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Asymmetric Cannibalism in Retail Assortments

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SH.A.R.P. (Bultez and Naert, 1988; hereafter, B & N) optimizes the allocation of a retail outlet's selling area, across the items composing its assortment. Demand interdependencies—essentially, the cannibalism occurring within each homogeneous product category—are modelled through an attraction-type specification of the items' shares in the assortment's total sales volume. The latter, however implies fully symmetric patterns of substitution.

Here, by adopting the asymmetric variant of the attraction model proposed by Vanden Abeele, Gijsbrechts, and Vanhuele (1987; hereafter, VGV), we generalize SH.A.R.P. so as to integrate the diversity of substitution effects that may be inflected by brand loyalty, preference for a specific variety, or package-size purchasing habits. The extended model (SH.A.R.P. II) is empirically tested on a sample of data collected during an experiment on the canned dog food category of a large Belgian hypermarket. Normative results point to the benefits that can be derived from using SH.A.R.P. II. We also demonstrate that consistent recourse to rules of thumb may not be of harm.

This joint contribution is the outcome of the four co-authors' close collaboration at the European Institute for Advanced Studies in Management (EIASM, Brussels). The first draft of this article was presented at the EIASM's workshop on "Marketing Decision Support Systems: From MIS to MESs?" (March 1988). Suggestions made by the editor and two anonymous referees helped to improve the readability of our article and also encouraged us to further examine the relevance of rules of thumb.

INTRODUCTION

A retail business is continuously confronted with a need to monitor the interdependencies generated within its multiproduct assortments. The word "cannibalism" denotes retailers' concern for the multiple forms that substitution effects may take within their departments: between brands, either within or across variety-types; and vice versa between variety-types, either within or across brand lines.

Positive interactions may also occur. Price promotions, retail advertising, and special displays boost the sales of the items featured, often contribute to the favorable positioning, and are likely to increase storewide traffic or even chain patronage. For some interesting approaches to these aspects of retailing, refer to Corstjens and Doyle (1981), Doyle and Saunders (1987), McGoldrick (1986), and Wilkinson et al. (1981, 1982).

Whether positive or negative, cross-product effects occurring within retail assortments are difficult to measure and make the planning and control of merchandising activities quite complex. Reibstein and Gatignon (1984) offer evidence of the problems met by marketing model builders when trying to tackle those issues. The overwhelming number of commodities carried compounds the retailer's task and prevents an easy assessment of direct-product cost and profitability; this explains the current craze for the so-called DPP systems, integrating cost and space allocation rules. (For example, see Proctor and Gamble's approach to "Total System Efficiency," which is presented to its customers as an impartial audit methodology for distribution logistics and retail operations.)

In this article, we focus on substitutions resulting from shelf-space allocation within self-service superstores. The high importance that retailers attach to space management derives from the fact that the product-display area available constitutes a strictly binding constraint. Moreover, evaluating the consequences of shelf rearrangements is not a trivial exercise because these influence both distribution costs and items' sales in various ways.

On the demand side, multiple results may be produced by manipulations of the number of shelf facings. Thus increasing the visibility of an item is likely to stimulate the demand for it, which in turn may lead to a reduction of the demand for another item that consumers regard as a close substitute. Such (partially) offsetting cannibalistic effects should be anticipated when designing new layouts. Describing such a process is the purpose of our contribution.

In practice, space allocations are determined by rules of thumb. Numerous commercial packages integrate them into proportionality-to-sales (volume vs. revenue) or proportionality-to-margins (gross vs. net) heuristics: sales is the criterion used by PROGALI (Malsagne, 1972); gross margin is the standard in OBM (Looyen, 1970); net margins are referred to in CIFRINO (1963). More sophisticated decision-support systems take into consideration handling costs and inventory control: e.g., COSMOS (1969), SLIM (BCD, 1972) and HOPE (Duban, 1973). But none of these computer programs explicitly relates sales responses to facings.

Academics, on the other hand, have concentrated their efforts on the modeling of a relationship between *direct*-demand elasticity and product visibility.[1] Yet, most academics have neglected the *cross*-elasticities that best reflect cannibalistic interactions. For example, the optimal allocations defined by Lynch (1974) and Hansen and Heinsbroek (1979) are based on independent item sales-response functions.

The first substantive attempt at incorporating shelf-space cross-elasticities[2]—although at a global level, across departments—is the study by Corstjens and Doyle (1981). The very fact that their optimization procedure relies on a geometric programming algorithm, yielding numerical results but offering little generalizable analytical insight, clearly points to a lack of parsimonious models that could capture all possible interactions in a limited number of parameters.

Capitalizing on their experience of market-share functions, Bultez and Naert (hereafter, B & N) (1988) recently demonstrated how convenient and effective the attraction model could be in describing and controlling interactions between substitute items within homogeneous product assortments.[3] Nevertheless, in their search for parsimony (leading them to calibrate all interdependencies through a single parameter), B & N have implicitly assumed away sources of heterogeneity that may differentially shape substitution patterns.

Brand loyalty or preference for a product variety may introduce asymmetries for which an attraction model cannot account. The present article remedies this inadequacy, which becomes especially critical when working at the most disaggregate level (items defined by two or more dimensions: brand × variety-type).[4]

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[1] Complementary literature reviews can be found in Heinsbroek (1977), Corstjens and Doyle (1981), and B & N (1988).

[2] As argued by B & N (1988, pages 213–214 and 229), the theoretical breakthrough by Anderson and Amato (1974) suffers from a lack of realism. Anderson's (1979) model is restricted to a 2-item assortment.

[3] Named after B & N (1975) and Little et al. (1975); it is also known as the MCI (Multiple Competitive Interactions) model, following Nakanishi and Cooper (1974).

[4] Adhoc aggregation—e.g., over package sizes, if consumers are not indifferent to them—may overshadow asymmetries.

Here, we adapt SH.A.R.P. by incorporating into it a variant of the "competitive cluster asymmetry," proposed by Vanden Abeele, Gisbrechts, and Vanhuele (hereafter, VGV) (1987). Parsimony hardly suffers from this sophistication, since only one parameter gets added (per source of asymmetry).

In section 1, the theoretical background of SH.A.R.P. is reviewed. Section 2 shows how asymmetry can be introduced. Generalized cross-elasticities and optimization results are then derived in sections 3 and 4, respectively. In sections 5 and 6, a calibration method and an implementation procedure are discussed and illustrated through an application in a real setting. A sensitivity analysis evaluates how much ignoring asymmetry would cost the retailer. Section 7 raises the question of the dependability of the rules of thumb so commonly used in practice.

1. OPTIMAL SHELF-SPACE ALLOCATION: THEORETICAL BACKGROUND

B & N (1988) examine how a retailer should allocate a limited-product display area (S) among the items listed in one of the assortments carried so as to maximize profit (P). The assortment can be defined along two dimensions, illustrated in Table 1: its depth (D), determined by the set of brands offered to the buyer; and its width (W), delimited by the variety of item types—i.e., the range of tastes, package forms, and sizes constituting the various brands' product lines. Thus each item is identified by both its brand name (d, b, or $c \in D$) and its variety-type (w, v, or $j \in W$).

Table 1 actually reflects a space-allocation plan: entries in the assortment-display matrix define the partition of the display area between items.[5] A row (column) margin gives the total space-share reserved for the corresponding brand (variety-type).

An item's sales volume (q_{bv}) depends not only on its own visibility, hence on the space allocated specifically to itself (s_{bv}), but also on the other items display space (s_{cj}, for all other brands, $c \neq b$, and all other variety-types, $j \neq v$). Its contribution to profit is obtained by subtracting the cost its handling entails (C_{bv}) from the total gross margin generated by its sales ($g_{bv} \cdot q_{bv}$, where g_{bv} stands for the unit margin on brand b's item v).

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[5] For notation convenience, but without loss of generality, let us consider that if a brand's line does not include a specific variety-type, all variables characterizing the corresponding item are all set equal to zero.

TABLE 1

Assortment Display Matrix

WIDTH

DEPTH

W D						Total
	....	j	......	v	....	
⋮		⋮		⋮		⋮
b	....	σ_{bj}	......	σ_{bv}	....	$S_{b.}$
⋮		⋮		⋮		⋮
c	....	σ_{cj}	......	σ_{cv}	....	$S_{c.}$
⋮		⋮		⋮		⋮
Total	....	$S_{.j}$	......	$S_{.v}$	....	1

$\sigma_{bv} = s_{bv}/S =$ share of the total space available allocated to brand b's item v;

$S_{b.} = \sum_{v \in W} \sigma_{bv} =$ brand b's total space share;

$S_{.v} = \sum_{b \in D} \sigma_{bv} =$ fraction of the total space left to items of the v-th variety-type.

$$\sum_b S_{b.} = \sum_v S_{.v} = \sum_b \sum_v \sigma_{bv} = 1.$$

The SH.A.R.P. rule is then derived as the solution to the following space-allocation problem:

$$\underset{\{s_{bv}|b \in D,\ v \in W\}}{\text{MAX}} \qquad P = \sum_{b \in D} \sum_{v \in W} [g_{bv}\, q_{bv} - C_{bv}]$$

subject to: $q_{bv}\,(\ldots, s_{bj}, \ldots, s_{bv}, \ldots, s_{cj}, \ldots, s_{cv}, \ldots);$

$C_{bv}\,(s_{bv}, q_{bv});\ \sum_b \sum_v s_{bv} \leqslant S;\ s_{bv} \geqslant 0,$ for all $b \in D$ and $v \in W$.

Such a problem definition is quite general (except for the restriction that the assortment composition cannot be altered: the set of items considered is fixed, a priori); therefore the resulting allocation rule, which B & N have derived (1988, pages 214–215), has wide applicability. It essentially states that the space share ($\sigma_{bv} = s_{bv}/S$) to be assigned to an item should be proportional to the extent that:

1. Its visibility contributes to increasing the turnover of the most profitable products carried in the assortment considered (an overall effect reflected in the weighted mean of all the sales elasticity and cross-elasticities, with respect to the display of brand *b*'s item *v*, i.e., $\overline{\eta}_{bv}$);
2. its increased stocking on the shelf reduces handling and replenishment operations (a cost decrease measured by $\gamma_{bv}\, c_{bv}$).

More explicitly, the solution to the above allocation problem reads:

$$\sigma_{bv} = (\overline{\eta}_{bv} + \gamma_{bv}\, c_{bv})/(\overline{N} + \overline{G}), \qquad [1]$$

where: $c_{bv} = C_{bv}/\Pi$ represents the ratio of the handling cost resulting from carrying brand *b*'s item *v* to the assortment overall profitability;[6]

$\gamma_{bv} = -(\partial C_{bv}/\partial s_{bv}|dq_{bv} = 0)(s_{bv}/C_{bv})$ stands for the absolute value of the partial elasticity of the item's handling cost with respect to the space allocated to it (at a constant sales level);

[2]

$\overline{\eta}_{bv} = \Sigma_{c \in D}\, \Sigma_{j \in W}\, r_{cj}\, \eta(q_{cj}, s_{bv})$ is the weighted mean of all items' sales elasticities, $\eta(q_{cj}, s_{bv})$, with respect to the space allocated to brand *b*'s item *v*,

with $r_{cj} = \pi_{cj}/\Pi$ as the relative contribution of brand *c*'s item *j* to the assortment profitability;

and

$\overline{N} = \Sigma_{b \in D}\, \Sigma_{v \in W}\, \overline{\eta}_{bv}$ and $\overline{G} = \Sigma_b\, \Sigma_v\, \gamma_{bv}\, c_{bv}$ are the normalizing terms.

Assuming that items are substitutable, cross-elasticities are negative and equation [2] can be rewritten,

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[6] Π differs from the actual profit (P) in that the marginal handling cost gets substituted for its average (per unit sold). Of course, the items' sales volumes have to be measured in homogeneous "equivalent" units (taking into account differences in product forms and package sizes).

$$\bar{\eta}_{bv} = r_{bv} \cdot \eta(q_{bv}, s_{bv}) - \sum_{(c,j)} \sum_{\neq (b,v)} r_{cj} |\eta(q_{cj}, s_{bv})|,$$

in which the first term measures the positive impact produced by the increased visibility of item (b,v) on its own profitability, while the second term sums up its negative incidence on the profitability of others (the cannibalized ones). Thus $\bar{\eta}_{bv}$ really defines the overall effect on the entire assortment of a change in the visibility of item (b,v), generated through the resulting redistribution of the sales volume across items.

The apparent complexity of the terms entering [1] is due to the implicit nature of the sales and cost-response functions underlying the general nonlinear mathematical programming problem defined by B & N. Implementing [1] calls for the explicit specification of those functions.

Interested in rather homogeneous categories of substitutes, B & N suggest the attraction model for the description of the competition prevailing between brands, as well as of the cannibalism that may also occur within each brand's line,[7] i.e.:

$$m_{bv} = \alpha_{bv} s_{bv}^{\beta} / \sum_{c} \sum_{j} \alpha_{cj} s_{cj}^{\beta}, \qquad [3]$$

where: $m_{bv} = q_{bv}/Q$ denotes the share of brand b's item v in the product category total sales volume ($Q = \Sigma_b \Sigma_v q_{bv}$, supposed to be insensitive to space reallocations); α_{bv}, α_{cj} and β are non-negative parameters.

B & N extensively report on various tests of their model and allocation procedure. Their analyses, however, omit the distinction between the two dimensions of the retailer's assortment: items are either clustered together brandwise (they focus on the aggregate brand level: $S_{b.}$), or the brand lines are ignored (subscript i is substituted for $b \times v$: $s_i = s_{bv}$).

The intrinsically symmetric nature of model [3] would certainly apply in the absence of any hierarchical structure or partitioning in the retailer's assortment. It implies identical cross-elasticities of sales shares,[8]

$$\eta(m_{cj}, s_{bv}) = \eta(m_{bj}, s_{bv}) = \eta(m_{cv}, s_{bv}) = -\beta m_{bv}.$$

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[7] For a manufacturer, cannibalism usually means substitution between items within his own brand's line, but for a retailer it also covers the struggle between the competitive brands supplied by the various manufacturers with whom he is dealing.

[8] When the total-assortment sales volume, Q, can be considered insensitive to space reallocations (under the constraint of the limited area available, S), then: $\eta(q_{cj}, s_{bv}) = \eta(m_{cj}, s_{bv})$.

Thus when an item's visibility is increased, the sales share of any other item gets reduced by a fixed proportion, which only depends on the relative importance of the item whose visibility is enhanced (measured by m_{bv} here), and not on the alternative item affected (c,j). Therefore substitution between variety-types offered by the same brand $(j \neq v)$ and competition opposing different brands $(c \neq b)$ are treated similarly: both sorts of interdependencies are assumed to be of the same strength.

Indeed, such an assumption is quite restrictive since items tend to be clustered along the brand and/or variety dimensions. This clustering follows from the items' in-store physical display, their perceptual mapping by consumers, and from buyers' loyalty. For instance, should brand loyalty prevail, one would expect relatively stronger substitution effects between items of the same brand than between items offered by different brands,

$$|\eta(m_{bj}, s_{bv})| > |\eta(m_{cj}, s_{bv})|.$$

Applying [1] also pre-requires the specification of the product-handling cost equation. B & N postulate that it is a constant-elasticity function of the frequency of replenishment, i.e.:

$$C_{bv} = f_{bv}(q_{bv}/s_{bv})^{\gamma_{bv}}, \qquad [4]$$

with f_{bv} a positive scaling factor.

Although we take issue over [3], assumption [4] will not be questioned in the sequel. Simpler forms of [4] will even be advocated in section 6.

2. AN ASYMMETRIC VARIANT OF THE ATTRACTION MODEL

Model [3] can be presented under the relative attraction form,

$$m_{bv} = t_{bv}/\sum_{c} \sum_{j} t_{cj}, \qquad [5]$$

where each of the terms, t_{bv}, stands for the attractiveness of each respective item.

Let a_{bv} be the intrinsic attraction exerted by brand b's item v; i.e., the attraction it exerts independently of the choice context defined by other items bearing the same brand name, or of the same variety or package size,

$$a_{bv} = \alpha_{bv}\, s_{bv}^{\beta}.$$

If a_{bv} is substituted for t_{bv} in model [5], then model [3] is obtained. This shows that its symmetry derives from the fact that each of the terms, t_{bv}, uniquely depends on the context-free attraction exerted by the corresponding specific item (b,v). Consequently, the ratio of two items' sales shares is determined only by the ratio of their respective attractions,

$$m_{cj}/m_{bv} = t_{cj}/t_{bv} = a_{cj}/a_{bv}. \qquad [6]$$

It appears totally unrelated to the visibility of any other item, whether from the same brand line (either b or c) or from the same variety-type (either v or j). This symmetric substitution property of the classical attraction model implies that any third item's sales share will draw equally (proportionally) from all other items offered to the shopper's choice. Thus all items must be perfect substitutes and the assortment (as perceived by buyers) not hierarchically structured.

In order to circumvent this limitation of the classical attraction model, VGV (1987) develop a generalized formulation, by making the t_{bv} terms a function not only of the specific (b,v) item's attraction a_{bv}, but also of the attraction exerted by clusters of related items; i.e., a_{bj} and/or a_{cv}.

Here, adapting VGV's suggestion, we propose to adjust the a_{bv} terms by the total attraction exerted by the cluster of items that are directly competitive with (b,v): the items of the same brand's line (b) and/or of the same variety (v). More explicitly, we postulate that

$$t_{bv} = a_{bv}/(A_{b.}^{\theta_1} A_{.v}^{\theta_2}), \qquad [7]$$

where:

$$A_{b.} = \sum_{j} a_{bj} \text{ and } A_{.v} = \sum_{c} a_{cv}, \qquad [8]$$

account for the total attraction exerted by the items offered by brand b and by those of the v-th variety-type, respectively; $0 \leq \theta_k \leq 1$, are parameters regulating the degree of asymmetry caused by the k-th cluster of items ($k = 1$ <=> brand cluster; $k = 2$ <=> variety cluster).

Substituting [7] and [8] into [5], we obtain the asymmetric version of the model,

$$m_{bv} = a_{bv} A_{b.}^{-\theta_1} A_{.v}^{-\theta_2} / \sum_{d \epsilon D} \sum_{w \epsilon W} a_{dw} A_{d.}^{-\theta_1} A_{.w}^{-\theta_2}. \qquad [9]$$

Deflating the original attraction according to [7] and [8] now yields the following ratios of sales shares,

$$m_{cj}/m_{bv} = [a_{cj}/(A_{c.}^{\theta_1} A_{.j}^{\theta_2})]/[a_{bv}/(A_{b.}^{\theta_1} A_{.v}^{\theta_2})],$$

the meaning of which becomes clear if we consider the special case when brand loyalty is the single source of asymmetry ($\theta_1 = \theta$ and $\theta_2 = 0$), then

$$m_{cj}/m_{bv} = [a_{cj}/a_{bv}][A_{b.}/A_{c.}]^{\theta}.$$

The latter expression demonstrates the impact of our "deflation." Now the sales-share ratios are dependent on third items: in this particular case, they depend on the attraction exerted by all those offered by brands b and c.

In the case of full asymmetry ($\theta = 1$), the sales-shares ratio corresponds to the ratio of the items' shares within their own brand's sales,

$$m_{cj}/m_{bv} = m_{j|c.}/m_{v|b.}, \qquad [10]$$

where:

$$m_{v|b.} = a_{bv}/A_{b.}. \qquad [11]$$

Thus full asymmetry is shown to be equivalent to absolute intra-brand product-line cannibalism. As illustrated further in Figure 1.1, equations [10]–[11] imply that an increase of the visibility of brand b's item 1 will eat up the sales of brand b's item 2 exclusively; thus brand c's items are not affected at all: brand c's total sales share remains flat. At the other extreme, when asymmetry is reduced to nil ($\theta = 0$), the degree of cannibalism within the brand's line is as strong (or as moderate) as it is across brands. Figure 1.4 shows that in this case of symmetric substitution the increase in item (b, 1) sales come equally at the expense of item (b, 2) and of brand c.

In contrast, as Figures 1.2 and 1.3 indicate, when asymmetry is present, the sales share of item (b, 2) suffers more than brand c's items do.

3. GENERALIZED CROSS-ELASTICITIES

Economists traditionally rely on the concept of cross-elasticity to characterize product-demand interdependencies and especially their degree of substitutability.[9] The nature and relative strengths of competitive interactions are indeed reflected in the structure of the matrix of direct and cross-elasticities. Thus we shall measure asymmetry in the attraction model by reference to the cross-elasticity standard.

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[9] To typify the various competitive market situations, see e.g., Triffin (1940), Bishop (1952, 1955), and Hieser (1955). It should be noted, however, that their conception of asymmetry differs from ours: we consider that it implies $\eta(m_{bj}, s_{bv}) \neq \eta(m_{cv}, s_{bv})$, but they usually mean that $\eta(m_{bj}, s_{bv}) \neq \eta(m_{bv}, s_{bj})$. Note that were we to adopt their idea, the simple basic attraction model would have to be regarded as an asymmetric specification.

FIGURE 1.1

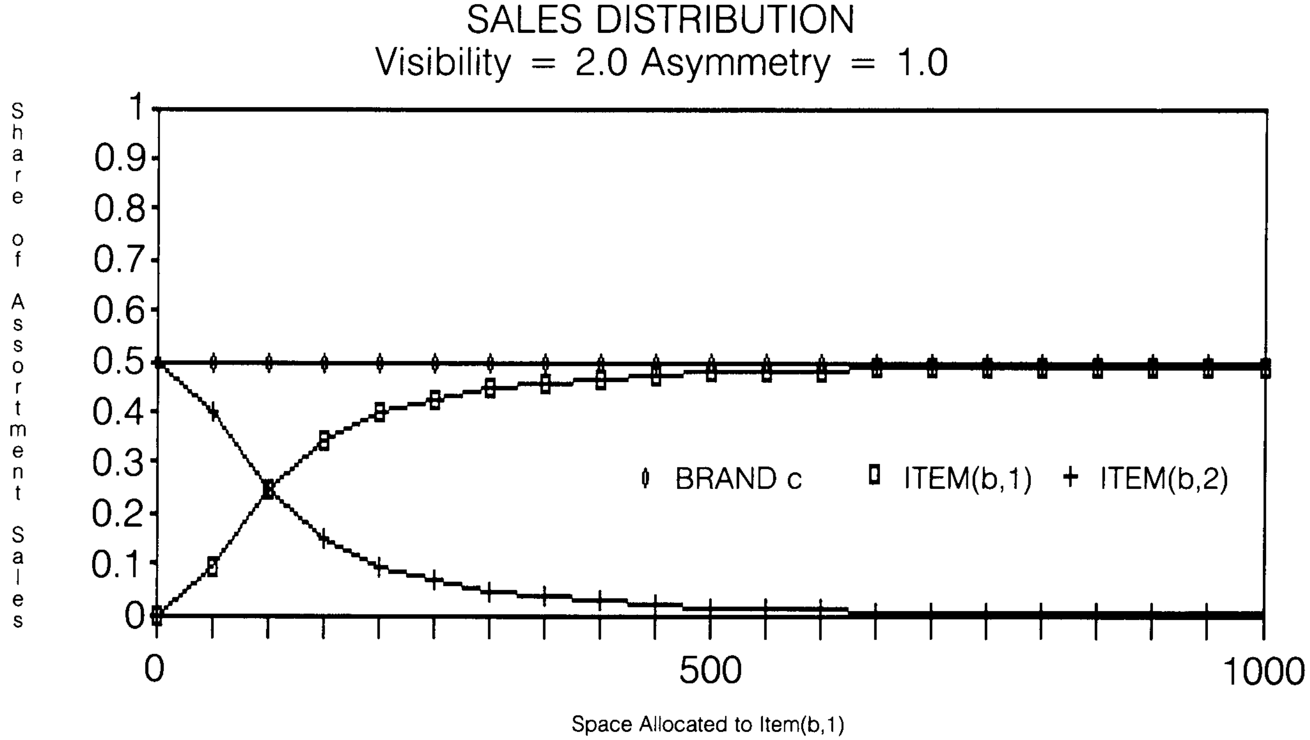

SALES DISTRIBUTION
Visibility = 2.0 Asymmetry = 1.0
Share of Assortment Sales
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0
500
1000
Space Allocated to Item(b,1)
BRAND c
ITEM(b,1)
ITEM(b,2)

FIGURE 1.2

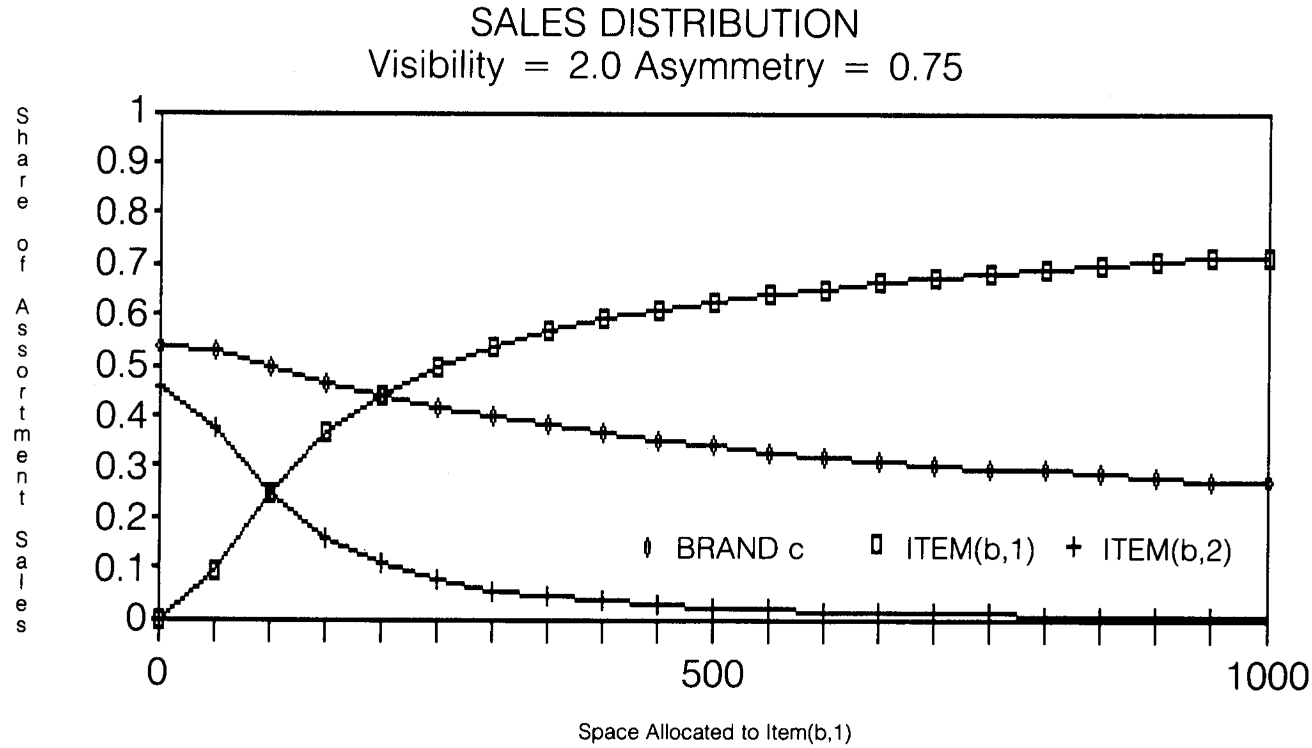

SALES DISTRIBUTION
Visibility = 2.0 Asymmetry = 0.75
Share of Assortment Sales
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0
500
1000
Space Allocated to Item(b,1)
BRAND c
ITEM(b,1)
ITEM(b,2)

FIGURE 1.3

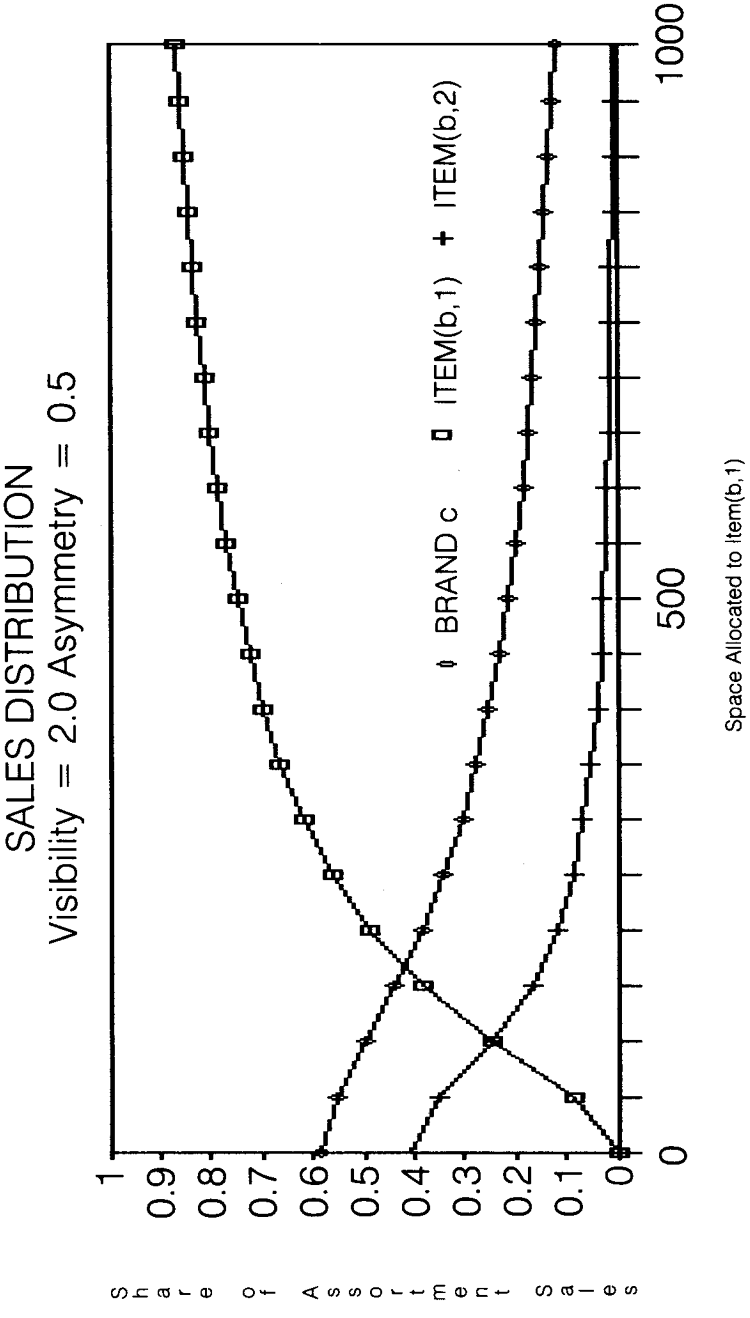
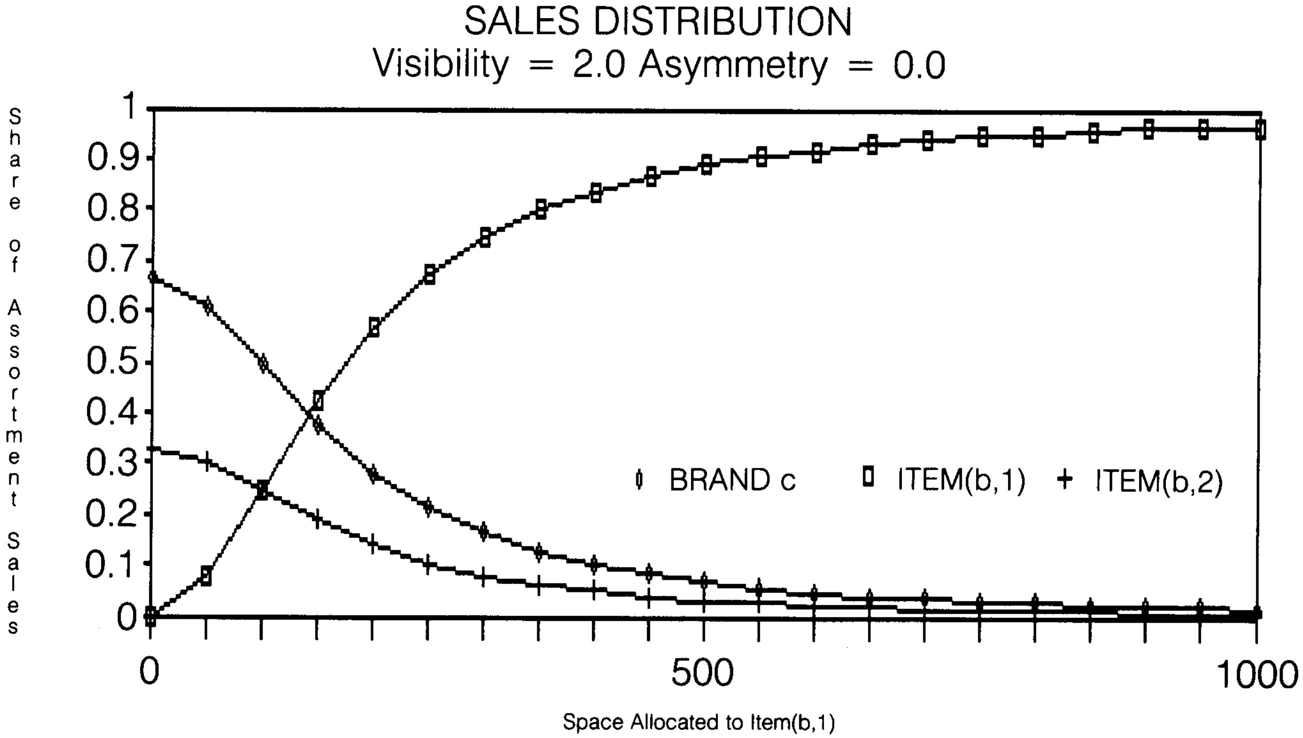

SALES DISTRIBUTION
Visibility = 2.0 Asymmetry = 0.5
BRAND c
ITEM(b,1)
ITEM(b,2)
Share of Assortment Sales
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
0
500
1000
Space Allocated to Item(b,1)

FIGURE 1.4


SALES DISTRIBUTION
Visibility = 2.0 Asymmetry = 0.0
Share of Assortment Sales
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0
500
1000
Space Allocated to Item(b,1)
BRAND c
ITEM(b,1)
ITEM(b,2)

Appendix A demonstrates that the generalized asymmetric attraction model [9] implies the following expression for the elasticity of item (c,j)'s sales share with respect to the space allocated to item (b,v):

$$\eta(m_{cj}, s_{bv}) = \beta[(\delta_{jv}\delta_{cb} - m_{bv}) - \theta_1\mu_{v|b.}(\delta_{cb} - M_{b.}) \\ - \theta_2\mu_{b|.v}(\delta_{jv} - M_{.v})], \qquad [12]$$

where: $\mu_{b|.v} = a_{bv}/A_{.v}$ and $\mu_{v|b.} = a_{bv}/A_{b.}$ represents the relative attraction of brand b's item v within the v-th variety-type, and the relative attraction of that same item within brand b's line, respectively;

$M_{b.} = \Sigma_w m_{bw}$ and $M_{.v} = \Sigma_d m_{dv}$ are brand b's aggregate sales share, and variety v's global sales share, respectively;

δ_{jv} and δ_{cb} are the so-called Kronecker's (binary) variables used to distinguish the various cases of competitive interactions which we may face; they are defined as follows: $\delta_{jv} = 1$ when $v = j$, then both items belong to the same variety-type; $= 0$ otherwise;

and analogously,

$\delta_{cb} = 1$ when $b = c$, then both items are offered by the same brand; $= 0$ otherwise.

Direct elasticities are displayed in the top part (①) of the tree-structure: the items examined are offered by the same brand ($\delta_{cb} = 1$) and belong to the same variety-type ($\delta_{jv} = 1$), hence: $\delta_{cb} \cdot \delta_{jv} = 1$.

Cross-elasticities measuring intra-brand cannibalism appear on the second set of branches; the intra-variety cannibalism is illustrated by the third group. Subset ④ corresponds to cannibalism between items of various types ($\delta_{jv} = 0$), offered by distinct brands ($\delta_{cb} = 0$).

Setting both asymmetry parameters equal to zero ($\theta_1 = \theta_2 = 0$), one of course recovers the direct and cross-elasticities that the simple attraction model [3] generates, i.e.:

$$\eta(m_{bv}, s_{bv}) = \beta(1 - m_{bv}); \; \eta(m_{cj}, s_{bv}) = -\beta \, m_{bv}.$$

Naturally, the intensity of cannibalism is regulated by the θs. This is best demonstrated when comparing intra-cluster (intra-brand or intra-variety) cross-elasticities to the across-cluster ones. For example, if brand loyalty dominates preference for specific variety-types so strongly that it constitutes the only source of asymmetry, then again $\theta_2 = 0$ and $\theta_1 = \theta \neq 0$, and Figure 2 indicates that,

FIGURE 2

Structure of Elasticities and Cross-elasticities

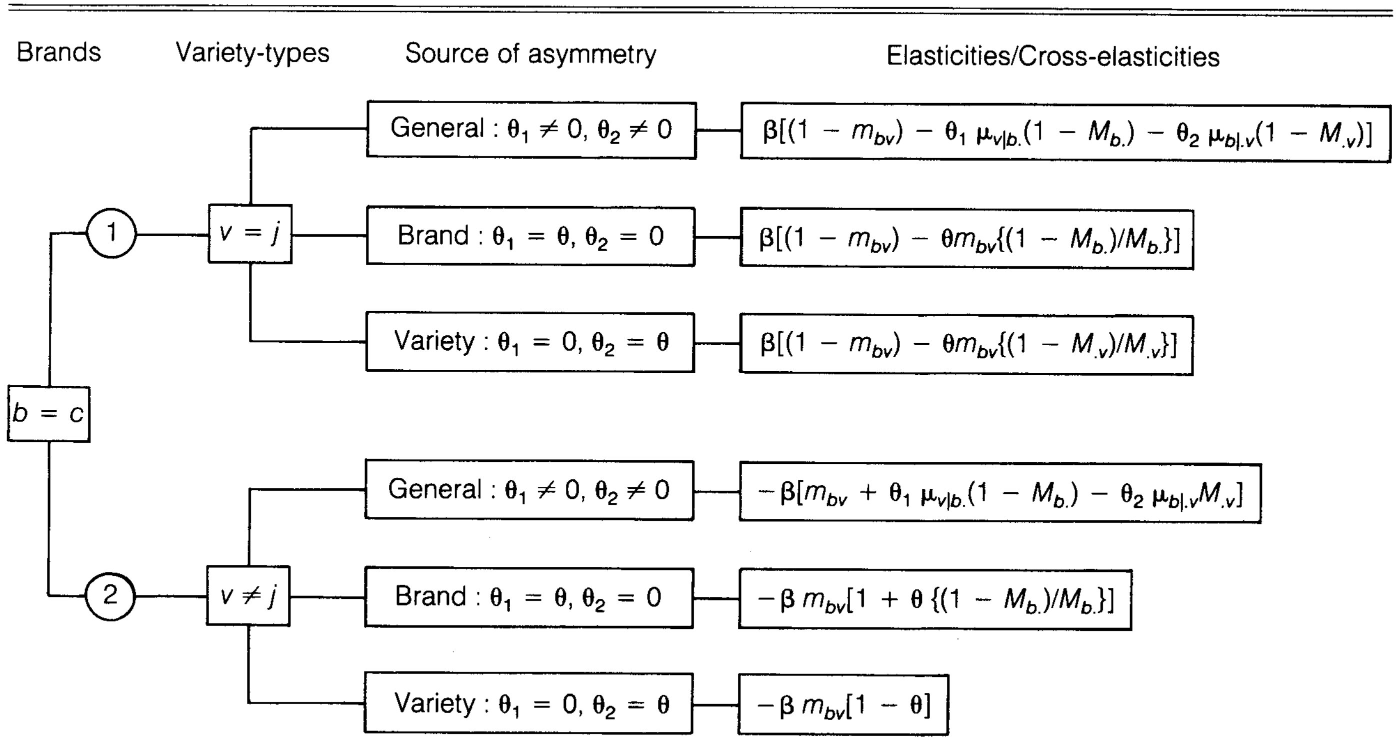
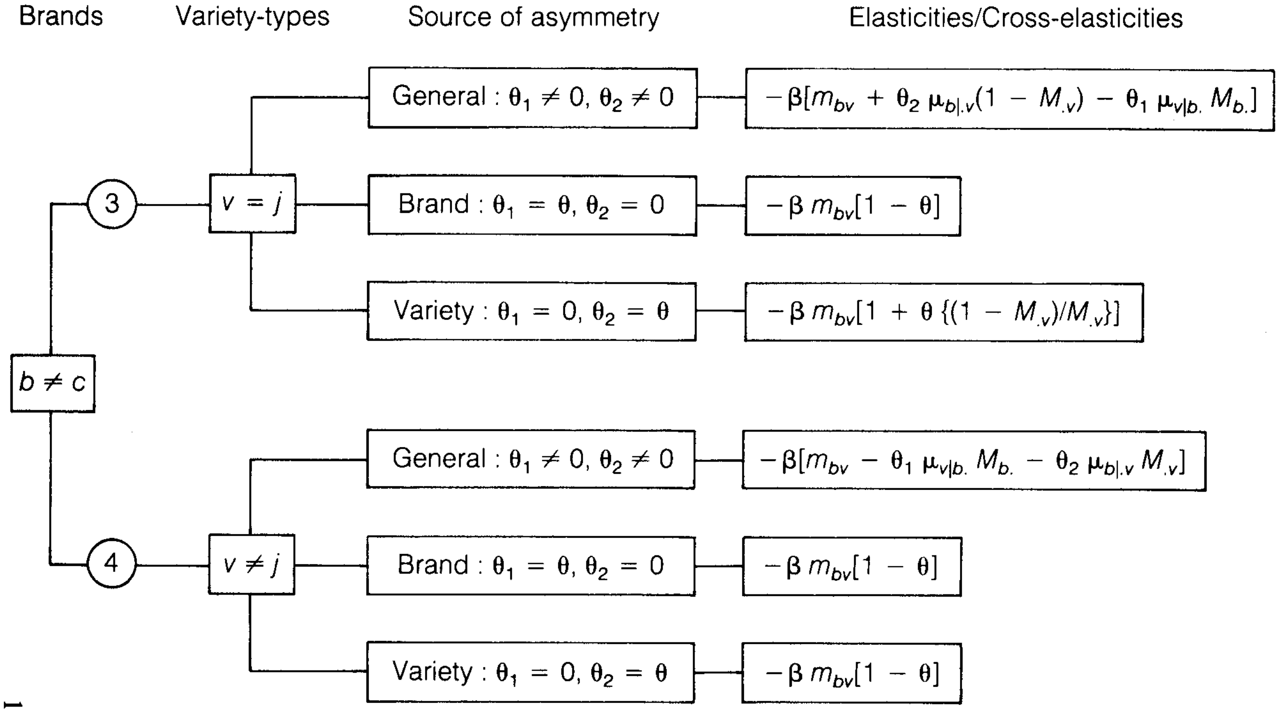


FIGURE 2 (Cont.)



$$\frac{|\eta(m_{bj}, s_{bv})|}{|\eta(m_{cj}, s_{bv})|} = \frac{1 + \theta\left[\dfrac{(1 - M_{b.})}{M_{b.}}\right]}{1 - \theta} = \rho_b.$$

Since $0 \leqslant \theta \leqslant 1$, then: $\rho_b \epsilon [1, +\infty[$ and therefore,

$$|\eta(m_{bj}, s_{bv})| \geqslant |\eta(m_{cj}, s_{bv})|,$$

which means that cannibalism between items of the same brand exceeds substitutability between brands.

Moreover,

$$\lim_{\theta \longrightarrow 1} [\eta(m_{cj}, s_{bv})] = 0,$$

and

$$\lim_{\theta \longrightarrow 1} [\eta(m_{bj}, s_{bv})] = -\beta[m_{bv}/M_{b.}] = -\beta\, m_{v|b.}.$$

Thus full asymmetry in cannibalism relationships leads to complete independence between items offered by different brands; substitution occurs only within brands (also refer to our comments on [10] and on Figure 1.1). From a pragmatic market-research point of view, it should be realized that such an extreme situation ($\theta = 1$) disqualifies the application to the whole product category of the attraction model (whether in its symmetric or asymmetric form). If θ gets close to one, the assortment should be split into distinct sub-categories that should then be analyzed separately.

The situation in which preference for specific variety-types takes precedence over consumers' brand attachment up to a point that it constitutes the only source of asymmetry ($\theta_1 = 0$ and $\theta_2 = \theta$) can be interpreted along the same lines.

4. SH.A.R.P. II

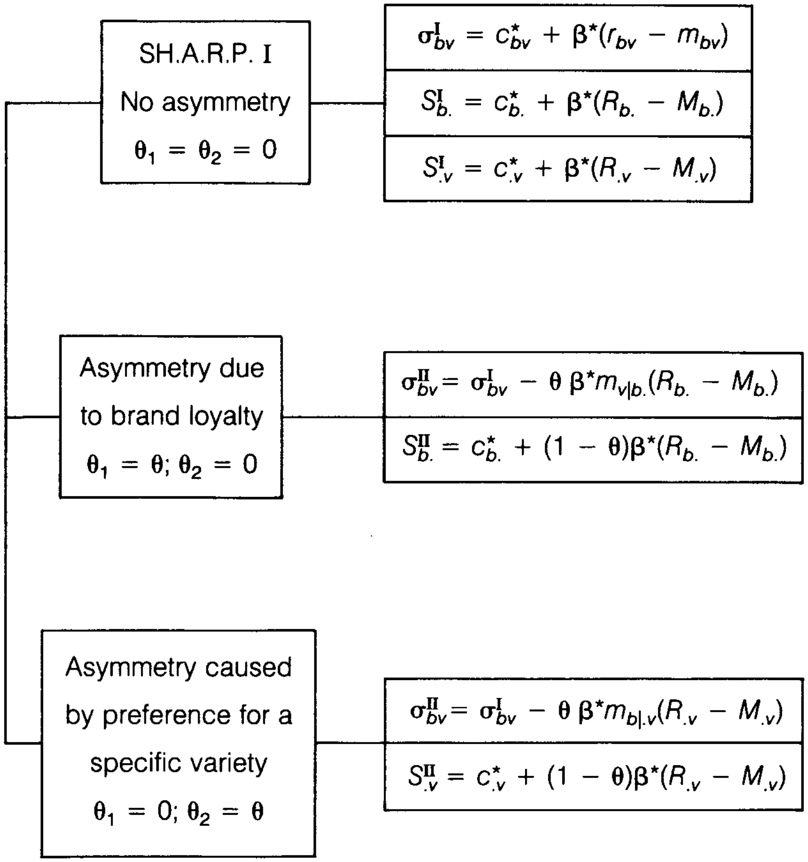
Extending SH.A.R.P. to take into account asymmetric cannibalism within the product categories carried by a retailer yields what we call SH.A.R.P. II, which actually subsumes B & N's original formula. Appendix B proves that the optimal space allocation under asymmetry should be determined according to the following SH.A.R.P. II rule,

$$\begin{aligned}\sigma_{bv}^{\mathrm{II}} = c_{bv}^{*} + \beta^{*}[(r_{bv} - m_{bv}) - \theta_1\mu_{v|b.}(R_{b.} - M_{b.}) \\ - \theta_2\mu_{b|.v}(R_{.v} - M_{.v})],\end{aligned} \qquad [13]$$

where: $c_{bv}^{*} = \gamma_{bv}\, c_{bv}/\overline{G}$ and $\beta^{*} = \beta/\overline{G}$.

FIGURE 3

Comparison of Allocation Rules



The various cases encompassed by [13] are reviewed in Figure 3. In accordance with Table 1, the total space share allocated to a brand's entire line, $S_{b.}$, is obtained by adding the shares left to items that make it up. Likewise, the portion of the gondola to be saved for a variety-type, $S_{.v}$, is arrived at by summing over brands that offer items of that same type.

The extent to which asymmetry alters the distribution of shelf space can be assessed by comparing the new formula (SH.A.R.P. II) with the original one (I). As a matter of illustration, let us concentrate on the aggregate level, when asymmetry results solely from brand loyalty ($\theta_1 = \theta$ and $\theta_2 = 0$). SH.A.R.P. II commends:

$$S_{b.}^{\text{II}} = \sum_v c_{bv}^* + \beta^*\left[\left(\sum_v r_{bv} - \sum_v m_{bv}\right) - \theta(R_{b.} - M_{b.})\sum_v \mu_{v|b.}\right].$$

Given that $\Sigma_v\, r_{bv} = R_{b.}$, $\Sigma_v\, m_{bv} = M_{b.}$ and $\Sigma_v\, \mu_{v|b.} = 1$ (as shown in Appendix B) and letting $c_{b.}^* = \Sigma_v\, c_{bv}^*$, it reduces to

$$S_{b.}^{\text{II}} = c_{b.}^* + \beta^*(1 - \theta)(R_{b.} - M_{b.}). \qquad [14]$$

Wrongly ignoring the asymmetry, SH.A.R.P. I advocates:

$$S_{b.}^{\text{I}} = c_{b.}^* + \beta^*(R_{b.} - M_{b.}). \qquad [15]$$

Not too surprisingly, the asymmetry parameter, θ, makes all the difference between [14] and [15]: it dampens the primary effect of the item's visibility on the buyer's choice (i.e., β^*). The larger the θ, the higher the relative importance attached to handling cost considerations in dividing up the product category's selling area. Intuitively, this can be justified by realizing that as θ approaches unity, brands' sales shares tend to stabilize: substitution gets restricted within the limits of each brand's line and no longer extends across brands. Therefore no discernible impact on the brands' sales distribution is produced by reallocating space (refer again to Figure 1).

5. AN EMPIRICAL ILLUSTRATION

Since its development in 1983, SH.A.R.P. I has been tested on four different product assortments and in five large grocery stores. We use here the data collected by Alen (1986) during the most recent of the experiments designed by B & N, for which no result has yet been reported. It took place over a 20-week period during the spring of 1986, in the largest Belgian hypermarket, the 16,440 square meter CORA-DISTRIMAS, located in Rocourt (Liège; with 54 checkout cash registers) and owned by the medium-sized FRADIS chain (ranked seventh among Belgian retailers). Highly decentralized as compared with its retailing competitors, FRADIS operates six hypermarkets and three groups of supermarket franchisees (Choc-Discount, Courthéoux, and Match). Its annual sales revenue amounts to 15 billion Belgian francs and its gross margin reaches 14 percent.

The experiment was run on the 20-item canned dog food assortment carried by CORA-DISTRIMAS. The product category examined was selected for its high sales-to-inventory turnover ratio (which facilitates the rapid observation of the impact produced by merchandising actions), and for the stability of its total sales volume (Q). It included six national brands and one private label, essentially offered in large package sizes (basically in 0.8 and 1.2 kgs; only two 0.4 kg items were listed). Weekly observations on sales volume, allocated shelf space, and promotions were gathered for all items.

The experimental treatments involved three distinct space arrangements, implemented over three consecutive 4-week periods. During the first period, an attempt was made for equi-repartition of the available shelf space; only items on promotion were allocated relatively more space. At the beginning of the second period, the visibility of some high-margin national brands was increased at the expense of the low-margin private label. In period three the prior allocation was reversed so as to favor low-priced items. Given that the purpose was to validate SH.A.R.P. I, possibilities of asymmetry were ignored and the analysis was restricted to aggregate effects at the brand level. No systematic variation along the package-size dimension was planned. In period two, however one such variation happened to occur. Weekly observations were averaged out per period; hence, we deal with a combination of two pre-test/post-test cross-sections of 20 items each: period two vs. period one, and period three vs. period two.

Since the promotional activities planned by manufacturers could not be controlled, a promotion covariate was introduced into the specification of the intrinsic attraction exerted by each item, i.e.:

$$a_{bv,\tau} = s^{\beta}_{bv,\tau}\, e^{\gamma p_{bv,\tau}}, \qquad [16]$$

with b = Bonzo, Chappi, Fido, Loyal, Pal, Pluto, or Produits Francs (brands);

v = 0.4, 0.8, or 1.2 kg (package sizes);

τ = 1, 2, or 3 (time periods);

and where the promotion variable, $p_{bv,\tau}$, is determined by both the monetary value of the price cut and the duration of the promotion.

In view of the nature of the product, we hypothesize a priori that asymmetry in cannibalism results from the shopper's propensity to buy a specific package size. Since the package size is likely to be related to the size of the dog, purchasers are expected to display some loyalty to a particular content volume. Note also that the major axes along which brands diversify their lines are precisely: package sizes for dogs, and food components (e.g., fish vs. meat) for cats.

Thus we asume $\theta_2 = \theta$ and $\theta_1 = 0$. Accordingly, the attractiveness terms [7] are defined by

$$t_{bv,\tau} = a_{bv,\tau}/A^{\theta}_{.v,\tau}, \qquad [17]$$

where $A_{.v,\tau} = \sum_{b} (s^{\beta}_{bv,\tau}\, e^{\gamma p_{bv,\tau}})$.

Let us observe that the variation over time of an item's sales share can be studied under the ratio form,[10]

$$m_{bv,\tau+1}/m_{bv,\tau} = (a_{bv,\tau+1}/a_{bv,\tau})(A_{.v,\tau}/A_{.v,\tau+1})^{\theta}(T_{..,\tau}/T_{..,\tau+1}).$$

Transforming this ratio into logarithms yields the following equation,

$$y_{bv,\tau} = \beta.[\ln s_{bv,\tau+1} - \ln s_{bv,\tau}] + \gamma.[p_{bv,\tau+1} - p_{bv,\tau}]$$
$$+\theta.[\ln A_{.v,\tau} - \ln A_{.v,\tau+1}] + \sum_{L=1}^{2} d_L \cdot \delta_{L,\tau}; \qquad [18]$$

where: $y_{bv,\tau} = \ln m_{bv,\tau+1} - \ln m_{bv,\tau}$, $\tau = 1, 2$;
d_L stands for $[\ln T_{..,L} - \ln T_{..,L+1}]$;
$\delta_{L,\tau}$ denotes a dummy variable equal to one if $L = \tau$, and to zero otherwise (introduced to capture the variation over time of the denominator of the attraction model).

In the absence of asymmetry ($\theta = 0$), equation [18] is linear in the parameters. In that special case our linearization reveals similarities to Nakanishi and Cooper's (1982) structural transformation of the MCI model into a dummy variable multiple-regression equation. But when θ is different from zero, [18] remains nonlinear, since the $A_{.v}$ variables are conditioned by all parameter values. To overcome this difficulty, we propose the following iterative estimation procedure.

Step 0: Assume $\theta = 0$ and estimate β and γ by applying the OLS method to the reduced form of equation [18]. Then set $h = 1$.

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[10] The terms defined by [17] could be further generalized using the differentiation factors and would then read

$$t_{bv,\tau} = \alpha_{bv}(a_{bv,\tau}/A^{\theta}_{.v,\tau}).$$

But estimating the item-specific constant factors, α_{bv}, would use up many degrees of freedom. In any case, working on the ratios of observations on successive periods $[\tau, \tau + 1]$ eliminates them,

$$t_{bv,\tau+1}/t_{bv,\tau} = (a_{bv,\tau+1}/a_{bv,\tau})(A_{.v,\tau}/A_{.v,\tau+1})^{\theta}.$$

Step h: Use the estimates obtained for β and γ in the previous step ($h - 1$) to determine the attraction of each item ($a_{bv,\tau}$) and the total attraction exerted by items of the same package size ($A_{.v,\tau}$), which are thus replaced by their estimated numerical values. Then re-estimate β and γ, together with θ, by applying OLS to the full form of equation [18]. Set $h = h + 1$ and return to step h. Stop when convergence occurs; i.e., when the difference observed between the new set of estimates just obtained and the one derived at the previous stage becomes negligible.

Simulation studies by Gijsbrechts and Vanden Abeele (1988) prove that an equivalent iterative process converges quickly to unbiased estimates.

Table 2 summarizes the results we obtained when applying this stepwise method to our sample. Standard errors (in parentheses) and t-ratios are reported below the OLS estimates. Package-size asymmetry shows up as significantly as the item-visibility effect. The alternative hypothesis, i.e., whether asymmetry could be caused by brand loyalty, was also tested; as expected, it was rejected.

The estimates of γ and β are shown to be remarkably stable. Furthermore, given the cross-sectional nature of our sample, the goodness-of-fit measure (R^2) indicates that the model offers a fairly reliable description of the cannibalism prevailing within the retailer's assortment.[11]

6. NORMATIVE IMPLICATIONS

Introducing a couple of simplifying assumptions can greatly help to reconcile theory with practice. These assumptions are necessary in order to relate allocation formula [13] to the rules of thumb commonly employed by retailers to determine shelf-arrangement. Referring to the replenishment cost function [4], let us regard it as a linear function of the frequency of the handling operations. This amounts to setting $\gamma_{bv} = 1$, then

$$C_{bv} = f_{bv}(q_{bv}/s_{bv}),$$

and it follows that

---

[11] Usually high R^2s are obtained in time-series analyses and lower ones for cross-sections. It should further be realized that we voluntarily opted for a transformation which should clear out the estimates of the spurious correlation that across-item differences (within a given period) might produce: [18] really catches hold of the causal relationship between over-time action changes and sales response.

TABLE 2

Econometric Analysis of Experimental Results

Package-size asymmetry

Table 2a. Pooled 3-period sample ($n = 40$)

	Promotion γ	Visibility β	Asymmetry θ	R^2
No asymmetry	0.0341	0.5240	0	0.6531
	(0.0046)	(0.1509)		
	7.4549	3.4718		
Final iteration	0.0399	0.3799	0.6120	0.7023
	(0.0048)	(0.1512)	(0.2321)	
	8.2396	2.5123	2.6369	

Table 2b. Period 2 versus period 1 ($n = 20$)

	Promotion γ	Visibility β	Asymmetry θ	R^2
No asymmetry	0.0257	0.6981	0	0.6333
	(0.0054)	(0.2625)		
	4.7598	2.6589		
Final iteration	0.0345	0.4814	0.8426	0.7231
	(0.0058)	(0.2467)	(0.2988)	
	5.9857	1.9518	2.8199	

$$c_{bv} = \gamma_{bv}(C_{bv}/\Pi) = f_{bv}(q_{bv}/s_{bv})/\Pi,$$

$$c^*_{bv} = c_{bv}/\sum_c \sum_j c_{cj} = \frac{f_{bv}(q_{bv}/s_{bv})}{\sum_c \sum_j f_{cj}(q_{cj}/s_{cj})} = \frac{f_{bv}(m_{bv}/s_{bv})}{\sum_c \sum_j f_{cj}(m_{cj}/s_{cj})}.$$

If we further recognize that product-handling operations essentially depend on the packaging of the goods and not so much on the brand concerned, then we can assume that $f_{bv} = f_v$ and

$$c^*_{bv} = \frac{f_v(m_{bv}/s_{bv})}{\sum_c \sum_j f_j(m_{cj}/s_{cj})}.$$

Relying on the multiple experiments they conducted, B & N (1988, page 224) recommend that the optimal allocation be approximated by reference to the results generated when the total space available is equally distributed among the items; i.e. when $s_{bv} = S/n = s$. In that case, the relative cost of handling item (b,v) becomes

$$c^*_{bv} = \frac{f_v m_{bv}}{\sum\limits_c \sum\limits_j f_j m_{cj}} = \frac{f_v m_{bv}}{\sum\limits_j f_j \sum\limits_c m_{cj}}$$

$$= \frac{f_v m_{bv}}{\sum\limits_j f_j M_{.j}} = \left(\frac{f_v}{\varphi}\right) m_{bv},$$

where $\varphi = \Sigma_j f_j M_{.j}$ stands for the weighted mean of the unit costs of replenishment.

The relative cost of handling of all items of the v-th variety type can then be written,

$$c^*_{.v} = \sum_b c^*_{bv} = \left(\frac{f_v}{\varphi}\right) \sum_b m_{bv} = \left(\frac{f_v}{\varphi}\right) M_{.v}. \qquad [19]$$

Substituting [19] into the formula for the space to be allocated to the v-th variety-type, defined by $S^{\text{II}}_{.v}$ in Figure 3, we get

$$S^{\text{II}}_{.v} = (f_v/\varphi)M_{.v} + (1 - \theta)\, \beta^*(R_{.v} - M_{.v}),$$

$$\text{where } \beta^* = \beta/\left(\sum_b \sum_v c_{bv}\right) = \beta\Pi/\left(\sum_b \sum_v f_v[q_{bv}/s]\right)$$

$$= \beta s\Pi/\left(Q \sum_v f_v M_{.v}\right) = \beta s\Pi/(Q\varphi).$$

Now if we let $\omega_1 = (1 - \theta)\, \beta^*$ and $\omega_{2v} = (f_v/\varphi) - \omega_1$, the following relationship is arrived at,

$$S^{\text{II}}_{.v} = \omega_1 R_{.v} + \omega_{2v} M_{.v}. \qquad [20]$$

Since [20] is derived from [13] on the basis of the equi-repartition of the selling area, it constitutes a reasonable approximation of the optimal allocation. As will be shown numerically hereafter, [20] will in fact provide a solution close to the best one.

Thus [20] throws light on a rather simple and interesting interpretation of the SH.A.R.P. II optimization result. It now appears as a *linear combination of the two most regularly used rules of thumb:* on the one hand, allocation strictly proportional to the relative contribution to the assortment profitability ($R_{.v}$); on the other hand, allocation determined in direct proportion to the relative sales volume, ($M_{.v}$). On examining the coefficients, one realizes that ω_1 will most often lie in the [0,1] range and in cases where variations in the packaging of the goods are negligible (when $f_v \cong \varphi$, then $\omega_{2v} = 1 - \omega_1$), they will add up to one so that [20] can almost be assimilated with a weighted mean of the two ratios.

The incidence of the degree of asymmetry is quite clear: the higher it is, the heavier the relative weight attached to the sales-share ratio (as θ increases, ω_1 goes down to zero, while ω_2 tends to one). This confirms the emerging pre-eminence of handling-cost effects as asymmetry becomes more pronounced.

Actual determination of the true optimum calls for sophisticated techniques designed for solving complex systems of nonlinear equations. To circumvent a similar obstacle in using SH.A.R.P. I, B & N have suggested a heuristic approach which, when adapted for our problem, consists of exploring system [13] iteratively, with the equi-repartition as the arbitrary initial allocation: the allocation derived at one stage is fed back into the formula to determine the next (better) allocation. However rudimentary such a heuristic might seem, it was proved to converge rapidly toward excellent solutions.

When extended and applied in our case, it performs reasonably well. Table 3 introduces a hypothetical, yet fairly realistic, example of a 12-item assortment, with four brands each presented in three different variety-types. The values chosen for the key-parameters, β and θ, are those estimated for CORA-DISTRIMAS (reported in Table 2).

In order to render these results intuitively obvious, the basic attraction coefficients (α_{bv}) that determine the items' sales shares when visibility is identical have all been set equal to 1.00. Only the unit gross margins are differentiated—and by a substantial amount. Although the cost figure used for all variety-types may appear large when compared with profit, it represents slightly more than 63 percent of the net margin (in practice it may well exceed 70 percent).

The optimal space allocations produced by three different sets of parameter estimates $[\hat{\beta},\hat{\theta}]$ are compared in Table 4. The top set (boldface) is generated by the unbiased estimates: $[\hat{\beta} = 0.38; \hat{\theta} = 0.61]$; the second and third allocations correspond to cases where the retailer ignores the existence of asymmetries and either uses all the information available: $[\hat{\beta} = 0.524; \theta = 0]$, or only a subsample: $[\hat{\beta} = 0.698; \theta = 0]$. Starting with the third allocation, one easily verifies the property that the most profitable items get the largest space shares; i.e., those offered by the fourth

TABLE 3

Values Assigned to the Fixed Parameters
Unit Gross Margins
(g_{bv})

	Variety-type (v)	Assortment's width			Brand's mean
	Brand (b)	1	2	3	($\bar{g}_{b.}$)
D	1	10	14	18	14
E	2	11	15	19	15
P	3	12	16	20	16
T	4	13	17	21	17
H					
Variety's mean ($\bar{g}_{.v}$)		11.5	15.5	19.5	Grand mean ($\bar{g}_{..}$) 15.5

1. Assortment: 12 items (4 brands × 3 variety types)
2. Preference factors: α_{bv} = 1.00, for all b's and v's
3. Replenishment costs: f_v = 5,000, for all v's
4. Unbiased estimates of the visibility and asymmetry parameters: β = 0.38 and θ = 0.61

brand (which gets 32.4 percent against 18.6 percent for Brand 1) and of the third variety-type (which gets 58.2 percent against 14.9 percent for type 1). A similar pattern of space repartition is observed in both other cases as well, but as the visibility parameter (β) decreases and asymmetry takes place, the dispersion of the space distribution gets reduced (smoothing effect). Thus the third variety-type obtains only 35.3 percent of the available space when asymmetry enters the picture, instead of the 45.8 percent it would be entitled to if asymmetry could be disregarded. To summarize, those results confirm that asymmetry makes the allocation "less radical."

Similar to what B & N observed with SH.A.R.P. I, the allocation derived from the first iteration already defines a satisfactory solution: it yields 70.4 percent of the total-profit increment to be expected from the optimization. Yet, from a pragmatic point of view, the question must be raised whether a retailer should care at all about asymmetry: does demand

TABLE 4

Comparison of Allocations

σ_{bv}

Variety-type (v) / Brand (b)	1	2	3	$S_{b.}$
1	**0.0727**	**0.0770**	**0.0816**	**0.2313**
	0.0501	0.0690	0.0987	0.2178
	0.0307	0.0515	0.1040	0.1862
2	**0.0765**	**0.0810**	**0.0858**	**0.2433**
	0.0540	0.0752	0.1085	0.2377
	0.0344	0.0603	0.1274	0.2221
3	**0.0804**	**0.0852**	**0.0903**	**0.2559**
	0.0585	0.0822	0.1193	0.2600
	0.0389	0.0715	0.1570	0.2674
4	**0.0847**	**0.0897**	**0.0951**	**0.2695**
	0.0634	0.0900	0.1312	0.2846
	0.0445	0.0858	0.1940	0.3243
$S_{.v}$	**0.3143**	**0.3329**	**0.3528**	**1.000**
	0.2260	0.3164	0.4577	
	0.1485	0.2691	0.5824	

asymmetry really make enough difference in the retailer's profit to be a part of decision making?

This issue can be addressed in two ways: (1), by evaluating the opportunity loss incurred by a "myopic" retailer (who completely neglects the asymmetry and relies on SH.A.R.P. I rather than on SH.A.R.P. II); and (2), by assessing to what extent the retailer may actually improve on rules of thumb by adopting SH.A.R.P. II. Table 5 synthesizes the first approach and the second one is dealt with separately, in the next section. The first row in Table 5 gives the maximum profit that would be reached when implementing the truly optimal allocation; i.e., based on the best unbiased estimates derived from the experiment. In contrast, other rows show at which levels the retailer's profit would be limited should he allocate the space available using the less reliable estimates. Thus if asymmetry prevails as significantly as in the CORA-DISTRIMAS case, the retailer's

TABLE 5

Sensitivity to Asymmetry and Estimation Bias

Assumed parameter estimates		Profit/cost	Opportunity loss
1a. Unbiased estimates using the full information available	$\beta = 0.38$; $\theta = 0.61$	190,471.0/120,469.5	**None**
1b. Estimates derived from the first two periods	$\beta = 0.48$; $\theta = 0.84$	190,343.1/120,603.8	**0.067%**
2. Myopism in the full information case	$\beta = 0.524$; $\theta = 0.00$	184,152.2/129,465.7	**3.431%**
3. Myopism in the limited information case	$\beta = 0.698$; $\theta = 0.00$	158,486.7/158,501.0	**20.181%**

NB: All profit levels are calculated under the assumption that $\beta = 0.38$ and $\theta = 0.61$ are the true parameter values.

myopism could cost between 3.5 percent and 20 percent of the net profit. The relative importance of this opportunity loss can best be measured by comparing it to the negligible incidence of the sampling error (reflected in the 0.07 percent difference observed when only the first half of the data collected during the experiment is used).

7. WHAT ABOUT RULES OF THUMB?

In practice, difficulties in the estimation of sales elasticities and cross-elasticities with respect to items' visibility and the lack of accurate measurement of direct product-handling costs lead retailers to rely on proportionality-to-sales or proportionality-to-gross margins norms for space allocation. However, B & N demonstrated the significance of the incremental benefits that could be derived from the implementation of SH.A.R.P. I (1988, Table 4, page 223) and concluded that: "Sharp retailers may well forget their rules of thumb."

Including asymmetry into the consumer response model adds realism to the approach but unfortunately does not simplify it. Is the resulting increase in sophistication really worth its cost? Section 6 has partially answered this question: SH.A.R.P. II clearly performs better than SH.A.R.P. I in the presence of asymmetry. Yet this is not surprising, since it was designed precisely for that purpose. Furthermore, equation [20] establishes that rules of thumb might well be reconciled with an optimization formula!

Testing their robustness against SH.A.R.P. II appears as an exercise of high managerial potential. Nonetheless, there is a need for the formalization of these rules as well as of the space allocation they determine. Since the proportionality-to-sales rule reveals a special case of the proportionality-to-margin rule (obtained by setting $g_{bv} = 1$, for all b's and v's), we focus on the latter.

The partition of available space ultimately resulting from the systematic reference to such a benchmark should, per definition, satisfy the following system of equations,[12]

$$\tilde{\sigma}_{bv} = g_{bv}\,\tilde{m}_{bv} / \sum_{c} \sum_{j} g_{cj}\,\tilde{m}_{cj}, \qquad [21]$$

where: $\tilde{m}_{bv}$ stands for the sales share that brand b's item v would capture in the long run, should the proportionality-to-margin rule be consistently applied over time;

$$\tilde{m}_{bv} = \tilde{t}_{bv} / \sum_{d} \sum_{w} \tilde{t}_{dw},\ \tilde{t}_{bv} = \alpha_{bv}\,\tilde{\sigma}^{\beta}_{bv} / \tilde{A}^{\theta}_{.v}\ \text{and}\ \tilde{A}_{.v} = \sum_{b} \alpha_{bv}\,\tilde{\sigma}^{\beta}_{bv};$$

for $b \in D$ and $v \in W$.

Appendix C establishes that the solution toward which decision rule [21] converges in steady state is given by,

$$\tilde{\sigma}_{bv} = \frac{[\lambda_{bv} / \Lambda^{(\theta/\bar{\beta})}_{.v}]}{\sum_{c} \sum_{j} [\lambda_{cj} / \Lambda^{(\theta/\bar{\beta})}_{.j}]}, \qquad [22]$$

with:

$$\lambda_{bv} = [\alpha_{bv}\,g_{bv}]^{1/(1-\beta)}, \qquad [23]$$

$$\Lambda_{.v} = \sum_{b} \alpha_{bv}\,\lambda^{\beta}_{bv} = \sum_{b} [\alpha_{bv}\,g^{\beta}_{bv}]^{1/(1-\beta)}, \qquad [24]$$

$$\bar{\beta} = 1 - (1 - \theta)\,\beta. \qquad [25]$$

The resulting sales shares and profit are also shown to be determined respectively by

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[12] In what follows, we assume that package-size preference is the prevailing source of asymmetry in order to facilitate the reference to the empirical part of our work. The alternative case of brand loyalty can nevertheless be studied analogously without trouble (only appropriate subscripts interchange is required).

$$\tilde{m}_{bv} = \frac{[\tilde{\sigma}_{bv}/g_{bv}]}{\sum_c \sum_j [\sigma_{cj}/g_{cj}]}, \qquad [26]$$

and

$$P = \frac{Q\left[1 - \frac{1}{S}\sum_b \sum_v (f_v/g_{bv})\right]}{\sum_b \sum_v [\tilde{\sigma}_{bv}/g_{bv}]}. \qquad [27]$$

If instead of relying on gross margins, the retailer allocates space according to sales volumes, [22] still applies but with the following redefinitions:

$$\lambda_{bv} = \alpha_{bv}^{1/(1-\beta)} \textit{ and } \Lambda_{.v} = \sum_b \alpha_{bv}^{1/(1-\beta)}.$$

The resulting sales shares and profit then reduce to: $\tilde{m}_{bv} = \tilde{\sigma}_{bv}$, and

$$P = Q\left[\sum_b \sum_v \tilde{\sigma}_{bv}\, g_{bv}\right] - \sum_b \sum_v \tilde{C}_{bv}.$$

But in this case, $\tilde{C}_{bv} = f_v(Q/S)$, hence we obtain:

$$P = Q\left[\bar{g} - \frac{1}{S}\sum_v N_v f_v\right] \qquad [28]$$

where $\bar{g} = \Sigma_b\, \Sigma_v g_{bv} \tilde{\sigma}_{bv}$ is the overall weighted mean of unit gross margins on the items carried and N_v the number of brands offered in variety v.

Equations [22]–[28] enable us to assess exactly the dependability of the rules of thumb in the presence of demand asymmetry caused by the consumer's preference for a specific variety-type.

Using the same parameter values as those chosen to illustrate the optimization, we apply [22]–[25] to derive the allocation based on the proportionality-to-margins rule. Results are displayed in Table 6. The resulting profit is revealed to be less than three percent below the maximum. If, on the contrary, the retailer relies on the proportionality-to-sales rule,

$$\tilde{m}_{bv} = \tilde{\sigma}_{bv} = 1/12,$$

because in our numerical example, all α_{bv} are assumed to be identical. Consequently, [28] gives

TABLE 6

Allocation Based on Rules of Thumb

	Allocation proportional to Gross Margin			
Variety-type (v) Brand (b)	1	2	3	$S_{b.}$
1	0.0465	0.0702	0.0952	0.2119
2	0.0543	0.0785	0.1039	0.2367
3	0.0624	0.0871	0.1128	0.2623
4	0.0710	0.0960	0.1221	0.2891
$S_{.v}$	0.2342	0.3318	0.4340	1.0000

Allocation proportional to:	Profit	Opportunity loss
gross margin	185,518.2	2.670%
sales	190,000.0	0.248%

$$P = 20{,}000\,[15.5 - (5{,}000 \times 12/10{,}000)] = 190{,}000,$$

which shows that the rule determines a quasi-optimal allocation.

Thus rules of thumb, which may depart markedly from the optimal allocation when substitution patterns are symmetric, perform remarkably well in the present case of asymmetric cannibalism. The reader should be warned, however, against drawing from this exemplative analysis an immediate conclusion favoring rules of thumb. It would at best be premature, because we should not lose sight of the fact that the formalization of retailers' practical allocation norms, through [21], implies that store managers are able to anticipate perfectly the sales redistributions generated by all possible changes in the shelf arrangement[13] (or that they consistently adjust the space allocation until sales stabilize at their steady-state level). Departures from this rational behavior deteriorate the peformance of rules of thumb.

Future investigations will, it is hoped, delimit their range of applicability. Here let us merely state that *asymmetry does not seem to rule out rules of thumb!*

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[13] B & N have also based the comparison of rules of thumb with SH.A.R.P. I on that same assumption.

8. CONCLUSIONS

Optimal space allocation should reflect both the cost and demand impacts of shelf rearrangements. SH.A.R.P. I does pay attention to both these components of the assortment profitability and strongly questions allocations based on retailers' rules of thumb. But SH.A.R.P. I disregards variants of cannibalism that produce asymmetry in demand substitution patterns.

SH.A.R.P. II, introduced in this article, tackles those variants and leads to a generalization of the optimal allocation algorithm proposed by B & N. The empirical illustration, although based on data collected during an experiment that was not specifically designed to detect asymmetries, confirms that:

1. Asymmetric forms of cannibalism are at work within retail assortments: their statistical impact is measurable and significant;
2. Overall, asymmetry dampens the influence of product visibility and thereby restricts cannibalism (globally);
3. Asymmetry affects space allocation to a non-negligible extent, making SH.A.R.P. I sub-optimal and calling for smoother shelf repartition.

It also induces us to revise our prior judgment on the inadequacy of retailers' simple allocation norms: referring to rules of thumb might not be as bad as once suggested by B & N. This last controversial finding requires further careful investigation before a firm conclusion can be reached. Systematic analyses of sensitivity to varying parameter values (especially to the preference coefficients, α_{bv}), as well as additional simulation and field experiments are obviously needed to determine the level of external validity that would allow rules of thumb to apply beyond the specific case considered here.

Although this research does not reach that point, SH.A.R.P. II does offer a proper methodological framework for determining what specific conditions are needed for rules of thumb to perform satisfactorily and, more importantly, why they are necessary.

APPENDIX A
DERIVATION OF THE GENERAL EXPRESSION FOR CROSS-ELASTICITIES

Starting from [9], let $t_{cj} = a_{cj} A_{c.}^{-\theta_1} A_{.j}^{-\theta_2}$, $T_{c.} = \Sigma_w t_{cw}$, $T_{.j} = \Sigma_d t_{dj}$, and $T_{..} = \Sigma_d \Sigma_w t_{dw}$. Accordingly, the sales share of brand c's item j is defined by $m_{cj} =$

$t_{cj}/T_{..}$. Then the marginal effect of a variation in the visibility of brand b's item v is determined by,

$$\frac{\partial m_{cj}}{\partial s_{bv}} = \frac{1}{T_{..}^2}\left[T_{..}\frac{\partial t_{cj}}{\partial s_{bv}} - t_{cj}\frac{\partial T_{..}}{\partial s_{bv}}\right]$$

or

$$\frac{\partial m_{cj}}{\partial s_{bv}} = \frac{1}{T_{..}}\left[\frac{\partial t_{cj}}{\partial s_{bv}} - m_{cj}\frac{\partial T_{..}}{\partial s_{bv}}\right] \qquad [A.1]$$

Differentiating the numerator of the sales share, m_{cj}, yields

$$\frac{\partial t_{cj}}{\partial s_{bv}} = A_{c.}^{-\theta_1} A_{.j}^{-\theta_2} \frac{\partial a_{cj}}{\partial s_{bv}} - \theta_1\, a_{cj}\, A_{.j}^{-\theta_2} A_{c.}^{-(1+\theta_1)} \frac{\partial A_{c.}}{\partial s_{bv}}$$

$$- \theta_2\, a_{cj}\, A_{c.}^{-\theta_1} A_{.j}^{-(1+\theta_2)} \frac{\partial A_{.j}}{\partial s_{bv}} \qquad [A.2]$$

We observe that: $A_{c.} = a_{cv} + \Sigma_{w \neq v}\, a_{cw}$, $A_{.j} = a_{bj} + \Sigma_{d \neq b}\, a_{dj}$, therefore:

$$\frac{\partial A_{c.}}{\partial s_{bv}} = \delta_{cb}\frac{\partial a_{bv}}{\partial s_{bv}}, \; \frac{\partial A_{.j}}{\partial s_{bv}} = \delta_{jv}\frac{\partial a_{bv}}{\partial s_{bv}},$$

$$and\; \frac{\partial a_{cj}}{\partial s_{bv}} = \delta_{cb} \cdot \delta_{jv} \cdot \frac{\partial a_{bv}}{\partial s_{bv}}.$$

Hence we realize that [A.2] can be rewritten,

$$\frac{\partial t_{cj}}{\partial s_{bv}} = A_{c.}^{-\theta_1} A_{.j}^{-\theta_2} \frac{\partial a_{bv}}{\partial s_{bv}} \{\delta_{cb} \cdot \delta_{jv}$$

$$- a_{cj}[(\delta_{cb}\, \theta_1/A_{c.}) + (\delta_{jv}\, \theta_2/A_{.j})]\}$$

Since: $a_{bv} = \alpha_{bv}\, s_{bv}^{\beta}$, we have:

$$\frac{\partial a_{bv}}{\partial s_{bv}} = \beta\, \alpha_{bv}\, s_{bv}^{\beta-1} = \beta(a_{bv}/s_{bv}).$$

Substituting the latter result and t_{cj}, where appropriate, we get:

$$\frac{\partial t_{cj}}{\partial s_{bv}} = \beta\frac{t_{cj}}{s_{bv}}\{\delta_{cb} \cdot \delta_{jv} - a_{bv}[\delta_{cb}(\theta_1/A_{c.}) + \delta_{jv}(\theta_2/A_{.j})]\} \qquad [A.3]$$

The second term in [A.1] can then be deduced directly from [A.3], since:

$$\frac{\partial T_{..}}{\partial s_{bv}} = \sum_d \sum_w \frac{\partial t_{dw}}{\partial s_{bv}},$$

and noting that:

$$\sum_d \sum_w t_{dw} \cdot \delta_{db} \cdot \delta_{wv} = t_{bv},$$

$$\sum_d \sum_w t_{dw} \cdot \delta_{db} \cdot (\theta_1/A_{d.}) = \theta_1 \sum_w (t_{bw}/A_{b.}) = \theta_1(T_{b.}/A_{b.}),$$

$$\sum_d \sum_w t_{dw} \cdot \delta_{wv} \cdot (\theta_2/A_{.w}) = \theta_2 \sum_d (t_{dv}/A_{.v}) = \theta_2(T_{.v}/A_{.v}),$$

we obtain:

$$\frac{\partial T_{..}}{\partial s_{bv}} = \frac{\beta}{s_{bv}} \{t_{bv} - a_{bv}[\theta_1(T_{b.}/A_{b.}) + \theta_2(T_{.v}/A_{.v})]\} \qquad [A.4]$$

Integrating [A.3] and [A.4] into [A.1] leads to,

$$\frac{\partial m_{cj}}{\partial s_{bv}} = \frac{\beta}{s_{bv}} \left\{ t_{cj}\left(\delta_{cb}\, \delta_{jv} - a_{bv}\left[\delta_{cb} \frac{\theta_1}{A_{c.}} + \delta_{jv} \frac{\theta_2}{A_{.j}} \right] \right) \right.$$
$$\left. - m_{cj}\left(t_{bv} - a_{bv}\left[\theta_1 \frac{T_{b.}}{A_{b.}} + \theta_2 \frac{T_{.v}}{A_{.v}} \right] \right) \right\} .$$

Realizing that $(\delta_{cb}/A_{c.})$ and $(\delta_{jv}/A_{.j})$ can be replaced by $(\delta_{cb}/A_{b.})$ and $(\delta_{jv}/A_{.v})$ respectively, and remembering that: $t_{cj}/T_{..} = m_{cj}$, $t_{bv}/T_{..} = m_{bv}$, $T_{b.}/T_{..} = M_{b.}$ and $T_{.v}/T_{..} = M_{.v}$, the derivative is reduced to:

$$\frac{\partial m_{cj}}{\partial s_{bv}} = \frac{\beta m_{cj}}{s_{bv}} \left\{ \delta_{cv}\, \delta_{jv} - a_{bv} \left(\delta_{cb} \frac{\theta_1}{A_{b.}} + \delta_{jv} \frac{\theta_2}{A_{.v}} \right) \right.$$
$$\left. - m_{bv} + a_{bv} \left(M_{b.} \frac{\theta_1}{A_{b.}} + M_{.v} \frac{\theta_2}{A_{.v}} \right) \right\} .$$

Multiplying both sides by (s_{bv}/m_{cj}), we define the elasticity,

$$\eta(m_{cj}, s_{bv}) = \beta \left\{ (\delta_{cb}\, \delta_{jv} - m_{bv}) - \theta_1\left(\frac{a_{bv}}{A_{b.}}\right) (\delta_{cb} - M_{b.}) \right.$$
$$\left. - \theta_2\left(\frac{a_{bv}}{A_{.v}}\right) (\delta_{jv} - M_{.v}) \right\}$$

Setting $a_{bv}/A_{b.} = \mu_{v|b.}$ and $a_{bv}/A_{.v} = \mu_{b|.v}$, expression [12] is deduced.

APPENDIX B
GENERALIZATION OF SH.A.R.P. UNDER ASYMMETRY

SH.A.R.P. II is derived by substituting the generalized cross-elasticities, defined in [12], in the optimization rule [1]. Those enter into the weighted mean [2], which gives:

$$\bar{\eta}_{bv} = \beta \sum_c \sum_j r_{cj}[(\delta_{jv}\,\delta_{cb} - m_{bv}) - \theta_1\mu_{v|b.}(\delta_{cb} - M_{b.}) - \theta_2\mu_{b|.v}(\delta_{jv} - M_{.v})].$$

Since the r_{cj}'s are the items' relative contributions to the assortment profitability, they add up to one ($\Sigma_c\, \Sigma_j\, r_{cj} = 1$), and setting $R_{b.} = \Sigma_j\, r_{bj}$ and $R_{.v} = \Sigma_c\, r_{cv}$, it follows that:

$$\sum_c \sum_j r_{cj}\, m_{bv} = m_{bv} \sum_c \sum_j r_{cj} = m_{bv};$$

$$\sum_c \sum_j r_{cj}\, \mu_{v|b.}(\delta_{cb} - M_{b.}) = \mu_{v|b.}\left(\sum_c \delta_{cb} \sum_j r_{cj} - M_{b.} \sum_c \sum_j r_{cj}\right)$$

$$= \mu_{v|b.}\left(\delta_{bb} \sum_j r_{bj} - M_{b.}\right) = \mu_{v|b.}(R_{b.} - M_{b.});$$

$$\sum_c \sum_j r_{cj}\, \mu_{b|.v}(\delta_{jv} - M_{.v}) = \mu_{b|.v}\left(\sum_j \delta_{jv} \sum_c r_{cj} - M_{.v} \sum_c \sum_j r_{cj}\right)$$

$$= \mu_{b|.v}\left(\delta_{vv} \sum_c r_{cv} - M_{.v}\right) = \mu_{b|.v}(R_{.v} - M_{.v});$$

$$\sum_c \sum_j r_{cj}\, \delta_{jv}\, \delta_{cb} = \sum_c \delta_{cb} \sum_j \delta_{jv}\, r_{cj} = \sum_c \delta_{cb}\, \delta_{vv}\, r_{cv} = \delta_{bb}\, r_{bv} = r_{bv}.$$

Therefore the weighted mean reduces to,

$$\bar{\eta}_{bv} = \beta[(r_{bv} - m_{bv}) - \theta_1\, \mu_{v|b.}(R_{b.} - M_{b.}) - \theta_2\, \mu_{b|.v}(R_{.v} - M_{.v})].$$

Summing over v and b then yields one of the normalizing terms in [1],

$$\bar{N} = \beta\left[\left(\sum_b \sum_v r_{bv} - \sum_b \sum_v m_{bv}\right) - \theta_1 \sum_b (R_{b.} - M_{b.}) \sum_v \mu_{v|b.} - \theta_2 \sum_v (R_{.v} - M_{.v}) \sum_b \mu_{b|.v}\right]$$

We see that, $\sum_b \sum_v r_{bv} = \sum_b \sum_v m_{bv} = 1;$

$$\sum_v \mu_{v|b.} = \sum_v (a_{bv}/A_{b.}) = A_{b.}/A_{b.} = 1;$$

$$\sum_b (R_{b.} - M_{b.}) = \sum_b\left(\sum_v r_{bv} - \sum_v m_{bv}\right) = 1 - 1 = 0;$$

$$\sum_b \mu_{b|.v} = \sum_b (a_{bv}/A_{.v}) = A_{.v}/A_{.v} = 1;$$

$$\sum_{v} (R_{.v} - M_{.v}) = \sum_{v} \left(\sum_{b} r_{bv} - \sum_{b} m_{bv} \right) = 1 - 1 = 0.$$

Hence we conclude that $\overline{N} = 0$ and the optimization rule [1] becomes

$$\sigma^{\text{II}}_{bv} = \{\beta[(r_{bv} - \underline{m}_{bv}) - \theta_1\ \mu_{v|b.}(R_{b.} - M_{b.}) - \theta_2\ \mu_{b|.v}(R_{.v} - M_{.v})] \\ + \gamma_{bv}\ c_{bv}\}/\overline{G},$$

a result equivalent to [13].

Note that the nil balance of the competitive interactions, reflected in $\overline{N} = 0$, results from the consistency of the attraction model: what is gained by an item comes at the expense of others within the same assortment.

APPENDIX C
IMPLICATIONS OF RULES OF THUMB

C.1. Allocation Rule

To avoid a long and tedious derivation, we limit ourselves here to proving that [22] solves [21].

Consider the attraction exerted by variety v. Substituting [22] into it yields

$$\tilde{A}_{.v} = \sum_{c} \alpha_{cv}[\lambda_{cv}/\Lambda_{.v}^{(\theta/\overline{\beta})}]^{\beta}/\beta,$$

where $\tilde{D}$ stands for the denominator of $\tilde{\sigma}_{bv}$.
It may also be rewritten

$$\tilde{A}_{.v} = \left[\sum_{c} \alpha_{cv}\ \lambda^{\beta}_{cv} \right] / [\Lambda_{.v}^{(\theta\beta/\overline{\beta})}\ \tilde{D}^{\beta}],$$

which according to [24] reduces to

$$\tilde{A}_{.v} = \Lambda_{.v}^{1-(\theta\beta/\overline{\beta})}/\tilde{D}^{\beta}$$

and using [25],

$$\tilde{A}_{.v} = [\Lambda_{.v}^{(1-\beta)/\overline{\beta}}]/\tilde{D}^{\beta}.$$

Hence the numerator of the sales share, $\tilde{m}_{bv}$, is determined as follows,

$$\tilde{t}_{bv} = \frac{\alpha_{bv}[\lambda_{bv}/\Lambda_{.v}^{(\theta/\overline{\beta})}]^{\beta}}{D^{\beta}} \frac{D^{\beta\theta}}{\Lambda_{.v}^{(\theta(1-\beta)/\overline{\beta})}}.$$

Grouping terms, one gets

$$\tilde{t}_{bv} = [\alpha_{bv}\ \lambda^{\beta}_{bv}]\ \Lambda_{.v}^{-(\theta/\overline{\beta})}\ \tilde{D}^{-\beta(1-\theta)}.$$

Then the numerator of the space share, $\tilde{\sigma}_{bv}$, is defined by

$$g_{bv}\ \tilde{t}_{bv} = g_{bv}\ \alpha_{bv}[\alpha_{bv}\ g_{bv}]^{\beta/(1-\beta)}\ \Lambda_{.v}^{-(\theta/\overline{\beta})}\ \tilde{D}^{-\beta(1-\theta)},$$

or

$$g_{bv}\,\bar{t}_{bv} = [\alpha_{bv}\, g_{bv}]^{1/(1-\beta)}\, \Lambda_{.v}^{-(\theta/\bar{\beta})}\, \bar{D}^{-\beta(1-\theta)},$$

which yields

$$g_{bv}\,\bar{t}_{bv} = \lambda_{bv}\, \Lambda_{.v}^{-(\theta/\bar{\beta})}\, \bar{D}^{-\beta(1-\theta)},$$

an expression equivalent to the numerator of [22] up to a constant scaling factor, which completes the proof.

C.2. Resulting Sales Shares

Starting from [21], we realize that

$$\bar{m}_{cj} = (\bar{\sigma}_{cj}/g_{cj})\left[\sum_{d}\sum_{w} g_{dw}\,\bar{m}_{dw}\right]$$

and summing over c and j, we get

$$1 = \left[\sum_{c}\sum_{j}(\bar{\sigma}_{cj}/g_{cj})\right]\left[\sum_{d}\sum_{w} g_{dw}\,\bar{m}_{dw}\right],$$

since $\Sigma_c\,\Sigma_j\,\bar{m}_{cj} = 1$. As a consequence,

$$\sum_{d}\sum_{w} g_{dw}\,\bar{m}_{dw} = \frac{1}{\sum_{c}\sum_{j}(\sigma_{cj}/g_{cj})},$$

which when substituted back into the expression of the sales share, yields an expression equivalent to [26].

C.3. Resulting Profit

The total profit generated by the assortment is equal to

$$P = \left[\sum_{b}\sum_{v} g_{bv}(\bar{m}_{bv}\,Q)\right] - \left[\sum_{b}\sum_{v}\bar{C}_{bv}\right].$$

Substituting [26] for $\bar{m}_{bv}$ one obtains

$$P = Q\sum_{b}\sum_{v}\left[\frac{\bar{\sigma}_{bv}}{\sum_{c}\sum_{j}(\bar{\sigma}_{cj}/g_{cj})}\right] - \sum_{b}\sum_{v}\bar{C}_{bv}$$

but as $\Sigma_b\,\Sigma_v\,\bar{\sigma}_{bv} = 1$ and assuming $\bar{C}_{bv} = f_v(\bar{q}_{bv}/\bar{s}_{bv})$, the profit expression reads

$$P = \frac{Q}{\sum_{c}\sum_{j}(\sigma_{cj}/g_{cj})} - \sum_{b}\sum_{v} f_v(Q\bar{m}_{bv}/S\bar{\sigma}_{bv})$$

or

$$P = Q\left\{\frac{1}{\sum_{c}\sum_{j}(\sigma_{cj}/g_{cj})} - \frac{1}{S}\sum_{b}\sum_{v} f_v\left[\frac{(1/g_{bv})}{\sum_{c}\sum_{j}(\sigma_{cj}/g_{cj})}\right]\right\},$$

which ultimately reduces to [27].

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