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A Model of Voluntary Childlessness

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A Model of Voluntary Childlessness

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Abstract

Demographers and sociologists have studied and asked for a theory of childlessness for more than two decades, however, this specific choice of zero fertility has not interested economists. Nowadays, facts show us that permanent childlessness can concern up to 30% of all women of a cohort. This paper gives an endogenous fertility model that looks in detail to the mechanisms leading to fluctuations in childlessness. Two mechanisms are considered. The first mechanism goes through the inter-generational evolution of preferences, that can be either exogenous or endogenous. I show that under some values of the parameters, oscillatory dynamics of childlessness may arise. The second mechanism goes through the female labor market; a more gender parity labor environment and an increase in the fixed cost of becoming parents could be an explanation for the dynamics of fertility and childlessness that we have observed in the United States since the early nineteenth century.

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1 Introduction

Over the last few decades, occidental countries have been facing a decrease in their fertility rates. In almost all European countries, total fertility rates are below the replacement fertility rate, 2.1, that is needed to maintain a stable population (2.00 in France, 1.38 in Germany, 1.51 in Greece, 1.37 in Portugal, 1.46 in Spain, 1.91 in Sweden[1]). Throughout this research, I look at one specific feature that is directly related to the fertility decline in developed economies: the increase in the number of couples remaining voluntarily childless. The literature on childlessness is large among demographers and sociologists but very few things have been done by economists.

Poston and Trent (1982) distinguish two types of childlessness, involuntary childlessness and voluntary childlessness. The first happens when the couple is unable to have children because of involuntary features, mainly biological constraints leading to subfecundity. The second type of childlessness, that is voluntary childlessness, can either be defined in a restrictive way such as couples who have never wanted children or in a broader way as couples for whom it just happened to remain childless (Toulemon (1996)). In this research, the definition of voluntary childlessness will include couples who simply do not want to have children as well as couples who remain childless after a series of postponements (delaying childbearing is a more common attitude than a single decision to remain childless for life). This position can be discussed because postponements lead to a decrease in women's fecundity which may end up in an involuntary cause of childlessness. However, as economists that study rational individuals, it is natural for us to define these women as voluntary childless because of the fact that women know from the beginning of their reproductive cycle that they are more fecund at 25 years old than what they are at 35 years old. Poston and Trent (1982) are among the few who have proposed a theoretical analysis of childlessness: their statement is that there is an U-shaped relationship between childlessness and the development level of countries. They argue that childlessness in developing countries is predominantly involuntary (specially caused by nutritional deficiency and diseases) while childlessness in developed countries is mainly voluntary. For example, the completed marital childlessness rate[2] in 1970 of West Germany is the same as the one of Mozambique, 16%, and Luxembourg has also the same rate than Sri Lanka, both 13%. In this research, I do not look at the issue of involuntary childlessness because I consider that it has stabilized to its natural level[3] for developed economies and that today's fluctuations are due to the voluntary component.

[1] *Eurostat* 2008.

[2] The completed marital childlessness rate gives the percentage of childless married women older than 40 years old at one moment in time. For more measures of childlessness, see Poston and Trent (1982).

[3] For the Hutterites women, married before 25 years old, the completed childlessness rate is 2.4%, see Tietze (1957).

The aim of this work is to understand the mechanisms that can be responsible for the dynamics of voluntary childlessness. In Houseknecht (1982), the author explains how voluntary childlessness is affected by three main variables all together: female education, female labor employment and culture. In this research, I concentrate in the last two variables and by culture I mean preferences over fertility. In the model, individuals have different tastes for children: this kind of heterogeneity will have an important role on the household fertility decision and in particular, for specifying the barrier between childlessness and parenthood. A first question that I try to solve is whether endogenous dynamics of preferences for children can explain the dynamics of childlessness. For this, I propose two models, one where the intergenerational transmission of preferences, from parents to children, is exogenous and another where it is endogenous; this last is in line with Bisin and Verdier (2001) where the traits of children depend both on the preferences of their parents and on the social environment. These models are able to replicate the dynamics of childlessness under some special, not very realistic, conditions.

In a second part, I study the role of female labor-force participation in a model with endogenous wages and simulate it. This is in line with the theoretical literature of fertility and female labor market such as Galor and Weil (1996) where changes in relative wages of women with respect to men's can explain the dynamics of fertility rates or Doepke et al. (2008) that looks at how the change in labor demand during World War II influenced the Baby-Boom period. In this last paper, the increased demand for female labor during the war touched women who were old enough to work and when women of the following generation entered the labor market, this was much more competitive, and consequently, these younger women chose to have more children. To my knowledge there is no model that gives a complete analysis of the economic reasons leading a women to remain childless, my contribution to the demographic economic literature is to provide a benchmark model that can account for the long run fluctuations of both fertility and childlessness. The first result is that a switch to a labor environment that gives more opportunities in the labor market to women can be a good explanation of the observed relationship between childlessness and completed fertility for the cohorts born at the beginning of last century. The second result is that an increase in the cost of becoming parents during the mid of this century, can also reproduce empirical evidence on childlessness and fertility for cohorts born between 1930 and 1944, and in particular the positive relationship between both variables.

This paper is organized as follows. Section 2 provides an analysis of the existent literature on childlessness and analyzes childlessness over time and across countries. Section 3 presents the basic model. Section 4 studies two models of inter-generational transmission of preferences. Section 5 introduces endogenous wages and Section 6 concludes.

2 Facts about childlessness

2.1 Childlessness and fertility

First of all, I would like to address the following question: is childlessness just a specific case of an endogenous fertility problem? Or, taking the question in an empirical perspective, is there a persistent link between completed fertility and childlessness? A first intuition would be that whenever fertility is high, childlessness is low and vice versa, in other words, we imagine a negative correlation between fertility and childlessness. In the following paragraphs I look whether this negative correlation exists or not.

United States: In Figure 1, I plot both completed fertility, as children ever born (CEB), and childlessness for different cohorts of women born between 1840 and 1959 in the United States. A clear and unique negative relationship between both variables is not always present; the correlation coefficient is -0.22, which is quite low. So the statement that "as fertility declines, voluntary childlessness should increase"[4] is not that obvious. The relationship between childlessness and CEB is actually positive for the cohorts of women born between 1930 and 1939 that represent the generations just in between the parents of the baby-boom cohorts and the baby-boom cohort itself.

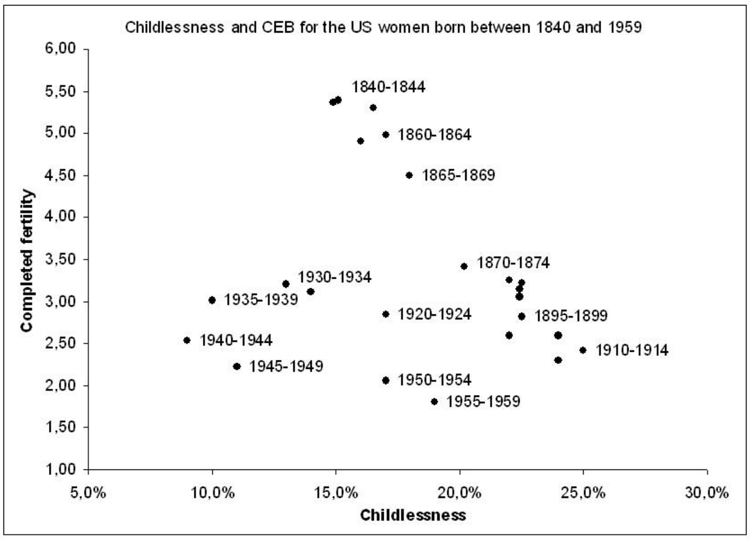

Figure 1: Relationship between childlessness and CEB for women born between 1840 and 1959 in the United States. Note: See Table 1 of the Appendix for details.

[4]Poston and Trent (1982), page 477.

Netherlands: For women born between 1900 and 1954 in the Netherlands, completed fertility has been decreasing and, except for the last cohort, childlessness has decreased as well. The correlation between both variables in this period of time is 0.60. This positive correlation shows that the choice of being childless is a different choice than the one of how many children a woman wants to have.

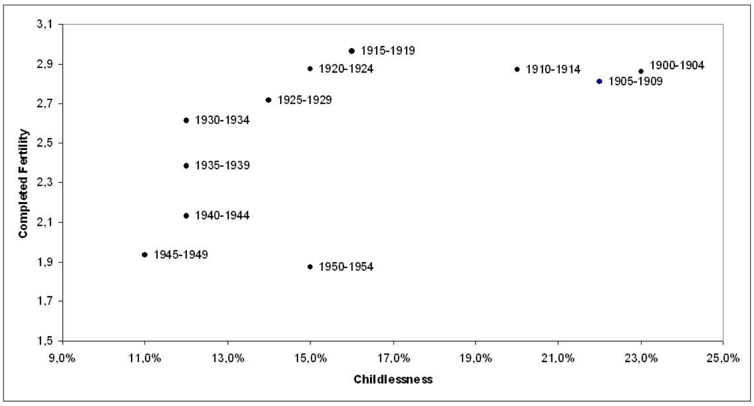

Figure 2: Relationship between childlessness and completed fertility for women born between 1900 and 1954 in Netherlands. Note: See Table 2 of the Appendix for details.

Cross country comparison: Another question to be asked is whether in countries with high fertility we find low childlessness and vice versa. Figure 3 gives the relationship between both variables for some OECD countries for women born in 1965. The correlation between the two variables is -0.27 and it shows that a cross country analysis tells us, again, that fertility and childlessness do not have a clear negative relationship as we could have expected.

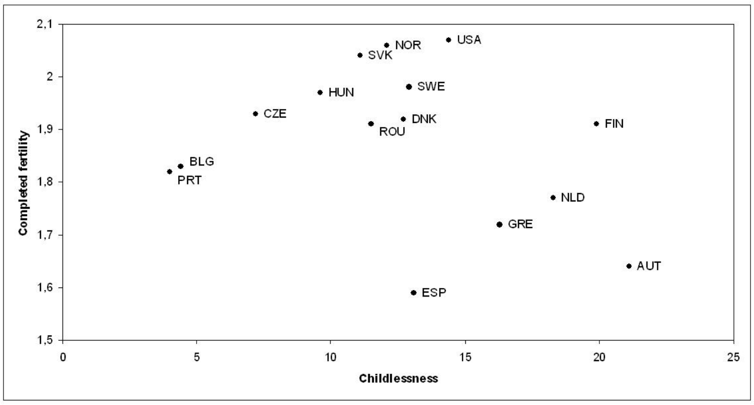

Figure 3: Relationship between childlessness and completed fertility for women born in 1965 in OECD countries. Note: See Table 3 of the Appendix for details.

The conclusion we can take from this brief exposition of facts is that fertility and childlessness are not correlated in a clear and unique way, both through time and across countries. This argument motivates and gives sense to this research that looks for a theory explaining voluntary childlessness.

2.2 Factors affecting childlessness

Through the last two decennies, there has been a general increase in childlessness in developed countries. To illustrate this fact, the *Census Bureau* reveals that, in the United States, childlessness among 40 to 44 years old women has increased from 9.5% in 1981 to 20.4% in 2006. In the United Kingdom, it has risen from around one in ten women born in the mid-1940s to around one in five women born in the late 1950s. The *Federal Statistical Office* of Germany reported in March 2004 that 30% of German women born between 1964 and 1967 in the former territory of the Federal Republic were childless.

This increase in childlessness in developed countries has been explained by a combination of economic, social and cultural reasons, in line with the factors leading to the second demographic transition (see van de Kaa (1987)). Some of these reasons are the decline of social pressures to get married and have children, more individualistic lifestyles centered around an individual's career rather than an interest in finding a partner and building a family, easier ways to get divorced or break up a relationship which directly leads to concerns about the durability of the relationships, financial concerns, dislike of children, the median age of motherhood has increased and consequently, the time available to bear children decreases and the difficulty of becoming pregnant increases, etc. From a social perspective, Blake (1979) analyzes the different attitudes towards childlessness in the United States for different groups. She separates the advantages (i.e. more intimacy among couples) from the disadvantages (i.e. feeling of unfulfillement or loneliness) that childlessness can bring to a couple. The results are that those who are more likely to consider the disadvantages of childlessness are: men, the elder, the less educated, the married and widowed, the Catholics and Jews, the poorer and those residing in small towns. An interesting idea present in Blake (1979) and in Houseknecht (1982) is that for lowly educated individuals, children are associated to social rewards for women and to instruments to give meaning to their life, but for highly educated individuals, the social cost of remaining childless is covered by economic benefits and career commitments. Along with these works, Noordhuizen et al. (2010) looks at the public acceptance of voluntary childlessness, most religious individuals been the ones who tolerated the less voluntary childlessness.

Concerning the role of education, Hoem et al. (2006), stresses that even if it is true that childlessness increases with the level of education, the field of education is a more important indicator for women's potential reproductive behavior: the field of education explains the variation in childlessness more than two times better than the education level. For example, women born between 1955 and 1959 in Sweden who have only reached secondary school have a similar level of childlessness (14.7%) as physicians (15.9%) that have a higher tertiary education. Another example is that women who are trained in personal services in hotels and restaurants, having an education of a two-years secondary level, have even higher childlessness (22%) than medical doctors with a research degree (18.9%) do[5]. Before them, Meier (1958) had already stressed the importance of sectors in the decision of remaining childless or not in a normative analysis of the question. In his research, he introduces the "new non-fertile social roles" for women whose work requires geographical mobility and consequently a non stable home[6]. Concerning population stability, this proportion of women could be adapted depending on whether population is too small or too big.

2.3 Childlessness across countries

Considering the international variability of childlessness, Poston and Trent (1982) show that the difference in childlessness rates between countries is very large. For example, around 1970, the completed marital childlessness rate of Singapore was 4% while for Indonesia it was 12% or the childlessness rate of Martinique was 26% and the one of Dominican Republic 3%. We already said in the introduction that the authors suggest that involuntary childlessness predominates among developing countries and voluntary childlessness predominates among developed countries. A high level of childlessness could be representative for either a developing or a developed country so that the authors suggest a theoretical framework where the relationship between childlessness and development follows a U-shaped pattern. As a country develops, involuntary childlessness decreases because of the causes leading to subfecundity decrease and once it reaches a minimum level (close to the natural biological level of sterility) childlessness increases because of voluntary reasons. The lowest childlessness rates correspond to an intermediate state of development before voluntary childlessness gains influence. The developed countries follow the increasing side of the U-curve because of structural factors, the same leading to fertility decreases, these being industrialization, urbanization, increase in educational attainment, increase in female labor force participation and advances in contraceptive technology. Consequently, dividing countries between developing and developed countries, development is negatively correlated with childlessness for the first and positively related for the seconds. In this paper, I focus on developed countries since I am concerned on voluntary childlessness (see Figure 3).

[5]The average percentage of childlessness among all women in Sweden born between 1955 and 1959 is 15.7%.
[6]For the list of the professions that are more likely to remain childless, see Meier (1958), table 1.

2.4 Childlessness over time

The evolution of the proportion of childless couples over time has been studied in Merlo and Rowland (2000), in details for Australia, and in Rowland (2007), for other developed countries. Merlo and Rowland (2000) gives the proportion of childless women between 45 and 49 years old, married and unmarried, for cohorts born between 1851 and 1951 in Australia. For cohorts born before 1890, the overall (married and unmarried) childlessness rate oscillates from 20% to 25%, this is interpreted in Rowland (2007) as the expected childlessness rates in a population with late marriages and health problems. The highest percentage of childless women are seen for the cohorts born between 1891 and early 1900 (being in their reproductive age during World Wars and the Great Depression): more than 30% of them remained childless. The main factor of such a high childlessness rate was a rise in marital childlessness (childlessness among never married women actually decreases for these cohorts with respect to the past). The lowest percentages of childlessness happens for the cohorts that produced the Baby Boom, born between 1931 and 1941: only 4.9% of wives remained childless (the childlessness rate for both married and non married was 8.8%). This period was also exceptional because it was marked by unusual proportions of couples getting married and having children; there was an idealization of the traditional family and role of women that can be explained by the fact these women did not have the same economic opportunities as men had. For recent cohorts, the Australian Bureau of Statistics (ABS) estimates that 28% of women that are currently in their reproductive age will remain childless, however, taking into account that the age of the first birth is being delayed, Rowland (2007) estimate that around one women in five who are currently in the reproductive age will remain childless in Australia.

Rowland (2007), using European, Australian, American, and Japanese data, shows that historical trends in childlessness rates are very similar across developed countries. The statistics for these countries reveal three main features. First, a peak in childlessness rates for the 1880-1910 birth cohorts: approximately one fifth to one fourth of women belonging to the 1900 birth cohort remained childless. Then, a pronounced decline in the proportions of childless women born between 1900 and 1940, until reaching minimum levels of 10% of childless women. And, third feature, a revival of childlessness among more recent cohorts born after the Second World War. In the United States, the revival of childlessness starts for women born around 1945 or, if we look at period data, around 1982. In Juhn and Kim (1999) one of the results is precisely that it is in the 1980s that women increased their relative supply of skills in the economy, the increase in childlessness being certainly related with the increase in professional attainment of women. The increase in childlessness among younger cohorts also comes with an increase in the public acceptance of voluntary childlessness: in the Netherlandts in 1965, only

22.7% of the Dutch population approved that a married couple decided not to have children, while in 1996, the proportion had increased to 89.8% (Noordhuizen et al. (2010)).

Data on childlessness tells us that the childlessness rate fluctuations are very similar in all Western countries. This fluctuations can be affected both by exogenous shocks, such as wars and depressions, and endogenous changes, such as the evolution of cultural and social norms, as well as changes in the female labor market. The following graphic illustrates the evolution of childlessness among women born in different cohorts at ages of 45-49 years old (it shows the evolution of childlessness among all women and not only married women).

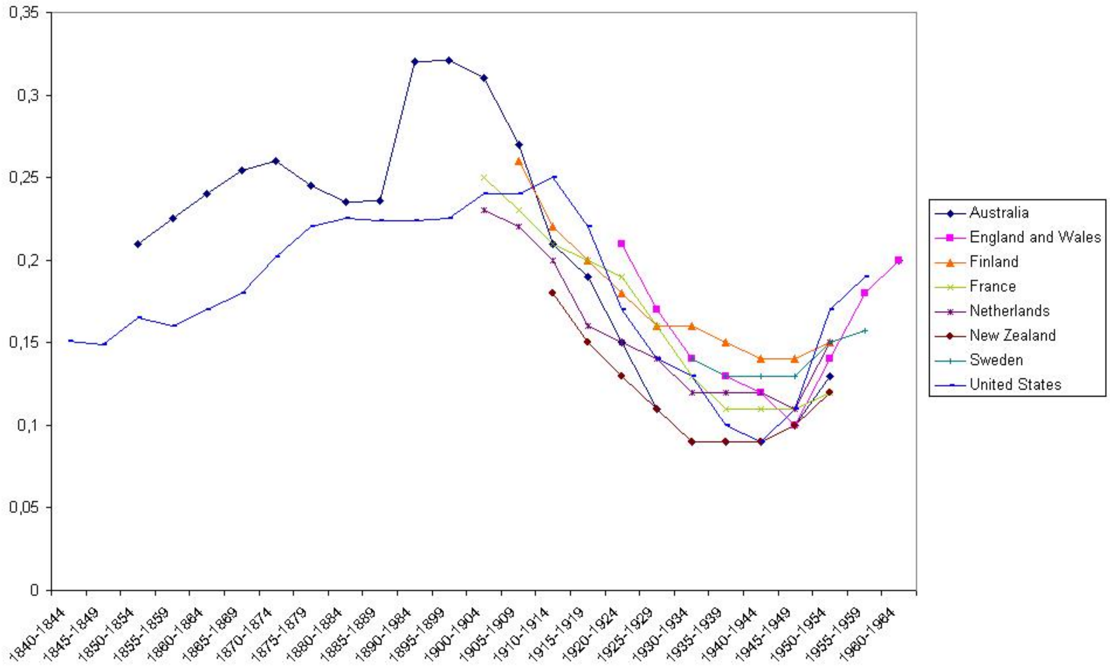

Figure 4: Evolution of childlessness for women cohorts born between 1840 and 1960

3 The model

I consider an overlapping-generations model with an infinite discrete time framework in which individuals live for two periods: childhood and adulthood. In the first period of life (childhood), individuals consume a fixed amount of time from their parents. In the second period of life (adulthood), people take decisions about consumption and fertility. Fertility is measured in terms of couples of children and a couple is made by a female and a male. During childhood, men and women are identical. When adults, they differ in their wages and the fact that only women has to give up some of her time to bear and raise children. The model introduces intragenerational heterogeneity by allowing preferences over fertility to vary across individuals and couples. In this first section, wages are exogenous, I extend this hypothesis in Section 5.

3.1 Utility function of households

Individuals are indexed by i and households by j. Members of a couple take joint decisions about consumption, c_t^j, and the number (of couples) of children they want to have, n_t^j. The joint utility function of a couple j at time t is the following:

$$U_t^j(c_t^j, n_t^j) = \ln\left(c_t^j\right) + \gamma^j n_t^j \tag{1}$$

This utility function of the couple is an additively separable function of their consumption and the number of children they have. It is logarithmic in the couple's consumption, in line with Barro and Becker (1986), where the utility of a parent from consumption is given by an increasing and concave utility function. The couple's utility also depends linearly on the number of children they have, this is similar to the framework used in Barro and Becker (1986) except that in their case, they incorporate the utility of children into the utility of parents while I suppose that parents are not altruistic in the utility of their children but just in the life of them[7]. The variable $\gamma^j > 0$ multiplying the fertility decision of the couple, is the willingness of having children of the couple j. This willingness is given by an average between the taste for children of the woman, γ^{if}, and the taste of the man, γ^{im} (each member of the couple has the same bargaining power):

$$\gamma^j = \frac{\gamma^{im} + \gamma^{if}}{2}$$

3.2 Constraint

Each adult has one unit of time; men use this time for working and women use it either for working or to bear and raise the children of the couple. The assumption that only women raise children is realistic: there is a biological fact that men do not get pregnant or breastfeed but there is also a social component revealed by the fact that among parents of children under 18 who are full-time workers, married mothers are more likely to provide childcare to the children and to do household activities than fathers[8]. Raising children implies an opportunity cost for women: the time spent with the children is no longer available to work and the higher her wage, the higher this opportunity cost.

[7]See Appendix C for a utility function that is logarithmic in n_t^j: $U_t^j(c_t^j, n_t^j) = \ln\left(c_t^j\right) + \gamma^j \ln\left(\mu + n_t^j\right)$.

[8]*U.S. Bureau of Labor Statistics* in the release *Married Parents' Use of Time Summary* (2008).

A non-childless couple faces both a time cost and a fixed cost. The time cost, $\theta \in [0,1]$, is related to the bearing and raising of a child, it includes the pregnancy and breast-feeding time as well as the home production tasks such as cleaning, cooking and transport. The fixed cost, k, can be interpreted as a start-up cost to having children, this can be buying a larger house, buying a car, preparing the first pregnancy or life insurances. It can also be seen as an obligation to protect and raise children, quoting Dasgupta (2005):

> "People do not have an obligation to become parents, of course, but they acquire one toward their children if they choose to become parents."

This is in line with what is stated in the United Nations Convention on the Rights of the Child text. A last interpretation, and close to the precedent, is the fact that the loss of freedom and flexibility of a couple are mainly related to the coming of the first child (Espenshade (1977)). This fixed cost is also present in Bick (2010) who also points out the fact that k could also be negative, meaning that a couple receives an utility gain for the births of its first child. Empirical evidence for the presence of this type of cost is given in Espenshade (1977), that clearly shows the difference in terms of costs of a first child compared to the second.

The household constraint takes the following form,

$$c_t^j = w^m + \left(1 - \theta n_t^j\right) w^f - kI\left(n_t^j\right) \tag{2}$$

where w^m and w^f are respectively the wages per unit of time for men and for women. The dichotomic variable, $I(n_t^j)$, differentiates the constraint between childless and non-childless couples in the following way,

$$\begin{cases} I(n_t^j) = 0 & \text{if} \quad n_t^j = 0 \Rightarrow c_t^j = w^m + w^f \\ I(n_t^j) = 1 & \text{if} \quad n_t^j > 0 \Rightarrow c_t^j = w^m + (1 - \theta n_t^j) w^f - k \end{cases}$$

3.3 The household problem

Couples solve the following problem,

$$\max_{c_t^j, n_t^j} \quad U_t^j(c_t^j, n_t^j) = \ln(c_t^j) + \gamma^j n_t^j$$

$$s.t. \qquad c_t^j = w^m + (1 - \theta n_t^j) w^f - kI(n_t^j)$$

$$\text{and} \qquad 0 \leq n_t^j \leq \frac{1}{\theta}$$

where

$$\begin{cases} I(n_t^j) = 0 & \text{if} \quad n_t^j = 0 \\ I(n_t^j) = 1 & \text{if} \quad n_t^j > 0 \end{cases}$$

There are three possible solutions to this problem for a couple j: two corner solutions and an interior solution. Assuming that $w^m > k$, the corner solutions (n^0, c^0) and $(n^{n_{\max}}, c^{n_{\max}})$ are given by,

$$\begin{cases} n^0 = 0 \\ c^0 = w^m + w^f \end{cases}$$

and

$$\begin{cases} n^{n_{\max}} = \frac{1}{\theta} \\ c^{n_{\max}} = w^m - k \end{cases}$$

Defining by $V_t^j(n_t^j)$ the indirect utility function of a couple j at time t, such as,

$$V_t^j(n_t^j) = \ln\big(w^m + (1-\theta n_t^j)w^f - k\big) + \gamma^j n_t^j$$

The maximization problem for the interior solution (n^{j*}, c^{j*}) is then given by,

$$\max_{n_t^j} \quad V_t^j(n_t^j) = \ln(w^m + (1-\theta n_t^j)w^f - k) + \gamma^j n_t^j$$

$$s.t. \qquad 0 < n_t^j < \frac{1}{\theta}$$

The solution to this problem is given by,

$$n^{j*} = \frac{w^m + w^f - k}{\theta w^f} - \frac{1}{\gamma^j} \tag{3}$$

and

$$c^{j*} = \frac{\theta w^f}{\gamma^j}$$

Proposition 3.1. *There exists a unique value of $\gamma^j \equiv \gamma^*$ for which couples are indifferent between being childless or not.*

Proof. See Appendix B. □

This allows us to define two types of couples,

1. The ones with high willingness for children, with $\gamma^j \geq \gamma^*$, that choose the interior solution.
2. The ones with low willingness for children, with $\gamma^j < \gamma^*$, that remain childless (these are often called DINKS in the media or the marketing literature, standing for "double income, no kids").

Effect of a change in wages on γ^*: Applying the implicit function theorem to the function Ξ defined as,

$$\Xi(\gamma, w^m, w^f, k) = v(\gamma, w^m, w^f, k) - z(\gamma, w^m, w^f, k)$$

we can check that $\frac{\delta\gamma^*}{\delta w^m} < 0$ meaning that an increase in the wage of men reduces the critical level γ^*. The relationship between γ^* and w^f is not clear and γ^* increases with the fixed cost k.

Note: If we set $k = 0$, we would would have childlessness for values such that $\gamma^j \leq \frac{\theta w^f}{w^m + w^f}$.

3.4 Comparative analysis

Effect of a change in w^m on fertility: From the fertility of the couple given by the interior solution (3), we can check that holding women's wage constant, fertility is linearly increasing in men's wage:

$$\frac{\delta n^{j*}}{\delta w^m} = \frac{1}{\theta w^f} > 0$$

The reason is due to the assumption that all childrearing is done by women, therefore, the increase in men's wage has a pure income effect on the fertility decision of a couple.

Effect of a change in w^f on fertility: Keeping men's wage constant, an increase in women's wage has both an income effect and a substitution effect on fertility: it raises the overall income of the couple but the time that is not dedicated to work becomes more expensive.

$$\frac{\delta n^{j*}}{\delta w^f} = \frac{k - w^m}{\theta (w^f)^2}$$

Fertility can then be either increasing or decreasing on women's wage:

- $k > w^m \Rightarrow \frac{\delta n_t^j}{\delta w^f} > 0$; in order to have $n_t^j > 0$ for this case, then $\gamma^j > \theta$ must hold because,

  $$\lim_{w^f \to \infty} n_t^j = \frac{1}{\theta} - \frac{1}{\gamma^j}$$

  This can be interpreted in the following way: when the willingness for children of a couple is higher than the time cost of raising children, and men's wage is not high enough to cover the fixed cost of having children, the income effect of an increase in the woman's wage will dominate the substitution effect and fertility will increase. This positive relation between women wages and fertility, due to the presence of a fix cost of having children, is a particularity of this model and it could partly explain the relatively higher fertility levels of lower income groups. When the time cost of the children is larger than the willingness for children, the couple will have no children for any increase in the woman's wage.

- $k < w^m \Rightarrow \frac{\delta n_t^j}{\delta w^f} < 0$; if the fixed cost of having children is covered by men's wage (which is the most likely to occur), we will have that a higher w^f will reduce the fertility of the couple since the higher wage increases the opportunity cost of having children more than the household income. This negative relationship between women wages and fertility is what we usually find in the literature (Galor and Weil (1996)). Without the fix cost k, and keeping the same structure of the model, this inverse relationship between fertility and women's wage would always hold.

The following figure illustrates the relationship between the wage of women and fertility in the case where $k < w^m$ and $\gamma^j < \theta$. We can see that fertility will be constant and equal to the corner solutions for either a very low wage for women ($n^j = n^{n_{\max}}$) or a high wage for women ($n^j = n^0$). At the interior solution, a higher female wage decreases fertility.

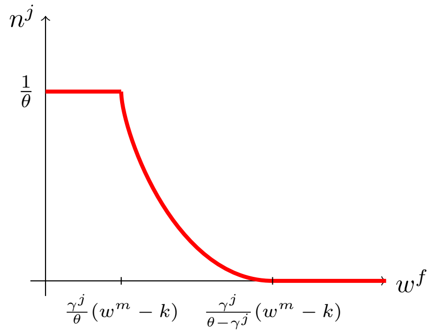

Figure 5: Couple's fertility as a function of women's wage

4 Dynamics

In this section I look at the dynamics of population groups assuming that there are two types of people, some with high taste for children and others with low taste for children. The main question here is whether a model of inter-generational transmission of preferences can explain the dynamics of childlessness.

4.1 Classifying marriages

I assume that there are only two values for the individual's taste for children: a low value, $\underline{\gamma}$, for the individuals with low taste for children and a high value, $\overline{\gamma}$, for the individuals with high taste for children. Since both members of a couple have the same bargaining power in the decision to have children, we will have three different types of couples:

1. $(\underline{\gamma}\ \underline{\gamma})$: couple $j = 1$, characterized by $\gamma^1 = \underline{\gamma} < \gamma^*$ so that it remains childless

2. $(\overline{\gamma}\ \underline{\gamma})$: couple $j = 2$, with $\gamma^2 = \frac{\overline{\gamma}+\underline{\gamma}}{2} \geq \gamma^*$ and a fertility rate $\overline{n}$

3. $(\overline{\gamma}\ \overline{\gamma})$: couple $j = 3$, with $\gamma^3 = \overline{\gamma} > \gamma^2$ and the highest fertility rate $\overline{\overline{n}}$.

For couples $j = 2$ and $j = 3$, the number (of pairs) of children they will have are the following,

$$\overline{n} = \frac{w^m + w^f - k}{\theta w^f} - \frac{2}{\overline{\gamma} + \underline{\gamma}}$$

$$\overline{\overline{n}} = \frac{w^m + w^f - k}{\theta w^f} - \frac{1}{\overline{\gamma}}$$

Total population at time t, denoted by P_t, will be defined by the sum of the individuals with high taste for children, $\overline{P_t}$, and the individuals with low taste for children, $\underline{P_t}$, as follows,

$$P_t = \underline{P_t} + \overline{P_t} \tag{4}$$

Random matching: I assume that couples match randomly. This assumption might seem unrealistic and too simplificative of the marriage market because high rates of homogamy are found for some social groups such as French aristocrats, individuals of a particular religion or among educated and non educated individuals (see Bisin and Verdier (2000)). Random matching would be a wrong way to model marriage if individuals would differ in one of these observable traits since it would clearly underestimate homogamous marriages. However, in our case, we have heterogeneity in taste for children, which is not an observable characteristic (unlike the social class, religion, ethnicity or education). To my knowledge, there is no evidence about the fact that individuals with high or low taste for children are matched together. Moreover, homogamous marriages mainly arise because of a certain value or characteristic the parents want to transmit to their children. I do not know any study showing that taste for children is a characteristic that parents want to transmit (it cannot be the case of childlessness of course). Consequently, the taste for children is not shown to be a characteristic that segregates the marriage market and this allows me to assume a random matching marriage market, which allows to compute the proportions of each type of marriage as follows[9]:

1. The proportion of marriages of type 1, at time t, is: $\left(\frac{\underline{P_t}}{\overline{P_t}+\underline{P_t}}\right)^2$
2. The proportion of marriages of type 2, at time t, is: $\frac{2\overline{P_t}\underline{P_t}}{\left(\overline{P_t}+\underline{P_t}\right)^2}$
3. The proportion of marriages of type 3, at time t, is: $\left(\frac{\overline{P_t}}{\overline{P_t}+\underline{P_t}}\right)^2$

[9]In Appendix D, I relax the hypothesis of random matching allowing for a degree of "assortativeness". I show that this does not change the results.

Denoting by n_t the average number of couples of children by couple at time t, this will take the following form:

$$n_t = \left(\frac{\overline{P_t}}{\overline{P_t}+\underline{P_t}}\right)^2 \overline{\overline{n}} + \frac{2\overline{P_t}\underline{P_t}}{\left(\overline{P_t}+\underline{P_t}\right)^2}\overline{n} + \left(\frac{\underline{P_t}}{\overline{P_t}+\underline{P_t}}\right)^2 0$$

and simplifying we get,

$$n_t = \frac{\overline{P_t}}{(\overline{P_t}+\underline{P_t})^2}\left(\overline{P_t}\overline{\overline{n}} + 2\underline{P_t}\overline{n}\right) \tag{5}$$

4.2 Dynamics of the population

In this subsection we compute two analyzes: in a first part we assume that exogenous probabilities relate the willingness for children of a couple to the taste for children that a child of this couple will have in the next period, and in a second part we endogenize these probabilities making the assumption that the taste for children depends both on the willingness of parents and on the average fertility rate of the population.

4.2.1 Exogenous probabilities

Lets denote the following probabilities:

a	the probability of having a child with $\overline{\gamma}$ in a marriage of type $(\overline{\gamma}\ \overline{\gamma})$
b	the probability of having a child with $\overline{\gamma}$ in a marriage of type $(\overline{\gamma}\ \underline{\gamma})$

The dynamics for the two groups are given by the following equations,

$$\overline{P_{t+1}} = 2a\overline{\overline{n}}\ \underbrace{\left(\frac{\overline{P_t}}{\overline{P_t}+\underline{P_t}}\right)^2 \frac{\overline{P_t}+\underline{P_t}}{2}}_{\text{number of marriages of type } \overline{\gamma}\overline{\gamma}} + 2b\overline{n}\ \underbrace{\frac{2\overline{P_t}\underline{P_t}}{\left(\overline{P_t}+\underline{P_t}\right)^2}\frac{\overline{P_t}+\underline{P_t}}{2}}_{\text{number of marriages of type } \overline{\gamma}\underline{\gamma}}$$

and

$$\underline{P_{t+1}} = 2(1-a)\overline{\overline{n}}\left(\frac{\overline{P_t}}{\overline{P_t}+\underline{P_t}}\right)^2 \frac{\overline{P_t}+\underline{P_t}}{2} + 2(1-b)\overline{n}\frac{2\overline{P_t}\underline{P_t}}{\left(\overline{P_t}+\underline{P_t}\right)^2}\frac{\overline{P_t}+\underline{P_t}}{2}$$

that can be simplified as follows,

$$\overline{P_{t+1}} = \frac{1}{\overline{P_t}+\underline{P_t}}\left(a\overline{\overline{n}}\,\overline{P_t}^2 + 2b\overline{n}\,\overline{P_t}\underline{P_t}\right) \tag{6}$$

and

$$\underline{P_{t+1}} = \frac{1}{\overline{P_t}+\underline{P_t}}\left((1-a)\overline{\overline{n}}\,\overline{P_t}^2 + 2(1-b)\overline{n}\,\overline{P_t}\underline{P_t}\right) \tag{7}$$

These two equations, describing the dynamics of the groups, can also be expressed by a single difference equation of order one;

$$z_{t+1} = \frac{a\overline{\overline{n}}z_t + 2b\overline{n}}{(1-a)\overline{\overline{n}}z_t + 2(1-b)\overline{n}} \equiv \phi(z_t) \tag{8}$$

where $z_t = \frac{\overline{P_t}}{\underline{P_t}}$ is the relative group of individuals with high taste for children. Computing the first and second order derivative of $\phi(z_t)$ we have that,

$$\phi'(z_t) = \frac{2\overline{\overline{n}}(a-b)\overline{n}}{\left((1-a)\overline{\overline{n}}z_t + 2(1-b)\overline{n}\right)^2}$$

and

$$\phi''(z_t) = \frac{-4\overline{\overline{n}}^2\overline{n}(a-b)(1-a)}{\left((1-a)\overline{\overline{n}}z_t + 2(1-b)\overline{n}\right)^3}$$

and we can easily see that,

$$\phi(0) = \frac{b}{1-b} > 0$$
$$\lim_{z_t \to \infty} \phi(z_t) = \frac{a}{1-a} > 0$$

The proportion of childless women at time t, denoted by χ_t, can be expressed in terms of z_t as follows,

$$\chi_t = \frac{1}{(z_t+1)^2} \tag{9}$$

and the average number (of couples) of children, n_t can also be rewritten as,

$$n_t = \frac{z_t}{(1+z_t)^2}\left(z_t\overline{\overline{n}} + 2\overline{n}\right) \tag{10}$$

Definition 4.1 (Steady State)**.** *We define a steady state, a state where the relative group of individuals with high taste for children, $z_t = \frac{\overline{P_t}}{\underline{P_t}}$, is constant over time, so that $z^{ss} = z_t = z_{t+1} = \dots$.*

From Equation (8), we see that there is a unique, positive, steady state, z^*, where $z^* = \phi(z^*)$, equal to;

$$z^* = \frac{-\left((1-b)\overline{n} - \frac{a}{2}\overline{\overline{n}}\right) + \sqrt{\left((1-b)\overline{n} - \frac{a}{2}\overline{\overline{n}}\right)^2 + 2(1-a)\overline{n}b\overline{\overline{n}}}}{(1-a)\overline{\overline{n}}}$$

For the study of the dynamics of z, we distinguish three cases: $a > b$, $a = b$ and $a < b$.

Case $a > b$: This is when the probability for a child to have a high taste for children, $\overline{\gamma}$, is higher for children coming from couples who have the highest willingness for children ($j = 3$) than for children coming from parents with a lower willingness ($j = 2$). We have that $\phi'(z_t) > 0$ and $\phi''(z_t) < 0$, so that the function $\phi(z_t)$ is strictly increasing and concave in $\mathbb{R}_+$. The dynamics of z are, consequently, monotonic and converge to a unique positive steady state, whatever the initial condition z_0. The following graphic illustrates this,

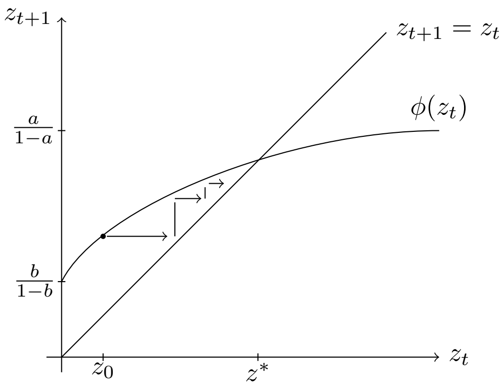

Figure 6: Monotonic dynamics in the case $a > b$

Monotonicity: Starting at a time $t = 0$, from any level $z_0 < z^*$, we have $z_t < z_{t+1}$, meaning the group of individuals with high taste for children increases relative to the group of people who dislike children until reaching the steady state level z^*. Reversely, from an initial value $z_0 > z^*$, we have $z_t > z_{t+1}$ and consequently the dynamics are decreasing. Since from any initial level z_0 we converge to the steady state level z^*, we can say that this steady state is globally stable in $\mathbb{R}_+$.

No extinction: The steady state value, z^*, is strictly positive, this means that, in the long run, none of the two groups will become extinct. If $z^* = 0$, this would imply that the population with high taste taste for children would disappear and if we had $z^* = \infty$, then the population disliking children would disappear. None of this cases is possible here, so that both groups will always be present.

Proposition 4.1. *If $a > b$, the dynamics of z_t are monotonic, and converge to a unique globally stable steady state.*

The intuition behind the proposition becomes clear if we consider respectively the proportion of children coming from marriages of type 2 (mixed marriages) and of type 3:

$$\frac{2\overline{n}}{\overline{\overline{n}}z_t + 2\overline{n}} \tag{11}$$

and

$$\frac{\overline{\overline{n}} z_t}{\overline{\overline{n}} z_t + 2\overline{n}} \tag{12}$$

Considering a case where z_0 is initially too low ($z_0 < z^*$), the proportion of children coming from marriages of type 2 is high compared to children coming from the other type of marriage. Since $a > b$, the proportion of children coming from the third type of marriage is then likely to increase, which will increase z (because those are the individuals with the highest probability to have a high taste for children) until it reaches the steady state.

Case $a = b$: We have that,

$$\phi(z_t) = \frac{a}{1-a}$$

so that $\phi'(z_t) = 0$. This says that the function ϕ is a constant and that the steady state is reached in one period: if z_0 is lower or higher than z^*, then in period one we will be at the steady state, which is globally stable as before and depends only in the value of a.

Proposition 4.2. *If $a = b$, the steady state is reached in one period.*

Case $a < b$: This is when the probability for a child to have a high taste for children, is higher for children of couples of type 2 than for children having parents with the highest willingness. We have that, $\phi'(z_t) < 0$ and $\phi''(z_t) > 0$. To analyze the stability of z^* in this case, I will proceed in two steps: first, I will compute the value $\bar{z}$ for which we have $\phi'(\bar{z}) = -1$ and then compare it to the steady state value z^* that we already computed. If $z^* > \bar{z}$ then $0 > \phi'(z^*) > \phi'(\bar{z}) = -1$ and consequently, z^* is locally stable. If $z^* < \bar{z}$ then $\phi'(z^*) < -1$ and z^* is unstable. If $z^* = \bar{z}$ then z^* is non-hyperbolic since $\phi'(z^*) = \phi'(\bar{z}) = -1$.

$$\phi'(\bar{z}) = -1$$

$$\Leftrightarrow \bar{z} = \frac{-2(1-b)\overline{n} + \sqrt{2\overline{n}(b-a)\overline{\overline{n}}}}{(1-a)\overline{\overline{n}}}$$

Now, comparing this with z^*, we have that,

$$\begin{aligned} z^* - \bar{z} = & \frac{-\left((1-b)\overline{n} - \frac{a}{2}\overline{\overline{n}}\right) + \sqrt{\left((1-b)\overline{n} - \frac{a}{2}\overline{\overline{n}}\right)^2 + 2(1-a)\overline{n} b \overline{\overline{n}}}}{(1-a)\overline{\overline{n}}} \\ & - \frac{-2(1-b)\overline{n} + \sqrt{2\overline{n}(b-a)\overline{\overline{n}}}}{(1-a)\overline{\overline{n}}} \end{aligned}$$

giving,

$$z^* - \bar{z} = \frac{\sqrt{\left((1-b)\overline{n} - \frac{a}{2}\overline{\overline{n}}\right)^2 + 2(1-a)\overline{n}b\overline{\overline{n}}} - \sqrt{2\overline{n}(b-a)\overline{\overline{n}}} + \frac{a}{2}\overline{\overline{n}} + (1-b)\overline{n}}{(1-a)\overline{\overline{n}}}$$

and by looking at the squared roots of the numerator, we can easily conclude that,

$$z^* > \bar{z}$$

Since the function $\phi'(z_t)$ is increasing when $a < b$ we can say that,

$$0 > \phi'(z^*) > \phi'(\bar{z}) = -1$$

consequently, the steady state z^* is locally stable.

Proposition 4.3. *If $a < b$, the dynamics are oscillatory and z^* is locally stable.*

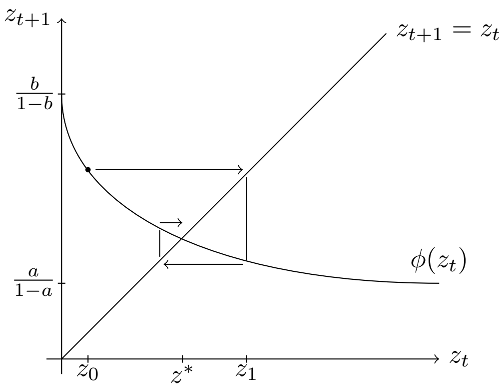

Figure 7: Oscillatory dynamics in the case $a < b$

The relationship $a < b$ means that children of small families are more likely to have a high taste for children than the ones born in bigger families. This might seem a little unrealistic but we could argue that if we consider a family with only one child, this child might feel lonely during his childhood and therefore will not want his own children to feel the same, so that he will want more children than his parents did. The reverse could happen in big families where children get fed up with noise and disorder. The intuition behind the oscillations can be due to the following mechanism: suppose that we start from a low level $z_0 < z^*$, as before we will have a higher proportion of children coming from the mixed couples, but now, those are the most likely to have a high taste for children, consequently in period 1 there will be many individuals with high taste for children and z_1 will be high. In period 2, the reverse will happen; a higher proportion of individuals coming from the third type of marriage, who are less likely to have a high taste for children, then z_2 will be low, and this will continue until reaching the steady state.

Comparison between the model and the facts: We can conclude that this model, with exogenous probabilities, only explains endogenous fluctuations in childlessness when $a < b$, which seems unrealistic. The most realistic case, $a > b$, implies monotonic dynamics that are clearly not present in the reality described in the first part of the paper. However, it still brings us a positive result from numerical simulations: a positive shock on w^m (w^f) decreases (increases) z^* and increases (decreases) both n^* and χ^*. This is interesting because this very simple model allows us to have a positive correlation between childlessness and fertility. The intuition is that an increase in the wage of men increases $\overline{n}$ and $\overline{\overline{n}}$ by the same amount, however, for couples of type 2, this increase is bigger in relation of the number of children they had before the shock than for couples of type 3, and since the fertility of the couples that are less likely to have children with high taste for children is the most affected one, then z^* and n^* decrease, and χ^* increases.

The next step will be to enlarge the model in order to see whether, with endogenous probabilities, we can go further in the analysis.

4.2.2 Endogenous probabilities

Here I enlarge the model assuming that the probability of having a child either with high taste for children when adult, $\overline{\gamma}$, or low taste, $\underline{\gamma}$, depends both on the willingness for children of the parents and on the average fertility of population. An empirical justification for this framework is given in Fernández and Fogli (2006) where the authors show that both family experience and cultural heritage are two determinant factors of the fertility choice. Accordingly, we consider the following probability functions,

$$a_t = (\overline{\gamma})^{\tau}(n_t)^{\eta}$$

and

$$b_t = \left(\frac{\overline{\gamma}+\underline{\gamma}}{2}\right)^{\tau}(n_t)^{\eta}$$

where $\tau \in [-1, 1]$ determines the weight of parental willingness for children on the taste of their own children and $\eta \in [-1, 1]$ can be interpreted as an externality of the average fertility influencing the taste for children, in other words, how the fertility behavior of one generation affects the taste for children of the next one. In Fernández and Fogli (2006) they show that "women whose parents were born in countries where women had more children, tend to have more children themselves", supporting the idea that $\eta > 0$, and "women from larger families tend to have more children", supporting $\tau > 0$, this last relationship between the taste of parents and the one of children is also sustained in Ben-Porath (1975). In Berent (1953) the author tests the hypothesis that family size runs through generations (which corresponds to $\tau > 0$ in our case); this hypothesis is verified in the population studied (married women in Great Britain), indeed, couples coming from higher families had themselves a higher fertility in average (Table 1 in Berent

(1953)). In Rowland (2007), it is also argued that, in Australia, the birth cohorts that had the lowest average family size also had the highest childlessness rate.

Replacing in a_t and b_t average fertility by its expression given in Equation (10) and then introducing these probability functions into the difference equation (8), we can obtain with some simple arrangements the following expression for the dynamics of z_t,

$$z_{t+1} = \frac{\left(\left(\frac{z_t}{z_t+1}\right)^2 \overline{\overline{n}} + \frac{2z_t}{(z_t+1)^2}\overline{n}\right)^\eta \left((\overline{\gamma})^\tau \overline{\overline{n}} z_t + 2\left(\frac{\overline{\gamma}+\underline{\gamma}}{2}\right)^\tau \overline{n}\right)}{\overline{\overline{n}} z_t + 2\overline{n} - \left(\left(\frac{z_t}{z_t+1}\right)^2 \overline{\overline{n}} + \frac{2z_t}{(z_t+1)^2}\overline{n}\right)^\eta \left((\overline{\gamma})^\tau \overline{\overline{n}} z_t + 2\left(\frac{\overline{\gamma}+\underline{\gamma}}{2}\right)^\tau \overline{n}\right)} \equiv \Phi(z_t) \tag{13}$$

This expression does not provide direct analytical results, but we can still take some intuitions from it by studying five cases: $\eta = 1$, $\eta = \frac{1}{2}$, $\eta = 0$, $\eta = -\frac{1}{2}$ and $\eta = -1$. The details of the computations can be found in Appendix E.

Case $\eta = 1$: A numerical analysis of this case tells us that the only steady state that seems to exist is the trivial one. For high values of τ, this steady state is stable (implying that $n^* = 0$ and $\chi^* = 1$) and it becomes unstable for lower values of τ. This means that when people are very influenced buy the others behavior, then everyone becomes childless.

Case $\eta = \frac{1}{2}$: For some positive values of τ, the dynamics are monotonic and converge to a positive and stable steady state. This can be interpreted in the same way as the case $a > b$ with exogenous probabilities. For low and negative values of τ, the only steady state is the trivial one.

Case $\eta = 0$: Here the probabilities are only affected by the preferences of the parents, this case is similar to the last model with exogenous probabilities since a_t and b_t remain constant over time and which of these is bigger depends on the parameter τ: $\tau > 0$ will lead to $a_t > b_t$ and $\tau < 0$ will lead to $a_t < b_t$. I show in Appendix E that the dynamics of z_t will converge monotonically to a positive steady state, and this will be globally stable. The intuition is the same as the one given for exogenous probabilities.

Case $\eta = -\frac{1}{2}$ and $\eta = -1$: The dynamics are oscillatory and converge to a unique positive steady state level that exists for values of τ not too small, otherwise, there is no steady state. This case differs to the previous results because we can have oscillations even if $a_t > b_t$. When the fertility behavior of the past generation affects negatively the tastes over fertility of the generation that follows, what happens is that children in large families get fed up with children although they originally may have a high taste for children. Starting with a high level of z_0, we would also have a high fertility level n_0 meaning that the adults at $t = 1$ would feel as being too many and consequently have a lower willingness to procreate which would decrease their fertility n_1.

Brief summary of the results:

- If $\tau > 0$ and $\eta > 0$: monotonic dynamics with a stable steady state.
- If $\tau > 0$ and $\eta < 0$: oscillatory dynamics with a stable steady state.
- If $\tau < 0$: only the trivial steady state and unstable ($\eta > 0$), no steady state ($\eta < 0$ close to zero), or a stable steady state with oscillatory dynamics ($\eta < 0$ and τ not too small).

To conclude this section, we can say that the dynamics of childlessness can be explained by the dynamics of preferences when the taste for children reacts negatively to the fertility rate of the past generation. This last hypothesis would however contradict one of the results of Fernández and Fogli (2006). In order to provide a more realistic result, we complete the model by introducing a labor market.

5 Extension with endogenous wages

Until now, we assumed that wages were exogenous. In this section, we relax this assumption by introducing a representative competitive firm and study how endogenous wages affect the dynamics of childlessness.

5.1 Production function

A representative competitive firm, producing the final good, Y_t, used for consumption at unit price, and using men's labor, L_t^m, and women's labor, L_t^f (both in units of time), as inputs, has the following production function,

$$F(L_t^m, L_t^f) = Y_t = \Big(\alpha(L_t^m)^{-\rho} + (1-\alpha)(L_t^f)^{-\rho}\Big)^{-1/\rho} \tag{14}$$

with $\alpha \in (0, 1)$ and $\rho > -1$, $\rho \neq 0$. Women's labor, L_t^f, can be divided in three: the labor of childless women, the labor of women having few children, $\overline{n_t}$, and the labor of women having many children, $\overline{\overline{n_t}}$. Each type of women will work a different amount of time. The amount of time that a woman spends working will then be,

$$\begin{cases} 1 & \text{if childless} \\ 1-\theta\overline{n_t} & \text{if she has } \overline{n_t} \text{ (couples of) children} \\ 1-\theta\overline{\overline{n_t}} & \text{if she has } \overline{\overline{n_t}} \text{ (couples of) children} \end{cases}$$

We will denote by L^{f1} the labor supplied by childless women, L^{f2} the supply of labor of women with $\overline{n}$ (couples of) children and L^{f3} the labor supply of women with $\overline{\overline{n}}$ (couples of) children. Total female labor supply, given by the total number of hours worked by all women, at time t, is then given by,

$$L_t^f = L_t^{f1} + L_t^{f2} + L_t^{f3}$$

Denoting respectively by w_t^m and w_t^f the wages per unit of time of men and women, at time t, the representative firm solves the following problem,

$$\max_{L_t^m, L_t^{f1}, L_t^{f2}, L_t^{f3}} \quad F\left(L_t^m, L_t^{f1}, L_t^{f2}, L_t^{f3}\right) - w_t^m L_t^m - w_t^f\left(L_t^{f1} + L_t^{f2} + L_t^{f3}\right)$$

and equalizing the marginal productivities of labor to their marginal cost, we can easily find the following expressions,

$$w_t^m = \alpha\left(\alpha + (1-\alpha)\left(\frac{L_t^f}{L_t^m}\right)^{-\rho}\right)^{-\frac{1+\rho}{\rho}}$$

$$w_t^f = (1-\alpha)\left(\alpha\left(\frac{L_t^m}{L_t^f}\right)^{-\rho} + (1-\alpha)\right)^{-\frac{1+\rho}{\rho}}$$

We can easily see that the wage of men increases as the female labor supply increases and decreases as men's labor increases. The same happens for the wage of women; an increase in female labor supply decreases women's wage and an increase in male's labor supply increases it.

At time t, total population, P_t, given by Equation (4), is composed by one half of men, P_t^m, and the other half of women, P_t^f. Total labor supplies (in units of time) for each type of person are then the following ones,

$$L_t^m = \frac{\underline{P_t} + \overline{P_t}}{2}$$

$$L_t^{f1} = \frac{\underline{P_t}^2}{2(\underline{P_t} + \overline{P_t})}$$

$$L_t^{f2} = \frac{\overline{P_t}\underline{P_t}}{\underline{P_t} + \overline{P_t}}(1 - \theta\overline{n_t})$$

$$L_t^{f3} = \frac{\overline{P_t}^2}{2(\underline{P_t} + \overline{P_t})}\left(1 - \theta\overline{\overline{n_t}}\right)$$

The total number of time supplied by women in terms of population groups is then given by,

$$L_t^f = \frac{\underline{P_t}^2}{2(\underline{P_t}+\overline{P_t})} + (1-\theta\overline{n_t})\frac{\overline{P_t}\underline{P_t}}{\underline{P_t}+\overline{P_t}} + \left(1-\theta\overline{\overline{n_t}}\right)\frac{\overline{P_t}^2}{2(\underline{P_t}+\overline{P_t})}$$

and defining by $l_t \equiv \frac{L_t^f}{L_t^m}$ the relative labor supplied by women, at time t, we can then write l_t in terms of relative population, z_t, such as,

$$l_t = \frac{1+2\left(1-\theta\overline{n_t}\right)z_t + \left(1-\theta\overline{\overline{n_t}}\right)z_t^2}{\left(1+z_t\right)^2} \tag{15}$$

We can then rewrite the returns to labor in terms of the relative labor of women such as,

$$w_t^m = \alpha\left(\alpha+(1-\alpha)l_t^{-\rho}\right)^{-\frac{1+\rho}{\rho}}$$

$$w_t^{1f} = (1-\alpha)\left(\alpha l_t^{\rho}+(1-\alpha)\right)^{-\frac{1+\rho}{\rho}}$$

5.2 Equilibrium definitions

Temporary Equilibrium: Given adult population groups $(\underline{P_t}, \overline{P_t})$ characterized by their respective willingness for children $(\underline{\gamma}, \overline{\gamma})$, a temporary equilibrium is a vector

$$\{c_t^j, n_t^j, \gamma^j, z_t, , P_t^m, P_t^f, P_t, L_t^m, L_t^{f1}, L_t^{f2}, L_t^{f3}, Y_t, w_t^m, w_t^f\}$$

satisfying the following conditions:

- the level of the couple's consumption, c_t^j, and the fertility of the couple, n_t^j, is such that each couple j maximizes its utility $U_t^j(c_t^j, n_t^j) = \ln c_t^j + \gamma^j n_t^j$ subject to the constraints $c_t^j = w^m + (1-\theta n)w^f - kI(n_t^j)$ and $0 \le n_t^j \le \frac{1}{\theta}$;
- couples match randomly and the willingness for children of the couple γ^j is given by an average of the tastes of its members so that there are three types of couples characterized by different willingnesses: $\underline{\gamma}$, $\frac{\underline{\gamma}+\overline{\gamma}}{2}$ and $\overline{\gamma}$;
- the relative size of population, at time t, z_t, is given by $z_t = \frac{\overline{P_t}}{\underline{P_t}}$;
- total population, P_t, at time t, has equal number of women and men: $P_t^m = P_t^f = \frac{P_t}{2}$ and is given by $P_t = \underline{P_t} + \overline{P_t}$;
- labor inputs $L_t^m, L_t^{f1}, L_t^{f2}$ and L_t^{f3} and output level Y_t are such that the competitive firm maximizes its profits given by: $Y_t - w_t^m L_t^m - w_t^f(L_t^{f1}+L_t^{f2}+L_t^{f3})$, and produces,

$$Y_t = \left(\alpha\left(L^m\right)^{-\rho} + (1-\alpha)\left(L^{f1}+L^{f2}+L^{f3}\right)^{-\rho}\right)^{-1/\rho}$$

- wages per unit of time, w_t^m and w_t^f, are such that the labor market clears:

$$L_t^m = \frac{\underline{P_t} + \overline{P_t}}{2}$$

$$L_t^{f1} = \frac{\underline{P_t}^2}{2(\underline{P_t} + \overline{P_t})}$$

$$L_t^{f2} = \frac{\underline{P_t}\,\overline{P_t}}{\overline{P_t} + \underline{P_t}} (1 - \theta \overline{n_t})$$

$$L_t^{f3} = \frac{\overline{P_t}^2}{2(\underline{P_t} + \overline{P_t})} \left(1 - \theta \overline{\overline{n_t}}\right)$$

Intertemporal equilibrium: Given initial young population groups $(\underline{P_0}, \overline{P_0})$ an intertemporal equilibrium is a sequence of temporary equilibria such that population groups follow the following expressions,

$$\overline{P_{t+1}} = \frac{1}{\overline{P_t} + \underline{P_t}} \left(a \overline{\overline{n_t}}\, \overline{P_t}^2 + 2b \overline{n_t}\, \overline{P_t}\, \underline{P_t} \right)$$

and

$$\underline{P_{t+1}} = \frac{1}{\overline{P_t} + \underline{P_t}} \left((1-a) \overline{\overline{n_t}}\, \overline{P_t}^2 + 2(1-b) \overline{n_t}\, \overline{P_t}\, \underline{P_t} \right)$$

6 Calibration and simulations for the United States

In this section, I study the effects on childlessness, fertility, female labor market participation and wage gap between men and women of a change in two parameters: the weight of women in the production of the final good and the fixed cost of going from childlessness to parenthood. For this, I fix two parameters and calibrate the rest of them in order to match United States data.

6.1 Calibration

The following two parameters are a priori fixed: b and ρ. I set the probability $b = 0.8a$, this is arbitrary but the only value that is affected by changing this restriction is the probability a that increases if the ratio $\frac{b}{a}$ decreases. The other variables remain unchanged, however, the dynamics are slower when the ratio $\frac{b}{a}$ is small. The substitution parameter is also a priori fixed to $\rho = -0.75$, implying an elasticity of substitution between female labor and male labor of 4. This choice is coherent with the estimates of Acemoglu et al. (2004). Changing ρ affects the distribution parameter α: the lower the substitution between the inputs, the higher will be the weight of men inside the firm.

Equations used for the calibration: At the steady state, we have a system of the following eight equations (with eight unknowns),

$$z = \frac{a\overline{\overline{n}}z + 2b\overline{n}}{(1-a)\overline{\overline{n}}z + 2(1-b)\overline{n}}$$

$$\begin{cases} w^m = \alpha\left(\alpha + (1-\alpha)l^{-\rho}\right)^{-\frac{1+\rho}{\rho}} \\ w^f = (1-\alpha)\left(\alpha l^{\rho} + (1-\alpha)\right)^{-\frac{1+\rho}{\rho}} \end{cases}$$

$$\begin{cases} \overline{n} = \frac{w^m + w^f - k}{\theta w^f} - \frac{2}{\overline{\gamma} + \underline{\gamma}} \\ \overline{\overline{n}} = \frac{w^m + w^f - k}{\theta w^f} - \frac{1}{\overline{\gamma}} \\ n = \frac{z}{(1+z)^2}\left(z\overline{\overline{n}} + 2\overline{n}\right) \end{cases}$$

$$l = \frac{1 + 2\delta\left(1 - \theta\overline{n}\right)z + \delta\left(1 - \theta\overline{\overline{n}}\right)z^2}{(1+z)^2}$$

and

$$\chi = \frac{1}{(1+z)^2}$$

I use the study of Turchi (1975)[10] to calibrate the fixed cost, k, of going from childless to parents. The average number of hours per year of child care for one child is estimated to be 515.22 while for two or three children this is 350.01 hours per year per child. The difference between both gives the fixed cost per year for the first child in terms of hours. Considering that childrearing is done for 18 years and that a period is 25 years, we have the following restriction for the fixed cost:

$$k = 0.0068\frac{w^m + w^{f2}}{2}$$

The other five parameters; θ, α, $\overline{\gamma}$, $\underline{\gamma}$ and a, are set to match five moments, taken from US data; n, χ, $\overline{\overline{n}}$, w^f and l. The following table gives us the value of the moments and the calibrated parameters:

Parameters	Moments	Source
$a = 0.670$	$n = 1$	
$\underline{\gamma} = 0.141$	$\chi = 0.146$	U.S. Census Bureau 2008
$\overline{\gamma} = 0.192$	$\overline{\overline{n}} = 1.97\overline{n}$	U.S. Census Bureau 2008
$\theta = 0.333$	$l = 0.667$	Erosa et al. (2005)
$\alpha = 0.587$	$w^f = 0.78w^m$	Erosa et al. (2005)

The rest of the variables take the following values:

$z = 1.617$	$w^m = 0.565$	$\overline{n} = 0.817$	$k = 0.00342$	$\gamma^* = 0.159$

[10] Turchi (1975), Table 3-5, page 92.

Notes:

- Relative labor supply l: quoting Erosa et al. (2005), "We document that the average number of hours of work per person is about 40% larger for men than for women between the ages of 20 and 40. By age 40, this difference in hours of work translates into a stock of accumulated experience that is about 50% larger for men than for women." (page 3), this implies that $l = 0.667$.

- Opportunity cost θ: A value of $\theta = 0.333$ implies a maximum fertility of around 6 children for a woman. According to Livi-Bacci (1977), considering the Hutterites' hypothetical number of children per woman, this is 8.2 if the women gets married at 25 years old[11]. Our calibration for θ may then be higher than the one expected but if we look at the data, only 0.5% of all women have seven or more children in the United States[12].

6.2 Simulation

Using the parameters calibrated in the last subsection, the dynamics of the relative population z_t are monotonic and the correlation between average fertility and childlessness along the transition path is negative. This means that if we consider an initial condition $z_0 = 3$ with a high proportion of individuals with high taste for children, the dynamics of z will be decreasing and we will see the following relationship between childlessness and fertility:

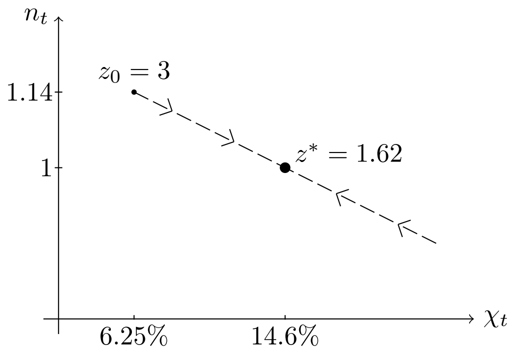

Figure 8: Correlation between n_t and χ_t along the transition path.

What the model tells us is that starting from a high proportion of individuals with high taste for children, the proportion of children coming from the mixed type of couples is low and the proportion of children coming from households of type 3 is high. Since probabilities are such that $b < a$, the proportion of mixed couples increases and the proportion of couples of type 3 decreases, the

[11] Livi-Bacci (1977), Table 1.2.

[12] U.S. Census Bureau for 40-44 years old women in 2006.

same happens for the proportion of children coming from each type of couple respectively. An increase in the proportion of children coming from couples of type 2 increases childlessness. In the labor market, the increase in childlessness increases the amount of time supplied by women, this decreases the wage of women and increases the wage of men; this increases the fertility rate of mothers. The overall effect on average fertility is negative due to the increase in childlessness and the increase in the proportion of couples of type 2.

Note: If we calibrate and then simulate with $\rho = 1$, instead of $\rho = -0.75$, meaning that L^m and L^f are complements (instead of substitutes), then the parameter that changes the most is the distribution parameter α, which increases to 0.742: men have a higher weight than women in the production of the final good when men and women are complements. The dynamics remain monotonic and the relationship between n and χ along the transition path is also negative.

6.3 Simulation with shocks

More gender parity (decrease in α): A negative shock on the distribution parameter α of the production function means that female labor has a bigger weight in the production of the final good of the representative firm. Empirical evidence for this type of shock is supported by the results of O'Neill and Polachek (1993). The economy is in the balanced growth path when the shock arrives and assuming that α decreases of 10%, the initial condition is $z^*_{\alpha=0.65} = 1.55$. Figure 9 shows how the shock affects average fertility and childlessness: both variables decrease after the shock. The intuition behind is the following: the shock increases wages

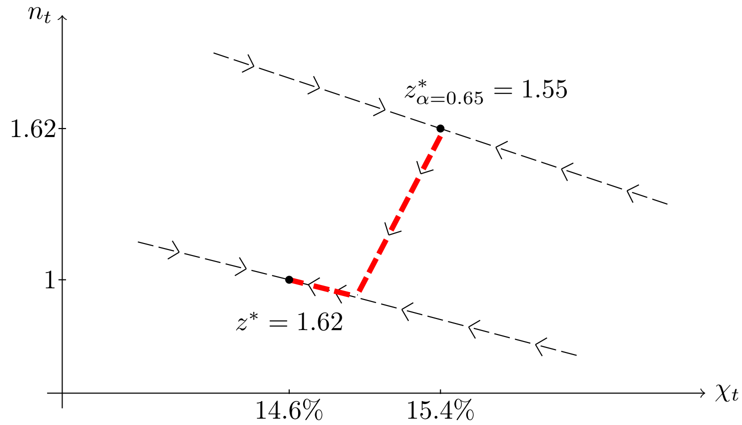

Figure 9: Effect of a decrease in α on n_t and χ_t.

of women and this decreases the fertility of mothers, $\overline{n}$ and $\overline{\overline{n}}$, at the same time, it decreases the proportion of children coming from couples of type 2 which are the most likely to end up being childless (the number of children coming from couples of type 3 increases). In other words, small families shrink more than big families and since these lasts are less likely to become childless, we have that childlessness decreases. The reason for the last increase in average fertility is the increase in the proportion of women having the highest fertility, $\overline{\overline{n}}$. Coming back to the United States' relationship between childlessness and fertility illustrated in Figure 1, this could be an explanation of what happened for the cohorts born at the beginning of the nineteenth century until the cohorts born before 1935.

In the labor market, the shock has a negative impact on men's wage and a positive impact on women's wage. Consequently, the wage gap between men and women at the new steady state is lower than the one before the shock, this is illustrated in Figure 10.

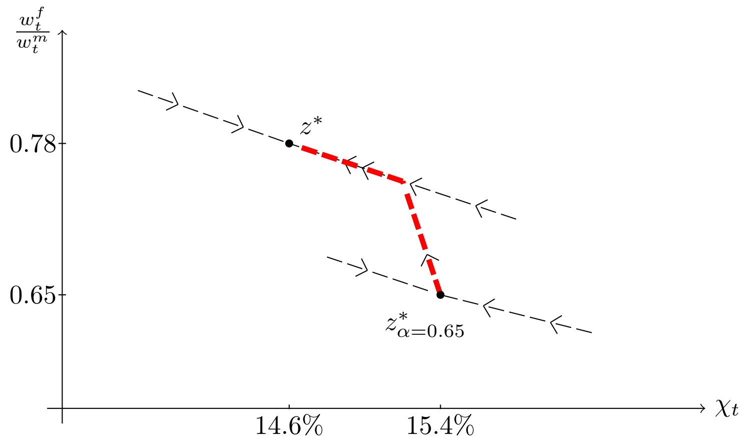

Figure 10: Effect of a decrease in α on $\frac{w_t^f}{w_t^m}$ and χ_t.

Figure 11 shows how the shock affects female labor participation relative to men's, l; this increases at the steady state. This is explained by a decrease in the fertility of mothers which is the consequence of an increase in their wages (implying a higher opportunity cost to have children) and an increase in their time available to work. The fluctuations are due to the opposite effects on relative labor of the decrease in the fertility of mothers (this increases l) and the increase in the proportion of households of type 3 along with the decrease in childlessness (decreases l). This increase in lifetime market participation of women is well documented in O'Neill and Polachek (1993).

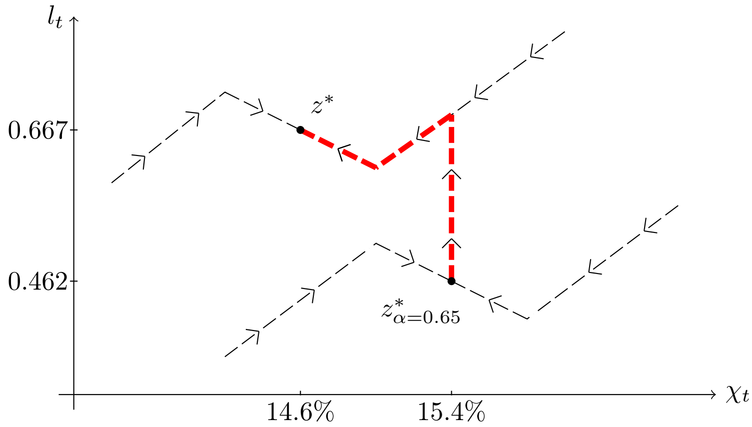

Figure 11: Effect of a decrease in α on l_t and χ_t.

The same type of effects would also appear by introducing a "mommy discrimination" parameter[13] making the hourly wage of a mother lower than the hourly wage of a childless women. A decrease in this mommy discrimination parameter decreases the wage gap between mothers and non mothers and has the same effect on average fertility and childlessness (Figure 9), on the fertility of mothers (both decrease), on relative labor (Figure 11) and on the wage gap between men and women (Figure 10) than a negative shock on α. The only difference between this shock and the last one is that the wage of men increases and the wage of childless women decreases at the steady state[14].

Increase in the fixed cost of children (increase in k): An increase in k could explain the dynamics of childlessness and fertility for the cohorts born between 1930 and 1944, for whom we observe a positive relationship between fertility and childlessness . This shock mainly affects the fertility of mothers negatively. Both average fertility and childlessness are lower after the shock. The lower childlessness rate is again due to an increase in the proportion of children coming from couples of type 3 that are less likely to become childless. This means that once you pay the fixed cost, because it is bigger, you are more likely to have more children who are

[13]The amount of labor supplied by women would then become $L_t^f = L_t^{f1} + \delta L_t^{f2} + \delta L_t^{f3}$ where δ reflects the fact that the hourly wage of a married mother is lower than the one of a childless married women. The existence of this type of discrimination is confirmed in Mincer and Polachek (1974) and explained in Erosa et al. (2005) by the fact that childless women have a higher attachment to labor, consequently, they invest more time to it and become more experienced. It is also argued that a reason for this wage gap between mothers and non mothers is due to the career interruptions that women have to take each time they have a child and that this reduction in labor supply is done at an age where the returns to labor are high.

[14]In order to simulate this shock, we need to use the utility function given in Equation 16 because it is less sensible to changes in women wages. With the linear utility function in n_t^j what happens is that the introduction of the mommy discrimination δ pushes all women to becoming childless. In the simulations I use $\mu = 0.3$.

less likely to become childless. If we consider that the fixed cost can be interpreted as the price of a house, the increase in the real index of housing prices between 1955 and 1970 (see Skinner (1991), Figure 1) can be an explanation of the positive relationship between childlessness and fertility for cohorts of women born between 1930 and 1944. The effect on the labor market variables is negligible, the variable that is affected the most is the amount of labor supplied by women that increases, since mothers have less children. Figure 12 gives an illustration of the effect of this shock on childlessness and average fertility.

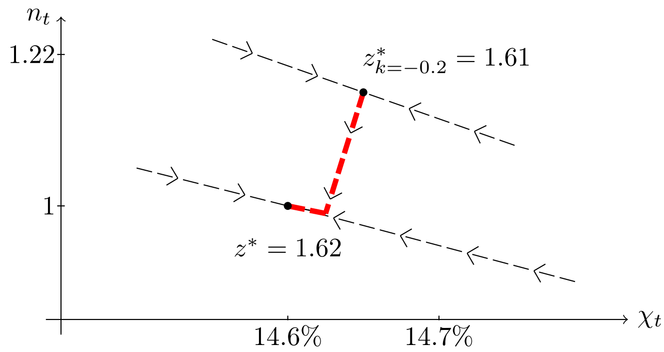

Figure 12: Effect of an increase of k on n_t and χ_t.

7 Conclusion

The aim of this work was to build a theoretical framework that could account for the fluctuations observed for childless women and to understand the economic mechanisms behind. After having reviewed different strands of the literature about childlessness, I have proposed three different models of childlessness, the first two aimed at studying the relationship between the dynamics of preferences and of childlessness and the last one tries to go further adding endogenous wages. The main results of this research are that shocks in the labor market that increase the labor opportunity of mothers can be at the origin of the fluctuations both in childlessness and in average fertility that we have observed in the United States since the beginning of the nineteenth century. The model also brings an explanation for the positive relationship between childlessness and fertility for the cohorts born during the second world war due to a possible increase in the fixed cost of becoming parents. A nice extension of the model would be to include the possibility of men and women to remain single since single women are much more likely to remain childless than married women (57.6% compared to 14.6% according to the U.S. Census Bureau, 2008).

A Tables for completed fertility and childlessness

Table 1: Childlessness rate and CEB in the United States for women born between 1840 and 1959.

Birth Cohort	Childlessness rate	CEB
1840-1844	15.1%	5.39
1845-1849	15.9%	5.36
1850-1854	16.5%	5.30
1855-1859	16.0%	4.90
1860-1864	17.0%	4.97
1865-1869	18.0%	4.50
1870-1874	20.0%	3.41
1875-1879	22.0%	3.25
1880-1884	22.5%	3.22
1895-1899	22.4%	3.15
1890-1894	22.4%	3.05
1895-1899	22.5%	2.82
1900-1904	24.0%	2.59
1905-1909	24.0%	2.30
1910-1914	25.0%	2.41
1915-1919	22.0%	2.59
1920-1924	17.0%	2.85
1925-1929	14.0%	3.11
1930-1934	13.0%	3.20
1935-1939	10.0%	3.01
1940-1944	9.0%	2.54
1945-1949	11.0%	2.22
1950-1954	17.0%	2.05
1955-1959	19.0%	1.80

Sources: Data for childlessness is taken from Rowland (2007) and for CEB from Jones and Tertilt (2006). Note: cohorts for CEB are built one year later than the ones for childlessness (i.e. 1-5 and 6-0 instead of 0-4 and 5-9).

Table 2: Childlessness rate and CEB in Netherlands for women born between 1900 and 1959.

Birth Cohort	Childlessness rate	Completed fertility
1900-1904	23.0%	2.86
1905-1909	22.0%	2.81
1910-1914	20.0%	2.87
1915-1919	26.0%	2.96
1920-1924	15.0%	2.87
1925-1929	14.0%	2.72
1930-1934	12.0%	2.61
1935-1939	12.0%	2.38
1940-1944	12.0%	2.13
1945-1949	11.0%	1.94
1950-1954	15.0%	1.88

Sources: Data for childlessness is taken from Rowland (2007) and for completed fertility from INED, for the cohorts 1900 to 1914, completed fertility is available only once every five years, for the others, averages from single years are computed.

Table 3: Childlessness rate and CEB for women born between 1900 and 1959 in 15 OECD countries.

Country	Childlessness rate	Completed fertility
Netherlands (NLD)	18.3%	1.77
United States (USA)	14.4%	2.07
Austria (AUT)	21.1%	1.64
Norway (NOR)	12.1%	2.06
Sweden (SWE)	12.9%	1.98
Denmark (DNK)	12.7%	1.92
Slovakia (SVK)	11.1%	2.04
Portugal (PRT)	4.0%	1.82
Romania (ROU)	11.5%	1.91
Spain (ESP)	13.1%	1.59
Hungaria (HUN)	9.6%	1.97
Greece(GRE)	16.3%	1.72
Czech Republic (CZE)	7.2%	1.93
Bulgaria (BLG)	4.4%	1.83
Finland (FIN)	19.9%	1.91

Source: OECD.

B Proof of Proposition 3.1

The interior solution is optimal (sufficient condition) if the utility of having children is higher than the one of remaining childless, that is, if the following condition is satisfied,

$$\ln\left(\frac{\theta w^f}{\gamma^j}\right) + \gamma^j\left(\frac{w^m + w^f - k}{\theta w^f} - \frac{1}{\gamma^j}\right) \geq \ln\left(w^m + w^f\right)$$

that can be rewritten in the following way,

$$\ln\left(\frac{\theta w^f}{\gamma^j\left(w^m + w^f\right)}\right) \geq 1 - \gamma^j\frac{w^m + w^f - k}{\theta w^f}$$

Denoting,

$$v(\gamma^j) = \ln\left(\frac{\theta w^f}{\gamma^j\left(w^m + w^f\right)}\right)$$

and

$$z(\gamma^j) = 1 - \frac{\gamma^j(w^m + w^f - k)}{\theta w^f}$$

the interior solution is optimal if $v(\gamma^j) \geq z(\gamma^j)$. Studying the function $v(\gamma^j)$, we have that $v'(\gamma^j) < 0$ and $v''(\gamma^j) > 0$, so that the function $v(\gamma^j)$ is decreasing and convex. The limits are the following,

$$\lim_{\gamma^j \to 0^+} v(\gamma^j) = +\infty$$

$$\lim_{\gamma^j \to +\infty} v(\gamma^j) = -\infty$$

and

$$v(\gamma^j) = 0 \Leftrightarrow \gamma^j = \frac{\theta w^f}{w^m + w^f}$$

For the function $z(\gamma^j)$, we have that $z'(\gamma^j) < 0$ and $z''(\gamma^j) = 0$, so that $z(\gamma^j)$ is linearly decreasing. We then have the following,

$$z(0) = 1$$

$$\lim_{\gamma^j \to +\infty} z(\gamma^j) = -\infty$$

and

$$z(\gamma^j) = 0 \Leftrightarrow \gamma^j = \frac{\theta w^f}{w^m + w^f - k}$$

Since $v(\gamma^j)$ is decreasing and convex and $z(\gamma^j)$ is decreasing but linear, we have that,

$$\lim_{\gamma^j \to +\infty}\left(v(\gamma^j) - z(\gamma^j)\right) > 0$$

so that for large values of γ^j, the interior solution is optimal. Note that at the value $\gamma^j = \frac{\theta w^f}{w^m+w^f-k}$, which corresponds to $n^* = 0$, the corner solution is optimal, since,

$$
\begin{aligned}
v\left(\frac{\theta w^f}{w^m+w^f-k}\right) - z\left(\frac{\theta w^f}{w^m+w^f-k}\right) &= \ln\left(\frac{\theta w^f}{\frac{\theta w^f}{w^m+w^f-k}(w^m+w^f)}\right) - 0 \\
&= \ln\left(\frac{w^m+w^f-k}{w^m+w^f}\right) \\
&< 0
\end{aligned}
$$

Consequently, we know that the two functions, $v(\gamma^j)$ and $z(\gamma^j)$, will intersect twice, once before the value $\gamma^j = \frac{\theta w^f}{w^m+w^f-k}$ and once after. For $\gamma^j < \frac{\theta w^f}{w^m+w^f-k}$, the constraint int $n_t^j \geq 0$ is not respected, so we only need to consider the values for $\gamma^j \geq \frac{\theta w^f}{w^m+w^f-k}$. This allows us to conclude that there will be a value of γ^*, where $v(\gamma^*) = z(\gamma^*)$, where couples are indifferent between having children or being childless.

C Another utility function

The following utility function could also be used,

$$
U_t^j(c_t^j, n_t^j) = \ln\left(c_t^j\right) + \gamma^j \ln\left(\mu + n_t^j\right) \tag{16}
$$

where μ can be interpreted as a substitution parameter between consumption and fertility for the couple. The first order conditions for n_t^j and c_t^j are the following ones:

$$
n_t^j = \begin{cases} \frac{1}{\theta} & \text{if } w^f \leq \frac{\gamma^j(w^m-k)}{1+\theta\mu} \\ \frac{\gamma^j}{1+\gamma^j}\frac{w^m+w^f-k}{\theta w^f} - \frac{\mu}{1+\gamma^j} & \text{if } \frac{\gamma^j(w^m-k)}{1+\theta\mu} < w^f < \frac{w^m-k}{\frac{\theta\mu}{\gamma^j}-1} \\ 0 & \text{if } w^f \geq \frac{w^m-k}{\frac{\theta\mu}{\gamma^j}-1} \end{cases}
$$

and

$$
c_t^j = \begin{cases} w^m - k & \text{if } w^f \leq \frac{\gamma^j(w^m-k)}{1+\theta\mu} \\ \frac{w^m+w^f(1+\theta\mu)-k}{1+\gamma^j} & \text{if } \frac{\gamma^j(w^m-k)}{1+\theta\mu} < w^f < \frac{w^m-k}{\frac{\theta\mu}{\gamma^j}-1} \\ w^m + w^f & \text{if } w^f \geq \frac{w^m-k}{\frac{\theta\mu}{\gamma^j}-1} \end{cases}
$$

The interior solution is optimal if the following inequation is satisfied:

$$
\begin{aligned}
\ln\left(\frac{w^m+w^f(1+\theta\mu)-k}{1+\gamma^j}\right) &+ \gamma^j \ln\left(\frac{\gamma^j}{1+\gamma^j}\frac{w^m+w^f-k}{\theta w^f} + \frac{\mu\gamma^j}{1+\gamma^j}\right) \\
&\geq \ln\left(w^m+w^f\right) + \gamma^j \ln\mu
\end{aligned}
$$

In order to go further with the analytical results, I chose the utility function given in Equation (1) and not the one proposed in this Appendix. This does not change the qualitative aspect of the simulations and we would have the same conclusions with one utility function or the other. However, the linearity of the utility function in n_t^j makes it more sensitive to changes in the female wages than what it would be with a utility function such as Equation (16).

D Relaxing random matching assumption

If we do not believe in the assumption of random matching, I look at the changes that will take place in the model by introducing some assortative matching between individuals of the same type. This means that individuals with the same tastes for children will be more likely to be together than in the random matching framework. Letting λ denote the degree of "assortativeness", the proportions of each type of couple, ate time t, are the following:

1. Type 1: $\left(\frac{\underline{P_t}}{\overline{P_t}+\underline{P_t}}\right)^2(1-\lambda)+\frac{\underline{P_t}}{\overline{P_t}+\underline{P_t}}\lambda$
2. Type 2: $\frac{2\overline{P_t}\underline{P_t}}{\left(\overline{P_t}+\underline{P_t}\right)^2}(1-\lambda)$
3. Type 3: $\left(\frac{\overline{P_t}}{\overline{P_t}+\underline{P_t}}\right)^2(1-\lambda)+\frac{\overline{P_t}}{\overline{P_t}+\underline{P_t}}\lambda$

It is easy to notice that $\lambda = 0$ corresponds to the random matching case and $\lambda = 1$ means that there are no mixed couples, so that individuals from different types do not form a couple (perfect assortative matching case). We can rewrite the proportions in terms of z_t as follows:

1. Type 1: $\left(\frac{1}{1+z_t}\right)^2(1-\lambda)+\frac{1}{1+z_t}\lambda$
2. Type 2: $\frac{2z_t}{(1+z_t)^2}(1-\lambda)$
3. Type 3: $\left(\frac{z_t}{1+z_t}\right)^2(1-\lambda)+\frac{z_t}{1+z_t}\lambda$

The dynamics of z_t can then be rewritten such as,

$$z_{t+1} = \frac{a\overline{\overline{n}}(z_t+\lambda)+2b\overline{n}(1-\lambda)}{(1-a)\overline{\overline{n}}(z_t+\lambda)+2(1-b)\overline{n}(1-\lambda)} \equiv \phi_a(z_t) \tag{17}$$

The case of perfect assortative matching, $\lambda = 1$ implies that we do not have any dynamics: we are always at the steady state equal to $\frac{a}{1-a}$. The first derivative of $\phi_a(z_t)$ is given by,

$$\phi_a'(z_t) = \frac{2\overline{\overline{n}}(1-\lambda)\overline{n}(a-b)}{\left((1-a)\overline{\overline{n}}(z_t+\lambda)+2(1-b)\overline{n}(1-\lambda)\right)^2} \geq 0 \iff a \geq b$$

and the second derivative by,

$$\phi_a''(z_t) = \frac{4\overline{\overline{n}}^2(1-\lambda)\overline{n}(a-b)(1-a)}{\left((1-a)\overline{\overline{n}}(z_t+\lambda)+2(1-b)\overline{n}(1-\lambda)\right)^3} \leq 0 \iff a \geq b$$

We also have that,

$$\phi_a(0) = \frac{a\overline{\overline{n}}\lambda + 2b\overline{n}(1-\lambda)}{(1-a)\overline{\overline{n}}\lambda + 2(1-b)\overline{n}(1-\lambda)} \geq \frac{b}{1-b} \iff a \geq b$$

and

$$\lim_{z_t \to \infty} \phi_a(z_t) = \frac{a}{1-a} > 0$$

This means that we will also have a unique positive steady state in the case of assortative matching. Convergence will be faster and the same type of dynamics will arise as in the random matching case.

E Computations for endogenous probabilities

Case $\eta = 1$: We can rewrite the expression given in equation (13) in the following way,

$$z_{t+1} = \frac{(\overline{\gamma})^\tau \overline{\overline{n}} z_t^2 + 2\left(\frac{\overline{\gamma}+\underline{\gamma}}{2}\right)^\tau \overline{n} z_t}{(z_t+1)^2 - \left((\overline{\gamma})^\tau \overline{\overline{n}} z_t^2 + 2\left(\frac{\overline{\gamma}+\underline{\gamma}}{2}\right)^\tau \overline{n} z_t\right)} \equiv \Phi_{\eta=1}(z_t)$$

This dynamic of z_t has two steady states that can be computed analytically. One is the trivial solution, $z^* = 0$, and the other one is the following,

$$z^* = \frac{-1 + 2\left(\frac{\overline{\gamma}+\underline{\gamma}}{2}\right)^\tau \overline{n}}{1 - (\overline{\gamma})^\tau \overline{\overline{n}}}$$

for which the sign is unknown but it is likely to be negative.

Case $\eta = \frac{1}{2}$: We have,

$$z_{t+1} = \frac{\left(z_t^2\overline{\overline{n}} + 2z_t\overline{n}\right)^{\frac{1}{2}}\left((\overline{\gamma})^\tau \overline{\overline{n}} z_t + 2\left(\frac{\overline{\gamma}+\underline{\gamma}}{2}\right)^\tau \overline{n}\right)}{(z_t+1)\left(\overline{\overline{n}} z_t + 2\overline{n}\right) - \left(z_t^2\overline{\overline{n}} + 2z_t\overline{n}\right)^{\frac{1}{2}}\left((\overline{\gamma})^\tau \overline{\overline{n}} z_t + 2\left(\frac{\overline{\gamma}+\underline{\gamma}}{2}\right)^\tau \overline{n}\right)} \equiv \Phi_{\eta=\frac{1}{2}}(z_t)$$

Other than the trivial steady state, we can have, here, two other steady states that are the roots of the following second order linear equation,

$$\overline{\overline{n}}\left(1 - (\overline{\gamma})^{2\tau}\overline{\overline{n}}\right) z^2 + 2\overline{n}\left(1 - 2(\overline{\gamma})^\tau \left(\frac{\overline{\gamma}+\underline{\gamma}}{2}\right)^\tau \overline{\overline{n}}\right) z - 4\left(\frac{\overline{\gamma}+\underline{\gamma}}{2}\right)^{2\tau} \overline{n}^2 = 0$$

which discriminant is,

$$\Delta = 4\overline{n}^2 \left(1 + 4\overline{\overline{n}} \left(\frac{\overline{\gamma} + \underline{\gamma}}{2}\right)^{\tau} \left[\left(\frac{\overline{\gamma} + \underline{\gamma}}{2}\right)^{\tau} - (\overline{\gamma})^{\tau}\right]\right)$$

If $\Delta > 0$, the two real roots are given by the following expressions:

$$z_1^* = \frac{-\overline{n}\left(1 - 2\left(\overline{\gamma}\right)^{\tau}\left(\frac{\overline{\gamma}+\underline{\gamma}}{2}\right)^{\tau}\overline{\overline{n}}\right) - \overline{n}\sqrt{1 + 4\overline{\overline{n}}\left(\frac{\overline{\gamma}+\underline{\gamma}}{2}\right)^{\tau}\left[\left(\frac{\overline{\gamma}+\underline{\gamma}}{2}\right)^{\tau} - \left(\overline{\gamma}\right)^{\tau}\right]}}{\overline{\overline{n}}\left(1 - \left(\overline{\gamma}\right)^{2\tau}\overline{\overline{n}}\right)}$$

and

$$z_2^* = \frac{-\overline{n}\left(1 - 2\left(\overline{\gamma}\right)^{\tau}\left(\frac{\overline{\gamma}+\underline{\gamma}}{2}\right)^{\tau}\overline{\overline{n}}\right) + \overline{n}\sqrt{1 + 4\overline{\overline{n}}\left(\frac{\overline{\gamma}+\underline{\gamma}}{2}\right)^{\tau}\left[\left(\frac{\overline{\gamma}+\underline{\gamma}}{2}\right)^{\tau} - \left(\overline{\gamma}\right)^{\tau}\right]}}{\overline{\overline{n}}\left(1 - \left(\overline{\gamma}\right)^{2\tau}\overline{\overline{n}}\right)}$$

Stability at $z^* = 0$: We can study the stability at the trivial steady state by looking at the first derivative of $\Phi_{\eta=\frac{1}{2}}(z_t)$ at $z_t = 0$, even though it is not defined for $z_t = 0$. We define $\Phi'_{\eta=\frac{1}{2}}(0)$ the following limit:

$$\Phi'\left(0^+\right) \equiv \lim_{z_t \to 0^+} \Phi'(z_t)$$

Using the definition of the derivative at one point, we have that,

$$\begin{aligned}\Phi'\left(0^+\right) &= \lim_{z \to 0^+} \frac{\Phi(z) - \Phi(0)}{z - 0} \\ &= \lim_{z \to 0^+} \frac{\Phi(z)}{z}\end{aligned}$$

and studying the function $\frac{\Phi(z)}{z}$, we have that,

$$\Phi'(0^+) = \lim_{z \to 0^+} \frac{\Phi(z)}{z} = +\infty$$

Consequently, the trivial steady state is locally unstable.

Case $\eta = 0$: The dynamics are given by the following expression,

$$z_{t+1} = \frac{\overline{\overline{n}} z_t (\overline{\gamma})^{\tau} + 2\overline{n}\left(\frac{\overline{\gamma}+\underline{\gamma}}{2}\right)^{\tau}}{\overline{\overline{n}} z_t \left(1 - (\overline{\gamma})^{\tau}\right) + 2\overline{n}\left(1 - \left(\frac{\overline{\gamma}+\underline{\gamma}}{2}\right)^{\tau}\right)} \equiv \Phi_{\eta=0}(z_t)$$

Note that this function is negative if $\tau < 0$ because a_t and b_t become higher than 1 and the denominator becomes negative, consequently, we can only study this function for $\tau > 0$ (the function is not defined for $\tau = 0$). The trivial steady state is no longer present but there are two real steady states given by,

$$z_1^* = \frac{-\left(2\overline{n}\left[1-\left(\frac{\overline{\gamma}+\underline{\gamma}}{2}\right)^{\tau}\right]-\overline{\overline{n}}\left(\overline{\gamma}\right)^{\tau}\right)-\sqrt{\Delta}}{2\overline{\overline{n}}\left(1-\left(\overline{\gamma}\right)\right)}$$

and

$$z_2^* = \frac{-\left(2\overline{n}\left[1-\left(\frac{\overline{\gamma}+\underline{\gamma}}{2}\right)^{\tau}\right]-\overline{\overline{n}}\left(\overline{\gamma}\right)^{\tau}\right)+\sqrt{\Delta}}{2\overline{\overline{n}}\left(1-\left(\overline{\gamma}\right)\right)}$$

where,

$$\Delta = \left(2\overline{n}\left[1-\left(\frac{\overline{\gamma}+\underline{\gamma}}{2}\right)^{\tau}\right]-\overline{\overline{n}}\left(\overline{\gamma}\right)^{\tau}\right)^{2}+8\overline{n}\left(1-\left(\overline{\gamma}\right)^{\tau}\right)\left(\frac{\overline{\gamma}+\underline{\gamma}}{2}\right)^{\tau}\overline{\overline{n}}$$

The first derivative of $\Phi_{\eta=0}(z_t)$ is given by,

$$\Phi'_{\eta=0}(z_t) = \frac{2\overline{n}\left[\left(\overline{\gamma}\right)^{\tau}-\left(\frac{\overline{\gamma}+\underline{\gamma}}{2}\right)^{\tau}\right]\overline{\overline{n}}}{\left[\overline{\overline{n}}z_t\left(1-\left(\overline{\gamma}\right)^{\tau}\right)+2\overline{n}\left(1-\left(\frac{\overline{\gamma}+\underline{\gamma}}{2}\right)^{\tau}\right)\right]^{2}} > 0$$

and the second derivative by,

$$\Phi''_{\eta=0}(z_t) = \frac{-4\overline{n}\left[\left(\overline{\gamma}\right)^{\tau}-\left(\frac{\overline{\gamma}+\underline{\gamma}}{2}\right)^{\tau}\right]\overline{\overline{n}}^{2}\left(1-\left(\overline{\gamma}\right)^{\tau}\right)}{\left[\overline{\overline{n}}z_t\left(1-\left(\overline{\gamma}\right)^{\tau}\right)+2\overline{n}\left(1-\left(\frac{\overline{\gamma}+\underline{\gamma}}{2}\right)^{\tau}\right)\right]^{3}} < 0$$

so that the dynamics of z_t will converge to the positive steady state, and this will be globally stable.

Case $\eta = -\frac{1}{2}$: The dynamics are given by the following expression,

$$z_{t+1} = \frac{\left(1+z_t\right)\left(\overline{\overline{n}}z_t^2+2z_t\overline{n}\right)\left(\left(\overline{\gamma}\right)^{\tau}\overline{\overline{n}}z_t+2\left(\frac{\overline{\gamma}+\underline{\gamma}}{2}\right)^{\tau}\overline{n}\right)}{\overline{\overline{n}}z_t+2\overline{n}-\left(1+z_t\right)\left(\overline{\overline{n}}z_t^2+2z_t\overline{n}\right)\left(\left(\overline{\gamma}\right)^{\tau}\overline{\overline{n}}z_t+2\left(\frac{\overline{\gamma}+\underline{\gamma}}{2}\right)^{\tau}\overline{n}\right)} \equiv \Phi_{\eta=-\frac{1}{2}}(z_t)$$

Now, $\Phi_1(z_t)$ is defined in $\mathbb{R}_+^*$.

Case $\eta = -1$: Now, the dynamics are given by this last expression,

$$z_{t+1} = \frac{z_t\overline{\overline{n}}\left(\overline{\gamma}\right)^{\tau}+2\overline{n}\left(\frac{\overline{\gamma}+\underline{\gamma}}{2}\right)^{\tau}}{\frac{z_t\left(z_t\overline{\overline{n}}+2\overline{n}\right)^2}{(1+z_t)^2}-\left[z_t\overline{\overline{n}}\left(\overline{\gamma}\right)^{\tau}+2\overline{n}\left(\frac{\overline{\gamma}+\underline{\gamma}}{2}\right)^{\tau}\right]} \equiv \Phi_{\eta=-1}(z_t)$$

and the first order derivative can be expressed as follows,

$$\Phi'_{\eta=-1}(z_t) = \frac{\frac{z_t\overline{\overline{n}}+2\overline{n}}{1+z_t}\left[z_t^2\overline{\overline{n}}\left(\overline{\gamma}\right)^\tau \frac{2\left(\overline{n}-\overline{\overline{n}}\right)-z_t\overline{\overline{n}}}{1+z_t} - 2\overline{n}\left(\frac{\overline{\gamma}+\underline{\gamma}}{2}\right)^\tau\left(\frac{z_t\overline{\overline{n}}+2\overline{n}}{1+z_t}+2z_t\overline{\overline{n}}\right)\right]}{\left(\frac{z_t\left(z_t\overline{\overline{n}}+2\overline{n}\right)^2}{(1+z_t)^2} - \left[z_t\overline{\overline{n}}\left(\overline{\gamma}\right)^\tau + 2\overline{n}\left(\frac{\overline{\gamma}+\underline{\gamma}}{2}\right)^\tau\right]\right)^2} < 0$$

which is negative because $\frac{2\left(\overline{n}-\overline{\overline{n}}\right)-z_t\overline{\overline{n}}}{1+z_t} < 0$.

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