Cosigned Or Group Loans

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Abstract

We analyze lending contracts when social sanctions are used to enforce repayments and borrowers differ in their unobserved sanctioning abilities. Symmetric group loans are preferred to cosigned loans when borrowers are relatively equal, and cosigned loans are preferred when borrowers are unequal. This explains why microlenders that target the poor (e.g., the Grameen Bank) use symmetric group loans while other untargeted lenders use cosigned loans. Complicated menus of loan contracts that induce borrowers to self select can do no better than these simple loan contracts unless borrowers are very productive. In particular, we explain why group lending arrangements offering different loan terms to members of the same group are seldom observed.

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1 Introduction

The Grameen Bank and its group lending contract has received substantial academic attention in recent years. Grameen makes symmetric group loans: identical loans are made to a group of borrowers and all are punished if one does not repay. It is now well established that symmetric group loans can do better than individual loans

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(Ghatak and Guinnane (6)). But symmetric group loans are just one way of lending to individuals who have insufficient collateral of their own. Asymmetric group loans, in which group members are given different loan terms, are also a possibility — but these are seldom observed except in their most extreme form: cosigned loans. In a cosigned loan, a borrower provides a cosigner who does not receive a loan but is punished if the borrower does not repay. Such arrangements are ubiquitous.[1]

In other words, symmetric group loans and cosigned loans are extremes on the continuum of joint liability lending. In this paper we first compare these two commonly observed loan contracts. Under what circumstances will we observe symmetric group loans? And conversely, when will we observe cosigned loans? Then we ask why only the extremes are observed, i.e., why group lending schemes seldom offer different loan terms to members of the same group.[2]

Our paper builds on Besley and Coate (4)'s influential model of how socially sanctioned punishments can be used to enforce repayment. These sanctions include social ostracism, shame, and exclusion from informal insurance networks, and are widespread in villages and other close-knit communities.[3] Borrowers potentially

[1]Co-signed loans have been around at least since 19th century Germany (Banerjee et al (3)). The distinction between cosigned loans and group loans is made in the literature but not theorized (Ghatak and Guinnane (6)).

[2]Leading microlenders such as the Grameen Bank, BancoSol in Bolivia, the Bank for Agriculture and Agricultural Co-operatives in Thailand and the Kenya Rural Enterprise Program all make group loans that are symmetric.

[3]Social sanctions form the basis of informal contract enforcement (see Greif (7) for a review). Recent evidence suggests social sanctions can help explain the high repayment rates of microlenders. Karlan (8) finds that Peruvian groups with social ties are more likely to repay than groups without social ties. He also finds that groups with social ties are more likely to ostracize defaulters. Ahlin and Townsend (1) find that Thai villages where borrowers report that they will be excluded from informal village credit markets if they do not repay the microlender have higher repayment rates than villages where borrowers do not report these socially sanctioned punishments.

differ both in their sanctioning abilities and in their susceptibility to social sanctions. We refer to borrowers as strong and weak, where strong borrowers have higher sanctioning ability/lower susceptibility to sanctions than weak borrowers. Consequently weak borrowers have a higher willingness to repay, since they are threatened with tougher sanctions ex post. In practice, however, sanctioning abilities and susceptibilities are difficult for an outside lender to observe. It is in such a private information environment that we analyze cosigned and group loans.

When would we expect cosigned loans to be used instead of group loans? If one of the borrowers does not have an investment opportunity, then there is no point in lending to both, and so cosigned loans are trivially the best option. But we show that even when both borrowers have investment opportunities, cosigned loans are preferred to group loans when borrowers are sufficiently unequal in sanctioning ability. Conversely, if borrowers are relatively equal, then symmetric group loans are preferred. Since microlenders target the poor, their borrowing pool is relatively equal in its sanctioning ability compared with an untargeted lender's. Just as the theory predicts therefore, we see microlenders make symmetric group loans. We therefore address a puzzle posed by Ray (13): microcredit schemes build on horizontal links between villagers (group loans) instead of vertical links (cosigned loans).

Why are only the extremes of symmetric group loans and cosigned loans observed in practice? After all, in making a symmetric group loan, a bank is forced to "level down" the loan size to a point at which even the member who faces the least significant social sanction still repays. In contrast, with an asymmetric group loan the bank could provide more funds to borrowers who face larger sanctions in the event of default. Ideally the bank would like to design a lending scheme that induces borrowers to (at least partially) reveal their sanctioning abilities and susceptibilities, and so lend more than in a symmetric group loan. We show, however, that unless borrowers are very productive no group lending scheme will achieve this. So conditional on making a group loan, the bank will make a symmetric group loan.

In contrast to the difficulties of trying to treat borrowers differently under private information while giving both a loan, if the bank instead only makes a single cosigned loan then private information ceases to have any bite at all. Private information is so much less of an impediment in cosigned lending than in group lending for the following reason. With cosigned loans, an individual will clearly never cosign a loan that he anticipates being defaulted on. But all cosigners are themselves loan recipients in an asymmetric group loan. So they may all may end up with positive utility even if default ensues.

In contrast to the adverse selection or moral hazard problems that have been the focus of the microcredit literature, our paper deals with limited enforcement.[4] Even though many believe that enforcement difficulties are a crucial reason for financial constraints in developing countries, there have only been a few papers on this topic.[5] As mentioned, Besley and Coate (4) is the closest study to ours. They study lending contracts with symmetric borrowers, and so neither asymmetric group loans nor cosigned loans arise in their model. In contrast, the potential borrowers in our model have unequal and unobserved sanctioning abilities. This allows us to study a richer

[4] This literature on adverse selection and moral hazard includes Armendariz and Gollier (2), Banerjee et al (3), Ghatak (5), Laffont (11), Rai and Sjöström (12), and Stiglitz (14), among others. In all of these papers, borrower returns are contractible, i.e., borrowers will repay as long as they have enough funds to do so. In our paper, by contrast, borrowers must be *induced* to repay by threatening punishment, e.g., the seizure of collateral by the bank or social sanctions imposed by other villagers. (Note also that the private information in these papers is on the riskiness of borrower projects, effort levels, or ability to repay; while the private information in our paper is on the borrower's willingness to repay).

[5] Besley and Coate (4) and Laffont and N'Guessan (10) study limited enforcement in microcredit contracts. Ligon et al (9) provide evidence for how limited enforcement constrains insurance in South Indian villages.

set of contracts for which symmetric group loans and cosigned loans are special cases.

We proceed as follows. In section 2 we describe the basic model and show that either cosigned loans or asymmetric group loans are efficient when the bank has full information on borrower types. In sections 3 and 4, we assume that the bank cannot observe borrower types. In section 3, we compare the simple loan contracts we observe (cosigned loans and symmetric group loans) and establish that cosigned loans are preferred whenever borrowers are sufficiently heterogeneous. In section 4 we establish circumstances under which more general loan contracts are ineffective in preventing strong borrowers from pretending to be weak and defaulting on the bank. In section 5 we illustrate our results with a simple numerical example. We conclude in section 6.

2 The economy

There are two agents, $i \in \{1, 2\}$, and a bank. As in much of the literature, we will assume a non-convex production possibility set to motivate credit constraints. Let α be the unequal investment level. Each agent can invest $x_i \geq \alpha$ in a project with certain rate of return $\rho > 1$. If an agent has $x_i < \alpha$ he must use a costless storage technology. Let $f(x_i)$ denote output from input x_i:

$$f(x_i) = \begin{cases} \rho x_i & \text{if } x_i \geq \alpha \\ x_i & \text{if } x_i < \alpha \end{cases} .$$

Aside from their unequal abilities to impose social sanctions on each other, which we discuss in detail below, agents are *ex ante* identical. They have no funds of their own to invest. Each agent has collateral c. The bank can threaten to seize this collateral if the borrower does not repay; in such cases, the bank can sell the collateral for an amount c.

To make the problem of interest, we assume throughout that

$$c < \alpha. \tag{1}$$

That is, borrowers do not possess enough collateral to raise α directly.

Agents can also impose sanctions on each other. We model these sanctions in the same fashion as Besley and Coate (4) as an exogenous social norm. Specifically, it is socially acceptable for one agent to sanction another if the other agent's action causes harm, and not otherwise. In the context of credit, this means that agent 1 can sanction agent 2 if agent 2's action causes the bank to seize collateral from agent 1; and vice versa.

We denote the combined social sanctioning ability of the two agents by s. By analogy with physical collateral, and in line with common usage, we often refer to s as *social collateral*. In this paper our main focus is on the consequences of differences in sanctioning ability across the two agents: one agent can impose sanctions of μs, while the other can impose sanctions of $(1-\mu)\, s \geq \mu s$, where $\mu \in [0, 1/2]$ is a measure of how similar the two agents are in terms of sanctioning abilities. Throughout, we refer to the agent with sanctioning ability μs as the *weak* agent, and to the agent with sanctioning ability $(1-\mu)\, s$ as the *strong* agent.[6]

It is natural to assume that the sanction imposed on an agent is no more than the harm he has caused:

$$\mu s \leq c \text{ and } (1-\mu)s \leq c \text{ for all } \mu. \tag{2}$$

Throughout the paper we assume $s \leq c$, which guarantees that (2) holds.

One interpretation of μ is as a measure of sanctioning ability. For example, a villager may have a plot of land that is upstream from another, and so can sanction

[6]Notice that the strong and weak agents have the same endowment of collateral c. Provided collateral endowments are observable by the bank, this assumption could be straightforwardly relaxed without qualitatively changing our results.

the downstream villager by restricting irrigation water. In such a situation, we would refer to the upstream villager as strong and the downstream villager as weak. Two alternative interpretations of μ (that are equally valid for the results that follow) are:

1. Agents differ in their susceptibility to sanctions. For example, agent 1 may be more susceptible to social ostracism than agent 2. In other words, even though their sanctioning abilities are exactly the same, agent 1 is effectively punished more than agent 2 by the same sanction. For example, agent 1 may have a shop at the village center, and so may indeed be more prone to social ostracism (loss of sales in his shop) than another villager.

2. Agents have the same sanctioning ability (s) but different skills in renegotiating the imposition of social sanctions. Suppose agent 2 fails to repay and the bank seizes c from agent 1 as a consequence. Agent 1 is now in a position to impose a sanction s on agent 2. Then the agents have the incentive to renegotiate: they would be collectively better of if no sanction were imposed. This is a standard split-the-surplus game. If μ denotes agent 1's bargaining power, then the outcome is for agent 2 to pay ("bribe") agent 1 an amount μs in return for not imposing the sanction. Conversely, if agent 1's bargaining power is $1-\mu$ then agent 2 pays a bribe of $(1-\mu)\,s$. The net effect is that a weak (strong) agent faces a welfare loss of $(1-\mu)\,s$ (μs) if he causes harm to the other agent.

The bank offers loan contracts $(x_i, R_i, \gamma_i(\cdot))$ for each $i \in \{1,2\}$, where x_i is the loan size, R_i is the repayment amount, and $\gamma_i(t_1, t_2)$ indicates if collateral c is seized from agent i when borrowers transfer t_1 and t_2 to the bank: if $\gamma_i(t_1, t_2) = 1$ then collateral is seized, while if $\gamma_i(t_1, t_2) = 0$ no collateral is seized.[7] In other words, the

[7]More generally we could allow the bank to seize a fraction of collateral c but that would not change our results.

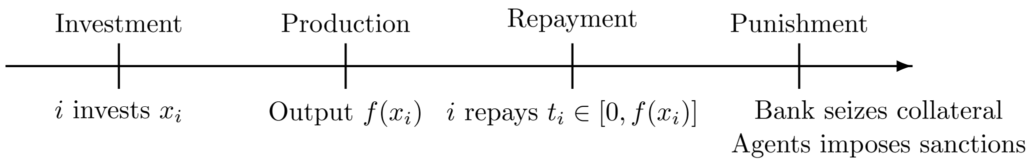

Figure 1: Basic Timeline

bank can impose a punishment of c directly on borrowers. The timing is as shown in figure 1.

First consider *individual loans*. The seizure rule for agent 1 is independent of whether agent 2 repays, and vice versa:

$$\gamma_i(t_1, t_2) = \begin{cases} 1 & \text{if } t_i < R_i \\ 0 & \text{otherwise} \end{cases}.$$

Consequently the maximum that can be recovered from each agent is $R_i \leq c$. A loan of at least α to either agent is infeasible by assumption (1). So lending is impossible with individual loans.

Next we distinguish between two types of joint liability lending: *cosigned loans*, where only one agent receives a loan and the other is a cosigner, and *group loans* where both agents receive a cosigned loan (and consequently both are cosigners). In both these loan contracts, the bank can seize collateral worth c but agents can also impose punishments on each other worth μs and $(1-\mu)s$.

A cosigned loan to agent 1 with agent 2 as cosigner has the following seizure rule, where collateral is seized from both if the loan is not repaid:

$$\gamma_1(t_1, t_2) = \gamma_2(t_1, t_2) = \begin{cases} 1 & \text{if } t_1 < R_1 \\ 0 & \text{otherwise} \end{cases}.$$

The seizure rule for a group loan to agents 1 and 2 stipulates that if *either* agent does not repay, then collateral is seized from both:

$$\gamma_1(t_1, t_2) = \gamma_2(t_1, t_2) = \begin{cases} 1 & \text{if } t_1 < R_1 \text{ or } t_2 < R_2 \\ 0 & \text{otherwise} \end{cases}.$$

Without loss, for the remainder of this section we assume that agent 1 is the weaker agent, and agent 2 is the stronger agent. So if the bank offers a group loan with $R_1, R_2 > 0$, then the repayment game is given below:

		Borrower 2's repayment	
		R_2	0
Borrower 1's repayment	R_1	$-R_1, -R_2$	$-R_1 - c, -c - \mu s$
	0	$-c - (1-\mu)s, -R_2 - c$	$-c, -c$

If the required repayments R_1 and R_2 satisfy

$$R_1 \leq c + (1-\mu)s \text{ and } R_2 \leq c + \mu s \tag{3}$$

then it is an equilibrium for both borrowers to repay. (In this case there is also an equilibrium in which both borrowers default. We shall restrict attention to a *weak* repayment constraint: whenever there are multiple equilibria, we focus on the repayment equilibrium.)[8]

Any loan (x_i, R_i) the bank makes to agent $i \in \{1, 2\}$ must give non-negative utility,

$$f(x_i) - R_i \geq 0 \tag{4}$$

and repayments must be feasible

$$R_i \leq f(x_i) \tag{5}$$

[8] Multiple equilibria arise in the same way in Besley and Coate (4), who also focus on the equilibrium in which both repay. Alternatively, if borrowers were to impose social sanctions on each other when both default, then the repayment game would have a unique equilbrium.

We now turn to the bank's contract design problem. We take the bank's objective to be the maximization of aggregate borrower welfare, subject to the constraint that is makes non-negative profits. Both altruistic lenders subject to a tight funding constraint (such as development banks), and competitive profit-maximizing banks can be expected to behave broadly in this manner.

Formally, the bank chooses (x_i, R_i) for each i to maximize aggregate welfare

$$\sum_{i=1,2} (f(x_i) - R_i)$$

subject to a break even constraint

$$\sum_{i=1,2} (x_i - R_i) \leq 0 \tag{6}$$

and subject to the repayment constraints (3), the individual rationality constraints (4), and the limited liability constraints (5).[9]

The solution to this problem is given by the proposition below. Even though individual lending is impossible, lending is feasible with group loans or cosigned loans. Given full observability of social sanctions, the efficient group loan will generally be asymmetric.

Proposition 1 ***(Benchmark: bank observes borrower types)***
(i) The bank lends to both agents (asymmetric group loans) if

$$\alpha \leq \min\left\{c + \frac{s}{2}, \left(2 - \frac{1}{\rho}\right) c + \left(1 - \frac{1-\mu}{\rho}\right) s\right\} \tag{7}$$

(ii) The bank lends only to the weak agent (cosigned loan) if

$$\min\left\{c + \frac{s}{2}, \left(2 - \frac{1}{\rho}\right) c + \left(1 - \frac{1-\mu}{\rho}\right) s\right\} < \alpha \leq c + (1-\mu)\, s \tag{8}$$

[9] Notice that co-signed loans are just an especially asymmetric group loan where $R_1 = 0$ or $R_2 = 0$.

(iii) No lending is possible if

$$c + (1 - \mu)\, s < \alpha \tag{9}$$

All formal proofs are relegated to the appendix. Proposition 1 establishes that the form of lending depends on relationship between the minimum investment α and the collateral endowments c, μs and $(1 - \mu)\, s$. When the minimum investment α is small enough, $\alpha < c + \mu s$, then it is feasible to recover α from both agents and so group loans are efficient. The strong agent is asked for a smaller repayment than the weaker agent: $R_1 = c + \mu s$ and $R_2 = c + (1 - \mu)\, s$. There is some freedom in how to set the loan sizes: depending on the project return ρ, it may be possible to give equal loans and still satisfy the individual rationality constraint of both borrowers. In general, however, the loan granted to the weaker borrower will be larger, to reflect his larger repayment.

The more interesting case is when $c + \mu s < \alpha < c + (1 - \mu)s$. Now the bank can recover α from the weak agent but not from the strong agent. The bank would like to lend to both using group loans. But that would mean losing money on the strong agent. Therefore group loans are feasible only if the bank can make enough money on the larger loan to the weak agent without violating the weak agent's individual rationality constraint. Note that the smaller μ is, the greater the discrepancy in the repayment sizes. Consequently when μ is sufficiently small, the weak agent is no longer willing to cross-subsidize the strong agent, and so it becomes impossible to lend at least α to each. In such a situation (condition (8)), the bank will only lend to the weak agent using the strong agent as cosigner.

Finally, if it is impossible to recover α from even the weak agent, i.e., if $\alpha > c + (1 - \mu)s$, then lending is clearly infeasible.

Group loans generate a higher surplus than cosigned loans: the total surplus using group loans is $(\rho - 1)(2c + s)$ while the total surplus with a cosigned loan is only

$(\rho - 1)(c + (1-\mu)s)$. But group loans are only feasible if the minimum investment size α is sufficiently low relative to the collateral the agents possess. With group loans the total lending exceeds $2c$ (the total collateral in the village), while with cosigned loans the total lending is less than $2c$.

So far we have established that asymmetric group loans with the weak borrower receiving a larger loan than the strong borrower are efficient if feasible. But in practice we observe microlenders make *symmetric* group loans. Even though villagers surely differ in their sanctioning abilities, lenders do not appear to take these differences into account. We shall discuss their reasons for doing so in section 4.

3 Symmetric group loans vs. cosigned loans

In practice banks are outsiders with limited information on the social standing of specific villagers and their sanctioning abilities/susceptibilities. In this section (and the next) we assume that the bank is uninformed in this regard. We do assume, however, that the bank knows the basic social structure of the community from which the borrowers are drawn. More formally, the bank knows the parameter μ, but does not know whether the sanctioning ability of borrower 1 is μ or $1-\mu$.[10,11] In other

[10] Or equivalently as we discussed in the previous section, (a) whether agent 1 is more or less susceptible than agent 2 to sanctions, or (b) whether agent 1 has higher or lower bargaining power in the renegotiation of sanctions than agent 2.

[11] What contract will the bank use if it does not even know μ? The only contract that is immune to default is to offer a cosigned loan of size $c + s/2$: the bank can be sure that at least one of the two borrowers faces a social sanction of $s/2$, no matter what the true value of μ is. In contrast, no group loan is default free for all possible values of μ. If the bank has some idea of μ's value, it may be prepared to make a loan which is defaulted on sometimes. When the bank's prior on μ is sufficiently concentrated around the true value, our results will be qualitatively unaffected. A full consideration of this case is beyond the scope of the paper.

words, the bank knows if the community is relatively equal, or relatively unequal, in terms of sanctioning ability — but does not know whether a particular agent is relatively strong or weak.

In general, the bank's contract design problem is to offer a menu of loan contracts that induces agents to reveal their types (and hence reveal how much they are willing to repay).[12] Let WS (respectively, SW) denote the state where borrower 1 is weak (strong) and borrower 2 is strong (weak). Since there are two possible states, WS and SW, the menu will include two possible contracts:

$$\left\{\left(x_1^{WS}, x_2^{WS}, R_1^{WS}, R_2^{WS}\right), \left(x_1^{SW}, x_2^{SW}, R_1^{SW}, R_2^{SW}\right)\right\} \tag{10}$$

Note that since individual lending is impossible under assumption (1), without loss *all* loan contracts are assumed to entail seizure of both agents' collateral if either agent $i = 1, 2$ fails to make the payment R_i. (This is clearly the case for group loans. For cosigned loans, it is true since there is only one required repayment — which if not made, triggers seizure of both agents' collateral.)

We will restrict attention to anonymous menus — that is, those in which if the names of the two borrowers were interchanged, the menu of possible contracts would be unchanged. Formally, an anonymous menu is one that satisfies $x_1^{WS} = x_2^{SW}$, $x_1^{SW} = x_2^{WS}$, $R_1^{WS} = R_2^{SW}$, and $R_1^{SW} = R_2^{WS}$.

When the menu of contracts offered is non-degenerate — that is, the contracts are not identical — how do the two borrowers decide which contract to accept? It is natural to suppose that the strong agent will have more of a say in selecting a menu option than the weak agent. We model this by assuming that with probability $1 - \mu$

[12] When the riskiness of borrower investments is unobserved, Ghatak (5) has shown that group lending induces borrowers to match assortatively. But in our model, it is the willingness to repay that is unobserved. If there are many strong and weak borrowers, then strong will match with weak in an attempt to default on group loans. So assortative matching does not help overcome this asymmetric information problem.

it is the strong agent who chooses which contract to accept, while with probability μ it is the weak agent who makes the decision. (Our results would be qualitatively unchanged if instead the strong agent chose the contract with a probability given by any non-linear but increasing function of $1-\mu$.) However, once the contract has been selected, the non-selecting agent has veto power and can decline the selected contract. When this happens, no loan is made at all. Consequently the selecting agent will only choose a contract that satisfies the other agent's individual rationality constraint.

In practice, the most common loan contracts are the symmetric group loan, and a cosigned loan. In this section, we analyze the performance of these two contracts. In section 4, we then consider whether the bank could offer an alternate loan contract that outperforms symmetric group loans and cosigned loans. As we will see, under a wide range of parameter values symmetric group loans and cosigned loans are in fact the most efficient lending contracts available to the bank.

3.1 The self-selection property of cosigned loans

Formally, a cosigned lending policy under asymmetric information is a particularly simple menu in which there are two menu items: one in which agent 1 takes a loan cosigned by agent 2, and the other in which agent 2 takes a loan cosigned by agent 1. In terms of the notation defined in (10), and under our anonymity assumption,

$$x_1^{WS} = x_2^{SW}$$

and

$$R_1^{WS} = R_2^{SW}$$

with

$$x_1^{SW} = x_2^{WS} = R_1^{SW} = R_2^{WS} = 0.$$

That is, $x_1^{WS} = x_2^{SW}$ is the loan size offered, $R_1^{WS} = R_2^{SW}$ is the required repayment, and the cosigning agent neither receives a loan nor is required to make a repayment.

As we saw in the section 2, when the bank's objective is to maximize aggregate borrower welfare, then conditional on making a cosigned loan he prefers to lend to the weaker borrower. The reason is simple — the weaker borrower can be called upon to repay $c + (1-\mu)s$, while the stronger borrower can only be induced to repay $c + \mu s$. Consequently a larger loan can be made to the weak borrower.

A striking property of cosigned loans is that the bank's ignorance of the relative sanctioning abilities of the two borrowers does not impede this targeting of the weak borrower. This can be seen as follows. The bank would ideally like to make a cosigned loan of $x = c + (1-\mu)s$ to the weak borrower, with a repayment of $R = x$. Consider what happens if it offers the cosigned loan menu defined above, with the loan sizes $x_1^{WS} = x_2^{SW}$ and repayments $R_1^{WS} = R_2^{SW}$ both set to the preferred level $c + (1-\mu)s$.

Without loss, suppose that agent 1 is the weak agent — i.e., the state is WS. Under this menu, if agent 2, the strong agent, is the one who selects the contract, he is happy to select the WS contract: agent 1 gets the loan of $c + (1-\mu)s$, while agent 2 is the cosigner. The reason is that under this selection, agent 1 will indeed repay, and so agent 2 does not lose his collateral. In contrast, if agent 2 selects the contract SW, then agent 1 foresees that agent 2 will default on the repayment $c + (1-\mu)s$ — and so agent 1 vetoes the selection, since it leaves him with negative welfare. Finally, if agent 1 is the contract-selecting agent, then the same arguments imply that he prefers to choose the WS contract in which he receives the loan, and agent 2 will not veto this choice since his expected payoff is 0.

To summarize, the bank is able to make a cosigned loan of $c + (1-\mu)s$ to any agent who can find a cosigner, and be sure that only a weak agent will take such a loan — and the weak agent will not default. Recall, moreover, that from Proposition

1 cosigned loans achieve the constrained first best when the minimum loan size α is relatively large compared to the collateral endowments c and s. Consequently, under these same conditions the bank's ignorance of agents' sanctioning abilities does not reduce social welfare:

Lemma 1 ***(Self selection of cosigned loans)***
If (8) holds, then self selection using cosigned loans allows an uninformed bank to lend as much as if it were fully informed.

As we will see in section 4, when the solution to the full information problem is for the bank to employ an asymmetric group loan (i.e., when (8) does not hold), the situation is very different: the bank's lack of knowledge of sanctioning abilities constrains its ability to lend efficiently. Specifically, unless the agents' project return ρ is very high, the bank is unable to effectively separate the two agents. Instead, the bank is forced to use either a cosigned loan or a symmetric group loan, even though neither is efficient under full information.

3.2 The choice between group loans and cosigned loans

So far we have discussed one commonly observed contract, namely cosigned loans. We now turn to the other commonly observed contract, group loans in which loans and repayments are identical across members. We have termed such contracts symmetric group loans. Formally, symmetric group loans are a degenerate menu of loan contracts in which both borrowers are offered identical loans,

$$\begin{aligned} x_1^{WS} &= x_2^{SW} = x_1^{SW} = x_2^{WS} \\ R_1^{WS} &= R_2^{SW} = R_1^{SW} = R_2^{WS}. \end{aligned}$$

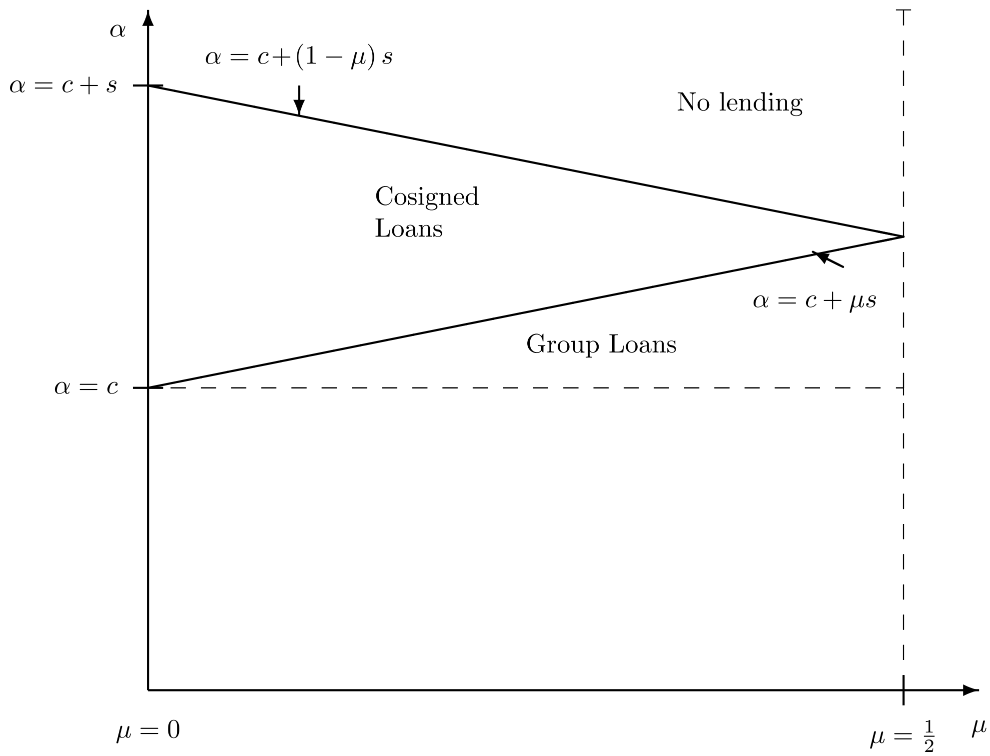

Figure 2: Private Information

A comparison between the two types of loans as the minimum project size α and the inequality parameter μ varies is depicted in Figure 2.

The comparison is summarized as:

Proposition 2 ***(Symmetric group loans vs. cosigned loans)***

(i) *The bank lends to both agents (symmetric group loans) if*

$$\alpha \leq c + \mu s \tag{11}$$

(ii) *The bank lends only to the weak agent (cosigned loan) if*

$$c + \mu s < \alpha \leq c + (1 - \mu)s \tag{12}$$

(iii) *No lending is possible if*

$$c + (1 - \mu)s < \alpha \tag{13}$$

As in the full-information problem (see Proposition 1), the bank's choice of contract depends on the relative size of the minimum investment α and the collateral endowments c and s. The bank can recover $c + \mu s$ from each borrower with symmetric group loans. Whenever symmetric group loans are feasible, they will be preferred to cosigned loans because $2c + 2\mu s$, the total lending using symmetric group loans is higher than $c + (1 - \mu)s$, the total lending with cosigned loans. When group loans are infeasible, i.e., when agents are sufficiently heterogeneous or the minimum investment size is sufficiently large, then the bank will just give a cosigned loan to the weak agent. Even though the bank cannot tell the agents apart, by offering a cosigned loan that the strong agent is unwilling to repay (and consequently the weak agent is unwilling to cosign), the bank effectively selects the weak agent.

Restating Proposition 2 slightly gives:

Corollary 1 ***(Inequality)***
For all sufficiently unequal agents the bank will use cosigned loans in preference to symmetric group loans. Conversely, if $\alpha \le c+s/2$ then for all sufficiently equal agents the bank will use symmetric group loans in preference to cosigned loans.[13]

Notice that the bank always prefers to give cosigned loans for μ sufficiently small. This is unlike the full information case: from Proposition 1, absent private information when the minimum project α is small[14] the bank prefers asymmetric group loans to cosigned loans *even when the borrowers are very unequal.*

So far we have compared two simple loan contracts, cosigned loans and symmetric group loans. This comparison is suggestive, but leaves open the question of whether the bank could offer an even better loan contract. We take up this question in detail in the next section. Note, however, that there are a couple of circumstances where we can immediately conclude that the bank's choice does indeed boil down to one between a symmetric group loan and a cosigned loan. First, when potential borrowers are exactly equal ($\mu = 1/2$) then we are back to the full-information problem. $>$From Proposition 1, the bank will either make a group loan or a cosigned loan; and since borrowers have the same exposure to social sanctions, the optimal group loan is symmetric. Second, from Lemma 1 we know that under some circumstances the bank can do no better than cosigned loans even if it had full information. In particular, if the weak borrower's repayment capability exceeds α, and μ is sufficiently small (α sufficiently large) for asymmetric group loans to be impossible, cosigned loans achieve the first best.

[13] If $\alpha > c + s/2$ then symmetric group loans are never feasible.

[14] Specifically, below

$$\min\left\{c + \frac{s}{2}, \left(2 - \frac{1}{\rho}\right)c + \left(1 - \frac{1}{\rho}\right)s\right\}.$$

4 Why asymmetric group loans are never observed

Above we provided a justification for why we should observe cosigned loans (Lemma 1 shows that sometimes the bank can do no better). But we have not justified why only symmetric group loans are observed. Recall that Proposition 1 clearly shows that in general asymmetric group loans are preferred to symmetric ones when the bank is informed. In this section, we demonstrate the difficulties of implementing asymmetric group loans when the bank is uninformed.

We consider two ways in which the bank may be able to improve upon the performance of the symmetric group loan and cosigned loan contracts discussed in section 3. First, a bank may be able to induce borrowers to reveal their types truthfully by offering a suitably designed menu of loan contracts. The menu of cosigned loans described in section 3.1 is one example of a menu that induces truthful self selection. There, however, information revelation is obtained at the cost of giving only one of the borrowers a loan. This encourages truthful selection because the payoff to a weak agent to cosigning a loan for the strong agent is very low — he both loses his collateral when the strong agent defaults, and does not receive any loan. As such, he will always veto a strong agent who attempts to take a cosigned loan for himself. Below, we examine whether the bank can induce truthful information revelation using alternate menus, i.e., those which involve making asymmetric group loans.

Second, the bank may simply offer asymmetric group loans where borrowers do not repay all of the time. Thus far we have restricted attention to loan contracts that are default-free. But by offering an asymmetric group loan with one of required payments above $c + \mu s$, the bank faces default whenever the borrower it asks for this high repayment turns out to be strong. This occurs with probability 1/2. More generally, by randomizing the requested repayments after the initial loan is made, the bank could effectively choose to face any default rate.

The results of this section are easily summarized: there are only limited circumstances in which the bank can offer loan contract that dominates the simple and commonly observed alternatives of cosigned loans and symmetric group loans. In particular, unless the project return ρ is high, symmetric group loans and cosigned loans are the best contracts at the bank's disposal.

4.1 Alternate menus

Let us start with the question of whether we can do better by offering non-degenerate menus of loan contracts (other than cosigned loans). We show that such menus can be useful, but only in very limited circumstances. In particular, unless projects are very productive then restricting attention to simple contracts, i.e., symmetric group loans or cosigned loans, is without any loss of surplus.

To understand why menus are of limited use in making group loans, it is useful to start by constructing an example in which a menu does in fact play a useful role. The example will make clear what conditions must be satisfied for a menu to be welfare improving. As we will argue, these conditions are unlikely to be met. Loosely speaking, in order to prevent strong agents from pretending to be weak and defaulting, the menu options must provide the strong agent with favorable terms. Consequently the weak agent must be offered unfavorable terms in order for the bank to break even. The weak agent will refuse to participate unless he is very productive.

Suppose the bank is operating in a somewhat unequal community, with $\mu = 1/4$, and that the project size α lies somewhere between $c + s/4$ and $c + 3s/8$. From Proposition 2 we know that cosigned loans are preferred to a symmetric group loan.

However, under some circumstances the following *menu* of loans does even better.

The bank offers a menu $\left\{\left(x_1^{WS}, x_2^{WS}, R_1^{WS}, R_2^{WS}\right), \left(x_1^{SW}, x_2^{SW}, R_1^{SW}, R_2^{SW}\right)\right\}$, where the loan contract $\left(x_1^{WS}, x_2^{WS}, R_1^{WS}, R_2^{WS}\right)$ is characterized by

$$\begin{aligned} x_1^{WS} &= \alpha \text{ and } R_1^{WS} = c + 3s/4 \\ x_2^{WS} &= 2c + 3s/4 - \alpha \text{ and } R_2^{WS} = c + s/4 \end{aligned}$$

while the loan contract $\left(x_1^{SW}, x_2^{SW}, R_1^{SW}, R_2^{SW}\right)$ is just the opposite, i.e., $x_1^{SW} = x_2^{WS}$ etc. Notice that $x_2^{WS} > \alpha$ since $\alpha < c + 3s/8$.

To see how this menu works, suppose for now that the strong borrower selects the WS contract in state WS and the SW contract in state SW. Under the terms of these contracts, the strong borrower is required to make a smaller repayment to the bank than the weak borrower is. As a result, the borrowers will repay their loans. Since the strong borrower chooses the contract with probability 3/4, the bank is repaid at least $\frac{3}{4}(2c + s) + \frac{1}{4}(2c) = 2c + 3s/4$ in expectation, and breaks even. Moreover, the bank lends a total amount of $2c + 3s/4$ which is higher than the total lending of $c + 3s/4$ under the cosigned loan contract, which in turn dominates the symmetric group loan contract.

A key feature of the loan menu is that despite repaying less, the strong borrower receives a large loan. It is this feature of the contract that gives the strong borrower the incentive to choose the intended loan contracts, i.e., contract SW in state SW and contract WS in state WS. Under this choice, the strong borrower's utility is

$$U^S = \rho\left(2c + 3s/4 - \alpha\right) - \left(c + \frac{s}{4}\right).$$

If instead the strong borrower deviates, and instead chooses the contract SW in state WS, then he is asked to make a repayment that exceeds the punishment he faces for non-repayment. Consequently both borrowers default, and the strong borrower's utility is

$$\tilde{U}^S = \rho\alpha - c.$$

That is, the strong borrower ends up "repaying" less, which is attractive, but at the cost of receiving a smaller loan, which is unattractive. His utility level under the intended contract WS is higher (i.e., $U^S \geq \tilde{U}^S$) whenever

$$\rho \geq \frac{\frac{s}{8}}{c + \frac{3s}{8} - \alpha},$$

a condition which is satisfied whenever the project return ρ is large enough.

In this menu it is the strong borrower who is given the incentives to choose the intended contract. These incentives are provided at the expense of the weak borrower, who receives a smaller loan and must make a larger repayment. His utility is $\rho\alpha - (c + 3s/4)$, which is positive whenever the project return ρ is large enough. In this case he will not veto the strong borrower's choice of the intended loan contract.[15]

This example shows that when the project return ρ is high enough, it may be possible for the bank to design a menu of contracts that allows it to lend a greater amount than is possible using either cosigned loans, or symmetric group loans. However, it also makes clear the main limitation on the use of such menus: Unless the project return ρ is high enough, it is impossible to simultaneously induce one borrower to select the "right" loan, while still meeting the individual rationality constraint of the other. Since a strong villager has less of an incentive to repay, he can only be asked for a small repayment. But the strong borrower must also receive a large loan — for otherwise, he can pretend to be the weak borrower, and simply default on the bank. This leaves the weak borrower with a large repayment and a small loan — an unattractive proposition unless his project return ρ is in fact very high.

This feature of the example generalizes to:

Lemma 2 ***(A condition that rules out menus)***

[15] It is easily shown, however, that when the weak borrower chooses the contract he will select the unintended contract. The borrowers default. By construction, the bank still breaks even, in spite of a 1/4 probability of default.

Fix $\mu \in [0, 1/2]$. Then there exists a $\bar{\rho}$ such that if $\rho \leq \bar{\rho}$, there exists no menu in which the bank is able to lend more than is possible using cosigned loans or a symmetric group loan, while itself breaking even.

Specifically, if ρ is low enough such that for both $\eta = \mu, 1 - \mu$,

$$\left(\frac{2}{1+\eta} - \rho\frac{1-\eta}{1+\eta}\right)\alpha > c + \rho\left(\frac{1-\eta}{1+\eta} - \frac{1}{\rho\,(1+\eta)}\right)\eta s \tag{14}$$

and one of

$$\alpha < \frac{c + (1-\eta)\,s}{\rho} \tag{15}$$

$$\alpha \geq c + \left((1-\eta) - \frac{\eta}{\rho}\right)\frac{s}{2} \tag{16}$$

hold, then no menu with the above properties exists.

Lemma 2 gives conditions under which no menu of contracts is useful. The combination of conditions is relatively hard to interpret. Fortunately, if we accept a weaker set of conditions we have:

Proposition 3 ***(No menu can do better if ρ small)***
If $\rho < 2 - c/\alpha$, then there exists no menu in which the bank is able to lend more than is possible using cosigned loans or a symmetric group loan.

Observe that the bound on ρ in Proposition 3 is tighter the further α is from c. That is, the larger the funding shortfall that must be met by the use of social sanctions, the less scope there is to make use of more complicated lending arrangements. Instead, the lending bank's choice reduces to one between making a cosigned loan to one borrower, or a symmetric loan to both.

This negative result contrasts starkly with Lemma 1. Unlike menus of asymmetric group loans, cosigned loans are a useful selection device because they are so extreme. The cosigner receives nothing, and so if a weak agent anticipates default, then he has no incentive to cosign a loan for a strong agent since he will certainly lose c. If the bank offers a menu of asymmetric group loans, however, the cosigner is given a loan of α. Even if a weak agent anticipates that both will default, he will now make a potentially positive surplus of $\rho\alpha - c$. So a weak agent may willingly go along with an asymmetric group loan if his output exceeds the loss of collateral. For this reason, separating borrower types is so much more difficult if the bank is trying to lend to both borrowers than if the bank is only trying to lend to one borrower.

4.2 Loans with default

We now to a consideration of loan contracts with a positive probability of default. The leading example is an asymmetric group loan, with one of the requested repayments above $c + \mu s$, the strong borrower's willingness to repay. A second possibility is to explicitly randomize the requested repayments.

The main rationale for why *asymmetric* group loans would never be offered is straightforward, and easy to see. Suppose a loan contract calls for repayments $R_1 > 0$ and $R_2 > 0$. As we have seen, the borrowers will only make these repayments in both states SW and WS if

$$R_1, R_2 \leq c + \min \{\mu s, (1 - \mu) s\}.$$

So if the bank wants to avoid the possibility of default, then to maximize the original loan size it can restrict its attention to contracts in which both borrowers repay the same amount.

Although default is costly for the bank — and thus ultimately for the borrowers — there remains the possibility that the constrained optimal loan contract is one in which default occurs. For example, the bank might offer a contract in which $R_1 = c + \mu s$ and $R_2 = c + (1 - \mu) s$. Under this contract, the bank will be repaid when borrower 2 is weak (state SW) but will not be repaid when borrower 2 is strong (state WS). Consequently the bank's total expected repayment is $2c + s/2$. Whenever the borrowers are even remotely close to having equal sanctioning ability, i.e., $\mu > 1/4$, then this is a lower repayment than is obtainable under the symmetric group loan contract in which each is asked to repay $c + \mu s$. In general, if $\mu > 1/4$ then a symmetric group loan is preferred to any other single loan contract (i.e. to any *degenerate* menu of loan contracts).

Moreover, even when borrowers are very unequal (i.e., $\mu < 1/4$), there is little scope for the bank to offer a single loan contract other than the symmetric group loan. The reason is that clearly the only way such a contract can succeed in generating a higher expected repayment is if one of the borrowers is sometimes asked to make a large repayment. Moreover, if the bank asks a strong borrower for the larger repayment, both borrowers default. So an asymmetric loan contract must end up sometimes taking large repayments from the weak borrower. The weak borrower will only agree to such a contract when the original loan is correspondingly large. But the circumstances in which the bank can afford to make two loans of more than the minimum size α, with at least one of them large, and cover these costs even in the face of default, are extremely limited.

To summarize:

Proposition 4 ***(No Single Loan Contracts Can Do Better)***
If $\mu > 1/4$, then offering any single loan contract other than a symmetric group loan is suboptimal. If $\mu < 1/4$, then there exists a $\hat{\rho}$ such that for all rates of return $\rho < \hat{\rho}$ the same is true.

5 An example

In this section, we provide a simple numerical example to illustrate our results. Suppose that both agents have collateral $c = 75$. They have projects with minimum scale $\alpha = 100$, and rate of return $\rho = 6/5$. The total social sanctions available are $s = 75$. As before we shall use (x, R) to denote a loan contract where x is loan size and R is repayment, and the seizure rule specifies that collateral will be seized from both if one does not repay.

Consider two economies, one which is more equal than the other. In the *equal* economy, $\mu = 1/3$. So the weak agent can sanction the strong agent in an amount of $\frac{1}{3}75 = 25$, and the strong can sanction the weak agent in the amount $\frac{2}{3}75 = 50$. So the strong agent's willingness to repay is 100 and the weak agent's willingness to repay is 125. With full information, the bank can make an asymmetric group loan: a loan of $(100, 100)$ to the strong agent and a loan of $(125, 125)$ to the weak agent.

In the *unequal* economy, $\mu = 1/5$. So the weak agent can sanction the strong agent in an amount of $\frac{1}{5}75 = 15$, and the strong can sanction the weak agent in the amount $\frac{4}{5}75 = 60$. So the strong agent is willing to repay up to 90 and the weak agent is willing to repay up to 135. With full information, the bank can again make an asymmetric group loan: a loan of $(100, 90)$ to the strong agent and a loan of $(125, 135)$ to the weak agent. Since the weak agent is sufficiently productive, he will cross-subsidize the strong, and the bank succeeds in lending at least $\alpha = 100$ to both. For both economies, condition (7) in Proposition 1 holds, i.e. these asymmetric group loans are efficient.

Now suppose that the bank cannot distinguish between the states WS and SW, i.e., cannot tell weak from strong, but can observe whether the economy is equal or unequal. Let us first restrict the bank to offering either symmetric group loans or cosigned loans, just as in section 3. In the equal economy, the bank can recover at least 100 from each agent. So the bank will make a symmetric group loan, i.e. loans of $(100, 100)$ to each agent. In the unequal economy, by contrast, the bank cannot make the same symmetric group loan because the strong agent will default. Instead, the bank offers a cosigned loan of $(135, 135)$. The weak agent is unwilling to cosign such a loan for the strong agent (because the strong agent will certainly default, and the weak agent will lose collateral). But the strong borrower is willing to cosign such a loan for the weak agent. Consequently, only the weak agent can invest. This illustrates Proposition 2. Symmetric group loans are preferred in relatively equal economies. In particular, condition (11) holds for the equal economy, and condition (12) holds for the unequal economy.

The question remains: can we do better than symmetric group or cosigned loans using menus of loan contracts. To illustrate the argument in section 4.1 consider the unequal economy. There the inefficiency is that only one of the two villagers is able to invest. Suppose the bank offers the full information contracts as a menu, i.e., offers the contracts $(100, 90)$ and $(125, 135)$, where $(100, 90)$ is intended for the strong agent and $(125, 135)$ is intended for the weak agent. The strong agent has an incentive to take the $(125, 135)$ loan meant for the weak agent and default; the weak agent will default as well. The bank will not break even with such a menu. So the bank must design a menu to reward the strong borrower in order to convince him to take a loan (and repay). But the bank is constrained: the contract intended for the weak agent must satisfy the weak agent's individual rationality constraint by asking for a repayment no higher than 120. This candidate menu is: a contract $(110, 90)$ intended for the strong agent and $(100, 120)$ intended for the weak agent. Does the strong borrower now have enough incentive to choose the larger loan? The answer is no. If the strong borrower chooses the $(100, 120)$ loan and defaults, her profits are $\frac{6}{5}(100) - 75 = 45$. If the strong borrower takes the $(110, 90)$ loan, her profits are $\frac{6}{5}(110) - 90 = 42$. Consequently, when the strong borrower chooses both agents will default. And clearly when the weak borrower chooses, he will take the larger loan too and both will default. So this candidate menu fails to separate the agents. More generally, since $\rho < 2 - c/\alpha$ for this example, Proposition 3 implies no other menu of loan contracts is of help. The bank can do no better than cosigned loans in the unequal economy.

6 Conclusion

We have analyzed lending contracts in a model where borrowers have unobserved social sanctioning capabilities (and consequently, unobserved willingness to repay). We have shown that simple and commonly observed loan contracts are constrained efficient unless projects are very productive. Symmetric group loans are constrained efficient when borrowers are relatively equal. Symmetry stems from the unobservability of the borrower's ability to sanction each other. When borrowers are relatively unequal, cosigned loans are efficient.

We have argued that group loans make most sense for targeted anti-poverty lenders. In that sense, we provide an explanation for why the commercial banking sector has not adopted group loans even though they have been used in microcredit for several decades. A testable implication of this paper is that mistargeting of microcredit to the rich will raise default rates. If microlenders allow rich borrowers to enter groups, the poor may very well continue to repay their loans but the rich who are stronger will default.

7 Appendix: Proofs

Proof of Proposition 1: First notice that

$$c + \mu s \le \min\{c + \frac{s}{2}, (2 - \frac{1}{\rho})c + (1 - \frac{1-\mu}{\rho})s\}$$

since $c + \mu s < c + s/2$ and $1 < (2 - 1/\rho)$. Also, $\mu < (1 - \frac{1-\mu}{\rho})$. Thus

$$\min\{c + \frac{s}{2}, (2 - \frac{1}{\rho})c + (1 - \frac{1-\mu}{\rho})s\} \le c + (1-\mu)s$$

because $c + \frac{s}{2} \le c + (1-\mu)s$

(i) There are two cases:

1. Case $\alpha \le c + \mu s$. The following group loan satisfies all the constraints,

$$\begin{aligned} x_1 &= R_1 = c + (1-\mu)s \\ x_2 &= R_2 = c + \mu s \end{aligned}$$

   and it is impossible to offer a larger loan to either agent without violating the breakeven constraint (6) or the repayment constraints (3).

2. Case $c + \mu s < \alpha \le \min\{c + \frac{s}{2}, (2 - \frac{1}{\rho})c + (1 - \frac{1-\mu}{\rho})s\}$. It is clearly efficient to set repayments as high as possible. So $R_1 = c + (1-\mu)s$ and $R_2 = c + \mu s$. The individual rationality constraints (4) make it efficient to give the stronger agent a smaller loan, so $x_2 = \alpha$ and $x_1 = 2c + s - \alpha$. Agent 2's individual rationality constraint is satisfied

$$\alpha > c + \mu s > \frac{c + \mu s}{\rho}$$

   And agent 1's individual rationality constraint is satisfied if

$$\rho(2c + s - \alpha) \ge c + (1-\mu)s$$

   or equivalently

$$\alpha \le (2 - \frac{1}{\rho})c + (1 - \frac{1-\mu}{\rho})s$$

(ii) If condition (7) does not hold, it is impossible to lend α to both agents. Why? If $\alpha > c + s/2$ then the breakeven constraint (6) and the repayment constraints (3) cannot all be satisfied if $x_1 + x_2 = 2\alpha$. If $\alpha \leq c + s/2$ but

$$(2 - \frac{1}{\rho})c + (1 - \frac{1-\mu}{\rho})s < \alpha$$

then the breakeven constraint (6) and the repayment constraints (3) can all be satisfied if $x_1 + x_2 = 2\alpha$, but it is impossible to satisfy the individual rationality constraints.

So the only option is to lend at least α to one agent. Since the bank can recover more from the weak agent, the most it can lend is

$$x_1 = R_1 = c + (1 - \mu)s$$

where agent 2 is the cosigner.**QED**

Proof of Proposition 2: (a) Group loans. Need $R \leq c + \mu s$ for weak agent to accept liability on any loan (x, R) offered to the strong agent. So the most the bank can lend using symmetric group loans is

$$x = R = c + \mu s$$

Such loans are feasible if (11) and the surplus is $(\rho - 1)(2c + 2\mu s)$.

(b) Cosigned loan. Need $R \leq c + (1 - \mu)s$ for strong agent to accept liability on any loan (x, R) offered to the weak agent. So the most the bank can lend is

$$x = R = c + (1 - \mu)s$$

This loan is accepted by both agents only if offered to the weak agent. Such a loan is feasible if (11) or (12) and the surplus is $(\rho - 1)(c + (1 - \mu)s)$.

(c) If group loans are feasible, they are always preferred to the cosigned loan. To see this, note that the surplus is higher with a cosigned loan if

$$(\rho - 1)(c + (1 - \mu)s) > (\rho - 1)(2c + 2\mu s)$$

or equivalently if

$$\mu < \frac{s-c}{3s}$$

But assumption (2) makes this impossible. **QED.**

Proof of Lemma 2: Suppose the menu induces borrowers to always default. Then the bank can lend at most $2c$ in total. Since $2c < 2\alpha$, the bank can only lend to one of the agents, and we have established that using cosigned loans to lend to the weak agent is efficient in such a case. So this menu can do no better cosigned lending. For a menu to do better than cosigned loans, at least one of the borrower's must select a contract in which repayment actually occurs. Let $1-\eta$ be the sanctioning ability of this borrower: note that $\eta \in \{\mu, 1-\mu\}$. Throughout the proof, we will refer to this borrower as the strong borrower, and to the other borrower as the weak borrower; this is solely for expositional convenience, and in fact the η in question may be greater than 1/2. Without loss, write the menu as $\left\{\left(x^S, x^W, R^S, R^W\right), \left(x^S, x^W, R^S, R^W\right)\right\}$, and assume that the menu is designed so that the strong borrower selects the contract which gives him a loan of x^S and repayment R^S. We refer to this contract as the intended contract.

We claim first that when the unintended contract is selected, the borrowers default; and that moreover, when it is the weak borrower's turn to choose the contract, he chooses the unintended contract. To establish this claim, note first that the menu can only dominate a symmetric group loan if one of the repayments R^S and R^W exceeds $c + \min\{\eta, 1-\eta\} s$. For repayment to occur when the strong borrower selects the intended contract, $R^S \leq c + \eta s$ and $R^W \leq c + (1-\eta) s$. Given these observations, either $R^W > c + \eta s$ or $R^S > c + (1-\eta) s$ — and so if the intended contract is not selected, the borrowers default.

Observe that the strong borrower chooses the intended contract if and only if

$$\rho x^S - R^S \geq \rho x^W - c. \tag{17}$$

What happens when the weak borrower makes the contract selection? He will choose the intended contract if

$$\rho x^W - R^W \geq \rho x^S - c.$$

Together, these two inequalities imply $c - R^W \geq R^S - c$. But this in turn implies the bank can lend no more than $2c$ in total, which again contradicts the assumption that the menu dominates cosigned lending. Thus the weak borrower will pick the unintended contract, completing the proof of the claim.

Given this preliminary observation, in order for the menu to deliver at least α to each borrower, and for the bank to break-even, the loan parameters x^S, x^W, R^S, R^W must satisfy — in addition to constraint (17) — the following set of inequalities:

$$\begin{aligned}
x^S &\geq \alpha \text{ and } x^W \geq \alpha \text{ (both borrowers receive at least } \alpha) \\
R^S &\leq c + \eta s \text{ (the strong borrower repays)} \\
R^W &\leq c + (1-\eta)\, s \text{ (the weak borrower repays)} \\
\rho x^S - R^S &\geq 0 \text{ (the strong borrower has positive utility)} \\
\rho x^W - R^W &\geq 0 \text{ (the weak borrower has positive utility)} \\
(1-\eta)\left(R^S + R^W\right) + 2\eta c &\geq x^S + x^W \text{ (the bank breaks even)}
\end{aligned}$$

These inequalities define a constraint set, X say. We will show that X must be empty.

Suppose to the contrary that X is non-empty. So it contains some element $\left(x^S, x^W, R^S, R^W\right)$. It is easily seen that it must then contain an element in which either (a) the weak borrower's IR constraint binds, $\rho x^W = R^W$, or (b) the weak agent receives the minimum feasible loan, $x^W = \alpha$. (If a contract satisfies neither condition, we can always reduce x^W and increase x^S while preserving the strong borrower's incentive to choose the right contract.)

First, suppose a menu exists that satisfies the stated constraints and in which the weak borrower receives zero utility, i.e., $\rho x^W = R^W$. For the bank to break even,

$$(1-\eta) R^S + (\rho(1-\eta)-1) x^W + 2\eta c \geq x^S,$$

and so the strong borrower's utility $\rho x^S - R^S$ from choosing the intended contract is certainly less than

$$(\rho(1-\eta)-1) R^S + \rho(\rho(1-\eta)-1) x^W + 2\rho\eta c.$$

A necessary condition for the strong borrower to choose the intended contract is thus

$$(\rho(1-\eta)-1) R^S + \rho(\rho(1-\eta)-1) x^W + 2\rho\eta c > \rho x^W - c$$

The repayment R^S must be less than $c + \eta s$, otherwise the borrower will not repay. So our contract must satisfy

$$\begin{aligned} (\rho(1-\eta)-1)(c+\eta s) + \rho(\rho(1-\eta)-1) x^W + 2\rho\eta c &> \rho x^W - c \\ \text{i.e. } c + \rho\left(\frac{1-\eta}{1+\eta} - \frac{1}{\rho(1+\eta)}\right)\eta s &> \left(\frac{2}{1+\eta} - \rho\frac{1-\eta}{1+\eta}\right) x^W \end{aligned}$$

But since $x^W \geq \alpha$, this contradicts assumption (14).

Second, suppose a menu exists that satisfies the stated constraints and in which the weak borrower receives the lowest feasible loan, $x^W = \alpha$. By assumption the weak borrower's IR constraint is satisfied, $\rho\alpha - R^W \geq 0$. So if the repayment that can be extracted from the weak borrower is high enough, i.e., $c + (1-\eta) s > \rho\alpha$, then we can always raise the repayment owed by the weak borrower so that his IR constraint binds, $\rho\alpha = R^W$ — but we have just ruled out this case. So if inequality (15) holds, the proof is complete.

The remainder of the proof deals with the case in which the weak borrower's maximum repayment is lower, i.e., $c + (1-\eta) s \leq \rho\alpha$. Note that we can assume without loss that the weak borrower is being asked to repay the maximum amount,

$R^W = c + (1-\eta)\, s$, since this leaves him with positive utility. So for the bank to break even, the loan to the strong borrower must satisfy

$$x^S \leq (1-\eta)\left(R^S + c + (1-\eta)\, s\right) + 2\eta c - \alpha.$$

The strong borrower's utility is thus less than

$$(\rho\,(1-\eta) - 1)\, R^S + \rho\,(1-\eta)\,(c + (1-\eta)\, s) + 2\rho\eta c - \rho\alpha$$

A necessary condition for the strong borrower to choose the intended contract is thus

$$(\rho\,(1-\eta) - 1)\, R^S + \rho\,(1-\eta)\,(c + (1-\eta)\, s) + 2\rho\eta c - \rho\alpha \geq \rho\alpha - c$$

As before, the strong borrower's repayment R^S must be less than $c + \eta s$. So our contract must satisfy

$$\begin{aligned} (\rho\,(1-\eta) - 1)\,(c + \eta s) + \rho\,(1-\eta)\,(c + (1-\eta)\, s) + 2\rho\eta c - \rho\alpha &\geq \rho\alpha - c \\ \text{i.e. } c + \left((1-\eta) - \frac{\eta}{\rho}\right)\frac{s}{2} &\geq \alpha \end{aligned}$$

which contradicts assumption (16). **QED**

Proof of Proposition 3: We will show that when $\rho < 2 - c/\alpha$, then for all $\eta \in [0,1]$ inequality (14) must hold, along with either inequality (15) or (16).

For inequality (14), we must show that

$$f(\eta) \equiv (2 - \rho\,(1-\eta))\,\alpha - c\,(1+\eta) - \rho\,(1-\eta)\,\eta s + \eta s > 0$$

Observe that

$$\begin{aligned} f'(\eta) &= \rho\alpha - c + \rho\eta s - \rho\,(1-\eta)\, s + s \\ f''(\eta) &= 2\rho s \end{aligned}$$

Thus f is convex quadratic function. Its minimum lies at

$$\eta^* = \frac{c - s - \rho\,(\alpha - s)}{2\rho s}$$

Since $\rho > 1$ and $\alpha - s > c - s$, $\eta^* < 1$. It follows that $f(\eta)$ is an increasing function over the the domain of interest, $\eta \in [0, 1]$. Finally, $f(0) = (2 - \rho)\alpha - c$, which is positive by assumption. So inequality (14) holds, as claimed.

Since $\rho < 2 - c/\alpha$, inequality (15) holds whenever

$$\begin{aligned} \alpha &< \frac{c + (1-\eta)s}{2 - c/\alpha} \\ \text{i.e. } \alpha &< c + (1-\eta)\frac{s}{2} \end{aligned}$$

Since trivially $c + (1-\eta)\frac{s}{2}$ exceeds $c + \left((1-\eta) - \frac{\eta}{\rho}\right)\frac{s}{2}$, it follows that at least one (and possibly both) of inequalities (15) and (16) must hold. This completes the proof. **QED**

Proof of Proposition 4: We have given the main intuition for this result in the text. The details are tedious but not fundamentally hard, and are omitted. The proof is available from the authors upon request.

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