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ABSTRACT

Empirical evidence indicates local jurisdictions are internally more heterogeneous than standard sorting models predict. We develop a dynamic multi-region model, with fluctuating regional house prices, where an owner-occupying household's location choice depends on its current wealth and its current "match" and involves both consumption and investment considerations. The relative strength of the consumption motive and the investment motive in the location choice determines the equilibrium pattern of residential sorting, with a strong investment (consumption) motive implying sorting according to the match (wealth). The model predicts a negative relation between the size of house price fluctuations and residential sorting in the match dimension. Also, movers should be more sorted than stayers. These predictions are consistent with evidence from US metropolitan areas when income, age and education are used as proxies for the match.

JEL Classification: D31, D52, R13, R21, R23

Keywords: residential sorting, house prices, incomplete markets, owner-occupation, household mobility

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1 Introduction

A central theme in regional and urban economics has been to examine how households sort themselves into neighborhoods and communities according income and other socioeconomic characteristics. Roughly speaking, the sorting approach predicts that local jurisdictions should be internally more homogeneous than the larger geographical or economic unit of which they are a part. Also, the jurisdictions should differ from each other. However, recent empirical evidence reveals that there is considerable heterogeneity within municipalities and local neighborhoods. According to Ioannides (2004), in a typical American neighborhood, neighbors tend to differ significantly in terms of income, age and education. Rhode and Strumpf (2003) report that heterogeneity across US municipalities and counties, measured with respect to income and a number of other socioeconomic variables (including age, education, race, nativity, religion, owner-occupation rate and party vote shares in presidential elections) did not increase over the period 1850-1990 although migration costs fell, which should have made sorting easier. Davidoff (2005) finds that while the extent of sorting is generally quite small, it also varies widely across metropolitan areas. The fraction of income variance explained by differences across jurisdictions is on average approximately six percent, and it ranges from less than one percent to almost 25 percent.

As an attempt to understand the observed pattern of residential sorting, this paper develops a dynamic sorting model, with an emphasis on housing as an important, and sometimes risky, asset. The main prediction of the model links the degree of sorting to the size of house price fluctuations: these two should be negatively correlated. Consistent with this prediction, we find that in those US metropolitan areas, where house price fluctuations have been large, municipalities tend to have a rather diverse population, with the shares of different income, age and education groups in each municipality roughly corresponding to the overall population structure in the metropolitan area. If price fluctuations are smaller, the municipalities tend to be internally more homogeneous, and they tend to differ more clearly from each other in terms of income, age and education, so that the degree of residential sorting in the metropolitan area is higher.

In addition to the main result, the model predicts that among onwer-occupying households, movers should be more sorted than stayers. Finally, there should be a non-linear relation between wealth and mobility, so that households with intermediate wealth levels are more mobile than the poor and the wealthy. We present some evidence in support of these predictions as well.

Our approach is based on the following main ideas: (i) For owner-occupying households, housing is both a consumption good and an asset, and residential location choices may involve not only consumption but also investment considerations; essentially, expected resale value matters. (ii) Regional house prices fluctuate, and the capital gains and losses made in the housing market play an important role in determining how a household's wealth evolves over time. (iii) Borrowing constraints may narrow the set of feasible housing options, and impair a household's ability to move.

To capture these ideas, we consider a dynamic multi-region economy, where some locations are more desirable than others. While most households derive a positive utility premium from residing in a desirable location, the size of the premium varies between households, depending on socioeconomic characteristics, such as household size, the age of household heads, education or income; in the model these characteristics are summarized by the household's current "match." With a certain probability, a region is hit by a shock, so that its desirability ranking changes and regional house prices rise or fall. The regional shock may reflect e.g. altering labor market conditions, changes in the supply of public goods and services, or the evolution in the tastes and the needs of the population. Alternatively, the house price dynamics may be interpreted as reflecting (in a reduced form) the interaction between housing demand and supply. According to this interpretation, an area is currently expensive, because housing supply has not yet increased to absorb a positive demand shock. The resulting pattern of mean-reverting (relative) regional house prices implies that while a currently popular and expensive location is, for most households, more attractive from the consumption point of view, a currently less popular and less expensive location offers better investment opportunities.[2]

[2]In Section 2.1, we present evidence on mean-reversion.

In each period a household chooses its location based on its current wealth and its current match. The pattern of residential sorting, that emerges in equilibrium, depends on the relative strength of the consumption motive and the investment motive. If regional shocks are large and/or persistent, the consumption motive dominates. The households make their location choices mainly by comparing current benefit streams. Then the equilibrium pattern of residential sorting essentially boils down to differences in wealth: as a general rule, a household resides in an unpopular location if and only if it is borrowing constrained, and cannot afford a more expensive house. Since current wealth depends, in part, on past luck in the housing market, households living in the same area may then have little in common, except for the value of their home.[3] Finally, since most households want to live in a popular location, regional price differences, as well as capital gains and losses realized in the housing market, are large, compared with typical household wealth.

When regional shocks are small and/or transient, the investment motive is stronger (in relative terms, compared with the consumption motive). Caring about their future prospects, many households, which would receive a larger immediate welfare stream from a desirable location, voluntarily choose a less desirable area, in the hope of making capital gains. Typically, a household resides in a desirable location, if and only if its current match with that location is truly good. Given the empirical interpretation of the match, households living within the same jurisdiction should then resemble each other with respect to various socioeconomic characteristics, such as household size, the age of household heads, income or education, and different jurisdictions should differ from each other with respect to the distribution of these observable characteristics. The fact that many households voluntarily choose a less desirable location is also reflected in house prices. In equilibrium, regional house price differences, and price fluctuations, are small, in comparison with typical household wealth.

Generally speaking, our framework combines themes, which are typically addressed in two separate branches of literature. (i) Most papers on residential sorting use static gen-

[3] In their study of US neighborhoods, Ioannides and Seslen (2002) find that income is a poor predictor of household wealth. Neighborhood wealth distributions tend to differ significantly from neighborhood income distributions.

eral equilibrium models. Earlier sorting models[4] often assumed that households differ with respect to one characteristic only (typically income), and predicted perfect stratification along that dimension, a prediction that did not agree with empirical evidence. The more recent two-dimensional sorting models by Epple and Platt (1998), Epple and Sieg (1999) and Epple, Romer and Sieg (2001) are more successful in explaining the data. In these models, households differ both with respect to income and with respect to tastes, and there is imperfect sorting along both dimensions. An alternative approach to account for the observed diversity of households within jurisdictions is based on the heterogeneity of the housing stock (e.g., Nechyba (2000)). In contrast to the present paper, the atemporal nature of these models means that housing and location choices do not involve investment considerations, and there is no feed-back from house price fluctuations to household wealth.[5] On the other hand, in the sorting literature, the attractivity of different jurisdictions typically arises endogenously as a part of the equilibrium (e.g., the supply of local public goods and services is determined in a political economy equilibrium), whereas we take the process that determines the desirability of different locations as given.

(ii) The second branch of literature analyzes housing wealth as an important component of a household's asset portfolio. While the double nature of housing, as a consumption good and as an investment, and house price fluctuations play an important role here, this literature essentially focuses on the optimization problem of an individual household, and the implications for residential sorting are not examined.[6]

A few recent papers take up a roughly similar mix of issues as we do here. In a two-period framework, Ortalo-Magné and Rady (2008) examine tenure choice and income heterogeneity in booming cities, where house prices rise, and home-owners, who make capital gains, may choose to stay put, even when newcomers typically earn higher incomes.

[4]Examples include Ellickson (1971), Epple, Filimon and Romer (1984), Epple and Romer (1989, 1991), Henderson (1991) and Wheaton (1993). For a survey, see Ross and Yinger (1999).

[5]A few papers (e.g., Bénabou (1996), Fernandez and Rogerson (1996)) analyze sorting in a dynamic context. Even in these models, however, the households are typically assumed to be renters, and they are also assumed to choose their location once and for all (in the first period), so that realized capital gains and losses do not shape the equilibrium pattern of residential sorting.

[6]Examples include Ranney (1981), Ioannides and Henderson (1983), Poterba (1984), Henderson and Ioannides (1987), Bruenecker (1997), Flavin and Yamashita (2002), Cocco (2005), Sinai and Souleles (2005) and Li and Yao (2007).

Their model emphasizes that wealth rather than income, or tastes, can be a key determinant of a household's residential location. Consistent with their theory, they find that there is a positive correlation between the income dispersion in a neighborhood and the dispersion of time since a household moved to the neighborhood. Also the related work by Ortalo-Magné and Rady (2006) on house price dynamics and housing choices shares common themes and features with our paper, although here households choose between different apartment types ("flats" and "houses") rather than between different locations. In particular, in Ortalo-Magné and Rady (2006), as well as in our paper, capital gains and losses made in the housing market are a key driver of household wealth dynamics, and borrowing constraints may limit the set of feasible housing options. Glaeser and Gyourko (2005) study the joint process of falling house prices and neighborhood change in declining cities. Due to the durability of housing, a negative shock leads to a sharp fall in housing prices, but only a slow and gradual decline in city size. Low housing costs in a city attract low-income households. In the model, however, households are assumed to be renters, so that realized capital losses do not affect their residential location choices.

The plan of the paper is as follows. The basic model is developed in Section 2. Section 3, which contains the main theoretical results, shows how the equilibrium pattern of residential sorting reflects the relative strength of the consumption motive and the investment motive of housing. The section also establishes a link between the size of house price fluctuations and the pattern of residential sorting, and analyzes the degree of sorting among movers and stayers. Section 4 extends the basic model by introducing more general match dynamics. It particular, this extension allows us to consider household specific differences in expected tenure length, and endogenous correlation between wealth and the match. Some empirical evidence is presented in Section 5. Finally Section 6 concludes.

2 The model

2.1 Some empirical background

We develop a dynamic model of residential sorting, based on the following main ideas: (i) For owner-occupying households, housing is both a consumption good and an asset, and residential location choices may involve not only consumption but also investment considerations; essentially, expected resale value matters. (ii) Regional house prices fluctuate, and the capital gains and losses made in the housing market play an important role in determining how a household's wealth evolves over time. (iii) Borrowing constraints may narrow the set of feasible housing options, and impair a household's ability to move.

It is natural to include these elements in a framework which tries to understand households' location choices and residential sorting. In most developed countries, owner-occupied housing is the single most important investment for a typical household. For example, in the late 1990's, single family owner-occupied housing composed 2/3 of household wealth in the UK, 1/3 of household wealth in the US, and 2/3 of the assets of a US household with median wealth.[7] Given the importance of housing as an asset, it is reasonable to assume that investment considerations may also play a role when people choose where to buy a home. One simple way to motivate this assumption is to conduct an internet search. Our Google search with key words "location", "home" and "resale value" produced nearly one million hits, with headlines such as "Buying a home with a resale value: location, location, location" abounding.

Second, house prices are often highly volatile, and in different regions property values tend to rise and fall asynchronously, so that relative regional prices may vary considerably over time. Figures 1 and 2 illustrate this finding with price data from five UK regions and four US metropolitan areas.[8] Relative prices can fluctuate significantly even at a more

[7] Banks et al. (2002), *Federal Reserve's 2001 Survey of Consumer Finances.*

[8] According to Shiller (1993, Ch 5 p. 79) real estate booms and busts in US cities have been regionally asynchronized and prize movements often dramatic. Del Negro and Otrok (2007) find that, with the exception of the boom of the early 2000s, US house price dynamics have been mainly driven by local or regional, rather than national, shocks. For further evidence on US prices, see also Case and Shiller (1989), Malpezzi (1999), Case, Quigley and Shiller (2005), or Himmelberg, Mayer and Sinai (2005). For British evidence, see Muellbauer and Murphy (1997), or Cook (2003).

local level. In London, for example, the borough of Greenwich was 3% more expensive than the borough of Hackney in 1995, but in 2001 prices were 20% higher in Hackney than in Greenwich[9]; see also Iacoviello and Ortalo-Magné (2003). For similar findings on the Boston metropolitan area, see Case and Mayer (1996).

The capital gains and losses made in the housing market can be remarkably large in comparison with typical household incomes and savings, and empirical studies reveal that falling home equity value may seriously constrain a household's ability to move.[10] To illustrate the size of the wealth shocks, Table 1 shows maximum and minimum house-price-to-income ratios in four major US cities over the period 1979-1996. In the UK, the average annual capital gain in the London market between 1983 and 1988 corresponded to 72% of the mean annual disposable household income in the UK over that period, and exceeded by the factor of 7.8 average yearly household savings. Between 1989 and 1992, the annual capital loss of a typical London homeowner was equivalent to 77% of average disposable household income, and 8.4 times average household savings.

As a general rule, these housing market risks are uninsurable. Shiller (1993, 2003), for example, lists home equity insurance as one of the key financial markets currently missing.[11] Nevertheless, location choices and the timing of transactions can affect the distribution of risks that a household faces. While house price fluctuations include an important random component, they also display certain regularities. In particular, regional house prices tend to exhibit mean-reversion in time horizons of one year and longer; possible explanations include lags in housing construction, mean-reversion in underlying economic fundamentals, and the interaction of borrowing constraints and wealth effects, which gives rise to temporary overshooting of prices.[12] There is also some evidence on long-run equilibrium relationships between house prices in different areas: if prices in a particular location are currently above the equilibrium level, they are likely to fall, in

[9] Source: Land Registry, http://www.landreg.gov.uk.

[10] See Chan (1996, 2001), Ferreira, Gyourko and Tracy (2008), Henley (1998).

[11] Shiller (1993), and Shiller and Weiss (1999) discuss the potential problems, both economic and psychological, involved in providing hedging against house price swings, as well as ways to overcome these problems. See Shiller (1993, 2003), and Iacovello and Ortalo-Magné (2003) for discussion on some real life experiments in the US and the UK.

[12] See Glaeser and Gyourko (2007), Ortalo-Magné and Rady (2005), Evenson (2003), Lamont and Stein (1999).

relative terms, some time in the future; if relative prices are above the equilibrium level, the opposite is likely to happen.[13]

2.2 The basics of the economy

The economy has two locations. Each location has an equal, fixed, stock of identical houses. Each house is occupied by a single household and no one household is ever homeless. All households are owner-occupiers and there is no rental housing. For convenience, assume that the stock of houses and the mass of households each comprises a continuum of size unity.

There are infinite discrete time periods indexed by $t = 0, 1, ...$ In each period, one of the locations is deemed to be "desirable" while the other one is "less desirable". When a period changes, the relative ranking of the locations is reversed with probability $\pi \in (0, 1)$.

We also consider a small region interpretation of the model, with a continuum of locations. Then in each period, one half of the locations are "desirable" while the remaining locations are "less desirable", and when a period changes, a measure π of the locations is hit by a regional shock. The long-run equilibrium of the model is essentially identical under both interpretations.[14]

The households differ in the utility premium they derive from residing in the desirable location. The household specific component of the premium is captured by the match, θ: a high realization of θ implies a good match with the currently desirable location, while a low (negative) realization implies a good match with the less desirable location.[15] The aggregate heterogeneity of households is unchanged over time, and θ has a stationary distribution, with a cumulative distribution function $G(\theta)$, on some support $[\theta_L, \theta_H]$. Without loss of generality, we assume that the median match $\theta_m = 0$, i.e. $G(0) = \frac{1}{2}$.

[13] That is, regional house prices are cointegrated. For evidence from British regions, see MacDonald and Taylor (1993), Alexander and Barrow (1994) or Cook (2003). For evidence from US census regions, as well as for a comparison between the US and the UK, see Meen (2002).

[14] A straightforward extension of the small region version of the model involves considering a case, where in each period a measure $\varphi \in (0, 1)$ of the locations is desirable, while the remaining locations are undesirable. The main results of the paper, stated as propositions, carry over to this extended framework.

[15] As will become clear below, even households with low realizations of θ may derive a positive premium from the desirable location. However, even if this is the case, households with low θ lose less if they reside in the undesirable location than households with higher realizations of θ.

A household with current match θ receives per period utility $\frac{1}{2}\varepsilon+\theta$, when living in the currently desirable location. The per period utility of anyone household living in the less desirable location is $-\frac{1}{2}\varepsilon$. Here the parameter $\varepsilon > 0$ measures regional welfare differences. ε also gauges the size of regional shocks: if a location is hit by a shock, the utility stream it offers to the (median) household changes from $\frac{1}{2}\varepsilon$ to $-\frac{1}{2}\varepsilon$, or vice versa.

Given these assumptions, all households with a current match $\theta > -\varepsilon$ derive a positive utility premium from residing in the desirable location. The measure of these households is $1 - G(-\varepsilon) > \frac{1}{2}$. In particular, if $\theta_L > -\varepsilon$ and $G(-\varepsilon) = 0$, all households would rather live in the popular area. Since the measure of houses in the desirable location is one half, housing is in short supply in the popular region.

A household's match may change over time. First, if the neighborhood or jurisdiction where the household resides is hit by a regional shock, the match between the household and the location is broken, and a new match is independently drawn from the distribution function $G(\theta)$.[16] Second, even if the overall popularity of the jurisdiction remains unaltered, between periods the match may change for some idiosyncratic, or household specific, reason[17], with probability $\lambda \in [0, 1]$, and the new match is independently drawn from the distribution $G(\theta)$. In Section 4, we drop the assumption of independent draws, and the match is allowed to follow a general Markov process, with possibly different transition dynamics after a regional and an idiosyncratic shock.

Finally, the households live forever and discount future utilities by a common factor $\beta \in (0, 1)$.

In any period, the aggregate welfare is maximized, if all households with $\theta > \theta_m = 0$ are allocated to the (currently) desirable location, those with $\theta < 0$ live in the less desirable location, and the group (always of measure zero, if G is continuous) with $\theta = 0$ is divided between the locations so that capacity constraints on housing are not violated. In other

[16] An underlying premise is that a location which was popular (unpopular) in period t and another location which is popular (unpopular) in period $t + 1$ are likely to be "desirable" ("undesirable") in different ways; thus it is plausible to assume that the match that the household had with the period-t desirable (undesirable) location does not carry over to the period-$(t + 1)$ desirable (undesirable) area.

[17] The match changes for similar reasons as in the search models by Wheaton (1990) and Williams (1995). Examples include change of household size or educational status and evolution in tastes when members of the household age.

words, there is perfect sorting according to the match. If this allocation rule is followed, the aggregate utility in any period is $w^* = \frac{1}{2}E[\theta \mid \theta \geq 0]$.

2.3 Wealth dynamics

In the market outcome, the location choice depends on not only the match, but also on wealth. In this section, we study how a household's wealth evolves over time.

A household cannot sell a home without buying another one, and vice versa.[18] We choose the minimum level of housing wealth as the origin and fix the value of a cheap home to 0. We also normalize the house price in a popular location to 1. This normalization means that house price swings are always of size unity. However, we shall below show how their magnitude can be measured in a meaningful way, by comparing them with the value of financial assets, and with average household wealth.

Consistent with empirical evidence, we assume that capital gains and losses made in the housing market are uninsurable.[19] The incomplete markets setting we consider here is the simplest possible one. In addition to owning a home, the households can carry wealth to the future by holding a single risk-free, non-interest bearing financial asset, which can be interpreted as outside money. The real supply of money is M/p, where M is the fixed nominal supply, and $1/p$ is the price of money, in terms of housing (in desirable locations).[20]

Denote financial asset holdings by a and let h be housing. h is equal to 1, if the household owns a house in a desirable location, and equal to 0, if the house is in an

[18] This follows from our basic assumptions: (i) no household can be homeless (being homeless would result in very large negative utility), (ii) there is no rental housing, and (iii) the measure of homes equals the measure of households.

[19] Clearly, also changes in the "match" are uninsurable.

[20] We could also easily introduce pure credit, or inside money, and allow the households to borrow up to a certain limit, without changing any of the results: in the steady state, the interest rate is zero, so that inside and outside money are perfect substitutes (see Ljungqvist and Sargent (2004, Ch. 17.10). Assume by contrast, that the interest rate is positive and only inside money is held in equilibrium. Then in any (non-degenerate) equilibrium of a pure credit economy, with zero net supply of financial assets, (see Huggett (1993)) some households must have negative positions. But, since the households have no income sources outside the housing market, a household with negative initial financial asset holdings exceeds any finite debt limit with a positive probability. Thus there cannot be a stationary equilibrium with a positive rate of interest.

undesirable location. We also define a household's total wealth (n), which consists of both financial wealth (money) and housing wealth

$$n_t = a_t + h_t. \tag{1}$$

In any given period t, the household's budget constraint is

$$h_t + a_t = a_{t-1} + (1 - s_t)h_{t-1} + s_t(1 - h_{t-1}), \tag{2}$$

where s_t is an indicator function which is equal to 1 if there is a regional shock between periods $t-1$ and t, and 0 otherwise. Combining (1) and (2) yields

$$n_{t+1} = n_t + s_{t+1}\left(1 - 2h_t\right). \tag{3}$$

The household's wealth position (n) changes if and only if the household makes a capital gain or suffers a capital loss in the housing market. This stark way to model wealth dynamics is motivated by the observation that wealth shocks realized in the housing market can be remarkably large compared with typical household incomes and savings.[21] If, prior to the regional shock, the household owned a property in a then unpopular location, $(h_t = 0)$ the household makes a capital gain and climbs one rung in the wealth ladder; if the house was in an expensive area $(h_t = 1)$ before the change of fortunes, the household suffers a loss and falls one rung down.

There is a lower limit for financial asset holdings $a_{\min}$, that a household is not allowed to exceed. A simple and fairly natural normalization is adopted here by fixing the minimum

[21] In an earlier version of the paper, we considered an extension of the model, with more general wealth dynamics at the aggregate level. In each period some households exit the economy (or die), while new households enter. The wealth of the exiting households is passed on to the newcomers, but the mapping is not one-to-one. Technically, it is assumed that each newcomer has an endowment of a representative consumption good, which it sells to the exiting households, and the size of the endowment varies between households. At the aggregate level, wealth dynamics, as well as the stationary wealth distribution, then depend on the wealth distribution of the new households, as well as on capital gains and losses made in the housing markets. We showed that the main empirical prediction of the paper emerges also from this extended model: the degree of residential sorting in the match dimension is negatively correlated with the size of house price fluctuations.

balance to be zero, $a_{\min} = 0$, but allowing a negative minimum balance would just involve a change of origin, without altering the analysis or any of the results.[22] Since the minimum wealth level is $n = 0$ (the minimum level of housing wealth is 0, and the minimum level of financial asset holdings is 0) and since households make capital gains and losses of size unity, we can now assume, without loss of generality, that wealth only takes non-negative integer values $n = 0, 1, 2, ...$ At wealth levels $n \geq 1$, a household may freely choose its housing location, and its wealth portfolio may consist of n units of financial assets and a cheap house ($h = 0$), or $n - 1$ units of financial wealth and an expensive home ($h = 1$). If $n = 0$, the household owns a house in an undesirable location, $h = 0$, and since it has no money, $a = a_{\min} = 0$, it cannot afford a house in a desirable location: choosing $h = 1$ would imply $a = -1 < a_{\min}$, and this is not allowed. The borrowing constraint that limits a household's location choices can be expressed as follows:

$$h_t = 0 \text{ if } n_t = 0. \tag{4}$$

2.4 The household's problem

At each time t a household chooses its location $h_t \in \{0, 1\}$ so as to maximize the expected discounted utility stream

$$E_\theta \sum_{t=0}^{\infty} \beta^t \left[h_t \left(\frac{1}{2}\varepsilon + \theta_t \right) - (1 - h_t) \frac{1}{2}\varepsilon \right],$$

subject to (3) and (4). The problem can be conveniently presented in a recursive form. Let $V(\theta, n)$ be the (ex post) value function of a household with current type θ and current wealth n. Also define the household's ex ante value function $V(n) = E_\theta [V(\theta, n)]$, which describes the household's expected prospects when the household faces a shock (idiosyncratic or regional) and does not yet know its new match. The value function

[22] This is because the interest rate is zero. See Aiyagari (1994) or Ljungqvist and Sargent (2004, Ch. 17.10). See also footnote 20 above.

$V(\theta, n)$ satisfies the Bellman equation

$$V(\theta,n) = \max_{h\in\{0,1\}} h\left(\tfrac{1}{2}\varepsilon+\theta\right) - (1-h)\tfrac{1}{2}\varepsilon + \beta\left\{(1-\pi)\left[(1-\lambda)V(\theta,n)+\lambda V(n)\right]\right. \\ \left. +\pi\left[(1-h)V(n+1)+hV(n-1)\right]\right\} \tag{5}$$

subject to (4). In the current period, the household's utility is $-\frac{1}{2}\varepsilon$ or $\frac{1}{2}\varepsilon+\theta$, depending on its location choice. Its prospects for the next period are discounted by β and are given inside the curly brackets. With probability $(1-\pi)(1-\lambda)$ the household is not exposed to any shocks, and it will face the same value function $V(\theta, n)$ as today. With the complementary probability $[1-(1-\pi)(1-\lambda)]$ the match is broken and the household's prospects are captured by the ex ante value function. If the match changes for household specific reasons, the wealth of the household remains unaltered and future welfare is given by $V(n)$. If there is a regional shock, not only the match changes, but also house prices rise or fall, and depending on housing location, the household makes a capital gain or suffers a capital loss, resulting in expected future welfare $V(n+1)$ or $V(n-1)$.

At each unconstrained wealth level $n \geq 1$, the household chooses the desirable location if and only if

$$\theta + \varepsilon > \pi\beta\left[V(n+1) - V(n-1)\right]. \tag{6}$$

The condition (6) involves a useful decomposition of the decision problem into the consumption motive, figuring on the left-hand side, and the investment motive, visible on the right-hand side. The strength of the consumption motive depends on the current match θ and the measure of regional disparities ε. If there were no need to care about the future, all households with $\theta > -\varepsilon$ would choose the currently desirable region, while only those with $\theta < -\varepsilon$ would (voluntarily) live in the less popular area. The downside of choosing a currently popular and expensive location is that a household may suffer capital losses, if regional house prices fall, and may then be borrowing constrained in the future, when the match θ with an expensive location is better than today. By contrast, opting for a currently less popular and less expensive area entails the chance of making capital gains. These considerations are captured by the investment motive. Due to the investment motive, even some households with $\theta > -\varepsilon$, i.e. households whose immediate

benefits are higher in the desirable location, may voluntarily choose the unpopular area.

At each wealth level n, there is then a critical value of the match

$$\theta_n^* = \begin{cases} \theta_H & \text{if } n = 0 \\ -\varepsilon + \pi\beta \left[V(n+1) - V(n-1) \right] & \text{if } n \geq 1 \end{cases} \tag{7}$$

and the household's location choice rule assumes a simple threshold form:

$$h(\theta, n) = \begin{cases} 1 & \text{if } \theta > \theta_n^* \\ 0 & \text{if } \theta \leq \theta_n^* \end{cases} \tag{8}$$

Figure 3 shows the critical match θ_n^* with different values of n when θ is uniformly distributed on $[-\frac{1}{2}, \frac{1}{2}]$, $\varepsilon = 1$, $\beta = .95$, and $\pi = .3$. Clearly, θ_n^* decreases with n, and wealthier households are ready to choose the desirable location even with a more modest match. This is a general property of θ_n^*, and it stems from the fact that the ex ante value function is concave. (Concavity is proved in the appendix.) Also, this finding has a natural interpretation. Assets are valued since they provide the option to make unconstrained choices in the future. However, if a household is wealthy, additional assets are of less value: the more assets the household has, the more distant is the prospect of being borrowing constrained at some point in the future. To put it differently, the investment motive is more important for poor households than for wealthy households.

The appendix shows that at very high wealth levels, the investment motive all but vanishes, and as a consequence $\lim_{n\to\infty} \theta_n^* = -\varepsilon$. That is, the majority of sufficiently wealthy households live in expensive locations. This property is needed, when we establish the equilibrium of the model. In particular, if $\theta_L > -\varepsilon$ – and all households prefer the desirable location from the consumption point of view – there is a finite wealth level $\overline{n}$, such that all households with $n \geq \overline{n}$ choose a desirable location. In Figure 3, $\theta_L = -\frac{1}{2} > -1 = -\varepsilon$, and $\overline{n} = 3$.

2.5 Equilibrium

The previous section showed how a household chooses its location, and its asset portfolio, based on its current wealth and its current match. On the other hand, a household's current wealth depends on its past fortunes in the housing market, and its past location and portfolio choices. Then the long-run wealth distribution is induced by the households' policy rule. Location choices and the stationary wealth distribution together constitute the long-run equilibrium of the model.

Denote by $f(n)$ the size of wealth class n. Given the households' location choice rule (8), $f_{n-1}^{1}(n) = (1 - G(\theta_n^*)) f(n)$ is then the frequency of households at wealth level n, with an expensive home ($h = 1$) and $n - 1$ units of financial assets; similarly, let $f_n^0(n) = G(\theta_n^*) f(n)$ denote the frequency of households at wealth level n, owning a cheap home ($h = 0$) and n units of financial assets. The appendix shows that the long-run equilibrium is characterized by the set of equations

$$f_n^1(n+1) = f_n^0(n), \ n = 0, 1, ... \tag{9}$$

and the wealth distribution is implicitly defined by the sequence

$$f(n+1)/f(n) = \gamma(n), \ n = 0, 1, ... \tag{10}$$

where $\gamma(n) \equiv \frac{G(\theta_n^*)}{1-G\left(\theta_{n+1}^*\right)}$. These equations hold both under the two-locations and the atomistic-locations version of the model: both model variants have the same long-run equilibrium.[23] The wealth distribution is single-peaked, with wealth classes in the middle having more mass than those on the tails, and the right tail can be approximated by a power series[24]; these properties are consistent with observed empirical wealth distribu-

[23] The environment that an individual household faces is identical in both model variants: there is a regional shock with probability π. Then the households' location choices, analyzed in Section 2.4, are identical in both cases.

[24] These properties hold, since $\gamma(0) = \frac{G(\theta_0^*)}{1-G\left(\theta_1^*\right)} = \frac{1}{1-G\left(\theta_1^*\right)} \geq 1$, $\gamma(n)$ is decreasing in n and $\lim_{n\to\infty} \gamma(n) = \frac{G(-\varepsilon)}{1-G(-\varepsilon)} < 1$.

tions. In the hump of the wealth distribution $\gamma(n) \approx 1$,[25] meaning that the critical match θ_n^* is relatively close to θ_m: interestingly, in the hump the households' location choice rule (summarized by θ_n^*) tends to be relatively close to the socially optimal rule, while in the tails location choices deviate more from the socially optimal policy. Equations (9) imply that the distribution of financial assets is identical in both locations – or location types. This symmetry property means that in steady state the asset side of the economy, as defined by the joint distribution of housing wealth and financial wealth, looks exactly the same at the end of any given period and at the beginning of the subsequent period even if the popularity ranking of the locations is reversed.

Let us turn to housing markets. Due to the borrowing constraint, households at the lowest wealth level, $n = 0$, can only afford a cheap home. This implies the restriction $f_{-1}^1(0) = 0$. On the other hand, the majority of sufficiently wealthy households chooses an expensive location. In particular, the fact that $\lim_{n\to\infty} \theta_n^* = -\varepsilon$ implies $\lim_{n\to\infty} \frac{f_{n-h}^h(n+1)}{f_{n-h}^h(n)} = \lim_{n\to\infty} \frac{f(n+1)}{f(n)} = \frac{G(-\varepsilon)}{1-G(-\varepsilon)} < 1$ for $h \in \{0,1\}$, so that the sequences $f_{n-h}^h(n)$, $h \in \{0,1\}$, converge. Using these results, and summing both sides of (9) over all wealth classes yields $\sum_{n=0}^{\infty} f_n^0(n) = \sum_{n=0}^{\infty} f_{n-1}^1(n)$. Finally, given that the aggregate mass of households is unity, it follows that

$$\sum_{n=0}^{\infty} f_{n-h}^h(n) = \frac{1}{2}, \; h \in \{0,1\}. \tag{11}$$

These equations indicate that the demand for housing, on the left-hand side, is equal to the supply of housing $\left(\frac{1}{2}\right)$, in both locations. Households' location choices together with the endogenously arising long-run wealth distribution guarantee that housing markets clear. Essentially, if few households willingly choose the less desirable location, in the long-run equilibrium many households end up living there because they are borrowing constrained.

In addition to the households' location choice rule and the wealth distribution, the third constituent of the equilibrium is the relative price of housing and financial assets, p. To solve for p, consider the asset market clearing condition $E[a] = \frac{M}{p}$, where the

[25] The mode of the distribution is a wealth level n_{mod} such that $\gamma(n_{\text{mod}} - 1) > 1$ and $\gamma(n_{\text{mod}}) < 1$.

left-hand side is the aggregate demand for financial assets and the right-hand side is the net supply, equal to real outside money.[26] Using (1) and the housing market equilibrium $E[h] = \frac{1}{2}$, the asset market equilibrium condition can be rewritten as $E[n] = \frac{1}{2} + \frac{M}{p}$, and the relative price of housing and financial assets is[27]

$$p = \frac{M}{E[n] - \frac{1}{2}}. \tag{12}$$

Notice that p also measures the monetary size of house price fluctuations.[28]

3 Residential sorting

3.1 Main patterns

This section studies how the equilibrium pattern of residential sorting, as well as the size of house price fluctuations, reflects the relative strength of the consumption motive and the investment motive of housing. We begin by analyzing social welfare. Addressing this normative issue will then allow us to characterize sorting, since in the present model high social welfare is associated with location choices based on the match, rather than wealth.

The expected prospects of households at wealth level n are given by the ex ante value function $V(n) = E_\theta[V(\theta, n)]$. To get a measure of social welfare, we sum over all wealth classes, using the sizes of the wealth groups as weights:

$$W = \sum_{n=0}^{\infty} f(n) V(n). \tag{13}$$

[26] The equilibrium we establish here essentially resembles the equilibrium of the simple Bewley-type model considered by Ljungqvist and Sargent (2004, Ch 17.10.4), where outside money and inside money (credit) are perfect substitutes, and the interest rate is zero.

[27] Notice that $\lim_{n\to\infty} \frac{n+1}{n} \frac{f(n+1)}{f(n)} = \lim_{n\to\infty} \gamma(n) = \frac{G(-\varepsilon)}{1-G(-\varepsilon)} < 1$. Thus the sum $E[n] \equiv \sum_{n=0}^{\infty} nf(n)$ converges, and $E[n]$ is always finite.

[28] If the households are allowed to borrow in terms of financial assets, and the borrowing limit, denoted in monetary terms, is $-B$, the asset market equilibrium condition reads $E[a] = \frac{M+B}{p}$, and $p = (M+B) / \left(E[n] - \frac{1}{2}\right)$.

It is shown in the appendix that

$$W = \frac{1}{2}\frac{E\left[\theta \mid h=1\right]}{1-\beta}. \tag{14}$$

Essentially, social welfare reflects the degree of residential sorting in the match dimension, summarized by the average quality of the match in the desirable location. The equality (14) is needed in the proofs of Propositions 1 and 2.

Proposition 1 *Social welfare increases, when (i) the size of regional shocks (ε) decreases, or (ii) regional shocks become more frequent (π increases).*

Proof See the appendix. ■

Proposition 2 *When (i) the size of regional shocks (ε) decreases or (ii) the regional shocks become more frequent (π increases), the degree of residential sorting in the match dimension increases in the following sense. (a) In each location $h \in \{0,1\}$, the average match $E\left[\theta \mid h\right]$ becomes more distinct from the economywide average $E\left[\theta\right]$. (b) The locations become more distinct from each other and the between-locations variance of the match increases. (c) The locations become internally more homogeneous in the sense that the within-location variance of the match decreases.*

Proof When conditions (i) and/or (ii) hold, it follows from Proposition 1 that $E\left[\theta \mid h=1\right]$ increases. (a) Then, since $\frac{1}{2}E\left[\theta \mid h=1\right] + \frac{1}{2}E\left[\theta \mid h=0\right] = E\left[\theta\right]$, and $E\left[\theta\right]$ is a constant, it follows that $E\left[\theta \mid h=0\right]$ decreases. Thus the difference $\left|E\left[\theta \mid h\right] - E\left[\theta\right]\right|$ increases for $h \in \{0,1\}$. (b) Item (a) implies that the between-locations variance $Var\left(E\left[\theta \mid h\right]\right) = \frac{1}{2}\left(E\left[\theta \mid h=0\right] - E\left[\theta\right]\right)^2 + \frac{1}{2}\left(E\left[\theta \mid h=1\right] - E\left[\theta\right]\right)^2$ increases. (c) The economywide variance of the match $Var\left(\theta\right)$ can be decomposed $Var\left(\theta\right) = Var\left(E\left[\theta \mid h\right]\right) + E\left[Var\left(\theta \mid h\right)\right]$. Since $Var\left(\theta\right)$ is a constant, it follows from item (b) that the within-locations component $E\left[Var\left(\theta \mid h\right)\right]$ must decrease. ■

To understand these results, recall that the basic allocation problem in the economy arises since there is not enough housing capacity in desirable locations to accommodate all households with a positive utility premium. Essentially, social welfare is high, if the

allocation problem is mainly solved through self-selection, based on the goodness of the match, while welfare is low, if few households willingly choose a less desirable location, and wealth determines who lives where.

Next remember that households' location choices reflect a trade-off between the consumption motive and the investment motive of housing. An increase in interregional welfare differences, and the size of regional shocks, ε, strengthens the consumption motive to choose a desirable location in the current period. On the other hand, it also reinforces the incentives to accumulate assets (investment motive), since a household stands to lose more if it faces the borrowing constraint at some point in the future. However, since future utility losses are discounted and only occur by chance, while the higher welfare stream is available right away, the effect on the consumption motive dominates. Hence, the larger the regional differences or shocks, the less likely an unconstrained household chooses a currently undesirable area.

A change in the frequency of regional shocks, π, affects only the investment motive, while leaving the consumption motive intact. The higher π, the more likely a household living in a popular area suffers a capital loss, while the more likely a household living in an unpopular area makes a capital gain. Then, at any unconstrained wealth level, a household is more willing to choose a currently undesirable location. The preceding discussion is summarized by

Lemma 1 *For all* $n \geq 1$, $\frac{d\theta_n^*}{d\varepsilon} < 0$ *and* $\frac{d\theta_n^*}{d\pi} > 0$.

Proof See the appendix. ■

Changes in the relative strength of the consumption motive and the investment motive also affect the wealth distribution.

Lemma 2 *When regional shocks become smaller (ε decreases) or more frequent (π increases), the wealth distribution shifts to the right, in the sense of first-order stochastic dominance. In particular, the size of the borrowing constrained group decreases.*

Proof Define the cumulative distribution function $F(n;\varepsilon,\pi) = \sum_{i=0}^{n} f(i)$. By Lemma 1, the θ_n^*-schedule shifts up when ε decreases or π increases. This then increases $\gamma(n) \equiv$

$\frac{G(\theta_n^*)}{1-G\left(\theta_{n+1}^*\right)}$, so that by (10) the ratio $f(n+1)/f(n) = \gamma(n)$ goes up for all $n = 0, 1, ...$ It follows that $dF(n;\varepsilon,\pi)/d\varepsilon \geq 0$ and $dF(n;\varepsilon,\pi)/d\pi \leq 0$, for each $n = 0, 1, ...$ ■

Combining these elements leads to the results stated in Propositions 1 and 2. When the investment motive is strong, and the households care a lot about their future prospects, housing markets are mainly cleared through self-selection, which results in a high degree of sorting in the match dimension, and high social welfare. When the consumption motive is strong, market clearing relies on a larger group of households being borrowing constrained; this gives rise to a low degree of sorting in the match dimension, and a low level of social welfare.

Above we examined how changes in the size and the frequency of regional shocks affect the mechanism through which housing markets clear. On the other hand, when a household chooses its housing location, it simultaneously chooses the composition of its wealth portfolio. Then Lemmas 1 and 2 can be (re)interpreted from the asset market point of view. In particular, a strong investment motive drives up the demand for financial assets, and their relative price $1/p$. As a result, the share of financial assets in total wealth, $E[a]/E[n] = (E[n] - \frac{1}{2})/E[n]$, increases, while the share of housing (in popular locations), $\frac{1}{2}/E[n]$, decreases. An upshot of the greater valuation of financial assets is that house price fluctuations (the size of which is normalized to unity) become smaller, compared with the value of financial wealth a, as well as total wealth n. Then a wealth shock has a smaller impact on a household's (relative) wealth position, and a typical household is better equipped to withstand capital losses:

Remark 1 *Assume that regional shocks become smaller (ε decreases) or more frequent (π increases). Then (i) the monetary size of house price fluctuations, p, decreases, and (ii) the price fluctuations become smaller compared with household wealth, measured by average wealth $E[n]$, median wealth, or any other quantile n_q of the wealth distribution, where $n_q = \min\ n$, s.t. $q \leq F(n)$.*

Combining Remark 1 with Proposition 2 allows us to establish a connection between the size of house price fluctuations and the degree of residential sorting.

Corollary 1 *The smaller or the more frequent the regional shocks are, (i) the smaller are house price fluctuations and (ii) the more residential sorting there is in the match dimension.*

Much of our empirical work reported in Section 5 is based on Proposition 2 and Corollary 1.

Next we proceed to analyzing sorting in the wealth dimension. Above we noted that the distribution of financial assets is identical in both location types. Then, given that $E[a \mid h=1] = E[a \mid h=0]$, interregional wealth differences derive entirely from different house values

$$E[n \mid h=1] - E[n \mid h=0] = E[h \mid h=1] - E[h \mid h=0] = 1. \tag{15}$$

To assess the magnitude of these interregional wealth differences in a meaningful way, we compare them with typical household wealth in the economy:

Proposition 3 *When regional shocks become larger (ε increases) or less frequent (π decreases), interregional wealth differences become larger compared with typical household wealth, as measured by average wealth, median wealth or any other quantile of the wealth distribution.*

Proof The result follows from equation (15) and Lemma 2. ■

The following proposition is about polar cases.

Proposition 4 *(a) When $\varepsilon \to 0$ or $\delta \equiv \frac{\pi\beta}{1-\beta(1-\pi)} \to 1$, there is perfect sorting in the match dimension and no sorting in the wealth dimension. In any given period, a household chooses a desirable location if and only if $\theta > \theta_m$. (b) If $\theta_L + \varepsilon > \pi\beta\frac{E[\theta]-\theta_L}{1-\beta}$, there is perfect sorting in the wealth dimension and no sorting in the match dimension. A household resides in a less desirable location if and only if it is borrowing constrained.*

Proof See the appendix. ■

The equilibrium pattern of residential sorting, with different values of ε, is illustrated in Figure 4. In each panel, the cumulative wealth distribution is measured on the hori-

zontal axis, and the cumulative match distribution on the vertical axis. Then area has a simple frequency mass interpretation (with one quarter of the area of the unit square corresponding to one quarter of the households etc.). The figure shows a clear pattern, with the degree of residential sorting in the match dimension decreasing, and the degree of wealthwise sorting increasing, as the size of the regional shocks grows. Also the magnitude of house price fluctuations, measured by $P \equiv \frac{1}{2}\frac{1}{E[n]}$, $P \in [0,1]$, grows together with the size of the shocks (see Remark 1). Panels a (no shocks) and d (large shocks) correspond to polar cases, with perfect sorting in the match dimension and in the wealth dimension, respectively (and no sorting in the complementary dimension). Panels b and c are intermediate cases, with shocks of intermediate size, and imperfect sorting along both dimensions. A similar set of figures could be also presented with respect to π.

3.2 Movers and stayers

In this section we establish a relation between wealth and household mobility, and study the degree of residential sorting among movers and stayers.

We begin by demonstrating a simple humpshaped relation between wealth and mobility. Take any given wealth class n. At the beginning of any period, the portion $1 - G(\theta_n^*)$ of households own a house in the desirable location; since equations (9) hold in the steady state, this is true even after a regional shock. Between any two periods, $(1-s)\lambda + s$ households are hit by a shock, which breaks their match. Then the share $((1-s)\lambda + s)\, G(\theta_n^*)$ of the households, which are in the popular area at the beginning of the period, get a realization $\theta < \theta_n^*$ and move to the unpopular area. Therefore, mobility from the desirable to the undesirable location in wealth class n is equal to $((1-s)\lambda + s)\, G(\theta_n^*)[1 - G(\theta_n^*)]$. Likewise, it is easy to conclude that mobility from the undesirable to the desirable location equals the same measure. Then overall mobility in wealth class n is $\mu(n) = ((1-s)\lambda + s)\, \widetilde{\mu}\left(G\left(\theta_n^*\right)\right)$, where

$$\widetilde{\mu}\left(G\left(\theta_n^*\right)\right) \equiv 2G(\theta_n^*)[1 - G(\theta_n^*)]. \tag{16}$$

Clearly, there is more mobility in those periods when the economy is hit by a regional

shock and $s = 1$. Under the atomistic locations interpretation, in any given period, mobility at wealth level n is $\overline{\mu}(n) = ((1-\pi)\lambda + \pi)\widetilde{\mu}(G(\theta_n^*))$. Notice also that in the two-region case, $\overline{\mu}(n)$ is the long-run average mobility at wealth level n.

Essentially, $\mu(n)$ or $\overline{\mu}(n)$, defines a humpshaped relation between wealth and mobility[29]:

Proposition 5 *Assume that $\overline{n} \geq 2$. Then mobility is increasing in wealth at low wealth levels, and decreasing in wealth at high wealth levels, so that households at intermediate wealth levels are more mobile than the poor and the wealthy.*

Proof Equation (16) implies that $\widetilde{\mu}(G)$ is a downward opening parabola, with its peak at $G(\theta_m) = \frac{1}{2}$. Also $\widetilde{\mu}(G) = 0$ at the extreme points $G = 0$ and $G = 1$. According to Proposition 1, θ_n^*, and thus $G(\theta_n^*)$, is decreasing in n. Also, $G(\theta_n^*) > \frac{1}{2}$ at low values of n, with $G(\theta_0^*) = 1$. On the other hand $G(\theta_n^*) < \frac{1}{2}$ at high levels of n, since $\lim_{n\to\infty}\theta_n^* = -\varepsilon$ and $G(-\varepsilon) < \frac{1}{2}$. In particular, if $\theta_L > -\varepsilon$ we have $G(\theta_n^*) = 0$ for all $n \geq \overline{n}$, where $\overline{n} < \infty$. ■

This pattern of mobility essentially reflects the varying strength of the investment motive at different wealth levels. Rich households, with a weak investment motive, typically want to live in a popular location, and only rarely find it optimal to move. Poor households tend to reside in a cheap location; for the borrowing constrained this is obviously the only alternative. At intermediate levels of wealth, the investment motive is neither extremely strong nor very weak; when the match is broken, these households often find it optimal to change location. Maximum mobility is attained, if the location choice rule θ_n^* corresponds to the socially optimal median rule θ_m. As discussed in Section 2.5, in the mode of the wealth distribution, households location choices tend to deviate relatively

[29] Notice that the measure $\mu(n)$ (or $\overline{\mu}(n)$) answers the following question: Assume that a household has wealth n in a given period t. What is the probability that the household moves during the period? An alternative question might be: What is the probability that the household lives in different locations in period t and in period $t+1$? The answer to this question is an alternative mobility measure $\widetilde{\mu}(n) = (1-s_{t+1})\lambda 2G(\theta_n^*)[1-G(\theta_n^*)] + s_{t+1}\left[G(\theta_n^*)G(\theta_{n+1}^*) + (1-G(\theta_n^*))\left(1-G(\theta_{n-1}^*)\right)\right]$. If there is no regional shock between periods t and $t+1$ (that is, $s_{t+1} = 0$), there is a humpshaped relation between wealth and mobility, as measured by $\widetilde{\mu}(n)$. If there is a regional shock ($s_{t+1} = 1$), the relation may take many possible forms, including humpshaped and monotonously increasing.

little from the socially optimal policy. Then, typically, the most mobile households are found in the hump of the wealth distribution, while the least mobile are in the tails.

Remarkably, the relationship between wealth and mobility established in Proposition 5 is essentially the same as empirically documented by Henley (1998) for the UK; see especially Figure 2 in Henley (1998). According to Henley (1998, p.425), "levels of housing wealth are an important factor in explaining mobility, and the relationship between the two is not linear." British households with large negative housing equity are virtually immobile. Also very wealthy households tend to move relatively little. Households with intermediate levels of wealth are the most mobile.

Next we proceed to comparing the degree of residential sorting among movers and stayers. In any given period, we classify as a mover a household which has moved during that period. The following results are proved in the appendix.

Proposition 6 *(a) In both location types, movers have a better match with their (new) home region than stayers, in the sense of first order stochastic dominance. (b) Movers are more sorted than stayers in the match dimension.*

When interpreting item (a) of the proposition, remember that a good match with a cheap location means that a household has a low realization of θ.

Item (a) reflects the fact that those who move from one location to another tend to have rather strong match-related reasons to make that choice, while those who stay put may do so largely because they have been lucky or unlucky in the housing market. For example, households which move from a desirable location to an undesirable location, choose a cheap area, although they could afford a more expensive house (their former home). By contrast, at least a part of the old residents live in a cheap location because they have been locked in by falling home equity values. Likewise, in an expensive region, newcomers from cheaper locations tend to have a good match with the area they have chosen, whereas old residents, who may have bought their home before the rise of local house prices, often stay put even with a more modest match.[30] Item (b) is a rather

[30] More generally, and more formally, the appendix shows that in cheap locations, the wealth distribution of movers first order stochastically dominates the wealth distribution of stayers, while in the expensive

straightforward corollary of item (a). Since movers are better matched with their home region than stayers in both location types, movers are obviously more sorted than stayers. The empirical work on movers and stayer reported in Section 5 is based on item (b).

4 More general match dynamics

In this section, we drop the assumption that, after a shock, the new match is independently drawn, and allow the match to follow a general Markov process. This extension introduces two new features to the model. First, the strength of the investment motive may reflect expected tenure length and household specific moving plans. Second, wealth and the match can be correlated.

There are $J \geq 2$ different match realizations. If the match changes for idiosyncratic, or household specific, reasons ($s = 0$), the transition probabilities from one match to another are given by a transition matrix Λ_0. If there is a regional shock ($s = 1$), the transitions are governed by a (possibly) different matrix Λ_1. To guarantee the existence of a stationary joint distribution for wealth and the match, we adopt the small region interpretation of the model, and assume that there is a continuum of atomistic locations. In each period, a measure π of the matches is broken due to regional shocks, and a measure λ for household specific reasons. Let $\pi = \xi\sigma$ and $\lambda = (1-\xi)\,\sigma$, where $\sigma \in (0,1)$ is the overall probability that the match is broken, and $\xi \in (0,1]$ measures the relative frequency of regional and idiosyncratic shocks. The parameter σ can be interpreted as reflecting the overall degree of turbulence in the economy. The stationary marginal distribution of the match is defined as the eigenvector associated with a unit eigenvalue of Λ', where $\Lambda \equiv (1-\xi)\,\Lambda_0 + \xi\Lambda_1$.[31] Notice that if the frequency of shocks (σ) changes, but the relative probabilities of regional and idiosyncratic shocks (ξ and $1-\xi$) remain constant, the stationary match distribution is unaltered.

Next we proceed to studying households' location choices. The value function $V(\theta, n)$

locations, the opposite is true.

[31] We assume that the matrix Λ is indecomposable, so that it induces a unique long-run match distribution, but otherwise we do not impose any restrictions on the structure of the stochastic matrices Λ_0 and Λ_1.

satisfies the Bellman equation

$$V(\theta,n)=\max_{h\in\{0,1\}} h\left(\tfrac{1}{2}\varepsilon+\theta\right)-(1-h)\tfrac{1}{2}\varepsilon+\beta\left\{(1-\sigma)V(\theta,n)\right.$$
$$\left.+\lambda E_{\widetilde{\theta}}\left[V\left(\widetilde{\theta},n\right)\mid\theta,s=0\right]+\pi E_{\widetilde{\theta}}\left[(1-h)V\left(\widetilde{\theta},n+1\right)+hV\left(\widetilde{\theta},n-1\right)\mid\theta,s=1\right]\right\}, \tag{17}$$

subject to (4). At any unconstrained wealth level $n\geq 1$, the household chooses a currently desirable location if and only if

$$\theta+\varepsilon>\pi\beta E_{\widetilde{\theta}}\left[V\left(\widetilde{\theta},n+1\right)-V\left(\widetilde{\theta},n-1\right)\mid\theta,s=1\right]. \tag{18}$$

Importantly, the investment motive, figuring on the right-hand side of (18) now depends on not only the household's wealth position, but also on the current match θ (and on the distribution of future matches $\widetilde{\theta}$, conditional on the current match). Intuitively, the connection between the match and the investment motive may be interpreted as reflecting the household's expected tenure length, and future moving plans. The investment motive tends to be weak, if the household is attached to the home area, and wants to live there even when the area is unpopular: it does not matter, if local house prices fall, since the household has no intentions to sell. In a similar vein, Sinai and Souleles (2005) argue, that onwer occupation is not risky, if a household intends to stay put for a long time. In our framework, attachment to home can be modelled by letting the match be correlated with regional shocks: the household is likely to draw a high realization of θ, when the home area is "desirable," and a low realization of θ, when the home area is "undesirable." Conversely, the investment motive tends to be strong, if the household buys a home knowing that it will probably not live there for a long time. Then a major function of the current house is to serve as a springboard to the future home. In particular, if the household is planning to move to a popular and expensive area in the future[32], it has an incentive to avoid housing market risks, which might jeopardize these plans. In sum, condition (18) indicates that a household is likely to buy a home in an expensive location (i) if it has a good match with that location, (ii) if it is wealthy and (iii) if it is planning to stay in the location for

[32]Formally, the household expects to draw a high realization of θ in the future.

a long time.

In addition to households' location choices, the second component of the long-run equilibrium is the endogenous stationary joint distribution of wealth and the match. Unlike in the basic model, wealth and the match are typically not independently distributed.[33] If households are attached to a home region, a positive correlation between the value of the match, θ, and household wealth naturally arises. This is illustrated in Figure 5. In equilibrium, those households, which derive the highest utility premium from residing in an expensive location, also tend to be wealthy. Typically, these households have seen the value of their house go up, as their home region has become more popular and more expensive. This coevolution of housing costs and household wealth is one of the advantages of owner occupation, discussed by Sinai and Souleles (2005).

While attachment to home, and the resulting positive correlation between wealth an the match, may seem a rather natural case to consider, the model is flexible enough to allow for many other alternatives as well. For example, if some households constantly derive a high utility premium from residing in a currently popular and expensive area, a different pattern arises. Those who insist on living in a fashionable location in every period, have to move against the tide, from an area of fading popularity and falling prices to an area of high prices. Then in equilibrium, the size of the utility premium and household wealth tend to be negatively correlated.

Overall, since expected tenure length and future moving plans may affect households' location choices, and since wealth and the match tend to be correlated, the equilibrium is typically more complex than in the basic model. Nevertheless, the main message of the paper carries over: The pattern of residential sorting reflects the relative strength of the consumption motive and the investment motive. In particular, there is a negative correlation between the size of house price fluctuations and the degree of sorting in the match dimension. This is illustrated in Figure 5, where panel a corresponds to a situation with small regional shocks, a strong (in relative terms) investment motive, small house

[33] Correlation arises, since (i) current wealth depends on past location and portfolio choices (and luck), (ii) past choices were influenced by past match realizations, and (iii) the current match is correlated with past match realizations. The vector difference equation, which implicitly defines the long-run joint distribution is presented in the appendix. The appendix also establishes the equilibrium of the model.

price fluctuations, and a high degree of sorting in the match dimension. In panel b regional shocks are larger, and the consumption motive dominates; then price fluctuations are more pronounced, and sorting takes place mainly in the wealth dimension.

More formally, the appendix proves that the main results of the paper, Propositions 1, 2 and 3, and Corollary 1, still hold, with the exception that π is substituted by σ. (A change in π would also alter the stationary match distribution.) If $\lambda = 0$, so that there are no idiosyncratic shocks, these results hold verbatim.

5 Empirical evidence

We conduct our empirical analysis by using data from the US metropolitan statistical areas (MSAs) and local municipalities (so called Minor Civil Divisions or MCDs). The data are from the 1990 decennial census. For a detailed description of the data and their sources, see the appendix.

There is much discussion on large house price fluctuations in various MSAs, while residential sorting across MCDs, in particular, has been found to be much weaker than many conventional sorting models predict (see Rhode and Strumpf (2003)). We first examine how house price variations in the MSAs are related to the degree of residential sorting. By Proposition 2 and Corollary 1, we expect that MCDs within MSAs that have experienced large house price fluctuations should have diverse populations in the sense that the shares of different demographic groups of the MCDs by and large correspond to the population structure of the underlying MSA. On the other hand, MCDs in areas where prices have been less volatile should have a less diverse population, with certain demographic groups under- or overrepresented, compared with the MSA average.[34]

As Rhode and Strumph (2003), we proxy household types by characteristics such as income, education and age. Several sorting measures for these proxies are possible. In the literature, it is common to apply the dissimilarity index and the Gini coefficient. These indices vary between zero (when each type is equally represented in each community)

[34]Here we adopt an interpretation of the model, where a location corresponds to a MCD, while the entire economy is the MSA.

and one (when the types are completely sorted across municipalities).[35] The dissimilarity index, D, and the Gini coefficient, GC, are defined as

$$D = \frac{1}{2} \frac{\sum_m \sum_i N_i \left| S_{mi} - S_m \right|}{N \sum_m S_m \left(1 - S_m\right)} \tag{19}$$

$$GC = \frac{1}{2} \frac{\sum_m \sum_i \sum_j N_i N_j \left| S_{mi} - S_{mj} \right|}{N^2 \sum_m S_m \left(1 - S_m\right)} \tag{20}$$

where S_{mi} is the share of age, education or income group m in the population of the MCD i, S_m is the corresponding share at the MSA level, N_i is the population of MCD i and N is the population of the MSA. An alternative sorting index derives from the Theil's entropy measure and is defined as

$$T = 1 - \frac{\sum_i \sum_m \frac{N_i}{N} S_{mi} \ln(S_{mi})}{\sum_m S_m \ln(S_m)} \tag{21}$$

Reardon and Firebaugh (2002) call T as the Theil's information theory index.[36] As with D and GC above, T varies between zero (when $S_{mi} = S_m$ for all i, m) and one (all municipalities contain members of one type only). The index T can be interpreted as one minus the ratio of the average within-municipality population diversity to the diversity of the total MSA population (see Reardon and Firebaugh (2002, p. 42)). Essentially, the indices D, GC and T rank the MSAs by the degree of residential sorting.

As a measure of house price fluctuations we use the standard deviation of the house price p_{it} of the MSA i over the sample period 1985-2000, where $p_{it} = \log(PI_{it}/PI_t)$, PI_{it} is the house price index in MSA i in period t, and PI_t is the US house price index in period t.[37] Basically, this measure ranks MSAs by the degree at which their house prices have fluctuated against the US average. As an alternative indicator of the size of house price swings we used the measure $\max_t(p_{it}) - \min_t(p_{it})$. The empirical results were qualitatively similar.

Table 2 reports sample correlations between the alternative sorting measures of dif-

[35] For additional properties of the indices see Rhode and Strumph (2003).

[36] The formula in (21) is the same as (15) in Rhode and Strumph (2003), but is equivalent to the index H of Reardon and Firebaugh (2002, Table 1).

[37] The price index data are from the Office of Federal Housing Enterprise Oversight.

ferent type characteristics and the measure of house price volatility. Consistent with our theory, each sorting measure is negatively correlated with house price volatility. Thus, MCDs within MSAs subject to high house price volatility tend to be less sorted than MCDs in MSAs with little house price variation, and vice versa. As a robustness check, we computed the standard deviation of p_{it} over the subsample 1985-1990, which predates our cross-section. Also this measure of house price volatility is negatively correlated with all the sorting indices.

A potential concern is that the observed correlation between residential sorting and house price volatility might arise from factors beyond the mechanism suggested by our theory. Therefore, to examine the robustness of the correlations, we run OLS regressions of different sorting measures on house price volatility and selected covariates.

We report our baseline regression results in Table 3. In column (1), (2) and (3), respectively, the dependent variable is income sorting, education sorting, and age sorting, all measured by the Theil information theory index in (21). In all the regressions, the coefficient estimate of house price volatility is negative and statistically significant at the 1% level (applying t-tests based on heteroscedasticity robust standard errors). Therefore, residential sorting and house price volatility appear to be correlated even if we partial out the effects of the applied control variables. If the dissimilarity index D or the Gini coefficient GC is used as the measure of sorting in the baseline regression, the coefficient estimate of house price volatility is still always negative and statistically significant at least at the 5% level.

The coefficient estimates of the control variables of Table 3 have meaningful interpretations whenever they are statistically significant. For example, the coefficient estimate of the number of MCDs is positive in all regressions, which is consistent with the idea that a large number of MCDs offer more opportunities for forming different homogeneous groups than a small number of MCDs.[38] On the other hand, the negative coefficient estimate of

[38]In particular, if the number of MCDs is less than the number of different types, it is not possible to achieve maximal sorting in the sense that each type resides in a separate region (cf. Eberts and Gronberg (1981)). In our case, the number of income groups (25) exceeds the number of MCDs in many metropolitan areas. As a robustness check, we recomputed the sorting indices with four income groups (formed by merging the original groups). In our baseline regression the coefficient of the number MCDs was no longer statistically significant. Otherwise, however, the results were qualitatively the same.

the average population size of MCDs is in line with the idea that a large population in an MCD can encompass a larger range of households than a small population, and thus, ceteris paribus, tends to reduce sorting across regions. We expect that sorting may be more beneficial in urbanized areas with high population density than in rural areas with low population density. We also expect that larger MSA areas are likely to provide more opportunities for beneficial sorting than small metro areas (cf. Hoxby (2000)). In line with these assertions, the coefficient estimates of the density of the MSA and the area size of the MSA are positive when they are statistically significant. Finally, the negative (and weakly significant) coefficient estimate of the number of families in MSA in column (1) suggests that it is harder to obtain homogeneous income groups from a large population than from a small population, ceteris paribus.

Recent literature indicates that physical and regulatory constraints, which hinder housing construction, may have significant implications for the house price dynamics and the development of the MSAs. Obviously, such constraints might induce correlation between residential sorting and house price volatility. To control for such effects, we augment our baseline regressions with variables that measure the degree of physical and regulatory constraints of the MSAs. The variables are the "land topographic unavailability measure" (UNDEV) of Saiz (2008) and the "Wharton Residential Urban Land Regulation Index" (WRLURI) of Gyourko, Saiz, and Summers (2008). The former (the latter) variable is available for 83 (208) MSAs in our original sample. We find that the two variables are correlated with our sorting measures and the measure of house price volatility, while it turns out that only WRLURI is a statistically significant explanatory variable in our baseline regressions. When WRLURI is added to the regressions, the coefficient estimate of house price volatility remains negative and statistically significant; in fact the coefficient increases in absolute value[39] in all the regressions (explaining sorting measures D, GC and T for income, age and education).[40]

[39] The coefficient estimate is larger, in absolute value, than the one obtained from our baseline regression with the original sample of 242 MSAs. It is also larger than the coefficient estimate obtained from the baseline regression with 208 MSAs.

[40] Intuitively, the part of house price volatility, which is orthogonal to WRLURI (the strictness of regulation), can be thought of as measuring the size of regional shocks (ε). We know that house price

Arguably, the characteristics of the built environment also affect the pattern of residential sorting (cf. Nechyba (2000)). If, say, the housing stock is very different in different parts of a MSA, one expects that the degree of sorting in the MSA should be relatively high, ceteris paribus. To control for these effects, we construct Theil indices for two aspects of the housing stock, the age of housing units, T_{HoAge}, and the number of housing units in a residential building[41], T_{HoUnit}. The interpretation of these Theil indices is as above: the larger the value of T_{HoAge} (or T_{HoUnit}), the more the MCDs within the MSA differ from each other. When we add T_{HoAge} and T_{HoUnit} to our baseline regressions (Table 3), the coefficients of these indices are of the expected sign, i.e. positive, but only T_{HoAge} is statistically significant, while T_{HoUnit} is not significant in any of the regressions. Importantly, the coefficient estimate of house price volatility still remains negative, and it is statistically significant at least at the 10% level in eight of our nine regressions (the expectation is the case, where the dependent variable is GC_{Income}). If we drop T_{HoUnit}, the coefficient of price volatility is almost[42] significant at the 5% level in all of the regressions (and if we further include the regulatory index WRLURI, the word "almost" can be dropped).

Finally, we add the share of rental housing to the set of control variables. A large rental sector in a MSA is associated with a higher degree of sorting in terms of age and education. However, the extent of income sorting is negatively correlated with the share of rental housing. This may reflect the presence of rent control in a number of metropolitan housing markets: under rent control, the allocation of housing is not determined by the willingness to pay, but by some other mechanisms, such as queueing (cf. Glaeser and Luttmer (2003)). When we augment our baseline regression with the share of rental housing, the coefficient estimate of house price volatility is still negative, and it is statistically significant at least at the 5% level.

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fluctuations tend to be larger in highly regulated MSAs, where housing supply is inelastic, than in lightly regulated MSAs ,where supply is more elastic. Then if housing construction is lightly regulated in a MSA, but nevertheless the MSA has experienced sizeable house price fluctuations, it is reasonable to assume that the MSA has been buffeted by large shocks. Our model predicts that under these circumstances, the degree of residential sorting should be low.

[41] This measure essentially tells whether there are detached houses, semi-detached houses or blocks of flats in an area.

[42] The t-statistic is just below the critical value 1.96, when the dependent variable is GC_{Income} or D_{Age}.

We turn to comparing residential sorting of movers to that of stayers across (so called) Public Use Microdata Areas (PUMAs) in the whole US.[43] This part is related to the work by Ortalo-Magné and Rady (2008), who study income distributions among movers and stayers. According to Proposition 6, movers should be more sorted than stayers. That is, if two mobile households choose the same jurisdiction, these newcomers typically share some common characteristics; they also tend to differ from other mobile households, which choose a different location. By contrast, stayers living within the same jurisdiction tend to have less in common with each other.

To investigate the above predictions, we classify an individual as a mover, if (s)he has resided in his/her current home for less than five years; otherwise the individual is a stayer. Then, for each characteristic (age, education, income) and each group (movers and stayers), we compute the Theil's information theory index across PUMAs in the whole US. Thus, N in (21) now stands for the US population, N_i is the population of PUMA i, S_m is the share of group m in the US, and S_{mi} is the share of group m in PUMA i. The PUMA data allows us to compute separate indices for owner-occupying households and households that live in rental housing.

The results for owner-occupying movers and stayers are reported in the first two columns of Table 4. Clearly, the degree of sorting is lower among stayers than movers. Based on our theory, we interpret the low degree of sorting among owner-occupying stayers as reflecting housing market related wealth shocks. Some households, which would like to move out of an area where property prices fall, may be unable to do so because they have negative equity. Alternatively, some stayers, who bought their home when prices were lower, may be unwilling to leave when the area becomes more expensive. If this wealth shock based mechanism were (a part of) the explanation, one expects that among renters (who do not face wealth shocks in the housing market) the pattern of sorting should be different. Interestingly, we find that among renters, stayers are more sorted than movers; see the third and fourth columns of Table 4. Finally, the last two columns show that results on all movers and stayers are qualitatively similar to those of owner-occupiers.

[43] Each PUMA has a population of approximately 100 000. For further information, see the appendix.

As an additional piece of evidence, we compare "short distance movers", i.e. households which have moved within the same metropolitan area, and "long distance movers", i.e. households, which have moved from another metropolitan area.[44] Because "long distance movers" have more likely moved between two uncorrelated markets (so that the prices of the old and the new home may have evolved very differently), they should be more sorted than "short distance movers". The Theil information theory indices reported in Table 5 indicate that among owner-occupiers "long distance movers" are indeed more sorted than "short distance movers", according to all three criteria.

Finally, if movers are more sorted than stayers, we expect that educational attainment, age and income are more dispersed across regions among movers than among stayers. Table 6 reports standard deviations over PUMA regions of the share of home-owners with a high school degree and the share of home-owners with at least a college degree, separately for movers and stayers. Clearly, both of the shares vary more across regions among movers than among stayers; and these differences are also statistically significant, as shown by the p-values of the Levene (1960) and the Brown-Forsythe (1974) tests for equal variance. Furthermore, Table 6 shows that owner-occupying movers' age and income vary more across PUMA areas than those of stayers. As a robustness check, Table 6 also makes the same comparisons for people that live in rental housing. Because renters do not face similar housing market related wealth shocks as owners, moving renters need not be more sorted than staying renters. Consistent with this, the results of Table 6 indicate that moving renters are, in the most part, no more sorted than staying renters (and, in fact, the reverse can also be true).

6 Conclusions

When a household buys a home in a certain location, the choice it makes has major implications for the composition of its wealth portfolio. If the household buys an expensive home, it has less net wealth left to allocate to other assets. Also, regional house prices

[44] We also use data on people that have moved *from* or *to* a non-MSA region. See the appendix for more details.

fluctuate over time, and as investments, different houses and locations offer different prospects. The success of today's investment will, in part, determine what kind of home the household will be able to buy in the future.

This paper examined how the asset aspect of housing affects the socioeconomic make-up of local jurisdictions. Our theoretical analysis suggests that a strong investment motive gives rise to internally homogeneous jurisdictions, where neighbors resemble each other. If expected resale value plays a major role in location choice, in equilibrium those households with the highest current utility premium will live in the most desirable and expensive locations, while households with a lower premium will choose locations which are currently less expensive but where property values may rise.

Even if expected resale value is not an important criterion in location choice, the asset aspect of housing still matters. When households rank locations based on current benefits, in equilibrium wealth determines who lives where. Typically a household resides in an unpopular location if and only if it is borrowing constrained and cannot afford a more expensive home. Since current wealth depends, in part, on past luck in the housing market, households residing within the same area may then have little in common, except for the value of their home.

To sum up, there is an inverse relation between the importance of investment considerations at the household level, and the importance of the wealth aspect of housing at the aggregate level. The less the households see the home as an investment, the more the asset aspect of housing moulds the socioeconomic make-up of jurisdictions and the pattern of sorting.

Empirically, the model predicts that the size of house price fluctuations should be negatively correlated with the degree of residential sorting. To examine this hypothesis, we computed measures of residential sorting for income, age and education. In a sample of US metropolitan areas, we documented a negative relationship between the degree of sorting and the size of house price fluctuations.

Theory Appendix

Location choice

The household's decision problem boils down to the choice of the sequence of optimal thresholds θ_n^*. Since $x_n \equiv G(\theta_n^*)$ is a monotonous function of θ_n^*, also x_n can be treated as a choice variable. Using the threshold rule (8) and integrating (5) over all θ shows that the household's decision problem can be summarized by the Bellman equation[45]

$$V(n) = \max_{x_n} u(x_n) + \beta \{(1-\pi) V(n) + \pi [x_n V(n+1) + (1-x_n) V(n-1)]\}, \quad (22)$$

subject to $x_0 = 1$, where

$$u(x_n) \equiv \left(\frac{1}{2} - x_n\right)\varepsilon + \int_{x_n}^{1} G^{-1}(x)\, dx$$

is the expected utility stream at wealth level n. Notice that $\frac{d^2 u(x_n)}{dx_n^2} = -\frac{1}{G'(\theta_n^*)} < 0$. Thus (22) defines a maximization problem with a concave objective function and linear constraints. As a consequence the value function $V(n)$ is concave.

We also show that $\lim_{n\to\infty}\theta_n^* = -\varepsilon$. If not, then $\lim_{n\to\infty}\theta_n^* = \widehat{\theta}^* > -\varepsilon$. Since θ_n^* is a non-increasing sequence, and, by assumption, the feasible values of θ_n^* lie on a finite interval, $\theta_n^* \in \left[\widehat{\theta}^*, \theta_H\right]$, we have $\lim_{n\to\infty}\left(\theta_{n+k}^* - \theta_n^*\right) = 0$ for all finite, positive integers $k \geq 1$. But then $\lim_{n\to\infty}(u_{n+k} - u_n) = 0$ for all $k \geq 1$. As a consequence, $\lim_{n\to\infty}[V(n+1) - V(n-1)] = 0$, and $\lim_{n\to\infty}\theta_n^* = -\varepsilon$. A contradiction.

Next, let $v(n) \equiv V(n+1) - V(n-1)$ and $\Delta x_n \equiv x_{n+1} - x_{n-1}$; since θ_n^* is a non-increasing sequence, $\Delta x_n \in [-1, 0]$. Also define the operator L

$$L[z(n)] \equiv (1-\pi) z(n) + \pi [x_{n+1} z(n+1) + (1 - x_{n-1}) z(n-1)],$$

where $z(n)$ is a generic function of n. Since $V(n)$ satisfies the recursive equation (22),

[45] Differentiating (22) with respect to x_n shows that the optimal thresholds are characterized by (7).

$v(n)$ satisfies the recursive equation

$$v(n) = \int_{x_{n+1}}^{x_{n-1}} G^{-1}(x)\,dx - \Delta x_n \varepsilon + \beta L[v(n)]. \tag{23}$$

Finally, the expression for θ_n^*, eq. (7), can be rewritten as

$$\theta_n^* = Q(n;\varepsilon,\pi) \equiv -\varepsilon + \pi\beta v(n) \text{ for } n \geq 1.$$

Proof of Lemma 1 (i) Define $q^\varepsilon(n) \equiv \frac{dv(n)}{d\varepsilon}$. Differentiating (23) with respect to ε yields $q^\varepsilon(n) = -\Delta x_n + \beta L[q^\varepsilon(n)]$. (Notice that indirect effects can be ignored due to the envelope theorem.) Let $q^\varepsilon_{\max} \equiv \max q^\varepsilon(n)$ and $n^\varepsilon \equiv \arg\max q^\varepsilon(n)$. Now $q^\varepsilon_{\max} \leq -\Delta x_{n^\varepsilon} + \beta q^\varepsilon_{\max}(1 + \pi\Delta x_{n^\varepsilon})$, and $q^\varepsilon_{\max} \leq \frac{-\Delta x_{n^\varepsilon}}{1-\beta(1+\pi\Delta x_{n^\varepsilon})} \leq \frac{1}{1-\beta(1-\pi)}$. Finally $\frac{d\theta_n^*}{d\varepsilon} = \frac{dQ(n;\varepsilon,\pi)}{d\varepsilon} = -1 + \pi\beta q^\varepsilon(n) \leq -1 + \pi\beta q^\varepsilon_{\max} \leq -\frac{1-\beta}{1-\beta(1-\pi)} < 0$.

(ii) Define $q^\pi(n) = \frac{d[\pi v(n)]}{d\pi}$. Then multiplying both sides of (23) by π, differentiating the resulting equation by π, and simplifying, yields $q^\pi(n) = (1-\beta)v(n) + \beta L[q^\pi(n)]$. Let $q^\pi_{\min} \equiv \min q(n)$ and $n^\pi \equiv \arg\min q(n)$. Now $q^\pi_{\min} \geq (1-\beta)v(n^\pi) + \beta q^\pi_{\min}(1 + \pi\Delta x_{n^\pi})$, and $q^\pi_{\min} \geq \frac{(1-\beta)v(n^\pi)}{1-\beta(1+\pi\Delta x_{n^\pi})} \geq \frac{(1-\beta)v(n^\pi)}{1-\beta(1-\pi)} > 0$. Finally $\frac{d\theta_n^*}{d\pi} = \frac{dQ(n;\varepsilon,\pi)}{d\pi} = \beta q^\pi(n) \geq \beta q^\pi_{\min} > 0$. ■

Stationary wealth distribution

In what follows we derive equations (9) and (10).

If there is a regional shock, all $f(n)$ households which were previously in wealth class n either go up to $n+1$ or fall to $n-1$, depending on their house location. They are replaced by $f^0_{n-1}(n-1)$ class $n-1$ households which have made a capital gain and $f^1_n(n+1)$ class $n+1$ households which have suffered a capital loss. The wealth distribution is stationary if and only if

$$f(n) \equiv f^0_n(n) + f^1_{n-1}(n) = f^0_{n-1}(n-1) + f^1_n(n+1) \tag{24}$$

for all n. We also consider the model version, with a continuum of atomistic regions. Between any periods, a measure π of the locations is hit by a regional shock, and the

wealth distribution is stationary if and only if

$$f(n) = (1-\pi) f(n) + \pi \left(f_{n-1}^{0}(n-1) + f_{n}^{1}(n+1) \right). \tag{25}$$

It is easy to conclude that (25) reduces to (24): as a consequence, both model variants have the same long-run wealth distribution and the same long-run equilibrium.

There are no wealth classes below 0 (i.e., $f(n) = 0$ for $n < 0$) and at wealth level 0 the households can only choose an unpopular location (i.e., $f_{-1}^{1}(0) = 0$). These restrictions and (24) then imply the set of equations (9). Finally, plugging the definitions $f_{n}^{0}(n) = G(\theta_{n}^{*}) f(n)$ and $f_{n-1}^{1}(n) = (1 - G(\theta_{n}^{*})) f(n)$ into (9) yields the sequence (10).

Proof of Proposition 1

(i) We begin by deriving equation (14), which is needed in the proof of the proposition. Using vector notation, equation (22) can be rewritten as follows

$$V = \max_{\{x_n\}} u + \beta \left[(1-\pi) I + \pi A \right] V \tag{26}$$

for $n \geq 1$ (and $x_0 = 1$) where V is the (ex ante) value function, stacked as a column vector, u is a column vector with elements $u_n = u(x_n)$, and A is a transition matrix, with elements $A_{i,j} = 1 - x_i$ if $j = i - 1$, $A_{i,j} = x_i$ if $j = i + 1$ and $A_{i,j} = 0$ otherwise. Premultiplying both sides of (26) by the stationary wealth distribution f' yields $f'V = f'u + f'\beta \left[(1-\pi) I + \pi A \right] V$. The distribution f is induced by the transition matrix A, and it satisfies the equation $f'A = f'$. But then $w \equiv f'u = (1-\beta) f'V = (1-\beta) W$. Finally

$$\begin{aligned} w &= \sum_{n=0}^{\infty} f(n) u(n) = \sum_{n=0}^{\infty} f(n) \left[(1-x_n) \left(\frac{1}{2}\varepsilon + E[\theta \mid \theta \geq \theta_{n}^{*}] \right) - x_n \frac{1}{2}\varepsilon \right] \\ &= \sum_{n=1}^{\infty} f_{n-1}^{1}(n) E[\theta \mid \theta \geq \theta_{n}^{*}] = \frac{1}{2} E[\theta \mid h = 1], \end{aligned} \tag{27}$$

where the third equality follows from the housing market equilibrium (11). Thus

$$W = \frac{w}{1-\beta} = \frac{1}{2}\frac{E\left[\theta \mid h=1\right]}{1-\beta}. \tag{28}$$

(ii) As proving the proposition with respect to π and ε involves the same steps, we introduce a generic parameter ρ, where $\rho \in \{\pi, \varepsilon\}$. Also, let x be the vector with the nth element x_n. Now

$$\frac{dw}{d\rho} \underset{(a)}{=} \frac{\partial w}{\partial \rho} + \frac{\partial w}{\partial x}\frac{dx}{d\rho} \underset{(b)}{=} \frac{\partial w}{\partial x}\frac{dx}{d\rho} \underset{(c)}{=} (1-\beta)\frac{\partial W}{\partial x}\frac{dx}{d\rho} \underset{(d)}{=} (1-\beta)\,V'\frac{df}{dx}\frac{dx}{d\rho}$$

Equality (a) involves a decomposition into the direct effect and the indirect effect. (b) follows from the fact that w does not depend directly on π and ε (see (27)), and thus $\frac{\partial w}{\partial \rho} = 0$. (c) follows from equality (28). (d) uses the definition of W, (13), and the envelope theorem: since the threshold θ_n^*, and thus also x_n, is optimally chosen in all wealth classes $n \geq 1$, a small policy change does not affect the value function $V(n)$.

By Lemma 2 we know that the wealth distribution shifts to the right, in the sense of first-order stochastic dominance, when π increases or ε decreases. As the value function $V(n)$ is increasing in n, this shift in the stationary distribution translates into higher social welfare:

$$\frac{dw}{d\pi} = (1-\beta)\,V'\frac{df}{dx}\frac{dx}{d\pi} \geq 0, \;\; \frac{dw}{d\varepsilon} = (1-\beta)\,V'\frac{df}{dx}\frac{dx}{d\varepsilon} \leq 0. \;\blacksquare$$

Proof of Proposition 4

(a) Match dimension. When $\varepsilon \to 0$, the basic allocation problem vanishes, and the result is obvious. Next consider the case $\delta \to 1$. The household chooses $\{x_n\}$, so as to maximize the value function V, where V satisfies the recursive equation $V = \delta A V + (1-\delta)\frac{u}{1-\beta}$. (This equation follows directly from (26).) Iterating forward, we get $V = (1-\delta)\sum_{t=0}^{\infty}(\delta A)^t \frac{u}{1-\beta}$. Next notice that $\lim_{t\to\infty} A^t = \mathbf{1}\otimes f'$ (where $\otimes$ is Kronecker product). Thus when $\pi \to 1$ and $\beta \to 1$, so that $\delta \to 1$, maximizing V becomes essentially equivalent to maximizing $f'u = w = \frac{1}{2}E\left[\theta \mid h=1\right]$. The objective function

$w = \frac{1}{2}E[\theta \mid h = 1]$ is maximized iff there is perfect sorting in the match dimension.

(b) Sorting in the wealth dimension. The putative equilibrium strategy is of the following form: $h(0, \theta) = 0$ for all θ (due to the borrowing constraint), $h(n, \theta) = 1$ for all θ and $n \geq 1$. Then in equilibrium $f(0) = f(1) = \frac{1}{2}$ and $f(n) = 0$ for all $n \geq 2$.

Given this strategy, it is easy to calculate the ex ante values of the program $V(n)$ at different wealth levels $n \geq 0$. In particular, one can show that $V(2) - V(0) = (1 - \delta)\frac{\varepsilon + E[\theta]}{1-\beta}$. Given the optimal location choice rule (6), the putative strategy is optimal for the household iff it always prefers the desirable location at wealth level $n = 1$, i.e., iff

$$\theta + \varepsilon > \pi\beta[V(2) - V(0)] = \pi\beta(1 - \delta)\frac{\varepsilon + E[\theta]}{1 - \beta} \text{ for all } \theta. \tag{29}$$

In particular, the condition (29) must hold for the lowest possible realization of the match θ_L. Inserting $\theta = \theta_L$, and slightly manipulating (29), yields the condition for residential sorting in the wealth dimension: $\theta_L + \varepsilon > \pi\beta\frac{E[\theta] - \theta_L}{1-\beta}$. ■

Proof of Proposition 6

(a) We define cumulative distribution functions $G(\theta \mid h, m)$ separately for four groups, conditioning on the households present location ($h \in \{0, 1\}$), and on whether the household has moved in the present period ($m = 1$, if the household has moved, and $m = 0$, if the household has not moved). So, for example, $G(\theta \mid h = 0, m = 1)$ is the distribution function for those households, which moved at the beginning of the period (from an expensive location) and currently live in a cheap location. We also define the functions

$$DG(\theta \mid h) \equiv G(\theta \mid h, m = 1) - G(\theta \mid h, m = 0), \ h \in \{0, 1\} \tag{30}$$

which allow us to compare (in the sense of first order stochastic dominance) the distributions of newcomers and old residents, who live in the same location (0 or 1).

To prove the proposition, we need to construct $G(\theta \mid h, m)$, $h, m \in \{0, 1\}$.

(i) As a first step, we characterize the match distributions of households living in the desirable and in the undesirable location, conditional on wealth class n. Given the thresh-

old location choice rule (8), the distribution in the desirable location $G(\theta \mid h=1,n) = G(\theta \mid \theta \geq \theta_n^*) = \frac{G(\theta)-G(\theta_n^*)}{1-G(\theta_n^*)}$ for $\theta \geq \theta_n^*$ (and 0 for $\theta < \theta_n^*$) is left-truncated, with truncation point θ_n^*, while the distribution in the undesirable location $G(\theta \mid h=0,n) = G(\theta \mid \theta < \theta_n^*) = \frac{G(\theta)}{G(\theta_n^*)}$ for $\theta \leq \theta_n^*$ (and 1 for $\theta > \theta_n^*$) is right-truncated with the same truncation point θ_n^*. It is easy to see that $\frac{\partial G(\theta|\theta \geq \theta_n^*)}{\partial \theta_n^*} \leq 0$ and $\frac{\partial G(\theta|\theta \leq \theta_n^*)}{\partial \theta_n^*} \leq 0$ for all θ. This property means that if we compare two wealth levels n_1 and n_2, such that $n_1 < n_2$, and consequently $\theta_{n_1}^* > \theta_{n_2}^*$, the higher threshold $\theta_{n_1}^*$ in group n_1 implies that the distribution $G(\theta \mid h, n_1)$ first-order stochastically dominates the distribution $G(\theta \mid h, n_2)$ for $h \in \{0,1\}$. More formally

$$G(\theta \mid h, n_1) \leq G(\theta \mid h, n_2) \text{ for all } \theta, \text{ when } n_1 < n_2 \text{ and } h \in \{0,1\}. \tag{31}$$

(ii) As a second step, we need to study the conditional wealth distributions, contingent on housing location and mobility. The main objective is to establish a first-order stochastic dominance relation between movers and stayers in each location.

Denote the mass of households with wealth n, and group (h,m), by $\varphi_m^h(n)$. Now

$$\begin{aligned}
\varphi_0^0(n) &= f_n^0(n)\,\psi(\theta_n^*) \equiv f_n^0(n)\left\{(1-s)\left[(1-\lambda)+\lambda G(\theta_n^*)\right]+sG(\theta_n^*)\right\} \\
\varphi_0^1(n) &= f_{n-1}^1(n)\,\widehat{\psi}(\theta_n^*) \equiv f_{n-1}^1(n)\left\{(1-s)\left[(1-\lambda)+\lambda\left(1-G(\theta_n^*)\right)\right]+s\left(1-G(\theta_n^*)\right)\right\} \\
\varphi_1^0(n) &= \varphi_1^1(n) = f_n^0(n)\left(1-G(\theta_n^*)\right)\left[(1-s)\lambda+s\right] = f_{n-1}^1(n)\,G(\theta_n^*)\left[(1-s)\lambda+s\right]
\end{aligned} \tag{32}$$

Also let $\widehat{\varphi}_m^h(n) \equiv \varphi_m^h(n) / \sum_i \varphi_m^h(i)$ be the relative share of wealth class n in group (h,m). Next, to compare the wealth distributions, we need the size ratios of adjacent wealth classes in different groups. Denote $\widehat{\gamma}_m^h(n) \equiv \widehat{\varphi}_m^h(n+1)/\widehat{\varphi}_m^h(n) = \varphi_m^h(n+1)/\varphi_m^h(n)$. Now using the equations (32) we get

$$\widehat{\gamma}_0^0(n)/\widehat{\gamma}_1^0(n) = \frac{\psi\left(\theta_{n+1}^*\right)}{\psi(\theta_n^*)}\frac{1-G(\theta_n^*)}{1-G\left(\theta_{n+1}^*\right)} \leq 1 \tag{33}$$

$$\widehat{\gamma}_0^1(n)/\widehat{\gamma}_1^1(n) = \frac{\widehat{\psi}\left(\theta_{n+1}^*\right)}{\widehat{\psi}(\theta_n^*)}\frac{G(\theta_n^*)}{G\left(\theta_{n+1}^*\right)} \geq 1 \tag{34}$$

These inequalities hold, since clearly $\psi\left(\theta_{n+1}^*\right)/\psi(\theta_n^*) \leq 1$, $\left(1-G(\theta_n^*)\right)/\left(1-G\left(\theta_{n+1}^*\right)\right) \leq$

1, $\widehat{\psi}\left(\theta_{n+1}^{*}\right)/\widehat{\psi}\left(\theta_{n}^{*}\right) \geq 1$ and $G\left(\theta_{n}^{*}\right)/G\left(\theta_{n+1}^{*}\right) \geq 1$. The inequality (33) allows us to compare the wealth distributions of mover and stayer households, which currently reside in the cheap location. The inequality tells that, for any adjacent wealth classes $(n+1)$ and n, the ratio $\widehat{\varphi}_{m}^{0}\left(n+1\right)/\widehat{\varphi}_{m}^{0}\left(n\right)$ is larger for movers than for stayers. But this means that in the cheap location newcomers are wealthier than the old residents, in the sense of first-order stochastic dominance. The inequality (34) then implies that in the expensive location the opposite is true, and old residents are wealthier than newcomers, in the sense of first-order stochastic dominance.

(iii) As a final step, we combine the results of steps (i) and (ii), and construct the conditional match distribution functions

$$G\left(\theta \mid h,m\right)=\sum_{n}\widehat{\varphi}_{m}^{h}\left(n\right)G\left(\theta \mid h,n\right),\ \text{for } h,m\in\left\{0,1\right\}. \tag{35}$$

That is, the conditional match distributions $G\left(\theta \mid h,m\right)$ are convex combinations of the location-contingent distributions $G\left(\theta \mid h,n\right)$ at different wealth levels n. In each group (h,m), the weight assigned to the distribution function $G\left(\theta \mid h,n\right)$ corresponds to the relative size of wealth class n in the group, $\widehat{\varphi}_{m}^{h}\left(n\right)$.

Using (30) and (35), we get

$$\begin{array}{l} DG\left(\theta \mid h=0\right)=\sum_{n}\left[\widehat{\varphi}_{1}^{0}\left(n\right)-\widehat{\varphi}_{0}^{0}\left(n\right)\right]G\left(\theta \mid h=0,n\right)\geq 0, \\ DG\left(\theta \mid h=1\right)=\sum_{n}\left[\widehat{\varphi}_{1}^{1}\left(n\right)-\widehat{\varphi}_{0}^{1}\left(n\right)\right]G\left(\theta \mid h=1,n\right)\leq 0 \end{array} \tag{36}$$

for all θ. The inequalities follow from stochastic dominance, results (31), (33) and (34). The expressions (36) mean that in a currently cheap location, the match distribution of old residents stochastically dominates the match distribution of newcomers, while the in a currently expensive location the opposite is true. Thus we have proved that in both areas movers (with $m=1$) tend to have a better match with the location than stayers $(m=0)$.

(b) To address the degree of residential sorting among movers and stayers, we further define $DG\left(\theta \mid m\right)\equiv G\left(\theta \mid h=1,m\right)-G\left(\theta \mid h=0,m\right),\ m\in\left\{0,1\right\}$. Then $DG\left(\theta \mid m=1\right)$ tells how the distribution of households which have moved from a cheap location to an

expensive location differs from the distribution of those households which have moved the other way round; also $DG(\theta \mid m=0)$ allows us to compare the distributions of immobile households living in different locations. Finally, to compare the degree of residential sorting between movers and stayer, we define the function $RS_{m/s}(\theta) \equiv DG(\theta \mid m=1) - DG(\theta \mid m=0)$. It is clear than both among movers and among stayers, those who live in the desirable location typically have a higher value of θ than those who reside in the less desirable location, that is $DG(\theta \mid m) \leq 0$ for all θ and for $m \in \{0,1\}$. Now we use the function $RS_{m/s}(\theta)$ to address the question: among which group (movers or stayers) are the households residing in different locations more distinct from each other. In particular, if $RS_{m/s}(\theta) \leq 0$ for all θ, movers are more sorted in this sense. But

$$\begin{aligned} RS_{m/s}(\theta) &= DG(\theta \mid m=1) - DG(\theta \mid m=0) \\ &= G(\theta \mid h=1, m=1) - G(\theta \mid h=0, m=1) \\ &\quad - [G(\theta \mid h=1, m=0) - G(\theta \mid h=0, m=0)] \\ &= DG(\theta \mid h=1) - DG(\theta \mid h=0) \leq 0, \end{aligned}$$

where the inequality follows from (36). ■

More general match dynamics

Let $v(\theta, n) \equiv V(\theta, n+1) - V(\theta, n-1)$ and $\Delta h(\theta, n) \equiv h(\theta, n+1) - h(\theta, n-1)$. Also define the operator $\widehat{L}$,

$$\begin{aligned} \widehat{L}[z(\theta,n)] \equiv (1-\sigma) z(\theta,n) + \lambda E_{\widetilde{\theta}}\left[z\left(\widetilde{\theta}, n\right) \mid \theta, s=0\right] \\ +\pi E_{\widetilde{\theta}}\left[h(\theta,n-1) z\left(\widetilde{\theta}, n-1\right) + (1-h(\theta,n+1)) z\left(\widetilde{\theta}, n+1\right) \mid \theta, s=1\right], \end{aligned}$$

where $z(\theta, n)$ is a generic function of θ and n. Since $V(\theta, n)$ satisfies the Bellman equation (17), the function $v(\theta, n)$ satisfies the recursive equation

$$v(\theta,n) = \Delta h(\theta,n)(\varepsilon+\theta) + \beta \widehat{L}[v(\theta,n)]. \tag{37}$$

For all θ and all $n \geq 1$, the household's location choice rule assumes the form $h(\theta, n) = 1$ iff $\theta \geq \widehat{Q}(\theta, n; \varepsilon, \sigma)$ (and 0 otherwise), where $\widehat{Q}(\theta, n; \varepsilon, \sigma) \equiv -\varepsilon + \pi\beta E_{\widetilde{\theta}}\left[v\left(\widetilde{\theta}, n\right) \mid \theta, s = 1\right]$.

Lemma 1' For all θ and $n \geq 1$, (i) $\frac{d\widehat{Q}(\theta,n;\varepsilon,\sigma)}{d\varepsilon} < 0$ and (ii) $\frac{d\widehat{Q}(\theta,n;\varepsilon,\sigma)}{d\sigma} > 0$.

Proof (i) Define $\widehat{q}^{\varepsilon}(\theta, n) \equiv \frac{dv(\theta,n)}{d\varepsilon}$. Differentiating (37) with respect to ε shows that $\widehat{q}^{\varepsilon}(\theta, n)$ satisfies the equation $\widehat{q}^{\varepsilon}(\theta, n) = \Delta h(\theta, n) + \beta\widehat{L}\left[\widehat{q}^{\varepsilon}(\theta, n)\right]$. Next define $\widehat{q}^{\varepsilon}_{\max} \equiv \max \widehat{q}^{\varepsilon}(\theta, n)$ and $\left\{\widehat{\theta}^{\varepsilon}, \widehat{n}^{\varepsilon}\right\} \equiv \arg\max \widehat{q}^{\varepsilon}(\theta, n)$. Then $\widehat{q}^{\varepsilon}_{\max} \leq \Delta h\left(\widehat{\theta}^{\varepsilon}, \widehat{n}^{\varepsilon}\right) + \beta\widehat{q}^{\varepsilon}_{\max}\left(1 - \pi\Delta h\left(\widehat{\theta}^{\varepsilon}, \widehat{n}^{\varepsilon}\right)\right)$, and $\widehat{q}^{\varepsilon}_{\max} \leq \frac{\Delta h\left(\widehat{\theta}^{\varepsilon}, \widehat{n}^{\varepsilon}\right)}{1-\beta\left(1-\pi\Delta h\left(\widehat{\theta}^{\varepsilon}, \widehat{n}^{\varepsilon}\right)\right)} \leq \frac{1}{1-\beta(1-\pi)}$. Finally, $\frac{d\widehat{Q}(\theta,n;\varepsilon,\sigma)}{d\varepsilon} = -1 + \pi\beta E_{\widetilde{\theta}}\left[\widehat{q}^{\varepsilon}\left(\widetilde{\theta}, n\right) \mid \theta, s = 1\right] \leq -1 + \pi\beta\widehat{q}^{\varepsilon}_{\max} \leq -\frac{1-\beta}{1-\beta(1-\pi)} < 0$. (ii) Define $\widehat{q}^{\sigma}(\theta, n) \equiv \frac{d[\sigma v(\theta,n)]}{d\sigma}$. Multiplying both sides of (37) by σ, differentiating with respect to σ, and simplifying, shows that $\widehat{q}^{\sigma}(\theta, n)$ satisfies the equation $\widehat{q}^{\sigma}(\theta, n) = (1-\beta)v(\theta, n) + \beta\widehat{L}\left[\widehat{q}^{\sigma}(\theta, n)\right]$. Next define $\widehat{q}^{\sigma}_{\min} \equiv \min \widehat{q}^{\sigma}(\theta, n)$ and $\left\{\widehat{\theta}^{\sigma}, \widehat{n}^{\sigma}\right\} \equiv \arg\min \widehat{q}^{\sigma}(\theta, n)$. Then $\widehat{q}^{\sigma}_{\min} \geq (1-\beta)v\left(\widehat{\theta}^{\sigma}, \widehat{n}^{\sigma}\right) + \beta q^{\sigma}_{\min}\left(1 - \pi\Delta h\left(\widehat{\theta}^{\sigma}, \widehat{n}^{\sigma}\right)\right)$, and $\widehat{q}^{\sigma}_{\min} \geq \frac{(1-\beta)v\left(\widehat{\theta}^{\sigma}, \widehat{n}^{\sigma}\right)}{1-\beta\left(1-\pi\Delta h\left(\widehat{\theta}^{\sigma}, \widehat{n}^{\sigma}\right)\right)} \geq \frac{(1-\beta)v\left(\widehat{\theta}^{\sigma}, \widehat{n}^{\sigma}\right)}{1-\beta(1-\pi)} > 0$. Finally, $\frac{d\widehat{Q}(\theta,n;\varepsilon,\sigma)}{d\sigma} = \xi\beta E_{\widetilde{\theta}}\left[\widehat{q}\sigma\left(\widetilde{\theta}, n\right) \mid \theta, s = 1\right] \geq \xi\beta\widehat{q}^{\sigma}_{\min} > 0$. ■

Stationary distribution. Let $\widehat{f}_n(\theta_j)$ denote the long-run frequency mass of households with match θ_j and wealth n, and let $\widehat{f}_n$ be a $J \times 1$ vector, with the jth element $\widehat{f}_n(\theta_j)$. Also let H_n, $n \geq 1$, be a $J \times J$ diagonal matrix, with the jth diagonal element $h(\theta_j, n)$ (and all off-diagonal elements equal to 0), and let $B_n = I - H_n$. The stationary distribution satisfies the following set of recursive equations

$$\widehat{f}'_n = (1-\sigma)\widehat{f}'_n + \lambda\widehat{f}'_n\Lambda_0 + \pi\left(\widehat{f}'_{n-1}B_{n-1} + \widehat{f}'_{n+1}H_{n+1}\right)\Lambda_1$$

for all $n = 0, 1, ...$ Simplifying yields

$$\widehat{f}'_n = (1-\xi)\widehat{f}'_n\Lambda_0 + \xi\left(\widehat{f}'_{n-1}B_{n-1} + \widehat{f}'_{n+1}H_{n+1}\right)\Lambda_1 \tag{38}$$

Notice in particular that the parameters ε and σ do not appear in (38), and thus the joint distribution of wealth and the match depends on these parameters only indirectly, through changes in policies.

Equilibrium. Postmultiplying both sides of equation (38) by the unit vector $\mathbf{1}$, and taking into account the fact that $\Lambda_0\mathbf{1} = \Lambda_1\mathbf{1} = \mathbf{1}$, yields a set of recursive equations for

the marginal distribution of wealth

$$f(n) = f_{n-1}^{0}(n-1) + f_{n}^{1}(n+1) \tag{39}$$

where $f(n) = \widehat{f}_n'\mathbf{1}$ is the frequency mass of households at wealth level n, $f_n^0(n) = \widehat{f}_n' B_n \mathbf{1}$ is the mass of households at wealth level n residing in an unpopular location, and $f_{n-1}^1(n) = \widehat{f}_n' H_n \mathbf{1}$ is the mass of households at wealth level n residing in a popular location. But equation (39) is identical to equation (24) so that equilibrium follows in the same way as in Section 2.5.

Lemma 2' Define the cumulative distribution function $\widehat{F}(\theta_j, n; \varepsilon, \sigma) = \sum_{i=0}^{n} \widehat{f}_i(\theta_j)$. Then $\frac{d\widehat{F}(\theta_j,n;\varepsilon,\sigma)}{d\varepsilon} \geq 0$ and $\frac{d\widehat{F}(\theta_j,n;\varepsilon,\sigma)}{d\sigma} \leq 0$ for all n and θ_j.

Proof Define a history as a collection of match realizations and regional shock realizations $\mathcal{H}_t = \{(\theta_\tau, s_\tau)\}_{\tau=0}^{t}$. Notice that histories are exogenous in the sense that they do not depend on the households' location choices. Denote a state by $y = (\theta, n)$. Consider two location choice rules h^0 and h^1 such that for some state $\widehat{y}$, $h^0(\widehat{y}) = 0$ and $h^1(\widehat{y}) = 1$, and for all other states $y \neq \widehat{y}$, $h^0(y) = h^1(y) = h(y)$ (where $h(y)$ is the common policy).

Next notice that there is a mapping from histories $\mathcal{H}_t$ to states y_t, conditional on policy h^i, $i \in \{0,1\}$ (and initial state). That is, at any date t, the household's wealth $n_t^i = n^i(\mathcal{H}_t)$ and the state $y_t^i = y^i(\mathcal{H}_t)$, where $i \in \{0,1\}$ refers to the policy that the household follows.

Consider two households. Household 0 follows policy h^0, while household 1 follows policy h^1. Assume the households have the same history $\mathcal{H}_t$. Define $\nu_t \equiv n_t^0 - n_t^1$ and notice that by equation (3) it obeys the law of motion $\nu_{t+1} = \nu_t + 2s_{t+1}(h^1(y_t^1) - h^0(y_t^0))$. Obviously,

$$\Delta\nu_t \equiv \nu_{t+1} - \nu_t = 2s_{t+1}(h^1(y_t^1) - h^0(y_t^0)) \in \{-2, 0, 2\}. \tag{40}$$

Assume that for some period t, $\nu_t = 0$ so that also $y_t^0 = y_t^1$. Given the properties of h^0 and h^1 it is evident that

$$\Delta\nu_t \in \{0, 2\}, \text{ if } \nu_t = 0. \tag{41}$$

($\Delta\nu_t = 2$ iff $y_t^0 = y_t^1 = \widehat{y}$ and $s_{t+1} = 1$). Next, assume the households have the same

initial wealth, $\nu_0 = 0$. From (40) and (41) it follows that $\nu_t = 2k,\ k \in \{0, 1, 2, ...\}$ for all $t = 0, 1, 2, ...$ The essential finding is that, given identical histories and equal initial wealth, household 1 cannot be wealthier than household 0.

Assume that there is a population of households following policy h^0, and another population following policy h^1. Also assume that all households, in either population, have the same initial wealth. As above, we refer to a household belonging to population 0 (1) as household 0 (1). Now, the proof of the lemma derives from the following observations. (i) After any given (common) history $\mathcal{H}_t$, household 0 is at least as wealthy as household 1. (ii) After any given (common) history $\mathcal{H}_t$, household 0 and household 1 have the same match. (iii) The probability distribution over the histories does not depend on policy. (iv) In any period t, and for any given current match, the wealth distribution under policy h^0 stochastically dominates the wealth distribution under policy h^1. (v) When $t \to \infty$, the joint distribution of wealth and the match converges to the stationary distribution. Thus stochastic dominance applies to the stationary distribution. Finally, Lemma 1' implies that when ε increases or σ decreases, the households may shift from policy h^0 to policy h^1, but the opposite shift (from policy h^1 to policy h^0) never happens. ■

Proposition 2' When ε decreases or σ increases, social welfare grows.

Proof Let us define a KJ state Markov chain y, where the $(nJ + j)$th state is given by the pair (θ_j, n). Notice that K (the number of wealth levels) is $\overline{n} + 1$, if $\theta_L > -\varepsilon$, and otherwise $K = \infty$. Let $\mathbf{h}$ be a $KJ \times 1$ vector, with the $(nJ + j)$th element $h(\theta_j, n)$. Further define a $KJ \times KJ$ diagonal matrix H, with the vector $\mathbf{h}$ on the diagonal (and all off-diagonal elements equal to zero), and let the $KJ \times KJ$ matrix $\widehat{A}$ be the transition matrix of the Markov chain y.

The value function can be presented as a $KJ \times 1$ vector $\widehat{V}$, where the $(nJ + j)$th element is the value of the household's program in state (θ_j, n). $\widehat{V}$ satisfies the Bellman equation

$$\widehat{V} = H(\mathbf{1}_K \otimes \boldsymbol{\theta}) + \left(\mathbf{h} - \frac{1}{2}\mathbf{1}_{KJ}\right)\varepsilon + \beta\left[(1-\sigma)I + \sigma\widehat{A}\right]\widehat{V}, \tag{42}$$

where $\boldsymbol{\theta}$ is the $J \times 1$ vector of types θ_j. The stationary distribution of y is a $KJ \times 1$ vector $\widehat{f}$. The distribution is induced by the transition matrix $\widehat{A}$ and it satisfies the equation

$\widehat{f}' = \widehat{f}'\widehat{A}$. Now define the measures of social welfare

$$\begin{aligned}\widehat{w} &\equiv \sum_n \sum_j \widehat{f}_n(\theta_j)\, h(\theta_j, n)\, \theta_j = \widehat{f}' H(\mathbf{1}_K \otimes \boldsymbol{\theta}) = \frac{1}{2} E[\theta \mid h = 1] \\ \widehat{W} &\equiv \sum_n \sum_j \widehat{f}_n(\theta_j)\, V(\theta_j, n) = \widehat{f}'\widehat{V}\end{aligned}$$

Next we premultiply both sides of (42) by $\widehat{f}'$. Then using the fact that $\widehat{f}' = \widehat{f}'\widehat{A}$, and noting that $\widehat{f}'\left(\mathbf{h} - \frac{1}{2}\mathbf{1}_{KJ}\right) = 0$, by the housing market equilibrium, yields

$$\widehat{W} = \widehat{w} + \beta\widehat{W} \Leftrightarrow \widehat{W} = \widehat{w}/(1-\beta). \tag{43}$$

Given the equation (43), and Lemma 2', Proposition 2' can be proved following the same steps as in the proof of Proposition 2. See part (ii) of the proof. ■

Proposition 3' When ε increases or σ decreases, the degree of residential sorting in the match dimension decreases in the sense explained in Proposition 3.

Proof The result follows from Proposition 2'. See the proof of Proposition 3. ■

Corollary 1' There is a negative relation between the size of house price fluctuations and the degree of residential sorting in the match dimension.

Proposition 4' When ε increases or σ decreases, the degree of residential sorting in the wealth dimension increases in the sense explained in Proposition 4.

Proof The results follows from Lemma 2'. See the proof of Proposition 4. ■

Data Appendix

Description of variables of Tables 2 and 3

Except the price variation measure (see footnote 37), the applied variables in Tables 2 and 3 are computed from extraction of data from the 1990 decennial Census, published in the ICPSR study 2889 (1990). The tables apply the data set 2 (DS2) where each variable is aggregated to the municipality (MCD) level. The final samples of observations cover

all MSAs for which we have house price data.

The sorting measures applied in Tables 2 and 3 are based on the following groups of types. We use five categories for age: (1) "children" (those of 0-15 years old), (2) "youth" (16-24 years old), (3) "adults, early career" (25-44 years old), (4) "adults, late career" (45-64 years old), and (5) "seniors" (those at least 65 years old). For education, we have three groups: (1) less than a high school degree, (2) at least a high school degree but not a college degree, and (3) a college degree or more. The Census defines the education groups for only those who are at least 25 years old. This age category is used to normalize the education groups within each region. Finally, for income we apply all the 25 income groups available in the ICPSR study 2889. The education and income categories applied here are similar to those of the dissimilarity indices and Gini coefficients considered by Rhode and Strumpf (2003, p. 1660) (see also their Data Appendix at www.unc.edu/~cigar/ or www.unc.edu/~prhode/).

To compute the control variables of Table 3 we use the following original variables of the data set (see ICPSR study 2889 (1990)): "v9" for "Number of municipalities"; "v103" and "v9" for "Average size of municipalities"; "v103" and "v121" for "Population density in MSA"; "v103" and "v121" for "Land area of MSA"; "v103" for "Number of Families in MSA." Among the additional control variables discussed in the text: the diversity measure of the age of housing units ("T_{HoAge}") assumes three classes: houses build (1) "at most 5 years ago", (2) "6-10 years ago", and (3) "at least 11 years ago". The corresponding measure for the number of housing units in a residential building ("T_{HoUnit}") is computed based on three classes: (1) "1-unit structures", (2) "2-4 unit structures" and (3) "5 or more unit structures". We apply "v1804" and "v1801" to compute the share of people that live in rental housing. Finally, the regulation variable ("WRLURI") is obtained from http://real.wharton.upenn.edu/~gyourko/Wharton_residential_land_use_reg.htm, while the variable "UNDEV" is obtained from Saiz (2008, Table 1).

Description of sorting measures of Tables 4, 5 and 6

The data applied in Tables 4, 5 and 6 are from the Census data provided at www.ipums.org. The web site provides detailed definitions for each variable in the data. For each observation unit (i.e., person) in the 1% sample from the 1990 Census, we downloaded household id (SERIAL), age (AGE), educational attainment (EDUC99), household income (FTOTINC), tenure (OWNERSHP), migration information (MIGRATE5, MIGMET5, MIGPLAC5) and location indicators (PUMA, STATEFIP, METAREA). These data include observations on $2,479,568$ persons from 1760 different PUMAs. The actual number of people in each PUMA is also obtained from www.ipums.org.

To compute the Theil information theory indices in Table 4, we classify each sample person into a mover (MIGRATE5 = 2) or a stayer (MIGRATE5 = 1). Furthermore, we classify a person as an owner, if OWNERSHP = 10 and a renter, if OWNERSHP = 20. Persons with missing observations on MIGRATE5 or OWNERSHP are excluded from the calculations. We apply similar categories as in Tables 2 and 3. For age, we estimate the shares of "children", "youth", etc. in each PUMA by computing the relative shares of the sample persons belonging to the relevant age category (for "children" the share of those 0-15 years old, etc.). For education, we restrict the sample to those at least 25 years old. The three education groups (consistent with those in Tables 2 and 3) are formed by (1) EDUC99 ≤ 9, (2) $10 \leq$ EDUC99 ≤ 11, and (3) $12 \leq$ EDUC99. Finally, to compute the index for income, we first restrict the sample to household heads only (SERIAL = 1). Then we employ FTOTINC to classify each household into one of the 25 income ranges used in the ICPSR data, and compute the corresponding relative shares in each PUMA. In all cases (age, education and income), the US level shares are obtained as a population weighted average of the PUMA shares.

To compute the Theil information theory indices in Table 5, we first restrict the sample into persons that are owner-occupiers (OWNERSHP = 10) and have moved recently (MIGRATE5 = 2). Within this subsample, we classify a person as a "short distance mover", if his current MSA is the same as five years ago, i.e., if METAREA and MIGMET5 match; otherwise the person is classified as a "long distance mover". In addition to data

on persons that have moved from one MSA region to another, we also use data on persons that have moved *from* or *to* a non-MSA region. If a person has moved from an MSA region to a non-MSA region, or vice versa, he or she is recorded as a "long distance mover", while a person that has moved between two non-MSA regions is recorded as a "long distance mover" only, if his or her current state of residence (STATEFIP) is different from that five years ago (MIGPLAC5). The indices are formed by applying the same convention of groupings as in Table 4.

The PUMA observations of the variables considered in Table 6 are computed for household heads only, while the applied groupings ("Owners", "Renters", "Movers", "Stayers") are defined in the same way as in Table 4. "High school degree, %" is the relative share of household heads at least 25 years old that have $10 \leq \text{EDUC99} \leq 11$, "College degree, %" is the corresponding share of those that have $12 \leq \text{EDUC99} \leq 17$. Finally, "Age" and "Income", respectively, refer to the average age (AGE) and income (FTOTINC) over the relevant households in each case.

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Tables

Table 1. Maximum and minimum house-price-to-income ratios, 1979-1996

	House-price-to-income ratio	
	min	max
Boston	5.4	12.0
New York	5.3	12.0
Los Angeles	6.7	11.1
San Diego	6.7	9.6

Source: Malpezzi, 1999

Table 2. Correlation between sorting measures and house price volatility

Sorting measure	Income	Education	Age
Dissimilarity index, D	-0.19	-0.14	-0.12
Gini index, GC	-0.16	-0.13	-0.12
Theil information theory index, T	-0.20	-0.17	-0.14

Notes: Correlations are reported between the house price volatility and sorting measures for income, education and age. The sorting measure varies by row. See the text for the formulae of the measures. Sample size is 242.

Table 3. OLS regressions of sorting measures on house price volatility and selected covariates

	Dependent variable: Sorting index of		
	Income	Education	Age
Independent variable	(1)	(2)	(3)
Constant	.026***	.035***	.015***
	(0.003)	(.004)	(.003)
House price volatility	−.039***	−.115***	−.045***
	(.013)	(.034)	(.016)
Number of municipalities	.007***	−.002	.0005
	(.002)	(.003)	(.002)
Average size of municipalities	−.004***	−.002**	−.0004
	(.0007)	(.001)	(.0006)
Population density in MSA	.033**	.039*	−.013
	(.014)	(.023)	(.010)
Land area of MSA	.002***	.006***	.0017*
	(.0007)	(.002)	(.0009)
Number of families in MSA	−.0136*	.017	−.003
	(.007)	(.90)	(.006)
R^2	.363	.195	.056

Notes: Dependent variable varies by column. "Income", "Education" and "Age" indicate the measures in (21) computed for income (with 25 groups), education (with three groups), and age (with five groups), respectively. Precise definitions of the groups in each case are given in the appendix. "House price volatility" is defined in the text. "Number of municipalities" is the number of MCDs divided by 100, "Average size. of municipalities" is the average population of MCDs (divided by 10000), "Population density in MSA" is the number of families in MSA per square kilometer (divided by 1000), "Land area of MSA" is the size of MSA area in squared kilometers (divided by 10000). "Number of families in MSA" is the size of MSA

population (in millions). The standard deviations of the applied variables are .013, .022, .012 for the dependent variables (from column (1) to column (3)), and .039, .581, 1.22, .076, 1.19, .200 for the regressors (top to bottom). The White's robust standard errors for the coefficient estimates are reported in parentheses. The ***, ** and * indicates statistical significance at 1%, 5% and 10% level, respectively. Sample size is 242.

Table 4. Sorting of movers and stayers

	Owners		Renters		All	
	Movers	Stayers	Movers	Stayers	Movers	Stayers
Income	.080	.043	.063	.130	.044	.036
Education	.063	.046	.063	.063	.054	.044
Age	.020	.013	.020	.036	.015	.011

Notes: The entries of the table refer to the Theil information theory index in (21) computed for the whole US using PUMA level data from the 1990 Census. Precise definitions of the groups in each of the cases (education, age, income) are given in the appendix.

Table 5. Sorting of long and short distance movers

	Long	Short
Income	.216	.099
Education	.089	.067
Age	.053	.021

Notes: The entries of the table refer to the Theil information theory index in (21) computed for the whole US using PUMA level data from the 1990 Census. Precise definitions of the groups in each of the cases (education, age, income) are given in the appendix.

Table 6. Comparing movers and stayers

	Mean		Std. deviation		Tests for equal variance	
	Movers	Stayers	Movers	Stayers	Levene	Brown-F.
Owners						
High School degree, %	0.45	0.46	0.12	0.07	0.00	0.00
College degree, %	0.38	0.26	0.17	0.13	0.00	0.00
Age	42.3	56.2	3.3	3.0	0.00	0.02
Income	46039	42021	15503	13113	0.00	0.00
Renters						
High School degree, %	0.48	0.45	0.09	0.11	0.00	0.00
College degree, %	0.27	0.17	0.13	0.10	0.00	0.00
Age	37.6	53.0	2.5	4.8	0.00	0.00
Income	23405	21760	6924	6839	0.28	0.47

Notes: The entries of the table are computed using PUMA level observations (total 1726). Each PUMA observation is obtained by averaging relevant observations (household heads) in the corresponding PUMA sample (from the 1990 Census). A household head is classified as a mover (a stayer), if he or she did not live (lived) in his or her current house five years ago. "High school degree, %" refers to the share of persons with a high school degree but not a college degree, "College degree, %" refers to the share of persons with at least a college degree, "Age" refers to the average age in years, while "Income" refers to the average annual income of household heads. (See the text for more detailed description of the variables.) "Levene" and "Brown-F.", respectively, refer to the p-values of the Levene (1960) and Browne and Forsythe (1974) tests for the equality of variances.

Figures

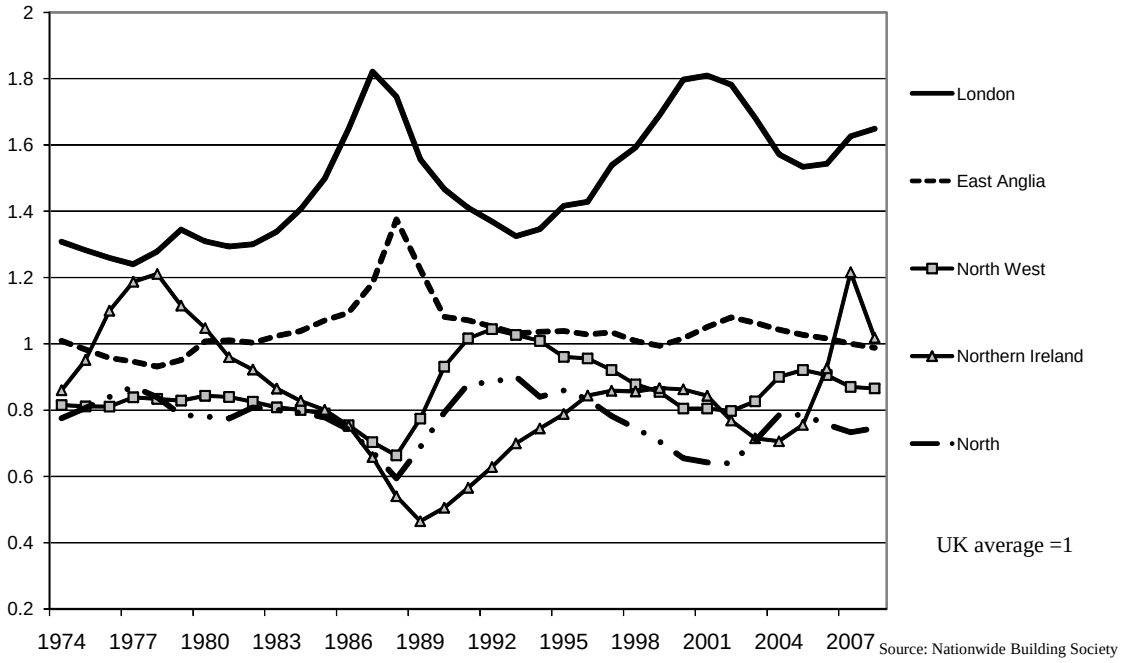

Figure 1: Relative house prices in the UK

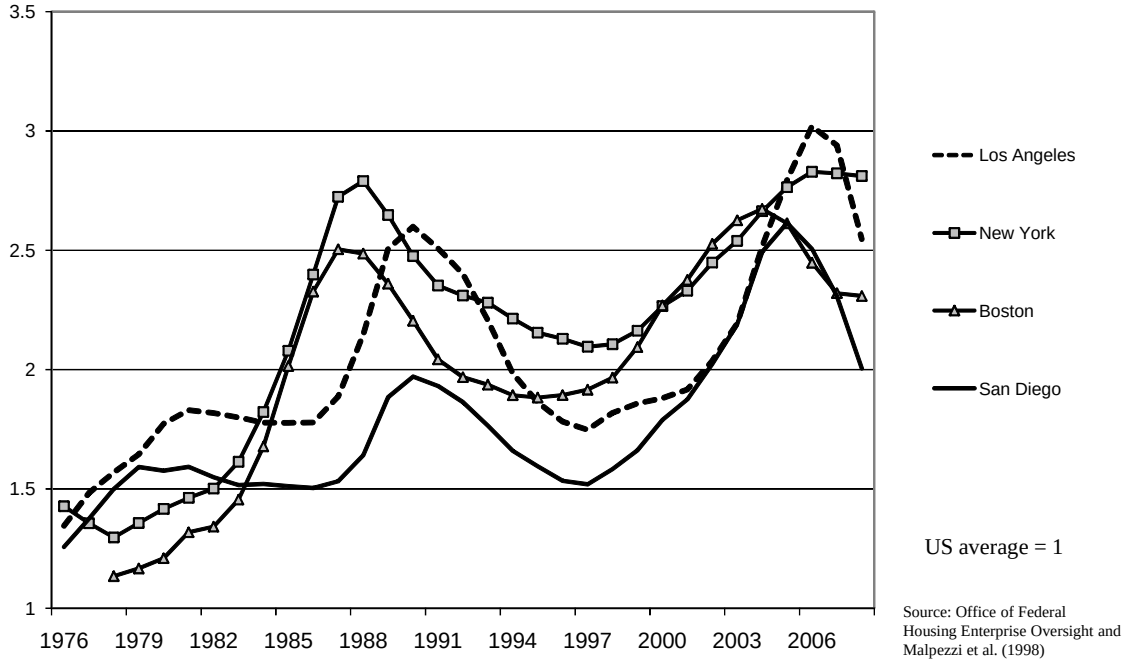

Figure 2: Relative house prices in the US

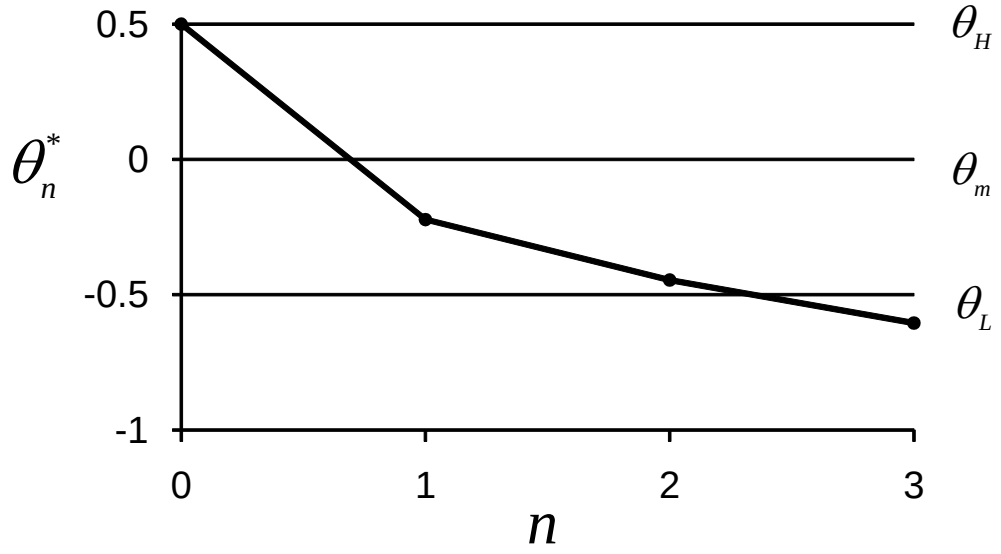

Figure 3: The θ_n^*-curve when θ is uniformly distributed on $[-\frac{1}{2}, \frac{1}{2}]$, $\varepsilon = 1$, $\beta = .95$, and $\pi = .3$.

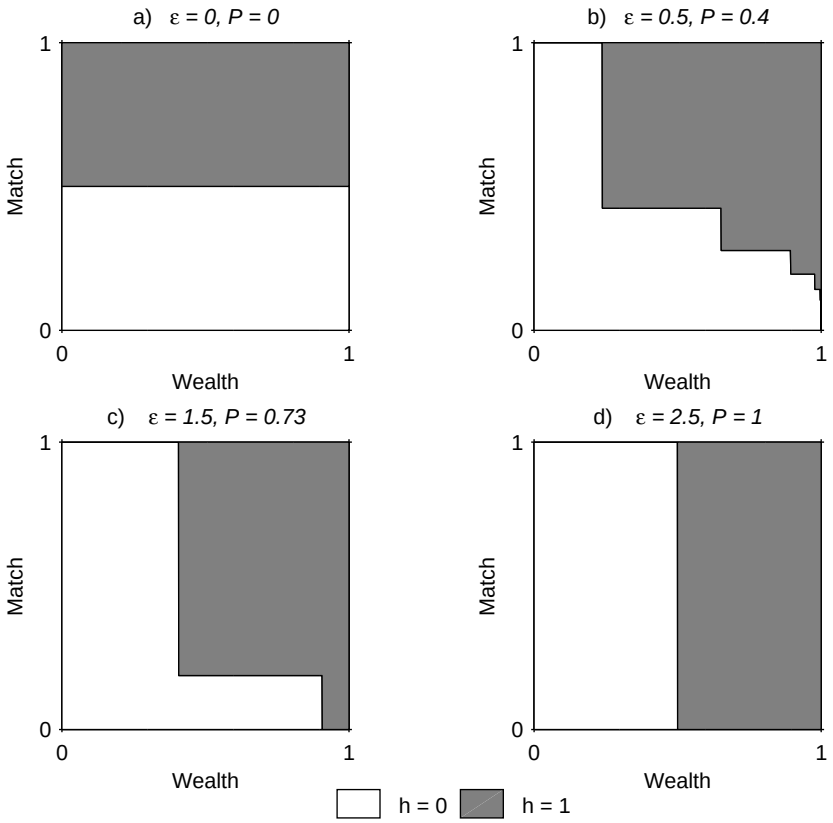

Figure 4: Equilibrium pattern of residential sorting with different values of the regional shock, ε, when the match, θ, is uniformly distributed on $[-\frac{1}{2}, \frac{1}{2}]$, $\beta = .95$, and $\pi = .2$. In each panel, the cumulative wealth (match) distribution is measured on the horizontal (vertical) axis. The measure of house price fluctuations is $P = \frac{1}{2}\frac{1}{E[n]}, P \in [0, 1]$.

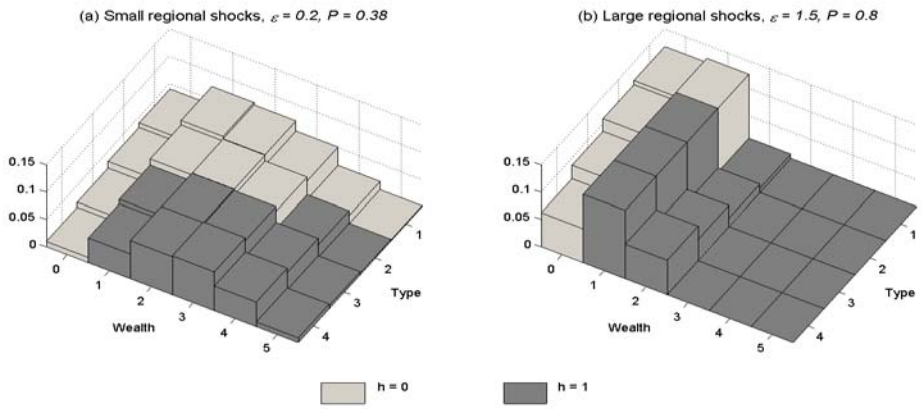

Figure 5: Equilibrium distribution of wealth, match and location, with two values of the regional shock, ε, when the match, θ, is governed by a four-state Markov process. The match realizations are $\theta_1 = -\frac{1}{2},\ \theta_2 = -\frac{1}{6},\ \theta_3 = \frac{1}{6},\ \theta_4 = \frac{1}{2}$ and the associated transition matrices are $\Lambda_0 = \begin{bmatrix} .5 & .5 & 0 & 0 \\ .5 & 0 & .5 & 0 \\ 0 & .5 & 0 & .5 \\ 0 & 0 & .5 & .5 \end{bmatrix}$ (when the match changes for household specific reasons) and $\Lambda_1 = \begin{bmatrix} 0 & 0 & .3 & .7 \\ 0 & .3 & .4 & .3 \\ .3 & .4 & .3 & 0 \\ .7 & .3 & 0 & 0 \end{bmatrix}$ (when the match changes due to a regional shock). In steady state the mass of each realization is $\frac{1}{4}$. The remaining parameters of the model (see text) are $\pi = .3$, $\lambda = .6$ and $\beta = .94$. The measure of house price fluctuations is $P = \frac{1}{2}\frac{1}{E[n]}, P \in [0, 1]$.

Aboa Centre for Economics (ACE) was founded in 1998 by the departments of economics at the Turku School of Economics, Åbo Akademi University and University of Turku. The aim of the Centre is to coordinate research and education related to economics in the three universities.

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