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The Option Value of Investments in Energy-Efficient and Renewable Energy Technologies

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The option value of investments in energy-efficient and renewable energy technologies

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Abstract: This paper takes up the need to engage in substantial investments in the energy producing capital stock to attack the climate change problem, caused by rising carbon dioxide concentrations in the atmosphere. However, the precise magnitudes of economic impacts of global warming as well as abatement and mitigation costs of climate change are not known and may be learned after time. Thus preferences to restrict or to loosen environmental objectives and related emission levels might change and are reflected in an uncertain "price" for environmental usage (e.g. emission tax, marketable permits). As a consequence investors face some uncertainty and need to take the option value of their investments in the energy producing capital stock into account and tend to delay their investment. Considering long usage periods and indivisibilities of power production investments adequate environmental policy has to be designed in a way to reduce uncertainty for investors.

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Keywords: Climate change; Option value; Investment uncertainty; Renewable Energy

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1 Introduction

The recognition that economic activities could change the mechanisms of our climate is no longer a new one. Since the UK government published the *The Economics of Climate Change:The Stern Review in 2006*[1] the number of articles on the economic aspects of climate change increased significantly. There has been a number of estimates to assess costs and benefits of policies intended to reduce greenhouse gas emissions. Alternative policy instruments and their merits to achieve emission reduction policies under efficient circumstances have been discussed as well.

However, there have been and there are still huge uncertainties about the effects of increasing accumulation of greenhouse gases in the atmosphere and associated potential damages. Secondly, due to the dynamic character of greenhouse gas accumulation a key feature of the global warming problem is its long-term character and the necessity to evaluate current economic decisions with respect to possible economic and environmental impacts of these decisions within decades or even centuries ahead. The complex dynamics of the climate and economic system together with the manifold uncertainties surrounding the process of emission accumulation as well as the environmental and economic impacts of an accumulated stock of emissions result in a particular demanding economic problem. Since there are many uncertain parameters in the functioning of the climate system as well as in the economic aspects of lowering emissions induced by economic activities, these uncertainties have to be transferred to the costs and benefits of climate change policies. An appropriate framework to analyze costs and benefits of climate change policies therefore has to integrate dynamic and stochastic aspects of global warming.

Viewed from relevant timescales in climate models the accumulation of greenhouse gases in the atmosphere can be seen as irreversible. In the economics of the environment the concept of irreversibility combined with uncertainty has been a subject of intense research starting with the work of Henry[2] and Arrow and Fisher[3]. Both have shown that when applying cost-benefit analysis to an environmental problem, the existence of an irreversibility effect leads to a bias in favor of environmental preservation. Translated to the climate change problem this would imply in the face of partially or totally irreversible accumulation of greenhouse gases to reduce emissions now in order to preserve the option to wait for more information and avoid costly adaptation expenditures later.

However, with regard to the climate change issue consequences are not straightforward, as environmental irreversibility have to be confronted with investment irreversibility. Stabilization of energy-related greenhouse gas emissions will require massive investments in infrastructure, power generation, energy research, traffic etc. Thus the former have to be balanced against investment expenditures in a capital stock to reduce

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[1] Stern (2006), N.: (2006): The Economics of Climate Change: The Stern Review, London: H.M. Treasury.

[2] Henry, C. (1974): Investment Decisions under Uncertainty: The Irreversibility Effect, American Economic Review, Vol. 64 , pp. 1006-1012.

[3] Arrow, K. J.and A.C. Fisher (1974): Uncertainty and the Evaluation of Public Investment Decisions, Quarterly Journal of Economics, Vol. 88, pp. 312-319.

emissions. Irreversible emission reduction investments, made prematurely, change the options available later[4].

Increased utilisation of renewable energy resources and further development of non-fossil-fuel energy technologies offer one of the most important opportunities for stabilizing energy-related greenhouse gas emissions with continued economic growth. From a global perspective economic growth, particular in developing countries, will require innovative and at the same time cost-effective carbon-emission-free technologies as well as an increase in energy efficiency to lower the carbon intensity of the energy mix. Power generation with carbon-emission-free technologies is characterized by huge shares of investment and low maintenance and operation costs. It may be realistic to assume that the energy production costs are distributed with a ratio of 80:20 over capital (investment) cost and maintenance, operation and fuel cost for the non-fossil energy technology, whereas the ratio for fossil energy technologies is 20:80 between investment and running costs (Gerlagh et. al. (2003)[5]). Irreversibility and the possibility of delay are of course important characteristics of investments in energy production. When an investor makes an irreversible investment expenditure, it exercises an option to invest. In other words, he gives up the opportunity of waiting for new information to arrive that might influence the desirability and timing of this expenditure. Due to the large percentage of investment costs within the total energy production costs for non-fossil energy technologies, the investment option value is more important in these technologies than investments in traditional fossil energy technologies. Due to their large share in energy production, fossil-fuel technologies have accumulated substantial experience, which is reflected in low energy production costs. Even though non-fossil-fuel technologies cumulated experience during the last decade, they still reflect a minor share in energy production due to much higher energy production costs. In contrast to the fossil-fuel energy technology there is more scope for learning and lowering costs per unit. Thus, the relevance of new information needs to be taken into account when considering investment opportunities in non-fossil-fuel technologies.

The application of welfare theoretical instruments is a prerequisite for an economic analysis of climate change policies. Due to the dynamic character of the climate system (as well as the need to take capital stock considerations into account) and the manifold uncertainties surrounding the accumulation of greenhouse gases, an integration of the dynamic and stochastic features is essential to capture the subject. As the climate change problem is one that extends over exceptionally long time horizons and uncertainties may unravel over time, climate change policies could be characterized as an ongoing process of acting and learning, which leaves opportunity for adjustments in the future. Sequential decision-making strategies that identify reasonable short-term policies in the face of long-term uncertainties are most appropriate. Therefore policy-makers need not make once-and-for all decisions and the decision to invest in a rising stock of forests is not a now-or-never response option to combat climate change, but could be postponed as new

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[4] Kolstad, C.D. (1996): Fundamental Irreversibilities in Stock Externalities, Journal of Public Economics, 60, 2, 221-233; Heal, G., Kriström, B. (2002): Uncertainty and Climate Change, Environmental and Resource Economics, Vol. 22, pp.3-39

[5] Gerlagh, R., van der Zwaan (2003), B.: Gross World Product and Consumption in a Global warming Model with Endogeneous Technological Change, in Resource and Energy Economics, Vol. 25, pp. 35-57.

information becomes available. The real options approach[6] is an appropriate framework to handle this economic problem, as it balances the value of waiting to invest in the forest stock against the irreversible accumulation of greenhouse gases.

The paper is structured as follows: In section 2 a simple model that is based on capital theoretic approaches and which encompasses the deterministic development of the capital stock and the stock of greenhouse gases is outlined. Section 3 shows the market solution of the outlined model. Section 4 extends the model by introducing uncertainty about the dynamics of greenhouse gas accumulation, the corresponding command optimum and its implications on carbon emission lowering investments.

2 The Model

Due to the long-run nature of the climate change problem we clearly have to work with a dynamic model that takes the cumulative nature of greenhouse gas emissions into account[7]. We want to restrict the analysis on the release of energy-related carbon emissions. One of the first applications of optimal control models to pollution stock problems could be found in Keeler et. al.[8]. The framework of control theory allows one to take the strong intertemporal aspects of the climate change problem into account. To keep it simple it is assumed that welfare of society at any point of time could be described by a function of the flow of consumption *C* and a pollution stock *S*, i.e. the time-invariant utility function would look like $U=U(C,S)$. It is assumed that the production of the consumption good increases the stock of pollutants as it adds emissions *E* to the stock of previously released emissions. In the climate change problem the accumulation of greenhouse gases in the atmosphere could be described by the following dynamics:

$$(1) \qquad \dot{S} = E - \kappa S \qquad \text{with } S(0) = S_0 \ .$$

In other words, the dynamics of the pollution stock is determined by a fixed "historical level" of the pollution stock S_0, the history of emissions *E,* and κ represents a natural absorption capacity of the ecological system, which contributes to the reduction of the pollution stock over time. Thus the lower is natural decay rate κ the more irreversible are the effects of emissions, and a constant climate regime with $\dot{S} = 0$ implies that the rate of carbon emissions is equal to the natural absorption capacity. Of course this

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[6] Dixit, A. and Pindyck, R. (1994): Investment under Uncertainty, Princeton.

[7] Nicodemus, G.(1997): Reale Optionswerte in der Klimaproblematik, Heidelberg.

[8] Keeler, E., Spence, M. and Zeckhauser R. (1971): The Optimal Control of Pollution, Journal of Economic Theory, Vol. 4, pp. 19-34.

implies that lower CO_2-stabilization-targets are more challenging and will require higher shares of carbon-emission-free energy technologies.

In order to facilitate exposition (with the same objective function as before) in the following, the welfare function should be separable in consumption and the pollution stock with the specific form

$$U(C,S) = U(C) - D(S) \ . \tag{2}$$

Further it should be assumed that $U'(0) = +\infty, U' > 0, U'' < 0$ for the utility function and the stock of pollutants should affect social welfare according to the convex damage function $D(S)$ with: $D'(0) = 0, D' > 0$ and $D'' > 0$.

As regards the production function it is assumed that the usual concavity properties hold. Under the assumption that labour supply is fixed, output Y could be described as a function of the capital stock K_Y that is required for the production of gross output

$$Y = f(K_Y) \tag{3}$$

As common standard the function that maps capital into production is increasing and concave. The productive capital stock depreciates with the constant rate φ and an increase requires investment I_Y:

$$\dot{K}_Y = I_Y - \varphi K_Y \quad \text{with} \quad K_Y = K_Y^0 \ . \tag{4}$$

The release of carbon emissions is determined by the consumption level of fossil fuels. Carbon intensity of energy consumption depends on the relevant percentages of the used fossil fuels but especially on the share of non-fossil-fuel technologies within the total mix of energy production technologies. Together with the aggregate carbon emission factor a the level of carbon emissions should be proportional to gross output, $E = a\ f(K_Y)$.

So far a reduction of emissions is only possible by reducing consumption or respectively production. However, there are other options to limit the release of greenhouse gas emissions through "pollution control expenditures". As regards the release of carbon dioxide fossil fuels could be used more efficiently by an improved energy production capital stock (e.g. more efficient power plants and heating systems, cars that consume less fuel, etc.) or by substituting fossil fuels by less carbon intensive fuels ("fuel switching", e.g. from coal to gas), or by increasing the share of non-fossil energy capacities. However, all options to reduce the carbon intensity of energy consumption require resources and therefore affect the accumulation of real capital. All investments I_U to lower the carbon intensity of energy production technologies will result in lowering the carbon emission factor *a*. In other words, the carbon emission factor is a function of the energy capital stock

$a = a(K_U)$. As long as the energy production capital stock K_U does not change, carbon emissions are proportional to the level of production. The energy producing capital stock is assumed to depreciate at a fixed rate and together with periodical gross investments in this capital stock we have:

$$(5) \qquad \dot{K}_U = I_U - \varphi\, K_U \quad \text{with} \quad K_U = K_U^0 .$$

The gradual transformation of the energy production system from fossil towards low-carbon-content and non-fossil energy technologies, aiming to lower the carbon emission factor, requires (additional) increasing investments. Thus

$$(6) \qquad E = a(K_U) f(K_Y) \ \text{with} \ a'(K_U) < 0 \ \text{and} \ a''(K_U) \geq 0 .$$

In other words, all output has to be allocated to: capital accumulation (to be used according to the production function (3)), consumption, or to lower the carbon emission factor by investments in the energy producing capital stock. As resources are allocated to two areas the allocation to the third is also determined. Total investment is now: $I = I_Y + I_U$.

The welfare measure can be expressed with δ denoting the social discount rate as the sum over all time of the maximized discounted flow of welfare associated with corresponding time paths for C and S:

$$(7) \qquad \Omega_0 \equiv \max \int_0^\infty e^{-\delta t} \left[U(C) - D(S) \right] dt .$$

With regard to the maximization of social welfare according to (7) the allocation of resources is now subject to

$$(8) \qquad \dot{K}_Y + \dot{K}_U = f(K_Y) - C - \varphi\,(K_Y + K_U)\,.$$

The following Lagrangean function could be formulated to find the necessary conditions for an optimal path[9] associated with maximization of social welfare under the above described restrictions:

$$(9) \qquad \Lambda = \Lambda(S, K_Y, K_U, C, I, \mu, \varphi, \lambda_K, \lambda_Y, \lambda_S, \lambda_U)$$

$$= \left[U(C) - D(S)\right] + \mu\left[E - \kappa S\right] + \psi\left[I - \varphi\, K\right]$$

$$+ \lambda_Y\left[f(K_Y) - C - I\right] + \lambda_K\left[K - K_Y - K_U\right] + \lambda_S\left[E - a(K_U)(C + I)\right].$$

The variables μ and ψ denote the costate variables measuring the shadow prices of their associated state variables in current values, and $\lambda_Y, \lambda_K, \lambda_S$ denote the Lagrangean (dynamic) multipliers. Maximization of (9) delivers the following necessary conditions:

$$(10a) \qquad \frac{\partial \Lambda}{\partial C} = U'(C) - \lambda_Y - \lambda_S\; a(K_U) = 0$$

$$(10b) \qquad \frac{\partial \Lambda}{\partial I} = \psi - \lambda_Y - \lambda_S\; a(K_U) = 0$$

$$(10c) \qquad \dot{S} = \frac{\partial \Lambda}{\partial \mu} = E - \kappa\; S$$

$$(10d) \qquad \dot{\mu} = \delta\mu - \frac{\partial \Lambda}{\partial S} = \delta\mu + D'(S) + \mu\;\kappa$$

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[9] Chiang, A.C. (1992): Elements of Dynamic Optimization.

(10e) $$\dot{K} = \frac{\partial \Lambda}{\partial \psi} = I - \varphi\, K_Y - \varphi\, K_U$$

(10f) $$\dot{\psi} = \delta\psi - \frac{\partial \Lambda}{\partial K} = \delta\psi + \psi\, \varphi - \lambda_K$$

(10g) $$\frac{\partial \Lambda}{\partial K_Y} = -\lambda_K + \lambda_Y f'(K_Y) = 0$$

(10h) $$\frac{\partial \Lambda}{\partial K_U} = -\lambda_K - \lambda_S a'(K_U)\, Y = 0$$

(10i) $$\frac{\partial \Lambda}{\partial E} = \mu + \lambda_S = 0\,.$$

The differential equations (10c-10f, canonical system) determine the optimal paths for the allocation of resources and the stock of greenhouse gas emissions. (10d) and (10f) denote the path of the costate variables measuring the shadow prices of their associated state variables. Both variables balance the current impact of allocating resources to the capital stocks (for non-energy production and energy-producing capital stock) with the rate of change of capital value (stock of pollutants), i.e. the overall future effects taken into account. While (10c) and (10e) represent the growth rate of the aggregated capital stock (production capital and energy-production capital stock) and the pollution stock, a capital theoretic interpretation of equations (10d) and (10f) shows that decisions have to be taken in a way that always both, the immediate and the future effects are taken into account. Secondly, from equation (10f) we can get $\lambda_K = \delta\psi + \varphi\psi - \dot{\psi}$, with λ_K representing the user cost of capital. In the command optimum maximization of social welfare requires (also) the optimal intertemporal allocation of capital between both usages: an additional unit of capital allocated to the non-energy producing capital stock K_Y must deliver the interest return plus the rate of change of capital value and taken the capital stock depreciation into account, i.e. for (10f) we can write:

$-\lambda_K - \varphi_Y\psi - \varphi_F\psi = \delta\psi - \dot{\psi}$. According to (10g) one additional unit of capital allocated to the capital stock K_Y delivers the marginal product $\lambda_K = \lambda_Y f'(K_Y)$ (c.f. 10g) or must be equal to $\lambda_K = \lambda_S a'(K_U)\, Y$ (c.f. 10h, 10i), i.e. resources expended on lowering the carbon emission factor through investments in modernization of the energy-producing capital stock must deliver "their marginal product" by lowering emissions. In order to encourage investments (towards less carbon-intensive energy technologies or non-fossil energy technologies) for the external benefit of lowering emissions, they have to be subsidized. Alternatively the release of carbon emissions has to be taxed to create an incentive for additional investment in the energy capital stock, e.g. to invest in non-fossil energy technologies. The level of carbon emissions is determined by the carbon emission factor of the energy-producing capital stock. In the long-run the optimal level of K_U (steady state with $\dot{\psi} = 0$) could be determined by:

$$(11) \qquad a'(K_U) = \frac{(\delta + \varphi)\psi}{\mu Y} .$$

The marginal benefit of lowering the carbon emission factor by increasing the environmental-friendly energy capital stock (and its impact on the accumulation of greenhouse gases in the atmosphere) has to be equal to the user cost of capital. Rearranging of equation (11) gives $\mu = \frac{(\delta + \varphi)\psi}{a'(K_U)\,Y}$. The discounted negative impact of carbon emissions on welfare is equal to the user cost of mitigation capital on emissions. The negative effect associated with the release of carbon emissions results in a reallocation of capital. Less capital is invested in the productive capital stock *(K_Y)* and additional capital is invested in lowering the carbon emission factor *(K_U)*.

3 Decentralized Market Solution

In the following it is asked what the decentralized market solution look like and how could the command optimum, described in section 2, be implemented. The market failure arising from the negative environmental externality is shown and a market intervention mechanism is pointed out to correct for the market failure.

To facilitate the exposition we assume that there are only two agents in the set-up, a representative consumer and a private company that produces *Y(t)* according to the production function $Y(t) = f(K_Y(t))$. The representative consumer maximizes discounted utility over an infinite time horizon according to his utility function *U=U(C(t))*:

$$(12) \qquad \max \int_0^\infty U(C) e^{-\delta t} dt$$

subject to the budget constraint:

(13) $$\dot{K}_Y = rK_Y + w - C\,,$$

with *r(t)* denoting the price of capital and *w(t)* the wage rate, which are given under the assumption of perfect competition. It is assumed that labor is supplied inelastically $(L = \bar{L} = 1)$. From the following current value Hamiltonian

(14) $$H = U(C) + \upsilon\left[rK_Y + w - C\right]$$

the necessary first-order conditions could be obtained:

(15a) $$\frac{\partial H}{\partial C} = U'(C) - \upsilon = 0$$

(15b) $$\dot{\upsilon} = \delta\upsilon - \frac{\partial H}{\partial K_Y} = \delta\upsilon - r\upsilon$$

(15c) $$\lim_{t\to\infty} \upsilon(t) K_Y(t) = 0\ .$$

The costate variable υ denotes the shadow price of marginal capital formation (saving) and is according to (15a) equal to the marginal utility of consumption.

A representative private company seeks to maximize its profit according to :

(16) $$f(K_Y) - rK_Y - \varphi\, K_Y - w \ \Rightarrow max\ !$$

Of course the firm also maximizes its profits over an infinite time horizon. Equation (16) denotes profit maximization at every time and corresponds to the maximization of discounted profits. The first order condition denoting the optimal allocation of capital could easily be obtained as:

$$f'(K_Y) = r + \varphi .$$

The marginal product of capital is equal to the market interest rate r net of capital depreciation φ. In the decentralized market solution there are no specific investments to lower the carbon intensity of energy production facilities. Investments in the non-energy capital stock are made according to the rule that user cost of capital are equal to the marginal product of capital received from the market.

Due to the positive externalities associated with investments in the energy capital stock aiming to reduce the carbon emission factor, it can be shown that the welfare-maximizing command optimum is not supported by a market economy without policy measures. In order to support the command optimum additional investments in the energy capital stock (e.g. non-fossil fuel technologies) have to be subsidized according to the positive externality they provide to society. As an alternative the release of carbon emissions has to be taxed to create an incentive for additional investment in the energy capital stock, e.g. to invest in non-fossil energy technologies.

Proposition: The implementation of an emission tax $\omega(t)$ such that

(17) $$\omega(t) = \frac{\mu(t)}{U'(C(t))}$$

with $\omega(t$ denoting the costate variable or shadow price of the stock of greenhouse gas emissions introduced in section 2, the welfare-maximizing command optimum could be supported by the decentralized market economy.

Proof: The maximization of the company now looks like:

(18) $$f(K_Y) - r(K_Y + K_U) - \varphi\, K_Y - \varphi\, K_U - w - \omega\, a(K_U) f(K_Y) .$$

The following necessary conditions could be received:

(19) $$f'(K_Y) - r - \varphi - \omega a(K_U) f'(K_Y) = 0$$

(20) $$-r - \varphi \ - \omega a'(K_U) f(K_Y) = 0 \ .$$

From (20) we could receive for the allocation of resources expended to increase the environmental-friendly energy capital stock: $\omega = \dfrac{r + \varphi}{a'(K_U) f(K_Y)}$. In other words, user cost of capital invested in the energy capital stock is equal to the marginal benefit of lowering carbon emissions and thus the accumulation of greenhouse gases in the atmosphere. This result was already obtained in section 2.

4 Uncertainty and Investments in Mitigation Capital

As already discussed in section 1, there is considerable uncertainty not only about the likely impacts of climate change, but also about the accumulation process of greenhouse gases in the atmosphere, the natural absorption capacity, the level of sequestration of greenhouse gases, etc. As a result of these uncertainties the shadow price, reflecting the marginal damage of carbon emissions associated with the stock of greenhouse gases, is also uncertain. In other words, the emission tax as presented in equation (17) does no longer support the command optimum. Moreover the precise magnitudes of economic impacts of global warming as well as abatement and mitigation costs of climate change are not known and will be learned after time. Therefore preferences to restrict or to loosen environmental objectives and related emission levels might change.

To reflect the uncertainty we introduce

(21) $$d\omega = \alpha\omega dt + \sigma \omega dz \ ,$$

denoting a geometric Brownian motion with drift[10], with α and σ are constants. Percentage changes in ω denote changes in optimal levels of the emission tax. The

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[10] (Dixit, A., Pindyck, R. (1994), pp.71)

change of the price can be subdivided in a deterministic and a stochastic component. The deterministic trend component α is determined by the sum of the social discount rate δ and the deterministic part of the natural absorption capacity κ. The deterministic part reflects the expectations on the future price and investors take it as a basis for their investment decisions in the energy capital stock K_U. The stochastic part reflects the uncertainty surrounding the future development of the emission tax due to the manifold uncertainties regarding climate change and the adequate policy.

Given the initial energy capital stock K_U the existing stock will decay as $dK_U = -\varphi\, K_U\, dt$ (or $K_U\, e^{-\varphi_F t}$) , when he makes no new investments. Thus the expected present value[11] of emission tax payments calculated using the discount rate ρ will be:

(22) $$V(K_U, \omega) = E\int_0^\infty -\,\omega(t)\, a(K_U\, e^{-\varphi\, t})Y\, e^{-\rho t}\, dt\ .$$

To characterize the value function one could look at it after time dt. If the firm makes no new investment, the energy capital stock will decay and the emission tax will have risen according to (21):

(23) $$V(K_U, \omega) = -(\omega\, a(K_U)\, Y)\, dt + e^{-\rho dt}\, E\left[V(K_U + dK_U, \omega + d\omega)\right]$$

To get the value of the initial “marginal unit” of the environmental-friendly energy capital stock, differentiation of (22) delivers:

(24) $$V_{K_U}(K_U, \omega) = E\int_0^\infty -\,\omega(t)\, a'(K_U\, e^{-\varphi\, t})\, Y\, e^{-(\varphi+\rho)t}\, dt$$

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[11] (Dixit, A., Pindyck, R. (1994), pp.374)

The partial derivative of the value function is positive, since $a'(K_U\, e^{-\varphi_F\, t}) < 0$, and denotes the reduction of emission tax payments. The investor faces the decision problem to invest in additional units of the energy producing capital stock when taking the stochasticity of (21) into account, which causes uncertainty regarding tax savings.

In the next step the value of the company's option to invest in additional investment projects with a given current energy production capital stock K_U and a given current emission tax ω should be considered. The value of this investment option can be denoted as $O(K_U(t), \omega(t))$. As long as the firm makes no new investments, we can have:

$$O(K_U, \omega) = e^{-\rho dt}\, E\left[O(K_U + dK_U, \omega + d\omega)\right] . \tag{25}$$

After having a Taylor expansion for this expression and having used Ito's Lemma we can get the following partial differential equation:

(26)

$$\alpha\, \omega O_\omega(K_U, \omega) - \varphi\, K_U\, O_{K_U}(K_U, \omega) + 1/2\, \sigma^2 \omega^2\, O_{\omega\omega}(K_U, \omega) - \rho\, O(K_U, \omega) = 0$$

As the company makes an investment it exercises the option to invest and the option value is gone; therefore $O_{K_U} < 0$. Thus the second term in (26) represents the positive value of increasing options as the decay of the energy capital stock offers new investment options. After a period of time *dt* the marginal investment option $-O_{K_U}(K_U, \omega) \equiv o(K_U, \omega)$ is again available. To determine the marginal investment option (26) is differentiated with respect to K_U :

(27)

$$\alpha \omega o_\omega(K_U, \omega) - \varphi\; K_U\; o_{K_U}(K_U, \omega) - \varphi\, o(K_U, \omega) + 1/2\, \sigma^2 \omega^2\; o_{\omega\omega}(K_U, \omega) - \rho\, o(K_U, \omega) = 0$$

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Reordering of the equation shows that the marginal investment option evolves (increases) according to the rate of return ρ plus the depreciation rate:

(27)'

$$\alpha\omega o_{\omega}(K_U,\omega)-\varphi_{K_U}K_U\, o_{K_U}(K_U,\omega)+1/2\,\sigma^2\omega^2\, o_{\omega\omega}(K_U,\omega)=(\rho+\varphi_{K_U})\,o(K_U,\omega)$$

.

As $o(K_U,\omega)$ is a function of two variables and (27) represents a partial differential equation with respect to both variables a general solution couldn't be found, it would have to be solved numerically. By assuming for $a(K_U)=K_U{}^{-b}$ with b>0 we can have for the expected present value (with $\omega(0)=\omega_0$):

(28) $$V_{K_U}(K_U,\omega)=bY\,K_U{}^{-(b+1)}\,\frac{\omega_0}{(\rho-\varphi-\alpha)}\ .$$

Since the expression $bY\,K_U{}^{-(b+1)}$ can be received by considering two situations in which initial values of K_U and ω differ from each other, the expressed marginal option value will be the same. Therefore we can further simplify $o(K_U,\omega)=f(x)$. Thus we can have the following differential equation:

(29) $$1/2\,\sigma^2x^2\,f''(x)+\alpha\,x f'(x)+\varphi(b+1)x\,f'(x)-(\rho+\varphi)f(x)=0\ .$$

A solution has to fulfill additionally the following conditions:

(29a) $f(0)=0$

(29b) $o(K_U,\omega)=V_{K_U}(K_U,\omega)-q$

with q denoting the marginal cost per energy capital stock unit.

(29c) $o_{\omega}(K_U,\omega)=V_{K_U\,\omega}(K_U,\omega)$

(29a) excludes a solution for $\omega=0$. According to (29b) the marginal option value is equal to the expected present value of an additional unit of the energy capital stock net of the direct cost for one unit. Reordering gives: $V_{K_U}(K_U,\omega)=q+o(K_U,\omega)$: the expected present value of an additional unit of the energy producing capital stock is equal to the direct cost of the unit plus the loss of the (marginal) option value to invest. (29c) is also known as “smooth-pasting condition”[12] and should be illustrated graphically (see figure 1).

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[12] Dixit, A.K. (1993): The Art of Smooth Pasting; Fundamentals of Pure and Applied Economics, Vol. 55, Chur.

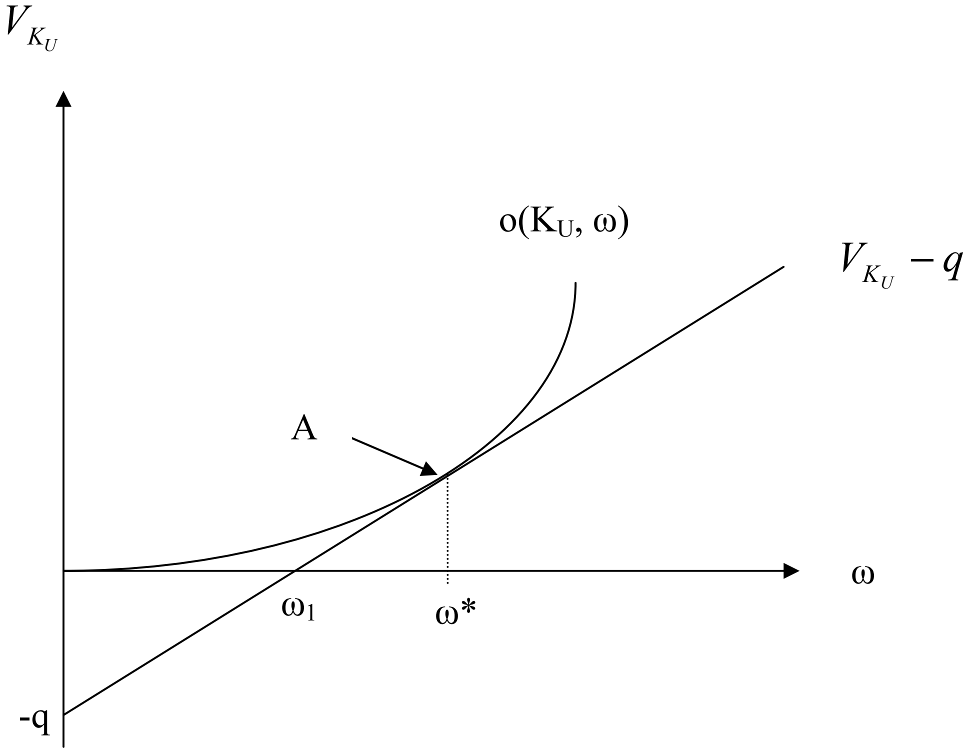

Figure 1: Smooth-Pasting Condition (Marginal investment condition)

The constant line denotes the profit flow of the marginal investment option net of direct capital cost q. The convex curve denotes the value of the marginal investment option. The point where the curve meets the constant line, in A, denotes the "investment trigger", i.e. the option should be exercised as the "smooth pasting condition" is fulfilled. For cases in which $o_{\omega}(K_U, \omega) < V_{K_U \omega}(K_U, \omega)$, it is worth to wait before exercising the option to invest in the energy capital stock, whereas a situation for which $o_{\omega}(K_U, \omega) > V_{K_U \omega}(K_U, \omega)$ investment should take place. It should be mentioned that the convex curve for $\omega > \omega^*$ should not be used for interpretation, but the constant line on the right hand side of ω^* $(\omega > \omega^*)$ helps to identify the profit flow of the investment. The curve denotes situations that are known as "bubbles" in financial markets.

In the following we are interested in finding how (1) increasing uncertainty and (2) the decay rate of the energy producing capital stock affects the decision to invest in K_U, which helps to lower the carbon emission factor. Obviously the answer to (1) depends on the size of the option value to invest. Intuitively we can expect that an increase in uncertainty (increasing σ^2) will increase the option value of investment and delay the investment in the environmental-friendly energy capital stock.

A general solution of the (linear) differential equation (29) could be obtained by a linear combination of two separate solutions. If we insert the function Ax^{β} in the homogenous differential equation (29), we can have the characteristic equation (or known as auxiliary equation)[13]:

$$(30) \qquad 1/2\,\sigma^2\,\beta\,(\beta-1)+\big(\alpha+\varphi(b+1)\big)\,\beta-\big(\varphi+\rho\big)=0$$

The fundamental quadratic equation can be written as a function of β and the parameters α, ρ and φ.: $\Phi(\beta)=1/2\,\sigma^2\,\beta\,(\beta-1)+\big(\alpha+\varphi(b+1)\big)\,\beta-\big(\varphi+\rho\big)=0$. Because of (29a) we have restricted a solution to this equation to the positive:

(31)

$$\beta_1=1/2-\big[\alpha+\varphi(b+1)\big]/\sigma^2+\left[\big[\alpha+\varphi(b+1)/\sigma^2-1/2\big]^2+\frac{2(\rho+\varphi)}{\sigma^2}\right]^{1/2}$$

To see how a change in the parameter σ influences the option value of an investment in the energy capital stock we differentiate the fundamental quadratic equation totally, i.e. $d\Phi=\left(\frac{\partial\varphi}{\partial\beta_1}\frac{\partial\beta_1}{\partial\sigma}+\frac{\partial\Phi}{\partial\sigma}\right)d\sigma=0$. As we have restricted the solution to the positive root $\beta_1 > 1$, $\frac{\partial\varphi}{\partial\beta_1} > 0$ and $\frac{\partial\Phi}{\partial\sigma}=\sigma\,\beta_1(\beta_1-1)>0$, we get for $\frac{\partial\beta_1}{\partial\sigma}<0$. Thus increased uncertainty (as σ increases, β_1 decreases and therefore the term (β_1 /(β_1 -1)) increases) about the negative economic impacts of climate change as well as the preferences of environmental policy and the associated adequate emission tax, increases the option value of an investment in the energy capital stock. In other words, investments to lower the carbon emission coefficient in the energy producing capital stock tend to be deferred (and will be lower) with increasing uncertainty about the likely environmental policy.

To find an answer to the second question, we also need to check the fundamental quadratic equation. A lower decay rate of the energy capital stock (longer usage period) will decrease β_1 and thus increases the marginal option value. The economic intuition is straightforward. Long usage periods or low decay rates, which are typical for the energy

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[13] Chiang, A.C. (1984): Fundamental Methods of Mathematical Economics (1984) p.505.

producing capital stock, will tend to increase the option values to invest and thus tend to delay new investments for cleaner, low-carbon or carbon-emission-free technologies.

Environmental policy recommendations can be derived immediately. Since the transformation of the energy system towards lower carbon emissions includes a transition from fossil towards non-fossil energy technologies, implying substantial investments, the government need to fix the price for environmental usage (e.g. emission tax or marketable permits) early and for a long period. In other words, environmental policy has to be designed in a way to reduce uncertainty for investors. Thus the option value for investments in the energy capital stock will be lowered and there is no need for investors to "hedge" for "policy risks". Environmental policy can provide an entry for directing investments towards innovative low-carbon or carbon-emission-free energy technologies. It can be expected that the costs associated with the use of carbon-free technologies decrease over time, as they offer a relatively large learning potential and thus could benefit society as whole.

5 Concluding Remarks

The gradual transformation of the energy production system from fossil towards low-carbon-content and non-fossil energy technologies, aiming to lower the carbon emission factor, will require substantial investments. Increased utilisation of renewable energy resources and further development of non-fossil-fuel energy technologies offer one of the most important opportunities to stabilize carbon emissions while keeping a certain level of economic growth. Viewed from a global perspective economic growth, particular in developing countries, will require innovative and at the same time cost-effective carbon-emission-free technologies as well as an increase in energy efficiency to lower the carbon intensity of the energy mix.

However, energy production with carbon-emission-free technologies is characterized by huge shares of investment and low maintenance and operation costs. At the same time there are still huge uncertainties about the effects of increasing accumulation of greenhouse gases in the atmosphere. The precise magnitudes of economic impacts of global warming as well as abatement and mitigation costs of climate change are not known and will be learned after time. Therefore preferences to restrict or to loosen environmental objectives and related emission levels might change and this might be reflected in an uncertain "price" for environmental usage (e.g. emission tax, marketable permits). As a consequence investors face some uncertainty and need to take the option value of their investments in the energy producing capital stock into account. Considering long usage periods and indivisibilities of power production investments environmental policy has to be designed in a way to reduce uncertainty for investors. The social planner (government) needs to fix the price for environmental usage (e.g. emission tax or marketable permits) early and for a long period to encourage early investments and to benefit from the learning curve through decreasing longer term carbon emission abatement costs.

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Der Optionswert von Investitionen in energie-effiziente und erneuerbare Energietechnologien

Um das Problem des Klimawandels - verursacht durch den Anstieg des Verbrauchs fossiler Energieressourcen und einer daraus resultierenden Zunahme der Konzentration von Kohlendioxid in der Atmosphäre - anzugehen, sind erhebliche Investitionen in den Energiesektor erforderlich. Jedoch ist die genaue Größenordnung der wirtschaftlichen Auswirkungen der globalen Erwärmung, und damit die Kosten des Klimawandels, mit Unsicherheit behaftet, wie auch die erforderlichen Investitionskosten innerhalb des energieerzeugenden und -verbrauchenden Sektors. Aus der Sicht des umweltpolitischen Entscheidungsträgers stellt die optimale Klimapolitik sich somit als sequentieller Entscheidungsprozess dar, der nach einer ständigen Anpassung der umweltpolitischen Vorgaben (Höhe der Emissionssteuer bzw. des Preises für Emissionszertifikate) verlangt. Dies hat zur Konsequenz, dass die Investoren den Optionswert einer realen Investition etwa in erneuerbare oder effizienzsteigernde Energietechnik einbeziehen müssen und somit die Investition hinausschieben. In Anbetracht der langen Nutzungsdauern und der Bedeutung von Unteilbarkeiten im energieerzeugenden Sektor, ist die Umweltpolitik so auszugestalten, dass die Unsicherheit aus Sicht der Investoren verringert wird.

Schlagworte: Klimawandel; Optionswerte; Investitionsunsicherheit; Erneuerbare Energien

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