination of the results in the frequency domain, time domain features of GDP of these economies are reported in Table 1 for preliminary statistics of the individual countries, and in Table 2 for correlations between the countries. These statistics are based on quarterly changes or growth in respective GDP. Table 1, provided to document relevant figures, is self-evident and needs no further explanations. In Table 2, the correlation between a pair of economies is the usual measure for the closeness between the pair. It is observed that the correlation of the output growth of the US and that of Canada is the highest, followed by Germany and the UK, and the output growth of the US is least correlated with that of Japan. The highest correlation of the output growth of Germany is with that of France, followed by that of Italy, the UK and the US. All four European economies are highly correlated, among them the UK has the weakest link but the link is still higher than that with the

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[2] The first G7 (six of G7 nations without Canada) Summit was held in November 1975 at Rambouillet, France. In June 1976, Canada joined the group at the San Juan Summit, in Puerto Rico, the United States, marking the birth of the G7. In May 1977 the European Community /European Union joined the group at the London Summit. In July 1989, delegations of 15 developing countries met with the G7 delegations on the eve of the so-called Summit of the Arch, at Paris. Since 1991, limited participation of the USSR and then Russia became more involved gradually and May 1998 witnessed the creation of G8 when Russia developed into a full member at the Birmingham Summit in the UK, though meetings of finance ministers were still mainly confined to the G7. In December 1999, finance ministers and central bank governors of the G20 held their inaugural meeting in Berlin, Germany.

US. Japan has the lowest correlation in output growth with all of the rest. It appears that geography still matters in the modern time.

{Table 1 about here}

{Table 2 about here}

3.2. *General outlook*

Cross spectra, coherence and phases in this empirical study are obtained from executing procedures in RATS, letting the time series in concern pass through a tent window in estimation. A tent window has the feature of curved edges to avoid generating spurious frequencies while ensuring consistency. To achieve consistency, A'Hearn and Woitek (2001) first fit an autoregressive model with the maximum lag being determined by the Akaike information criterion for univariate series, and a vector autoregressive model with the lag length being set to be 2 for bivariate series. They then derive the spectrum of the autoregressive model and the vector autoregressive model for the estimated smoothed spectrum of their time series. While not dismissing attempts of all alternative approaches, we prefer avoiding, whenever possible, any time domain estimation and fitting prior to frequency domain analysis that inevitably introduce additional errors and distortions. Therefore, we apply appropriate frequency domain procedures to produce consistent estimates and reduce potential spurious frequencies. The number of frequencies N′ is not always the same as the number of observations in the time domain N. The Fourier transform performs most efficiently when the number is in the form of 2^m that is equal to or greater than N, where m is an integer. So, in this study, with N being 107 (27 years' quarterly data), N′ is chosen as 128 and the window size M is chosen as 65 (N′/2+1) by the procedure using RATS. Following equation (11) through to equation (14), the significance test statistic is $\sqrt{\frac{3N}{2M}} = \sqrt{\frac{3\times 128}{2\times 65}} = 1.7187$ for $k \neq 0$, which is significant for all the frequency points except the zero frequency[3]. The estimated coherence and phases for the seven economies are exhibited in Figure 3. There are 21 pairs of economies and so are the graphs. Same as in Figure 1, coherence is plotted against the left hand side axis (in blue) with minimum being zero and maximum being one; and phases are plotted against the right hand side axis (in orange) with minimum being -1 (a half cycle lag) and maximum being 1 (a half cycle lead)[4]. While these graphs provide visual observations about the co-movement of economies in terms of closeness and phases on the whole spectrum, some kinds of summaries may be helpful for making overall assessments. Table 3 is provided to report précis of average coherence, overall leads/lags, and coincidence over the whole spectrum. In doing so, Table 3 does not take the full advantage of spectral analysis to examine co-movements with respect to short, medium and long cycle features. It is similar to time domain correlation analysis but included are the contemporary as well as led/lagged correlations, with the phase additionally indicating the degree of lead/lag relations. We further carve up the whole

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[3] The statistic is $\sqrt{\frac{3N}{4M}} = \sqrt{\frac{3\times 128}{4\times 65}} = 1.2153$ for $k = 0$ at the zero frequency point. The statistic would have been significant if the window width were chosen as 33. However, a joint consideration of the requirement that a window should not be too narrow to produce imprecise estimates and the fact that the statistic is all significant at all the non-zero frequency points means that a window width of 65 is the most favorable choice. Moreover, we do not evaluate the business cycle features at the zero frequency point alone; instead, we examine a frequency range from 0 to 0.05 for very long cycles and long-run trends. So, the possible defect at the zero frequency point is minimized.

[4] The phase is scaled down by π. So a half cycle lead is 1 as against π, and a half cycle lag is –1 against -π, to make interpretation simpler. It is possible that X leads Y and Y leads Z, but Z leads X instead of lags X. This happens when the sum of the leads (X leads Y and Y leads Z) is greater than 1 and smaller than 2.

spectrum into long-run trends, lower frequencies, higher frequencies, and middle ranges with the formula discussed in sub-section 2.3, paying specific attention to shorter cycles, longer cycles, and medium cycles respectively. These carve up summaries are reported in Tables 4 to 7 and will be analyzed next.

{Figure 3 about here}

{Table 3 about here}

We inspect the results in each of the rows of Table 3, i.e., inspect each economy's co-movements with the rest six in turn. The US economy shares the highest similarity with the Canadian economy, with the largest coherence, smallest phase lead/lag, and highest coincidence. Canada, on the other hand, has the highest coincidence and smallest lead/lag with the US, but shares the largest coherence, a little oddly, with the UK, though its coherence with the US is also fairly large (the second largest). The US appears to lag Canada in business cycle phases but the lag is only about 0.0025 cycles (recall that –1 is s half cycle lag). In Europe, Germany shares the largest coherence with Italy while Germany is in the closest phase as the UK in business cycles, measured by the degrees of lead/lag and coincidence. France is found to share the largest coherence with Italy too and is highly in pace with Germany in business cycle phases. The UK, though oddly shares the largest coherence with Canada, is highly in pace with Germany in business cycle phases. Italy is clearly linked to France and Germany in business cycle co-movements. Finally, Japan shares the least similarity with the rest of G7 economies in all the terms of coherence, phase lead/lag and coincidence. As these measures are blended over the whole range of the spectrum, there are some atypical results, e.g., the US seems to lag the UK, Japan and Germany, though the lags are very small.

3.3. *Results in four frequency ranges*

From the viewpoint of economic fundamentals and economies' adjustment to changes in the fundamentals, we would expect that economies share more coherence at lower frequencies, or in longer cycles on the whole. Table 4 reports these results on the long-run trend and very long cycles over ten years; whereas Table 5 reports the results on lower frequencies covering the traditional business cycle ranges of up to ten years. These results, compared with those in Table 3, are more consistent across the countries in explaining business cycle characteristics in the longer run that is more relevant in our studies.

It is clearly observed in Table 4 and Table 5 that coherences among these economies are considerably higher than those documented in Table 3. That is, there exists higher coherence in business cycle co-movements in longer cycles or at lower frequencies than that in the whole range. Moreover, the US leads all the other economies in business cycle phases to varied degrees in the traditional business cycle ranges, which appears to be sound, considering the dominant role of the US in the world economy and affairs. For example, the phase lead of the US to the UK is 0.1283 that is about a 0.064 cycle lead. For a five-year cycle, it is 0.32 years or a lead slightly longer than one quarter; and for a ten-year cycle, it is 0.64 years or about a three-quarter lead. The leads of the US to France, Germany and Italy are slightly longer; and the lead/lag between the US and Canada and that between the US and Japan are minimum.

Although the US and the UK share the largest coherence in very long cycles and long-run trends as evident in Table 4, they do not show the same in the traditional business cycle ranges. The largest coherence of the UK in the traditional business cycle ranges, as revealed in Table 5, occurs to be with France, followed by that with Germany, Italy and the US. Indeed, all the four European economies move together in terms of business cycle coherence and phases, though they also enjoy co-movements with the US and Japan to varied degrees in the long run. France has the highest coherence with the rest three European economies and the lowest coherence with Canada overall, and is least in pace with Japan and Canada in business cycle phases. Though Germany shares the largest coherence with France in the long-run and very long cycles, overall it shares the largest coherence, the smallest phase lead/lag and the highest coincidence with Italy, followed by its similarities with Japan in these terms, and has the most discrepancies with Canada. Even in the longer term, Japan has fairly small coherence with the US and appears to enjoy co-movements with the European economy. Canada shares business cycle similarities with the US to the extent greater than that with Europe and Japan. It is difficult to match Canada with Europe and Japan in business cycle patterns, except its somewhat oddly largest coherence with France in the long-run trend and very long cycles.

{Table 4 about here}

{Table 5 about here}

{Table 6 about here}

As expected, coherence between economies is considerably smaller at higher frequencies and in middle ranges, than that at lower frequencies or in longer cycles, consistently across all pairs except Japan with the US and the UK. Overall, coherence is slightly smaller at higher frequencies than that in middle ranges. Furthermore, there are no clear patterns of linkage between certain groups of economies in their business cycle co-movements, especially in middle ranges. In short cycles or at higher frequencies, the UK shares the least similarity with the US, with the smallest coherence, the largest phase lag and the lowest coincidence. Germany appears to bear much similarity with the UK, with the largest coherence and the highest coincidence, so does France with Italy. But Germany and France have the smallest coherence between them at higher frequencies or in shorter cycles. Some of these results, if mistaken as the whole story, would appear to be controversial. Nevertheless, the analysis indicates that the focus of international business cycle co-movements is not on short cycle features[5].

One of the implications that coherence is the highest and considerably higher at lower frequencies implies that economies tend to move together in longer cycles or at lower frequencies, even if they do not behave so in the short to medium terms. It in

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[5] This does not mean that one can simply use lower frequency data, such as annual data. As footnote 2 implies that the spectrum using annual GDP would be distorted and be different from the spectrum at annual and lower frequencies using quarterly GDP.

turn implies support for, and extension of, real business cycle theory with technology shocks or real shocks impacting economies across borders. This is due to the facts that there exist clearer patterns in international co-movements of national economies at lower frequencies or in longer cycles and that there are considerably weaker linkages and no identifiable patterns of linkages between certain groups of economies in their business cycle co-movements in middle ranges and shorter cycles.

4. Conclusions

In this paper, business cycle coherence and phases amongst GDP in the G7 have been inspected in the frequency domain. The present study has contributed to the existing literature by extending the decomposition of the trend component and the cycle component to the decomposition of all frequency components in a transformed domain on the one hand, and enriching empirical research on international business cycle on the other hand. The empirical investigation in the study has been reinforced by the frequency domain analysis method, which is more effective in presentation when cycles and phases are concerned. The paper has identified patterns in international business cycle co-movements among the G7, offering a general outlook of international business cycle co-movements and detailing the lower frequency, higher frequency and middle range characteristics of international linkages of output fluctuations. It opens up a new channel of research in our continuing search for knowledge and understanding about international business cycles.

Due to these distinctive features, this study is able to examine business cycles more effectively with regard to international linkages of economic activity and international transmission of output fluctuations, compared with previous research. The results and findings of this study can be summarized as follows. Firstly, there exist co-movements among G7 economies in output fluctuations overall, in terms of coherence; and the co-movements are not in the same pace, exhibited by phase leads/lags and phase differences. Secondly, co-movements among G7 economies are considerably stronger at lower frequencies or in longer cycles. There are also clearer patterns of linkages of international output fluctuations in longer business cycles. The US, with its dominant role in the world economy, leads all the other economies in business cycle phases to varied degrees. All the four European economies move together in terms of business cycle coherence and phases, though they also enjoy co-movements with the US and Japan to varied and lower degrees in the long run. While Japan appears to enjoy co-movements with the European economy, it still has fairly small coherence with the US even in longer cycles. Overall, Canada shares business cycle similarities with the US to the extent greater than that with Europe and Japan. Thirdly, co-movements between G7 economies are not only considerably weaker in shorter cycles and in middle ranges than in longer cycles, but also exhibit no clear patterns of linkage between groups of economies in international output fluctuations.

The above results and findings render two non-trivial implications. Firstly, as economies tend to move together in longer cycles or at lower frequencies, even if they do not behave so in the short to medium terms, analysis in different ranges of cycles is technically viable to identify useful patterns in international linkages of output fluctuations. Secondly, the results and findings are in support of real business cycle theory being extended to an international arena, with long effect real shocks impacting economies across borders, which also implies that shocks of short term nature, such as monetary shocks, play little role in inducing international co-movements of business cycles.

References

A'Hearn, B. and U. Woitek, 2001, More international evidence on the historical properties of business cycles, *Journal of Monetary Economics*, 47, 321-346.

Ambler, S., M. Cardia and C. Zimmermann, 2002, International transmission of the business cycle in a multi-sector model, *European Economic Review*, 46, 273-300.

Backus, D.K. and P.J. Kehoe, 1992, International evidence on the historical properties of business cycles, *American Economic Review*, 82, 864-888.

Backus, D.K., P.J. Kehoe and F.E. Kydland, 1992, International real business cycles, *Journal of Political Economy*, 100, 745-775.

Basu, S. and A.M. Taylor, 1999, Business cycles in international historical perspective, *Journal of Economic Perspectives*, 13, 45–68.

Beveridge, S. and C.R. Nelson, 1981, A new approach to decomposition of economic time series into permanent and transitory components with particular attention to measurement of the 'business cycles', *Journal of Monetary Economics*, 7, 151-174.

Clark, P.K., 1987, The cyclical component of the U.S. economic activity, *Quarterly Journal of Economics*, 102, 797-814.

Cochrane, J.H., 1988, How big is the random walk in GNP? *Journal of Political Economy*, 96, 893-920.

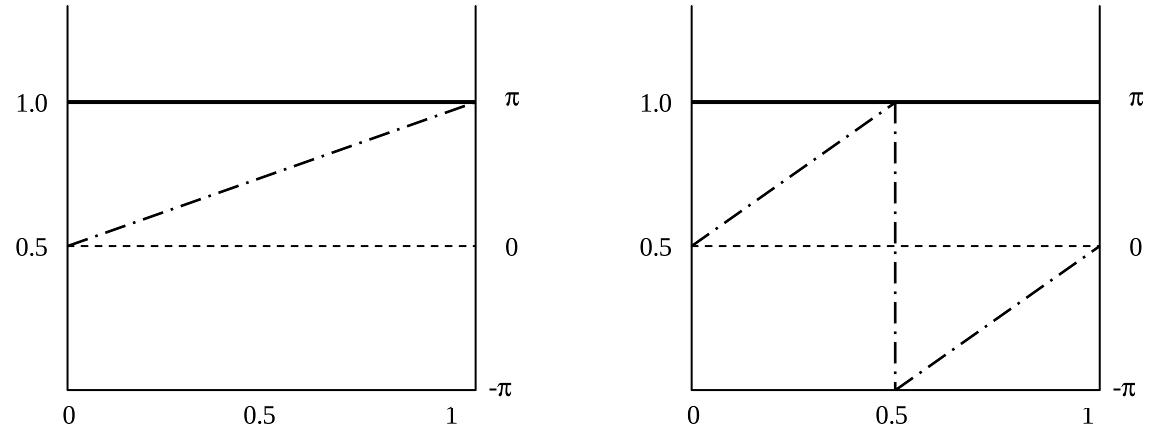
Kydland, F.E. and E.C. Prescott, 1982, Time to build and aggregate fluctuations, *Econometrica*, 50, 1345-1370.

Priestley, M.B., 1996, *Sprectral Analysis and Time Seties 9th priting* (1st priting 1981), Academic Press, London.

Schumpeter, J.A., 1939, *Business Cycles: A Theoretical, Historical and Statistical Analysis of the Capitalist Process*, McGraw-Hill Book Co., Inc., New York and London.

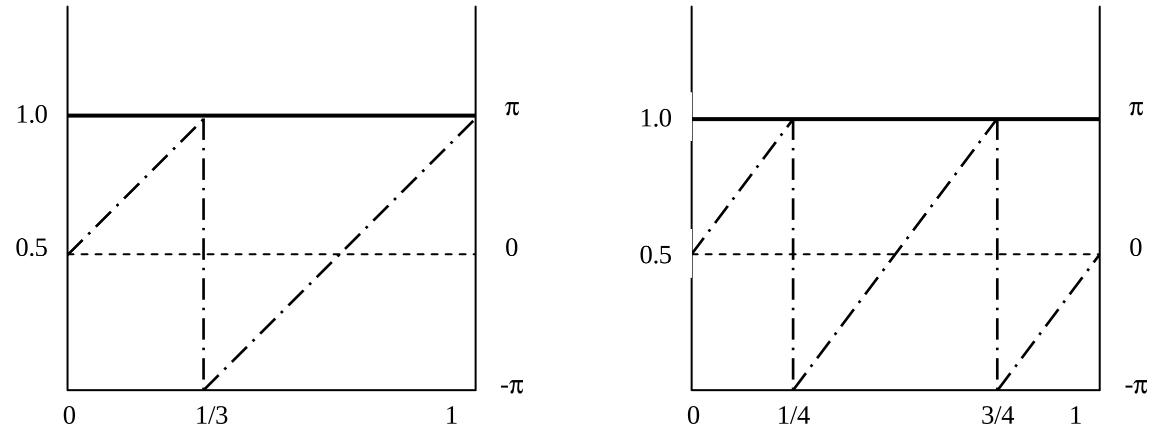
Watson, M.M., 1986, Univariate detrending methods with stochastic trends, *Journal of Monetary Economics*, 18, 49-75.

Ziemer, R.E., W.H. Tranter and D.R. Fannin, 1993, *Signals and Systems: Continuous and Discrete, 3rd ed*, MacMillan Publishing Company, NY.



(a)
Perfect coherence
one phase lag
Coherence: LHS
Phase: RHS

(b)
Perfect coherence
two phase lags
Coherence: LHS
Phase: RHS

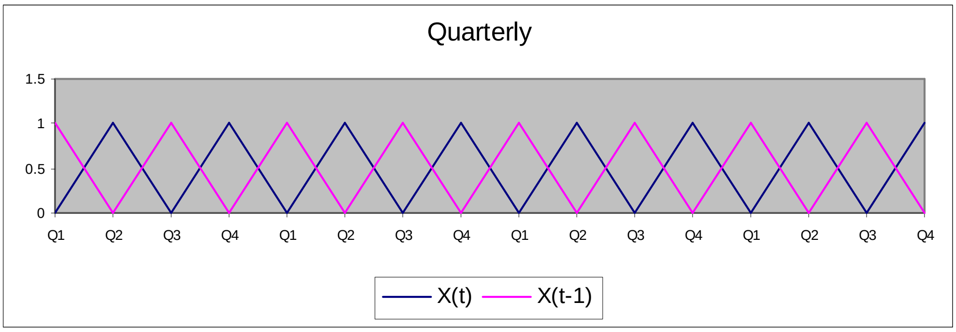


(c)
Perfect coherence
Three phase lags
Coherence: LHS
Phase: RHS

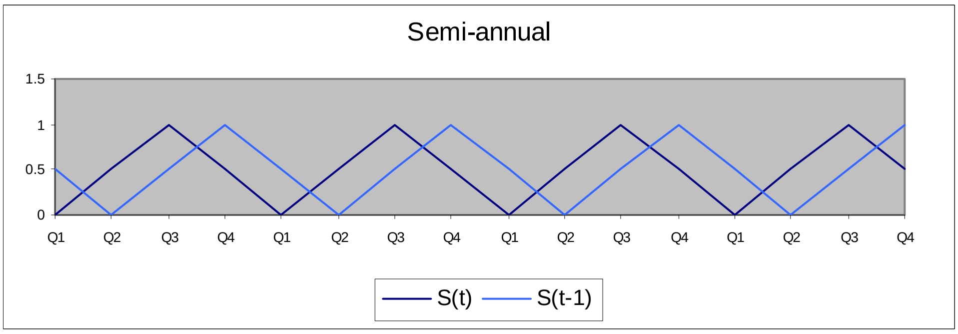
(d)
Perfect coherence
Four phase lags
Coherence: LHS
Phase: RHS

—— Coherence —·—·— Phase

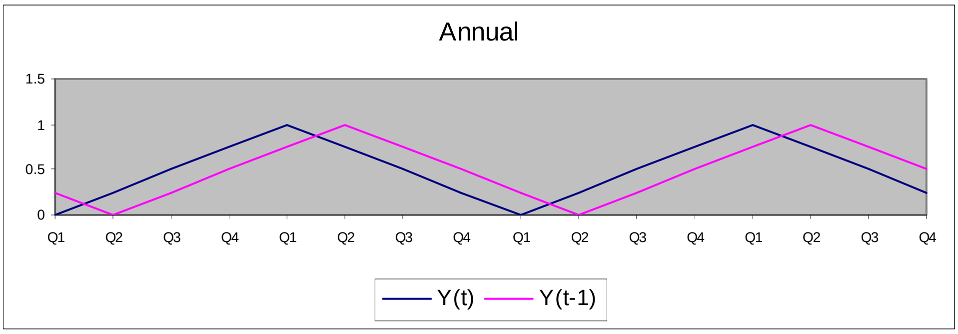
Figure 1. Coherence and phase



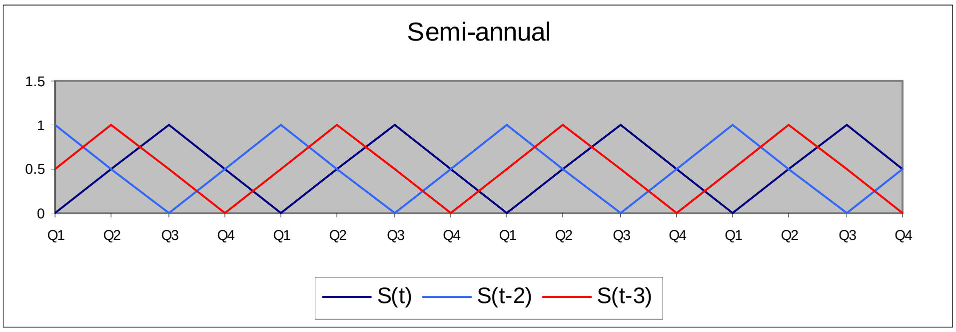
(a) lag in one quarter is half cycle



(b) lag in one quarter is 1/4 cycle

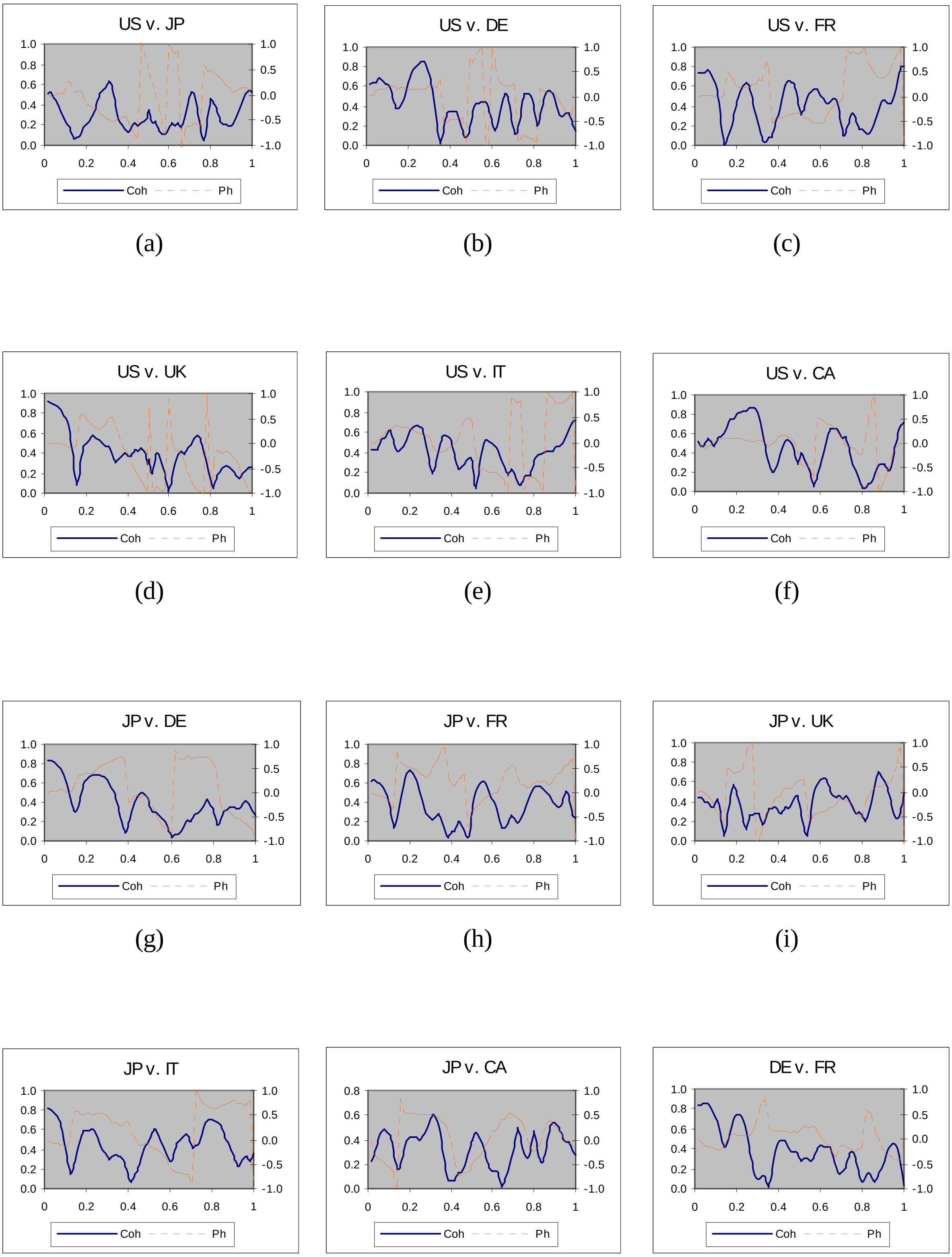


(c) lag in one quarter is 1/8 cycle



(d) a three-quarter lag is equivalent to a one-quarter lead in a four-quarter cycle

Figure 2. Exhibits of phase lags/leads



(a) (b) (c)

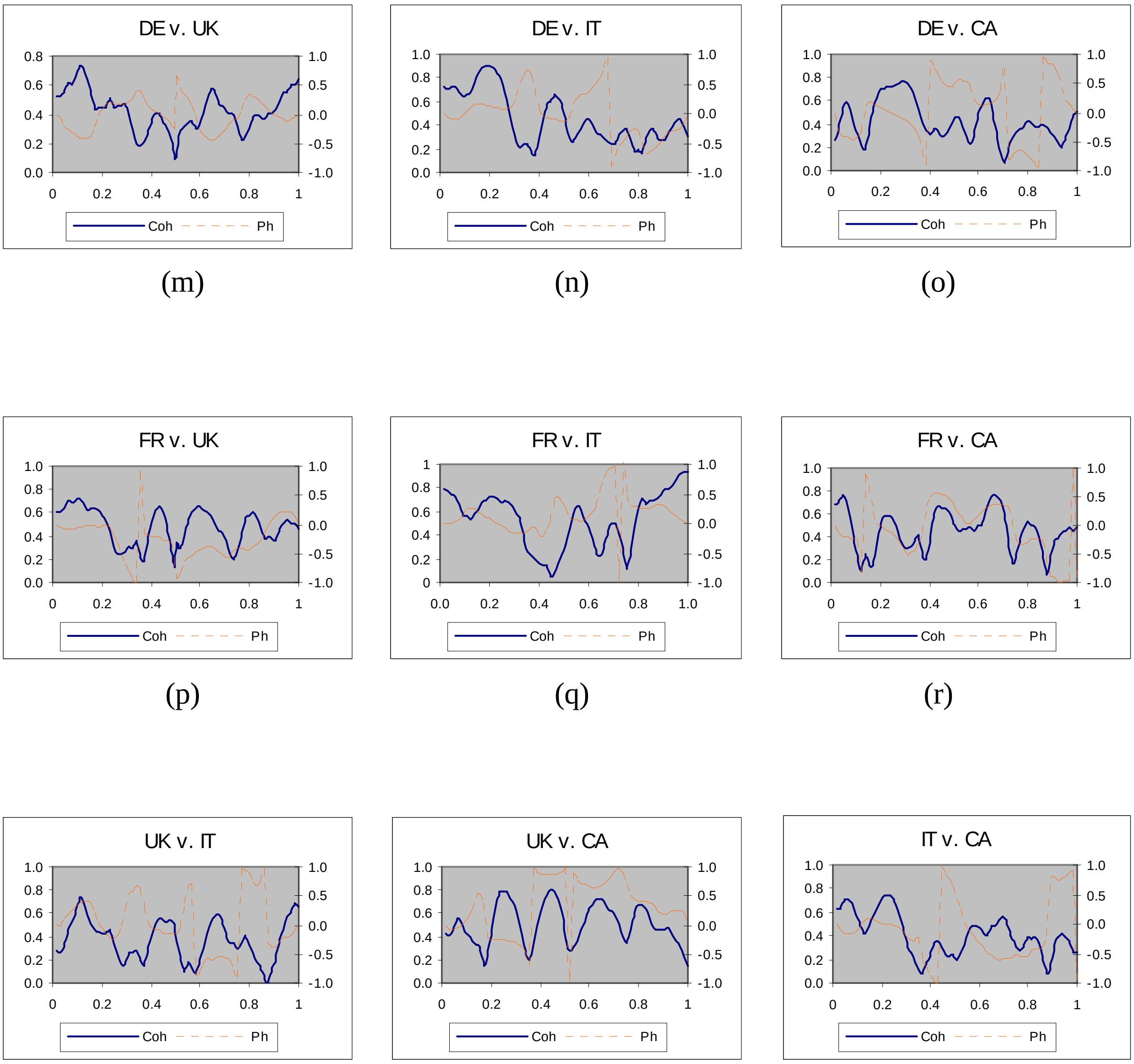
(d) (e) (f)

(g) (h) (i)

(j) (k) (l)

Figure 3. Coherence and phase lags/leads between G7 economies

(continued next page)



(m) (n) (o)

(p) (q) (r)

(s) (t) (u)

Figure 3. Coherence and phase lags/leads between G7 economies

(continued from previous page)

Table 1. Preliminary descriptive statistics

	US	JP	DE	FR	UK	IT	CA
Mean	0.7404e-2	0.6895e-2	0.4689e-2	0.5568e-2	0.4888e-2	0.5766e-2	0.6402e-2
Max	0.3780e-1	0.2582e-1	0.3395e-1	0.2431e-1	0.3549e-1	0.3337e-1	0.4059e-1
Min	-0.2060e-1	-0.3497e-1	-0.2275e-1	-0.1964e-1	-0.2765e-1	-0.2379e-1	-0.2930e-1
Median	0.8180e-2	0.7200e-2	0.4060e-2	0.6440e-2	0.5950e-2	0.4660e-2	0.6290e-2
Std	0.8666e-2	0.9384e-2	0.9576e-2	0.7564e-2	0.9239e-2	0.8553e-2	0.1050e-1

US: the United States; JP: Japan; DE: Germany; FR: France; UK: the United Kingdom; IT: Italy; CA: Canada. Apply to all the tables and figures.

Table 2. Correlation statistics

	US	JP	DE	FR	UK	IT	CA
US	1.0000	0.0570	0.2292	0.1765	0.2200	0.1574	0.4303
JP	0.0570	1.0000	0.1297	0.1760	0.1892	0.0585	-0.0141
DE	0.2292	0.1297	1.0000	0.3261	0.2732	0.3570	0.1713
FR	0.1765	0.1760	0.3261	1.0000	0.2966	0.4247	0.1853
UK	0.2200	0.1892	0.2732	0.2966	1.0000	0.1312	0.0564
IT	0.1574	0.0585	0.3570	0.4247	0.1312	1.0000	0.3661
CA	0.4303	-0.0141	0.1713	0.1853	0.0564	0.3661	1.0000

Table 3. Coherence and phases

	US	JP	DE	FR	UK	IT	CA
US	1.0000	0.2957	0.4245	0.4095	0.3987	0.4130	**0.4659**
	0.0000	-0.0418	-0.0365	0.1175	-0.2195	0.0433	**-0.0051**
	0.0000	0.3947	0.3857	0.4211	0.4621	0.4767	**0.2298**
JP	0.2957	1.0000	0.3967	0.3736	0.3729	**0.4473**	0.3251
	0.0418	0.0000	0.1104	0.2200	-0.0712	0.1977	0.0444
	0.3947	0.0000	0.4641	**0.3529**	0.3542	0.4770	0.3702
DE	0.4245	0.3967	1.0000	0.3768	0.4223	**0.4562**	0.4269
	0.0365	-0.1104	0.0000	0.0259	**-0.0062**	-0.0098	0.0352
	0.3857	0.4641	0.0000	0.2148	**0.2108**	0.3022	0.4220
FR	0.4095	0.3736	0.3768	1.0000	0.4763	**0.5310**	0.4530
	-0.1175	-0.2200	**-0.0259**	0.0000	-0.2580	0.1613	-0.0637
	0.4211	0.3529	**0.2148**	0.0000	0.3252	0.2408	0.3853
UK	0.3987	0.3729	0.4223	0.4763	1.0000	0.3732	**0.5044**
	0.2195	0.0712	**0.0062**	0.2580	0.0000	0.0259	0.3380
	0.4621	0.3542	**0.2108**	0.3252	0.0000	0.3889	0.4961
IT	0.4130	0.4473	0.4562	**0.5310**	0.3732	1.0000	0.4118
	-0.0433	-0.1977	**0.0098**	-0.1613	-0.0259	0.0000	-0.1072
	0.4767	0.4770	0.3022	**0.2408**	0.3889	0.0000	0.4125
CA	0.4659	0.3251	0.4269	0.4530	**0.5044**	0.4118	1.0000
	0.0051	-0.0444	-0.0352	0.0637	-0.3380	0.1072	0.0000
	0.2298	0.3702	0.4220	0.3853	0.4961	0.4125	0.0000

First row: coherence; second row: overall lags φ (economies in column titles lead economies in row titles, negative figures mean lags); third row: coincidence θ.
The largest coherence in each row is in bold, the smallest is underlined.
The smallest overall lag/lead in each row is in bold, the largest is underlined.
The highest coincidence in each row is in bold, the lowest is underlined.

The significance test statistic is $\sqrt{\frac{3N}{2M}} = \sqrt{\frac{3\times128}{2\times65}} = 1.7187$ for $k \neq 0$, which is significant for all the frequency points except the zero frequency.

Table 4. Coherence and phases (very long cycles and long-run trends)

	US	JP	DE	FR	UK	IT	CA
US	1.0000	0.4954	0.6367	0.7362	**0.9012**	0.4330	0.4917
	0.0000	-0.0175	0.0449	0.0077	0.0067	**0.0052**	-0.0639
	0.0000	0.0175	0.0449	0.0077	**0.0067**	0.0214	0.0639
JP	0.4954	1.0000	**0.8309**	0.6186	0.4272	0.7988	0.2967
	0.0175	0.0000	0.0212	-0.0250	**0.0007**	-0.0374	-0.2147
	0.0175	0.0000	0.0212	0.0250	0.0203	0.0374	0.2147
DE	0.6367	0.8309	1.0000	**0.8499**	0.5363	0.7215	0.3823
	-0.0449	**-0.0212**	0.0000	-0.0618	-0.0868	-0.0541	-0.2429
	0.0449	**0.0212**	0.0000	0.0618	0.0868	0.0541	0.2429
FR	0.7362	0.6186	**0.8499**	1.0000	0.6163	0.7648	0.7181
	-0.0077	0.0250	0.0618	0.0000	-0.0410	**-0.0045**	-0.1128
	0.0077	0.0250	0.0618	0.0000	0.0410	**0.0054**	0.1128
UK	**0.9012**	0.4272	0.5363	0.6163	1.0000	0.2872	0.4292
	-0.0067	**-0.0007**	0.0868	0.0410	0.0000	0.0353	-0.0596
	0.0067	0.0203	0.0868	0.0410	0.0000	0.0502	0.0596
IT	0.4330	**0.7988**	0.7215	0.7648	0.2872	1.0000	0.6558
	-0.0052	0.0374	0.0541	**0.0045**	-0.0353	0.0000	-0.0853
	0.0214	0.0374	0.0541	**0.0054**	0.0502	0.0000	0.0853
CA	0.4917	0.2967	0.3823	**0.7181**	0.4292	0.6558	1.0000
	0.0639	0.2147	0.2429	0.1128	**0.0596**	0.0853	0.0000
	0.0639	0.2147	0.2429	0.1128	**0.0596**	0.0853	0.0000

First row: coherence; second row: overall lags φ (economies in column titles lead economies in row titles, negative figures mean lags); third row: coincidence θ.

The largest coherence in each row is in bold, the smallest is underlined.

The smallest overall lag/lead in each row is in bold, the largest is underlined.

The highest coincidence in each row is in bold, the lowest is underlined.

The significance test statistic is $\sqrt{\frac{3N}{2M}} = \sqrt{\frac{3\times 128}{2\times 65}} = 1.7187$ for $k \neq 0$, which is significant for all the frequency points except the zero frequency.

Table 5. Coherence and phases (lower frequencies)

	US	JP	DE	FR	UK	IT	CA
US	1.0000	0.2016	0.5414	0.3948	0.5423	0.5201	**0.6215**
	0.0000	0.0577	0.1670	0.1523	0.1283	0.2565	**0.0550**
	0.0000	0.0994	0.1670	0.1561	0.1916	0.2565	**0.0749**
JP	0.2016	1.0000	**0.5502**	0.4675	0.3547	0.4592	0.3588
	-0.0577	0.0000	0.1656	0.2210	**-0.0352**	0.2261	-0.1172
	0.0994	0.0000	0.1659	0.3990	0.3747	0.3252	0.6009
DE	0.5414	0.5502	1.0000	0.6487	0.5864	**0.7669**	0.4422
	-0.1670	-0.1656	0.0000	-0.0960	-0.2665	**0.0675**	-0.1432
	0.1670	0.1659	0.0000	0.1390	0.2851	**0.1081**	0.2722
FR	0.3948	0.4675	0.6487	1.0000	**0.6615**	0.6322	0.3362
	-0.1523	-0.2210	0.0960	0.0000	-0.0483	0.1518	**0.0069**
	0.1561	0.3990	0.1390	0.0000	**0.0483**	0.1518	0.3572
UK	0.5423	0.3547	0.5864	**0.6615**	1.0000	0.5461	0.3779
	-0.1283	**0.0352**	0.2665	0.0483	0.0000	0.2557	0.1353
	0.1916	0.3747	0.2851	**0.0483**	0.0000	0.2641	0.2398
IT	0.5201	0.4592	**0.7669**	0.6322	0.5461	1.0000	0.5866
	-0.2565	-0.2261	-0.0675	-0.1518	-0.2557	0.0000	**-0.0225**
	0.2565	0.3252	0.1081	0.1518	0.2641	0.0000	**0.0800**
CA	**0.6215**	0.3588	0.4422	0.3362	0.3779	0.5866	1.0000
	-0.0550	0.1172	0.1432	**-0.0069**	-0.1353	0.0225	0.0000
	0.0749	0.6009	0.2722	0.3572	0.2398	0.0800	0.0000

First row: coherence; second row: overall lags φ (economies in column titles lead economies in row titles, negative figures mean lags); third row: coincidence θ.
The largest coherence in each row is in bold, the smallest is underlined.
The smallest overall lag/lead in each row is in bold, the largest is underlined.
The highest coincidence in each row is in bold, the lowest is underlined.

The significance test statistic is $\sqrt{\frac{3N}{2M}} = \sqrt{\frac{3\times 128}{2\times 65}} = 1.7187$ for $k \neq 0$, which is significant for all the frequency points except the zero frequency.

Table 6. Coherence and phases (higher frequencies)

	US	JP	DE	FR	UK	IT	CA
US	1.0000	0.3211	0.3771	0.3816	0.2215	**0.4413**	0.2785
	0.0000	0.2479	-0.2735	0.5693	-0.4208	0.1372	**-0.1236**
	0.0000	**0.2479**	0.3383	0.6943	0.5428	0.8386	0.5428
JP	0.3211	1.0000	0.3282	0.4422	0.4046	**0.4725**	0.3715
	-0.2479	0.0000	-0.2556	0.2387	**0.0004**	0.6062	0.0513
	0.2479	0.0000	0.5618	0.3637	0.3381	0.7312	**0.1627**
DE	0.3771	0.3282	1.0000	0.2199	**0.4370**	0.3101	0.3610
	0.2735	0.2556	0.0000	0.0259	0.0835	-0.3777	**0.0238**
	0.3383	0.5618	0.0000	0.2148	**0.1304**	0.3777	0.6045
FR	0.3816	0.4422	0.2199	1.0000	0.4773	**0.7295**	0.3963
	-0.5693	-0.2387	**-0.0259**	0.0000	-0.0738	0.2117	-0.5122
	0.6943	0.3637	0.2148	0.0000	0.2295	**0.2117**	0.6359
UK	0.2215	0.4046	0.4370	**0.4773**	1.0000	0.3382	0.4605
	0.4208	**-0.0004**	-0.0835	0.0738	0.0000	0.2442	0.2922
	0.5428	0.3381	**0.1304**	0.2295	0.0000	0.4936	0.2922
IT	0.4413	0.4725	0.3101	**0.7295**	0.3382	1.0000	0.3042
	-0.1372	-0.6062	0.3777	-0.2117	-0.2442	0.0000	**0.0737**
	0.8386	0.7312	0.3777	**0.2117**	0.4936	0.0000	0.6383
CA	0.2785	0.3715	0.3610	0.3963	**0.4605**	0.3042	1.0000
	0.1236	-0.0513	**-0.0238**	0.5122	-0.2922	-0.0737	0.0000
	0.5428	**0.1627**	0.6045	0.6359	0.2922	0.6383	0.0000

First row: coherence; second row: overall lags φ (economies in column titles lead economies in row titles, negative figures mean lags); third row: coincidence θ.
The largest coherence in each row is in bold, the smallest is underlined.
The smallest overall lag/lead in each row is in bold, the largest is underlined.
The highest coincidence in each row is in bold, the lowest is underlined.

The significance test statistic is $\sqrt{\frac{3N}{2M}}=\sqrt{\frac{3\times 128}{2\times 65}}=1.7187$ for $k \neq 0$, which is significant for all the frequency points except the zero frequency.

Table 7. Coherence and phases (middle ranges)

	US	JP	DE	FR	UK	IT	CA
US	1.0000	0.2938	0.3954	0.3987	0.3957	0.3690	**0.4876**
	0.0000	-0.2002	**0.0055**	-0.0838	-0.2454	-0.0545	0.0358
	0.0000	0.5735	0.4959	0.4077	0.5393	0.4150	**0.2038**
JP	0.2938	1.0000	0.3483	0.2965	0.3593	**0.4034**	0.2975
	0.2002	0.0000	0.2652	0.2318	-0.1191	**0.0278**	0.1077
	0.5735	0.0000	0.5404	**0.3625**	0.3835	0.4427	0.4113
DE	0.3954	0.3483	1.0000	0.3316	0.3607	0.4127	**0.4556**
	-0.0055	-0.2652	0.0000	0.1109	0.0330	0.1360	0.1130
	0.4959	0.5404	0.0000	**0.2115**	0.2363	0.3432	0.3975
FR	0.3987	0.2965	0.3316	1.0000	0.4127	0.3952	**0.4885**
	0.0838	-0.2318	-0.1109	0.0000	-0.4163	0.1555	0.1201
	0.4077	0.3625	**0.2115**	0.0000	0.4684	0.2981	0.3044
UK	0.3957	0.3593	0.3607	0.4127	1.0000	0.3478	**0.5652**
	0.2454	0.1191	**-0.0330**	0.4163	0.0000	-0.1358	0.4478
	0.5393	0.3835	**0.2363**	0.4684	0.0000	0.4053	0.6943
IT	0.3690	0.4034	**0.4127**	0.3952	0.3478	1.0000	0.3908
	0.0545	**-0.0278**	-0.1360	-0.1555	0.1358	0.0000	-0.2129
	0.4150	0.4427	0.3432	**0.2981**	0.4053	0.0000	0.4318
CA	0.4876	0.2975	0.4556	**0.4885**	0.5652	0.3908	1.0000
	-0.0358	-0.1077	-0.1130	0.1201	-0.4478	0.2129	0.0000
	0.2038	0.4113	0.3975	0.3044	0.6943	0.4318	0.0000

First row: coherence; second row: overall lags φ (economies in column titles lead economies in row titles, negative figures mean lags); third row: coincidence θ.
The largest coherence in each row is in bold, the smallest is underlined.
The smallest overall lag/lead in each row is in bold, the largest is underlined.
The highest coincidence in each row is in bold, the lowest is underlined.

The significance test statistic is $\sqrt{\frac{3N}{2M}} = \sqrt{\frac{3\times 128}{2\times 65}} = 1.7187$ for $k \neq 0$, which is significant for all the frequency points except the zero frequency.