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# Negative Correlation between Stock and Futures Returns: An Unexploited Hedging Opportunity?

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# Negative Correlation between Stock and Futures Returns: An Unexploited Hedging Opportunity?*

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**Abstract**

The negative correlation between equity and commodity futures returns is widely perceived by investors as an unexploited hedging opportunity. A Lucas (1982) two-country asset-pricing model is adapted to analyze the fundamentals driving equity and commodity futures returns. Using the model we argue that such a negative correlation could arise as a no-arbitrage equilibrium phenomenon and reflect traders' perceptions about the growth fundamentals in oil and GDP and does not necessarily indicate an arbitrage opportunity.

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# 1 Introduction

The Commodity Futures Modernization Act in 2000 gave large financial firms wide latitude in trading commodity derivatives.[1] The institutional fund managers shifted out of equities into commodity futures partly in the belief that it represents a previously unrecognized hedge for business cycle risk. Greer (2000) argues that commodity index funds as an asset class is underused while the index returns are negatively correlated with stocks and bonds over the period 1970-99. Gorton and Rowenhorst (2006) also found that the returns on long positions in commodity futures are negatively correlated with the returns from comparable bond and equity portfolios. Erb and Harvey (2006) report a similar historical record but caution against using historical correlations to make prospective portfolio allocations. Boyuksahin, Haigh and Robe (2010) provide detailed evidence of the correlation between equity and commodity returns and find that commodities did not provide enough diversification when it was needed. Likewise Daskalaki and Skiapoulos (2011) provide out-of-sample evidence that commodities as an asset class do not improve returns over portfolios which include only traditional asset classes.

A common question that arises in all these extant studies is: Does the negative correlation between commodity and equity returns provide an unexploited hedging opportunity? This question cannot be effectively answered without an asset pricing model that identifies the common macroeconomic fundamentals driving both commodity and equity returns. To the best of our knowledge, there is no theoretical treatment of the common macroeconomic fundamentals driving returns to both equities and commodity futures using general equilibrium principles.

In this paper, we adapt a Lucas (1982) international asset-pricing model to analyze the fundamentals driving equity and commodity futures returns. We show that in a frictionless complete market setting, even though households are fully hedged, a negative correlation could arise as a no-arbitrage equilibrium. Such a negative correlation by itself cannot be used as a hedging motive. In the model, the home country is exposed to two types of endowment risks. The first is the business cycle risk of its own output. The second is the commodity supply risk arising from the foreign endowment. We explicitly model the home resident's investments in commodity futures and equity. The model is kept quite simple and stylized where oil, used as a stand-in for commodities generally, is

[1] See, Basu and Gavin (2011) for a documentation of the rise commodity trading.

treated as a consumption item, and all returns are real. We demonstrate that the correlation between equity and futures real returns depends crucially on the variance-covariance matrix of these two economic fundamentals, oil and home output .

A central implication of our two-country asset pricing models is that the equity return is positively related to the growth rate of home output while the oil futures return is determined by the growth rate of GDP and the news about the future oil output. If home output and oil supply are positively correlated, then a greater supply of home output signals better news about the oil output. Thus during a business cycle boom (recession) equity return would be higher (lower) while futures return would be lower (higher). The immediate implication is that the equity and futures return negatively correlate as long as home output and foreign oil production positively correlate, i.e., if the systematic risk of oil and home output show positive comovement.

# 2 A Lucas Tree Model

There are two countries, home and foreign. At date $t$, the home country is endowed with $y_t^a$ units of its own good and the foreign country owns $y_t^b$ units of oil. The growth rates of these endowments evolve stochastically as a Markov process with a stationary distribution. Agents receive direct utility from the consumption of oil. At date $t$ ,there are two consumables, home good ($c_t^a$) and imported oil ($c_t^b$) respectively. Individuals in both countries are identical in terms of preferences. The instantaneous utility function of households is: $u(c_t^a)+v(c_t^b)$. In view of the complete market nature of the financial environment, all conceivable Arrow-Debreu securities can be traded. However, we will focus only on four financial instruments which traders hold in equilibrium: (i) equity claims ($z_{t+1}^a$) to future flows of home output which sell at the price $q_t^a$ today, (ii) equity claims ($z_{t+1}^b$) to future flows of oil which sell at the price $q_t^b$ today, (iii) claims to future delivery of oil at a price of oil contracted today which we call futures, (iv) a discount bond ($b_{t+1}$) held domestically which pays a risk free return $r_{t+1}$ in the following period. Let $f_t^j$ be the date t price for delivery of one barrel of oil at date $t+j$ and $n_t^j$ be the number of barrels of oils contracted at date t for delivery at date $t+j$, $k$ be the number of such traded futures which means $j=1,2...k$ and $s_t$ be the spot price of oil (or the real exchange rate) which by definition is equal to $f_t^0$. By definition, $n_t^0$ is the spot purchase of oil which is the same as $c_t^b$.

The flow budget constraint facing the home country is:

$$\begin{aligned} & c_t^a + s_t c_t^b + q_t^a(z_{t+1}^a - z_t^a) + q_t^b(z_{t+1}^b - z_t^b) + \sum_{j=1}^{k} f_t^j n_t^j + b_{t+1} \\ = \quad & z_t^a y_t^a + z_t^b s_t y_t^b + \sum_{j=0}^{k-1} f_t^j n_{t-1}^{j+1} + (1+r_t) b_t \end{aligned} \quad (1)$$

The home household receives direct utility from home goods, $c_t^a$ and oil $c_t^b$. In other words, the household maximizes the discounted stream of utilities :

$$E_0 \sum_{t=0}^{\infty} \beta^t [u(c_t^a) + v(c_t^b)]$$

subject to (1), where $0 < \beta < 1$.

Foreign country's problem is symmetric. It holds claims to home country's output and takes short position in issuing oil futures. In equilibrium, $n_t^j$ add up to zero across home and foreign because if the home takes a long position, the foreign must take a corresponding short position. Hereafter, we will specialize to the home country's problem. The first order conditions are:

Home equity:

$$z_{t+1}^a : u'(c_t^a) q_t^a = \beta E_t u'(c_{t+1}^a)\{q_{t+1}^a + y_{t+1}^a\} \quad (2)$$

Foreign equity:

$$z_{t+1}^b : u'(c_t^a) q_t^b = \beta E_t u'(c_{t+1}^a)\{q_{t+1}^b + s_{t+1} y_{t+1}^b\} \quad (3)$$

Futures:

$$n_t^j : f_t^j u'(c_t^a) = \beta f_{t+1}^{j-1} u'(c_{t+1}^a), \quad j = 1, 2, ...k \quad (4)$$

Bond:

$$b_{t+1} : 1 + r_{t+1} = \beta^{-1} E_t \frac{u'(c_t^a)}{u'(c_{t+1}^a)} \quad (5)$$

Spot :

$$f_t^0 = s_t = \frac{v'(c_t^b)}{u'(c_t^a)} \quad (6)$$

The equity price equations (2), (3) are standard. The futures price equation (4) basically means that if a trader buys a $j$ period futures today at the price $f_t^j$, he has the option to sell the same futures tomorrow at the price $f_{t+1}^{j-1}$ and make a notional capital gain or loss which explains the right hand term. Spot price (6) is given by the intratemporal marginal rate of substitution between $c_a$ and $c_b$.

## 2.1 Calculating Futures Price

Using (4) and (6), we can write the futures price with one period prior to maturity as:

$$f_{t+j-1}^1 u'(c_{t+j-1}^a) = \beta E_{t+j-1} f_{t+j}^0 . u'(c_{t+j}^a) \tag{7}$$

Next note from (6) that

$$f_{t+j}^0 = s_{t+j} = \frac{v'(c_{t+j}^b)}{u'(c_{t+j}^a)} \tag{8}$$

which upon substitution in (7) gives:

$$f_{t+j-1}^1 u'(c_{t+j-1}^a) = \beta E_{t+j-1} v'(c_{t+j}^b) \tag{9}$$

Using (9 recursively backward and also noting that at each date, the spot price is given by (6) one obtains,

$$f_t^j u'(c_t^a) = \beta^j E_t v'(c_{t+j}^b) \tag{10}$$

Since both countries have identical preferences and start with the same initial wealth positions, international asset markets will be used to pool risks. We price the assets assuming, as in Lucas, that perfect risk pooling occurs. This means that home country will hold half of its own output claims and half of oil claims. In equilibrium, $c_t^a = .5y_t^a$ and $c_t^b = .5y_t^b$.

Assume log utilities $u(c_t^a) + v(c_t^b) = \ln c_t^a$ $+\ln c_t^b$. It is easy to verify that the futures price is given by:

$$f_t^j = \beta^j E_t \left[ \frac{y_t^a}{y_{t+j}^b} \right] \tag{11}$$

## 2.2 Correlation between returns on equities and futures

The equilibrium equity prices are proportional to home production as follows:

$$q_t^a = q_t^b = \frac{\beta}{1-\beta} y_t^a$$

Note that the foreign equity price is also proportional to home production because the real exchange rate $s_t$ is $y_t^a / y_t^b$ which means $s_t y_t^b = y_t^a$.

This means that *ex post* returns on home and foreign stocks are equal. Call this equity return $R_{t+1}^E$. We have:

$$R_{t+1}^E = \beta^{-1} \frac{y_{t+1}^a}{y_t^a} \tag{12}$$

which means

$$\ln R_{t+1}^E = -\ln \beta + \ln(\frac{y_{t+1}^a}{y_t^a}) \tag{13}$$

In other words, the *ex post* equity return is proportional to the growth rate of home output.

The *ex post* return on futures (call it $R_{t+1}^F$) is: $f_{t+1}^{j-1} / f_t^j$ . Using (11), it follows that

$$R_{t+1}^F = \left[\frac{y_{t+1}^a}{y_t^a}\right] . \frac{E_{t+1}\left[\frac{1}{y_{t+j}^b}\right]}{E_t\left[\frac{1}{y_{t+j}^b}\right]} \tag{14}$$

For the sake of tractability, hereafter we specialize to a one period futures (which means that the futures horizon $j = 1$). Rewrite (14) as follows:

$$\begin{aligned} R_{t+1}^F &= \left[\frac{y_{t+1}^a}{y_t^a}\right] . \frac{E_{t+1}\left[\frac{y_t^b}{y_{t+1}^b}\right]}{E_t\left[\frac{y_t^b}{y_{t+1}^b}\right]} \\ &=> \quad \ln R_{t+1}^F = \ln \frac{y_{t+1}^a}{y_t^a} + \ln\left[\frac{y_t^b}{y_{t+1}^b}\right] - \ln E_t\left[\frac{y_t^b}{y_{t+1}^b}\right] \end{aligned}$$

Last equality comes from the fact that $\frac{y_{t+1}^b}{y_t^b}$ is already realized at date t+1 which means that $\ln E_{t+1}\left[\frac{y_t^b}{y_{t+1}^b}\right] = \ln\left[\frac{y_t^b}{y_{t+1}^b}\right]$.

Next assuming a lognormal distribution for the growth rate of oil output rewrite the above as[2]:

$$\ln R_{t+1}^F = \ln \frac{y_{t+1}^a}{y_t^a} - \left\{ \ln \left( \frac{y_{t+1}^b}{y_t^b} \right) - \ln E_t \left( \frac{y_{t+1}^b}{y_t^b} \right) \right\} - 0.5 var_t \ln \left( \frac{y_{t+1}^b}{y_t^b} \right) \quad (15)$$

The *ex post* one period futures return depends on the growth rate of home output (the first square bracket term) and the *news* about future production of oil (the second square bracket term). Everything else equal, better news about future oil production will depress the expected return to oil futures because the signal that oil production will rise also signals a lower future spot price. Likewise, a greater perceived uncertainty about oil production also depresses future spot price.

The correlation between equity and oil futures returns is ambiguous. It depends on the correlation between home production and oil production. To see this clearly, assume an iid process for oil production which means that $ln$ $(y_{t+1}^b/y_t^b)\tilde{\ } N(\mu, \sigma^2)$. One can rewrite (14) as:

$$R_{t+1}^F = \left[ \frac{y_{t+1}^a}{y_t^a} \right] \left[ \frac{y_t^b}{y_{t+1}^b} \right] \exp(\mu - .5\sigma^2)$$

which means

$$\ln R_{t+1}^F = (\mu - .5\sigma^2) + \ln \frac{y_{t+1}^a}{y_t^a} - \ln \frac{y_{t+1}^b}{y_t^b} \quad (16)$$

Using (13) and (16) one obtains:

$$cov_t(\ln R_{t+1}^E, \ln R_{t+1}^F) = var_t(\ln \frac{y_{t+1}^a}{y_t^a}) - cov_t(\ln \frac{y_{t+1}^a}{y_t^a}, \ln \varepsilon_{t+1}^b) \quad (17)$$

where $cov_t(.)$ and $var_t(.)$ are conditional covariance and variance respectively. If $cov(\ln \frac{y_{t+1}^a}{y_t^a}, \ln \frac{y_{t+1}^b}{y_t^b}) > 0$ and it exceeds $var(y_{t+1}^a)$, futures and equity returns will be negatively correlated. This will happen in a no arbitrage equilibrium.

[2] For any lognormal random variable $x$, following property holds: $\ln E(x^k) = kE(\ln x) + 0.5k^2 var(\ln x)$ where k is a constant.

### 2.3 A beta based intuition

The negative correlation between equity and futures returns can be understood as an inverse association between the systematic risks of futures and oil. Suppose a commodity investor wants to ascertain whether there is any predictable negative relationship between futures return and equity return. He runs a regression of futures return ($\ln R_{t+1}^F$) on the contemporaneous equity return ($\ln R_{t+1}^E$). This regression coefficient is simply $cov(\ln R_{t+1}^F, \ln R_{t+1}^E)/var(\ln R_{t+1}^E)$. Call this regression coefficient $\beta^F$ which can be interpreted as the beta of the futures given that $R_{t+1}^E$ is the market portfolio.

Based on (12), (16) and (17), notice that this regression coefficient is $1 - \left\{cov(\ln \frac{y_{t+1}^a}{y_t^a}, \ln y_{t+1}^b/y_t^b)/var(\ln \frac{y_{t+1}^a}{y_t^a})\right\}$. Next note that bracketed term is simply the regression coefficient of the log of oil growth on the log of home output growth. Alternatively, this regression coefficient can be interpreted as the beta of oil (referred as $beta^{oil}$) given that the home output captures all aggregate risk. Thus equation (17) basically means the following tight relationship between these two betas:

$$beta^F = 1 - beta^{Oil} \tag{18}$$

Note that $beta^F$ on the left hand side of (18) represents the systematic risk in the oil futures market. $beta^{Oil}$ on the right hand side summarizes the systematic risk in the real oil sector. The model predicts an inverse relation between $beta^F$ and $beta^{Oil}$. If the systematic risk of oil is quite substantial ($beta^{Oil} > 1$), a predictable relationship (a negative correlation) emerges between oil futures return and equity returns which means a negative $beta^F$. However, such a negative relationship cannot be exploited by investors because it arises as a no-arbitrage condition.

## 3 Conclusion

The negative correlation between equity and future returns is often interpreted as a potential hedging opportunity for investors. In this short paper, we establish that such a negative correlation can easily arise in equilibrium as a no arbitrage condition. We illustrate this point using a variant of Lucas (1982) international consumption CAPM model. The model shows that the correlation between equity and oil futures returns stems from the variance and covariance properties of GDP and oil production.

Our model uses general equilibrium perspective to understand the implications for correlation between equity and futures returns. The lesson that we learn from such a general equilibrium exercise is that commodity and equity markets are integrated and should not be studied in isolation. Thus a negative correlation between these two returns should not necessarily be misconstrued as a hedging opportunity to common macroeconomic shocks. Rather, it reflects the equilibrium response of equity and futures markets to fundamental shocks driving the economy.

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