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# Tax Evasion in a Principal-Agent Model with Self-Protection

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**Abstract**

Gatekeepers have an increasing role in taxation and regulation. While burdening them with legal liability for misconducts that benefit those who resort to their services actually discourages wrongdoings – as will be clarified in the paper – an alienation effect can also arise. That is, the gatekeeper might become more interested in covering up the illegal behavior and in cooperating with the perpetrator. Such perverse effects are difficult to detect and to measure. This paper studies the problem with respect to tax evasion by firms, by building upon the classical Allingham and Sandmo (1972) model and by providing a more detailed description of the "concealment costs" than that available in the literature, which often simply makes assumptions about their existence and their functional form. The relationship between a risk neutral firm owner aiming at evading taxes and a risk averse gatekeeper is described through a simple principal-agent framework. The paper highlights the role of legal rules pertaining to liability for tax evasion in shaping the parties choices, since concealment costs vary according to whether the risk neutral principal or the risk averse agent are held responsible when tax evasion is detected. The main result of the analysis is that there is a simple ex post test that can be run to infer whether harnessing the agent was socially beneficial.

**Keywords**: tax evasion, firm, agency, risk aversion
**JEL classification**: H26, H32, D81, K42

## 1 Introduction

There is by now a large positive and normative literature available on tax evasion – dealt at the levels of either individuals or firms.[1] But not too many papers deal with the role of gatekeepers,[2] like a lawyer or a tax manager, without whose presence and active support, it may not be possible to evade tax, whatever the underlying causes are. Sometimes this set-up may have been structurally determined. For instance, in big corporate structures, tax related activities are coordinated by the chief financial officer rather than the shareholders. Since

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[1] For an assessment by a founder of this stream of literature, see Sandmo (2005).

[2] According to Kraakman (1986), p. 53, gatekeepers are "private parties who are able to disrupt misconduct by withholding their cooperation from wrongdoers".

relative efficacies of tax enforcement on the latter (*i.e.*, the principal) and the former (*i.e.*, the agent) can have very different policy implications, it is important to view tax evasion, particularly by firms, within a principal-agent framework. Apart from the two studies that we mention shortly, to our knowledge no other paper in this area dealt with the issue of tax evasion by the firm within a principal-agent framework. In general, the literature on tax evasion by firms[3] preoccupies itself with the two choices of evasion and output, given the tax rate and probability of detection. There is a concealment cost associated with concealment technology when the firm is either risk-neutral or risk-averse. Even when the existing models do not operate under a principal-agent structure, in case of firms' tax evasion they internalize a costly concealment technology, which, however, is not specifically described. In agency models a significant concealment cost can instead be studied, since one can focus upon the compensation and the incentives needed for securing the cooperation of tax officers, employees, consultants, experts etc. An important role that these subjects can play, in exchange for remuneration, is that of exerting effort in order to hide the illegal aspects of the conduct and thus to reduce the probability of audit. This can be done, *e.g.*, by suitably blurring the signals of misconduct that usually trigger the intervention of the tax officials, thus helping the entrepreneur avoid the more standard hiding techniques and design new avoidance schemes.

The agency problem of the evading firm has been studied by Chen and Chu (2005) and Crocker and Slemrod (2005). Both papers focus on problems of asymmetry of information. Chen and Chu (2005) consider the relationship between the firm's owner and a risk-averse agent who is hired for productive purposes. The firm's profit stochastically depends on the agent's productive effort. If only the risk neutral principal is legally liable, no efficiency problem arises since the evasion risk is allocated to the party more able to bear it and the principal's choice about evasion is separable from the problem of designing the incentives for the agent. If, however, also the agent is liable, the principal must pay her a risk premium *ex ante*, since insuring the agent against sanctions is not legally permissible. Thus the agent's salary can differ from the one needed to provide the proper incentives for production, and a loss of control on the agent might arise.

Crocker and Slemrod (2005) assume that shareholders (the principal) contact with the vice-president or chief financial officer (the agent) to whom the decision about the firm's tax report is delegated. Both the principal and the agent are risk neutral. Only the manager knows the extent of legally permissible rules, and thus there is an information asymmetry between the parties. In the paper they characterize the optimal compensation contract for the tax manager and find out the response to changed enforcement policies. It turns out that sanctioning the agent is more effective than sanctioning the principal, since in the latter case the effect is only indirect – through contractual incentives upon the agent – and is weakened by the second best nature of the agency contract. A further result is that the agent's marginal compensation is increasing in her liability, since the principal aims at neutralizing the effect of sanctions upon the revelation of information possessed by the agent.

A second stream of literature to which this paper aims at contributing is that on self-protection (the terminology was invented by Ehrlich and Becker, 1972), *i.e.*, an activity that reduces the probability of occurrence of a bad outcome at a given cost (Sévi and Yafil, 2005). The effort performed by the agent in the model presented in this paper is actually aimed at producing self-protection.[4] It is now well-established that an increase in risk-aversion does

[3] For a survey, see Cowell (2004).

[4] As long as the agent is liable she protects herself. Also with reference to the liable principal one may speak of self-protection since the principal expends resources to pay the agent in order to reduce the probability of the bad outcome.

not necessarily lead to higher self-protection expenditures (Ehrlich and Becker, 1972, Sweeney and Beard, 1992, and Chiu, 2000). This is so because, as observed by Briys and Schlesinger (1990), expenditure on self-protection reduces the income in all states and therefore does not necessarily lead to less risky income prospects. Therefore, the attitude towards self- protection differs from the attitude towards self-insurance and market insurance in significant ways.

An agency model with self-protection has been studied by Privileggi et Al. (2001), who consider an (all or nothing) illegal activity that benefits the principal. The agent exerts effort in order to hide the wrongdoing and is risk-averse; the principal is risk-neutral. The main finding of the paper is that shifting the full liability upon the agent reduces the principal's profit. However, the agent's effort can either decrease or increase, and thus unwanted effects upon deterrence can arise. The resort to self-protection - in addition to self-insurance - by an individual evading taxpayer has also been studied by Lee (2001), who finds that - unlike in the standard model - under given assumptions the reported income is decreasing in the tax rate.

In this paper we present a principal agent model in which an agent is hired by an entrepreneur to exert effort in covering tax evasion. We study many regimes of legal liability for taxes and sanctions, that span from a full liability on the part of the principal to full liability on part of the agent including the intermediate cases of shared liability. While it is assumed that tax evasion always benefits the principal, the moral justifications for considering agent's liability – which actually is often provided to some extent by the law – might hinge upon the fact that the agent has a deep knowledge of the unlawful character of the conduct,[5] or is expected to follow strict deontological rules. Moreover, the agent might even have a deeper pocket than the principal, as sometimes happens for consultants with respect to small businesses.

When liability is shifted, either fully or partially, from the principal to the agent evasion becomes more costly, since the agent's cooperation must be paid. However, the agent might be pushed to exert a higher effort in covering it and thus the likelihood of discovering the evasion might decrease. Hence there is a problem of "excessive loyalty" of the agent with respect to the principal, often discussed in the psychological literature, *e.g.*, with respect to soldiers obeying orders that violate international conventions and human rights. While the economic literature on agency has mainly focussed on the opposite problem – *i.e.*, the agent's opportunistic behavior – in a more general perspective it is interesting also to apply the economic analysis to the non-optimal loyalty problem in all its forms (see, *e.g.*, Morck, 2009).

In the following it will be assumed as usual that the principal is risk neutral while the agent is risk-averse. Moreover, the principal cannot pursue her evasion design without securing the agent's cooperation, either for technical or legal reasons, since, *e.g.*, only the agent possesses the needed expertise or has the signatory authority for the needed documents. As long as only the principal is legally liable for tax evasion, an amoral agent can consider the cooperation in covering up as a kind of work without risk and thus accept a compensation just corresponding to her reservation utility. On the other hand, if the legal liability is shifted upon the risk-averse agent, things are clearly different. The agent is exposed to the risk of sanctions, at least as long as the principal cannot provide her an insurance. Since insurance against sanctions for illegal activities are often prohibited by the law, it will be assumed that the principal can neither insure the agent nor make the agent's remuneration conditional on the final result of the evasion.

A problem that arises when the agent is liable for sanctions is that the contract with the

[5]Gatekeepers liability in general can also refer to other cases – not studied here – in which the agent is not actively cooperating with the wrongdoer and might have imperfect information about the principal's conduct, but nevertheless can contribute to some extent at enforcing the law. For this broader perspective, see, *e.g.*, Hamdani (2003).

principal has an illicit content and thus is not legally enforceable. If, however, the parties have a long lasting relationship it seems plausible that they can trust each other and can conclude and honor a contract in which the agent performs the effort in covering up and the principal pays a remuneration that includes *ex ante* a premium for the risk.[6] Trust might be based on repeated interactions, that would arise if, *e.g.*, the agent is an employee or a consultant with a strong link with the enterprise (and thus the contract under consideration is a side one). Moreover, it is more likely if the principal knows the agent's preferences and can observe the agent's effort. It is interesting now to note how full information plays a crucial role in the model that we set up in this context. Trust is less plausible if information is imperfect. This assumption of perfect information will be maintained throughout the whole paper. As will become clear in the following, non trivial problems arise even if full information is considered.

In this paper we bridge some gaps in the previous literature, on the one hand by focussing on the specific contribution to tax evasion provided by agents (as in Crocker and Slemrod, 2005, and Privileggi et Al., 2001), while at the same time allowing for different attitudes toward risk of the parties (as in Chen and Chu, 2005), and including a full description of the firm's tax evasion problem, in which evasion is a continuous variable (while the wrongdoing's benefit was discrete in Privileggi et Al., 2001). Our structure that considers three variables (evasion, agent's remuneration and effort) gives sufficient complexity to the model and allows us to study the tax evasion choice without unnecessarily strong restrictions on risk preferences of the parties involved.

Our results provide relevant indications about the relationship between the agent's remuneration and the legal incentives in a risky environment. While the standard hint coming from the available literature is that the agent's remuneration is increasing in her liability [see, *e.g.*, Hamdani (2003), and, with reference to marginal bonuses, Crocker and Slemrod (2005)], we show that there are cases in which her compensation decreases in response to an expansion of her liability.

Another important effect of a full or partial shift of liability from the principal to the agent is the decrease of the principal's profit. Hence, the attractiveness of evasion decreases. The check on the principal's evasion arises because in such a framework the risk-averse agent must be exposed to a risk. But the beneficial effects on enforcement of the liability shift can be reduced or even overturned if the agent becomes more loyal, *i.e.*, chooses to work harder in order to increase the likelihood of escaping the punishment. However, also in our setting – which modifies the standard self-protection problem by introducing an agency relationship in which effort is performed – it is confirmed that the agent's risk-aversion does not necessarily imply a larger self-protection demand. We are in fact able at detecting a case in which after a liability shift no ambiguity is left and a favorable scenario from the social point of view arises. As long as the check introduced by the agent's liability involves huge costs for the principal and huge risks for the agent, they might prefer to boldly cut their misconduct. It is thus possible that, when liability is shifted – totally or partially – upon the agent, a smaller level of both tax evasion and effort is chosen, and the agent compensation decreases. Therefore, our main result is that whenever ex post it is observed that the remuneration is (weakly) lower in the agent's liability (full or partial) than in the principal's, then both evasion and effort under the former have to be lower. Hence, there is a simple ex post test that can be run to ascertain the effectiveness of a reform that harnesses gatekeepers with more legal liability. If a fall of their income arises, the reform is working in the right direction. Since information about

[6] Trust arising from repeated interactions might also in some cases render a *de facto* insurance of the agent viable. For a discussion of this possibility, and of the reasons by which it seems unlikely, see p. 161 in Chen and Chu (2005).

compensations and incomes can be provided by standard statistical data sources, one may run a kind of ex post test about the effectiveness of the liability shift.[7]

The paper is organized as follows: in Section 2, some alternative liability regimes are presented and their implications in terms of tax evasion and probability of detection are studied. Section 3 compares them and reports our main results in terms of policy implications. In Section 4 some numeric examples are discussed. Section 5 concludes, while all technicalities regarding the non-trivial constrained maximization techniques used in the text and the computation of the numeric examples are postponed in the Appendix.

# 2 The liability regimes

Liability for tax evasion can be placed by the legal system either only upon the principal or only upon the agent, or it might be shared among the two. The first case often occurs in practice, since tax evasion directly benefits the principal. Situations in which only the agent is liable seems rare in practice: nevertheless they are interesting as a limit case with respect to the shared liability. Note also that, as long as sanctions for tax evasion are a multiple of the due tax (See Yitzhaki, 1987), when liability for sanctions is fully transferred upon the agent the principal still bears some risk, as she must pay the due tax in case of audit.[8] For a clearer understanding of the effects of different liability regimes, we will thus explore also a further limit case, the one in which the whole risk is shifted upon the agent – *i.e.*, in case of detection the agent is held responsible for both the due tax and the sanction.[9]

In practice, however, intermediate cases prevail; that is, while the principal is held liable for tax evasion, additional specific responsibilities are also provided for the agent. This cases will be modelled, for the sake of simplicity, as a form of shared liability, since whenever the evasion is found out the whole sanctions applied can be seen as a total that is divided between the parties according to the rules designed for each of them.

Summing up, to ease the analysis, the cases described above are packed into three alternative legal regimes characterized as follows: i) only the principal is liable for both the due tax and the sanction (see Subsection 2.1); ii) the whole risk (both for the due tax and sanctions) is shifted upon the agent (see Subsection 2.2); iii) the principal is liable for the due tax but her liability for sanctions is limited to a share $0 \leq \lambda < 1$, while the agent bears the remaining part. This case (see Subsection 2.3) thus includes a full shift of the liability for sanctions (but not for the due tax) on the agent, which occurs when $\lambda = 0$.

## 2.1 Only the principal is liable

Let us consider the case in which the principal is legally liable for tax evasion if an audit occurs, while the agent is not.

The principal's expected net benefit is given by

$$\mathbb{E}\pi(x, r, E) = [1 - p(x)(1 + s)] tE - r, \tag{1}$$

where $p : \mathbb{R}_+ \to [0, 1]$ denotes the probability of audit, $x$ is the agent's effort in covering up, $s > 0$ is the sanction rate, $0 < t < 1$ is the tax rate, $E$ is the evaded amount, and $r$ is the

[7] Of course this is a *ceteris paribus* prediction and thus other factors that might explain income changes should be controlled for.

[8] The standard simplifying assumptions that audits perfectly reveal evasion is maintained in the paper.

[9] This full shift of the risk of tax evasion is more easy to conceive when sanctions are upon the evaded income, as in the original Alligham and Sandmo (1972) model. The results of this paper carry over to such a case.

agent's remuneration. It is also assumed that

$$[1 - p(x)(1+s)]\, tE - r > 0. \tag{2}$$

In other words, evasion implies a positive expected return (otherwise the principal chooses not to undertake any illegal action). Joint with the assumption that $r \geq 0$, (2) immediately implies that $E > 0$ must hold and feasible values of effort $x$ must satisfy $1 - p(x)(1+s) > 0$.

The principal wants to maximize (1) with respect to the three variables $x$, $r$ and $E$, subject to two constraints plus non-negativity of variables; that is, the principal's problem is:

$$\begin{aligned} &\max \{[1 - p(x)(1+s)]\, tE - r\} \\ &\text{s.t.} \begin{cases} u(r) - g(x) \geq u_0, \\ E \leq Y,\ x \geq 0,\ r \geq 0. \end{cases} \end{aligned} \tag{3}$$

In (3), $u : \mathbb{R} \to \mathbb{R}$ is the agent's utility function, $u_0$ is her reservation utility, $g : \mathbb{R}_+ \to \mathbb{R}_+$ denotes the cost of effort $x$ borne by the agent, while $Y > 0$ denotes the principal's income. $Y$ is assumed to be exogenous.[10] The first constraint allows for agent's participation.

**A. 1** *In the following we shall assume:*

**i)** $u'(w) > 0$ *and* $u''(w) < 0$, *for all* $w \in \mathbb{R}$,

**ii)** $0 < p(x) < 1$, $p'(x) < 0$ *and* $p''(x) > 0$ *for all* $x \geq 0$,

**iii)** $g'(x) > 0$ *and* $g''(x) \geq 0$ *for all* $x > 0$,

**iv)** $u(0) \leq u_0$, $g(0) = 0$, $g'_+(0) = 0$.

Assumption A.1(iv) is a technical condition which, joint with inequality (2), is sufficient to rule out corner solutions in problem (3).[11]

Necessary K.T. conditions[12] yield the following characterization of the solution $(x_P^*, r_P^*, E_P^*)$ for the problem (3) in which only the principal is liable:

$$E_P^* = Y, \tag{4}$$

$$u(r_P^*) - g(x_P^*) = u_0, \tag{5}$$

$$-p'(x_P^*)(1+s)\, tY = \frac{g'(x_P^*)}{u'(r_P^*)}. \tag{6}$$

Note that Assumptions A.1(i) and (iii) imply that the constraint defined by the four inequalities in (3) is convex; in addition, necessary condition (4) says that the maximization in (3) is actually performed only with respect to the two variables $x$ and $r$, for which the objective function turns out to be concave. This is sufficient to establish uniqueness of the solution $(x_P^*, r_P^*, E_P^*)$.

Condition (4) states that, at the optimum, the whole income $Y$ will be hidden, while conditions (5) and (6) say respectively that the agent must receive her reservation utility[13] and that the marginal benefit of effort for the principal, in terms of reduction of the expected loss, $-p'(x_P^*)(1+s)\, tY$, must equal the marginal rate of substitution between effort and compensation, $g'(x_P^*)/u'(r_P^*)$, for the agent.

[10] In order to ensure that the agent has a pocket deep enough for sustaining her liability, one might assume that she too has an exogenous income – coming, *e.g.*, from a legal activity. Since the inclusion of such constant term is immaterial with respect to our theoretical results, for simplicity we have dropped it.

[11] Actually, Assumption A.1(iv) is not necessary and in specific examples the condition that $g'_+(0) = 0$ may be dropped while interiority of solution can be obtained directly through a suitable choice of parameters' values.

[12] From the K.T. conditions it is easily seen that the first two inequalities in the constraint must be binding.

[13] The agent that cooperates in tax cheating might threaten the principal of disclosing the wrongdoing, and demand more than her reservation utility. We assume that the trust relationship between the parties is strong enough to exclude this possibility.

## 2.2 The agent bears all the risks

In order to reach a better understanding of the case in which liability involves the agent, let us begin by considering an extreme case, in which the whole risk is shifted upon her; *i.e.*, if an audit occurs, the agent bears all the consequences of tax evasion, and thus she has to pay the due tax plus the sanction on top of it.

The principal's problem becomes:

$$\max\,(tE - r)$$
$$\text{s.t.} \begin{cases} [1 - p(x)]\, u(r) + p(x)\, u[r - (1+s)\, tE] - g(x) \geq u_0, \\ E \leq Y,\ x \geq 0,\ r \geq 0. \end{cases} \tag{7}$$

Again, to ensure that the principal gets a positive return,

$$tE - r > 0 \tag{8}$$

must hold, which, as $r \geq 0$, implies that $E > 0$ holds true.

**Proposition 1** *Under Assumption A.1 and (8), the following conditions necessarily hold:*

$$r > p(x)(1+s)\, tE \qquad \text{for all } x \geq 0 \text{ and } E \leq Y \tag{9}$$

$$1 - p(x)(1+s) > 0 \qquad \text{for all } x \geq 0. \tag{10}$$

**Proof.** To establish (9) assume the contrary: $r \leq p(x)(1+s)\, tE$. Hence,

$$\begin{aligned} [1 - p(x)]\, u(r) + p(x)\, u[r - (1+s)\, tE] - g(x) &< u[r - p(x)(1+s)\, tE] - g(x) \\ &\leq u(0) - g(x) \\ &\leq u_0, \end{aligned}$$

where the first inequality follows from strict concavity of $u(\cdot)$, the second inequality from monotonicity of $u(\cdot)$ joint with the contradiction $r \leq p(x)(1+s)\, tE$ and the last inequality from Assumptions A.1 (iii) and (iv). Therefore, the agent's participation constraint – the first inequality in (7) – implies (9).

By (8),

$$tE - p(x)(1+s)\, tE = tE\,[1 - p(x)(1+s)] > tE - r > 0,$$

which implies (10). ■

Note that $E > 0$ in (9) implies $r > 0$.

Proposition 1 shows that in order to have a viable problem the risk-averse agent must receive a salary larger than her expected liability, while evasion must have an overall positive expected return.

As the principal's objective function is linear, convexity of the constraint is enough in order to have a concave problem. The agent's expected utility,

$$v(x, r, E) = [1 - p(x)]\, u(r) + p(x)\, u[r - (1+s)\, tE] - g(x)\,,$$

is concave in each relevant variable, $E$, $r$ and $x$, taken alone, but concavity jointly in all three variables could lack because of cross-effects. In Appendix 6.A sufficient conditions for assuring quasiconcavity of $v$ – and thus convexity of the constraint – are put forth and discussed. Essentially, convexity of the constraint in problem (7) implies that for every given evasion and effort combination, there is just one compensation $r$ representing the minimal cost the principal has to bear for implementing such combination without violating the agent's participation constraint. Large families of functions, such as HARA (Hyperbolic Absolute Risk Aversion) utility functions, hyperbolic probability functions and quadratic effort cost, satisfy these conditions.

From the K.T. conditions applied to (7) under Assumption A.1 it is straightforward to show that at the optimum, besides $E_A^* > 0$ and $r_A^* > 0$, $x_A^* > 0$ holds as well, while the first inequality once again is binding. The following characterization of the solution $(x_A^*, r_A^*, E_A^*)$ for the problem (7) in which only the agent is fully liable arises:

$$[1 - p(x_A^*)]\, u(r_A^*) + p(x_A^*)\, u[r_A^* - (1+s)\, tE_A^*] - g(x_A^*) = u_0, \tag{11}$$

$$-p'(x_A^*)\{u(r_A^*) - u[r_A^* - (1+s)\, tE_A^*]\} = g'(x_A^*) \tag{12}$$

$$\frac{[1 - p(x_A^*)]\, u'(r_A^*)}{p(x_A^*)\, u'[r_A^* - (1+s)\, tE_A^*]} \geq s. \tag{13}$$

Inequality (13), pertaining to the choice of the evasion amount, is the standard condition found in models of tax evasion, in which a risk-averse decision maker is considered. This is no surprise, since in a framework of full information the principal might also solve her problem by letting the agent to become the residual claimant with respect to the evasion activity. The agent would receive all the benefits and bear all the costs, while just paying a fixed fee to the principal. Under this alternative formulation the principal's role would just be that of choosing the largest fee compatible with the agent's participation constraint, while the agent would choose the same "efficient" $E_A^*$ and $x_A^*$ values already described. Note that (13) holds as a strict inequality when the second constraint, $E \leq Y$ in (7), is binding, *i.e.*, when $E_A^* = Y$.

As far as the choice of effort is concerned, condition (12) shows that now the relevant benefit is the reduction of the expected loss $-p'(x_A^*)\{u(r_A^*) - u[r_A^* - (1+s)\, tE_A^*]\}$ for the agent.

The agent's risk-aversion plays a decisive role in explaining the differences between this regime and the one considered in Section 2.1. If the principal could rely on a risk neutral agent, even when the risk is fully transferred the optimal agent's effort would be described by (6), the principal would choose the same evasion level as when she is liable, and the salary would differ from the case of principal's liability only by the amount of the expected loss.

## 2.3 Shared liability for sanctions

Let us come to the more realistic case in which the risk is never fully shifted onto the agent. Now, in case of audit the principal has to pay the due tax and bears a share $0 \leq \lambda < 1$ of sanctions, while the agent has to pay the remaining share $1 - \lambda$ of sanctions.[14]

The principal's problem then is:

$$\max \{[1 - p(x)(1 + \lambda s)]\, tE - r\}$$
$$\text{s.t.} \begin{cases} [1 - p(x)]\, u(r) + p(x)\, u[r - (1-\lambda)\, stE] - g(x) \geq u_0, \\ E \leq Y, \; x \geq 0, \; r \geq 0. \end{cases} \tag{14}$$

Now it will be assumed that, for a given $\lambda$,

$$[1 - p(x)(1 + \lambda s)]\, tE - r > 0, \tag{15}$$

so that once again the principal gets a positive return from tax evasion. Note that $r \geq 0$ implies $E > 0$ as before. Moreover, $1 - p(x)(1 + \lambda s) > 0$ must hold as well. As a matter

[14]The case in which $\lambda = 1$ implies full liability of the principal for both the due tax and the sanctions, and coincides with the model studied in Section 2.1.

of fact, the latter inequality can be further specified so that it becomes clear that condition (10) holds again. To see this note that, by applying the same reasoning as in Proposition 1, it can be easily shown that the following condition necessarily holds in order to let the agent participation constraint be satisfied:

$$r > p(x)(1-\lambda) stE, \tag{16}$$

which implies

$$\begin{aligned} [1-p(x)(1+s)] tE &= [1-p(x)(1+\lambda s)] tE - p(x)(1-\lambda) stE \\ &> [1-p(x)(1+\lambda s)] tE - r \\ &> 0, \end{aligned}$$

so that once again (10) holds true, that is the agent must receive a salary larger than her expected liability, while tax evasion must have an overall positive expected return.

**Proposition 2** *Under Assumption A.1 a solution $(x_\lambda^*, r_\lambda^*, E_\lambda^*)$ for problem (14) is completely characterized by the following necessary K.T. conditions:*

$$[1-p(x^*)] u(r^*) + p(x^*) u[r^* - (1-\lambda) stE^*] - g(x^*) = u_0 \tag{17}$$

$$\begin{aligned} -p'(x^*)\{(1+\lambda s) tE^* [(1-p(x^*)) u'(r^*) + p(x^*) u'(r^* - (1-\lambda) stE^*)] \\ +u(r^*) - u[r^* - (1-\lambda) stE^*]\} = g'(x^*) \end{aligned} \tag{18}$$

$$\frac{[1-p(x^*)] u'(r^*)}{p(x^*) u'[r^* - (1-\lambda) stE^*]} \geq \frac{(1-\lambda) s}{1-p(x^*)(1+\lambda s)} - 1. \tag{19}$$

The proof of Proposition 2 is based on standard tedious computations and thus it is omitted.[15]

Condition (18) pertaining to effort combines elements relevant for the principal – the agent's marginal utility of income in the two states, that drives the monetary remuneration – and elements relevant for the agent – the difference of the utilities in the two states of the world, which provides the personal motivation for exerting effort.

Like in the model discussed in the previous subsection, condition (19) pertaining to the tax evasion amount holds as a strict inequality when the second constraint, $E \leq Y$ in (14), is binding, *i.e.*, when $E_\lambda^* = Y$.

Condition (19) differs from its analogous, (13), that portrays the extreme case in which the agent bears all the risk. One can still interpret tax evasion as a choice of the agent, since the LHS in (19) has the standard form it takes in individual tax evasion problems, but one must assume that the relative prices are distorted. That is, now the slope in absolute value of the budget constraint facing the agent – the RHS of (19) – is lower because the agent is only partially liable for sanctions and does not have to pay the tax on the evaded amount.[16]

Once more, the agent's risk-aversion plays a decisive role in shaping the results. Under agent's risk neutrality a corner solution with full evasion – *i.e.*, $E_\lambda^* = Y$ – always arises since

[15] Note that the objective function in (14) is not concave and thus uniqueness of the solution $(x_\lambda^*, r_\lambda^*, E_\lambda^*)$ characterized in Proposition 2 cannot in general be either guaranteed or ruled out. However, this does not affect our results – specifically, Propositions 3 and 4. See the discussion in Appendix 6.B.

[16] Even assuming a full shift of liability for sanctions onto the agent, when $\lambda = 0$, condition (10) implies that the RHS in (19) satisfies $s/[1-p(x_\lambda^*)] - 1 < s$. In other words, the absolute value of the slope of the agent's budget constraint is smaller than $s$, as tax evasion must have positive expected total rate of return.

the LHS in (19) becomes $[1-p(x_\lambda^*)]/p(x_\lambda^*)$ and it is straightforward to show that condition (10) implies

$$\frac{1-p(x_\lambda^*)}{p(x_\lambda^*)} > \frac{(1-\lambda)s}{1-p(x_\lambda^*)(1+\lambda s)} - 1 \qquad \text{for all } 0 \leq \lambda < 1.$$

Moreover, condition (18) boils down to (6). Hence, with a risk neutral agent, the same tax evasion and effort would arise as in the case of principal's liability, while the agent's salary would be larger by a share $1-\lambda$ of the expected amount of sanctions.

# 3 A comparison

Since under Assumption A.1 $-g'(x)/p'(x)$ is increasing in the effort $x$, by considering the extreme case presented in Subsection 2.2, in which the agent bears all the risk, an effort smaller than under the exclusive principal's liability occurs if and only if

$$u(r_A^*) - u[r_A^* - t(1+s)E_A^*] \leq u'(r_P^*)(1+s)tY. \tag{20}$$

Since in the LHS the choice is made by a risk-averse agent, while in the RHS by a risk neutral one,[17] the well-known problem of self-protection is at stake; that is, in general one cannot say whether inequality (20) will be satisfied or not.[18] Hence, the only possible hint is that, whenever $E_A^* \to 0$, then the LHS $\to 0$ as well and thus a smaller effort will occur under agent's liability. One can thus guess that, as long as the agent's risk-aversion induces the choice of a small tax evasion level, at the limit this fact will also drive the effort downwards. Similar effects also arise under shared liability. In this case the condition to check becomes:

$$\begin{gathered}(1+\lambda s)tE_\lambda^* \{[1-p(x_\lambda^*)]u'(r_\lambda^*) + p(x_\lambda^*)u'[r_\lambda^* - (1-\lambda)stE_\lambda^*]\} \\ +u(r_\lambda^*) - u[r_\lambda^* - (1-\lambda)stE_\lambda^*] \leq u'(r_P^*)(1+s)tY\end{gathered}$$

where again the LHS $\to 0$ as long as a $E_\lambda^* \to 0$. This fact suggests the idea that the evasion level influences the amount of effort performed in the diverse liability regimes. In the following subsections this idea is checked within a more general scenario; as a matter of fact, besides the limit relationship between evasion level and effort performed just considered, we shall establish a sufficient condition linking the decrease of both the evasion level and effort to the decrease of the optimal remuneration.

## 3.1 Effects on the principal's profit

In order to ease the comparison between the case in which only the principal is liable and all the other cases, let us compare them under fixed values of effort, evasion and salary. The following Lemma is required in the proof of next Proposition 3.

**Lemma 1** *Whenever $(x_{A,\lambda}^*, r_{A,\lambda}^*, E_{A,\lambda}^*)$ solves either problem (7) or problem (14), then it is feasible for problem (3).*

**Proof.** Let

$$\varphi = \begin{cases} 1+s \text{ when the shift of risk is full (Subsection 2.2)} \\ (1-\lambda)s \text{ when the risk is shared (Subsection 2.3).} \end{cases} \tag{21}$$

[17] It has been shown before that when both parties are risk neutral the same effort arises.

[18] See Privileggi et Al. (2001).

The point $\left(x^*_{A,\lambda}, r^*_{A,\lambda}, E^*_{A,\lambda}\right)$ satisfies the first inequality in (3) – the participation constraint – as

$$\begin{aligned} u_0 + g\left(x^*_{A,\lambda}\right) &= \left[1 - p\left(x^*_{A,\lambda}\right)\right] u\left(r^*_{A,\lambda}\right) + p\left(x^*_{A,\lambda}\right) u\left(r^*_{A,\lambda} - \varphi t E^*_{A,\lambda}\right) \\ &< u\left[r^*_{A,\lambda} - p\left(x^*_{A,\lambda}\right) \varphi t E^*_{A,\lambda}\right] \\ &< u\left(r^*_{A,\lambda}\right), \end{aligned}$$

where strict concavity and strict monotonicity of $u(\cdot)$ establish the first and the second inequalities respectively. Since all other inequalities in the constraint (3) are also satisfied by $\left(x^*_{A,\lambda}, r^*_{A,\lambda}, E^*_{A,\lambda}\right)$, the proof is complete. ■

The proof of Lemma 1 shows that when the same values for effort, evasion and salary solving the problem under agent's liability are used when only the principal is liable, the agent's participation constraint relaxes, since the risk premium and the expected loss included in the salary become redundant when the agent does not face any risk.

**Proposition 3** *If the liability regime changes from full liability of the principal's to any other regime of agent's liability, then the principal's profit becomes always strictly smaller.*

**Proof.** Let $\left(x^*_{A,\lambda}, r^*_{A,\lambda}, E^*_{A,\lambda}\right)$ be the solution of either problem (7) or problem (14) for a given $0 \leq \lambda < 1$. Hence, by concavity of $u(\cdot)$ and either condition (11) or condition (17),

$$\begin{aligned} u\left[r^*_{A,\lambda} - p\left(x^*_{A,\lambda}\right) \varphi t E^*_{A,\lambda}\right] &> \left[1 - p\left(x^*_{A,\lambda}\right)\right] u\left(r^*_{A,\lambda}\right) + p\left(x^*_{A,\lambda}\right) u\left(r^*_{A,\lambda} - \varphi t E^*_{A,\lambda}\right) \\ &= u_0 + g\left(x^*_{A,\lambda}\right), \end{aligned}$$

which is equivalent to

$$-r^*_{A,\lambda} < -u^{-1}\left[u_0 + g\left(x^*_{A,\lambda}\right)\right] - p\left(x^*_{A,\lambda}\right) \varphi t E^*_{A,\lambda}, \tag{22}$$

where $u^{-1}(\cdot)$ denotes the (strictly increasing and strictly convex) inverse function of $u(\cdot)$.

Let

$$\sigma(E, x) = \begin{cases} tE \text{ when the shift of risk is full (Subsection 2.2)} \\ \left[1 - p(x)(1 + \lambda s)\right] tE \text{ when the risk is shared (Subsection 2.3),} \end{cases} \tag{23}$$

then the principal's optimal profit when the agent is either fully or partially liable is given by $\pi_{A,\lambda} = \sigma\left(E^*_{A,\lambda}, x^*_{A,\lambda}\right) - r^*_{A,\lambda}$. Now,

$$\begin{aligned} \pi_{A,\lambda} &= \sigma\left(E^*_{A,\lambda}, x^*_{A,\lambda}\right) - r^*_{A,\lambda} \\ &< \sigma\left(E^*_{A,\lambda}, x^*_{A,\lambda}\right) - u^{-1}\left[u_0 + g\left(x^*_{A,\lambda}\right)\right] - p\left(x^*_{A,\lambda}\right) \varphi t E^*_{A,\lambda} \\ &= \left[1 - p\left(x^*_{A,\lambda}\right)(1 + s)\right] t E^*_{A,\lambda} - u^{-1}\left[u_0 + g\left(x^*_{A,\lambda}\right)\right] \\ &\leq \max\left\{\left[1 - p(x)(1 + s)\right] tE - u^{-1}\left[u_0 + g(x)\right] : x, r, E \text{ satisfy (3)}\right\} \\ &= \left[1 - p\left(x^*_P\right)(1 + s)\right] tY - u^{-1}\left[u_0 + g\left(x^*_P\right)\right] \\ &= \left[1 - p\left(x^*_P\right)(1 + s)\right] tY - r^*_P \\ &= \pi_P, \end{aligned}$$

where in the second line we used inequality (22), in the third line the definitions of $\varphi$ and $\sigma$ as in (21) and (23) respectively, in the fourth line Lemma 1, in the fifth line we labeled as $\left(x^*_P, r^*_P, Y\right)$ the solution of the principal's problem when she is fully liable – characterized by conditions (4) to (6) – while in the last but one equality we rearranged terms in (5) and calculated $u^{-1}(\cdot)$ of both sides; finally we denoted by $\pi_P$ the principal's optimal profit when she is fully liable, and the proof is complete. ■

Hence, it turns out that when all the risk is borne by the principal, her profit is always larger, a result that extends that of Proposition 2 in Privileggi et Al. (2001) to the case of a continuous wrongdoing. When liability is totally or partially shifted onto the agent her risk-aversion works as a kind of restraint on the principal. If the impact of the liability shift is large – as a matter of fact, if no correction in the parties' behavior occurs profits might even become negative – a reduction of tax evasion may become necessary. A lower tax evasion, however, does not per se imply a larger expected tax revenue $G(E, x)$, as the latter is given by:

$$G(E, x) = tY - tE\left[1 - p(x)(1 + s)\right]; \tag{24}$$

specifically, also expected sanctions must be accounted for. Thus, one cannot *a priori* exclude that $G(E, x)$ decreases even if there is more tax compliance, because the countervailing effect of an $x$ increase might be stronger.

## 3.2 A simple test

Continuity of the possible evasion level implies that, when the liability is shifted from the principal to the agent, the principal does not need to renounce tax evasion altogether, but might nevertheless be forced to reduce it in order to avoid incurring a loss. This is because the agent requests an increase in her compensation. In fact, the principal's response to the liability shift might imply such a bold cut in tax evasion that actually a reduction of the agent's compensation may ensue.

**Proposition 4** *If it is observed that $r^*_{A,\lambda} \leq r^*_P$, then both $x^*_{A,\lambda} < x^*_P$ and $E^*_{A,\lambda} < E^*_P = Y$ must hold.*

**Proof.** From the proof of proposition 3 we know that

$$r^*_P = u^{-1}\left[u_0 + g\left(x^*_P\right)\right], \tag{25}$$

while from inequality (22) we have

$$r^*_{A,\lambda} > u^{-1}\left[u_0 + g\left(x^*_{A,\lambda}\right)\right] + p\left(x^*_{A,\lambda}\right)\varphi t E^*_{A,\lambda} > u^{-1}\left[u_0 + g\left(x^*_{A,\lambda}\right)\right]. \tag{26}$$

By using both (25) and (26) under the assumption $r^*_{A,\lambda} \leq r^*_P$ we get

$$u^{-1}\left[u_0 + g\left(x^*_{A,\lambda}\right)\right] < u^{-1}\left[u_0 + g\left(x^*_P\right)\right],$$

which, since both $u^{-1}(\cdot)$ and $g(\cdot)$ are strictly increasing, immediately yields $x^*_{A,\lambda} < x^*_P$, and the first part of the Proposition is established.

To show that $E^*_{A,\lambda} < E^*_P = Y$ holds as well, we first study the case in which the agent is only partially liable. Let $(x^*_\lambda, r^*_\lambda, E^*_\lambda)$ denote the solution of problem (14) for a given $0 \leq \lambda < 1$. Since $-g'(x)/p'(x)$ is increasing in the effort $x$, $x^*_\lambda < x^*_P$ implies $-g'(x^*_\lambda)/p'(x^*_\lambda) < -g'(x^*_P)/p'(x^*_P)$, and thus conditions (6) and (18), in turn, imply

$$\begin{aligned}(1+s)\,tYu'\left(r^*_P\right) > (1+\lambda s)\,tE^*_\lambda\left\{\left[1 - p\left(x^*_\lambda\right)\right]u'\left(r^*_\lambda\right) + p\left(x^*_\lambda\right)u'\left[r^*_\lambda\right.\right.\\ \left.\left. - (1-\lambda)\,stE^*_\lambda\right]\right\} + u\left(r^*_\lambda\right) - u\left[r^*_\lambda - (1-\lambda)\,stE^*_\lambda\right].\end{aligned} \tag{27}$$

By strict concavity of $u(\cdot)$, both

$$u'[r_\lambda^* - (1-\lambda) stE_\lambda^*] > u'(r_\lambda^*)$$

and (superdifferentiability property)

$$u(r_\lambda^*) - u[r_\lambda^* - (1-\lambda) stE_\lambda^*] > (1-\lambda) stE_\lambda^* u'(r_\lambda^*),$$

hold. Substituting the last two inequalities in (27) and rearranging terms yields

$$\begin{aligned}(1+s) tYu'(r_P^*) &> (1+\lambda s) tE_\lambda^* u'(r_\lambda^*) + (1-\lambda) stE_\lambda^* u'(r_\lambda^*) \\ &= (1+s) tE_\lambda^* u'(r_\lambda^*) \\ &\geq (1+s) tE_\lambda^* u'(r_P^*),\end{aligned}$$

where in the last inequality we used the assumption $r_\lambda^* \leq r_P^*$ and again concavity of $u(\cdot)$. Simplifying common (positive) terms in both sides yields $Y > E_\lambda^*$, as was to be shown.

To prove the case in which the agent is fully liable let $(x_A^*, r_A^*, E_A^*)$ be the solution of problem (7) and observe that the analogous of inequality (27) when $x_A^* < x_P^*$ obtained through (6) and (12) turns out to be simpler:

$$(1+s) tYu'(r_P^*) > u(r_A^*) - u[r_A^* - (1+s) tE_A^*],$$

so that only the superdifferentiability property of $u(\cdot)$ is required to obtain the result by following the same steps as above. ■

Proposition 4 provides the basics for running ex post tests aimed at assessing if a liability shift was beneficial from a social point of view. If a (weak) decrease in the gains of the agent is observed, notwithstanding the fact that her legal risks have increased, one can infer that she is risking less than beforehand and thus she must have a lower motivation for exerting effort to cover the evasion, which, in turn, is justified only if evasion has actually become smaller. Hence there is an unequivocal signal of the success of the policy, since a decrease of both tax evasion and effort imply that Government revenue in (24) will increase. An increase in the agent's compensation, on the other hand, is not a signal in the opposite direction: simply one cannot infer what is going on in terms of tax evasion and effort without further information.

# 4 Examples and comparative statics

In view of Appendix 6.A, in order to build some tractable examples we shall use a CARA negative exponential utility function of the form $u(w) = -e^{-\rho w}$ with (constant) absolute risk-aversion coefficient $\rho > 0$, as provided by (42) in Remark 2; note that $u : \mathbb{R} \to (-\infty, 0)$. Similarly, the probability function will be of the hyperbolic type as in (38): $p(x) = \alpha/(\beta x + 1)$, with $0 < \alpha < 1$ and $\beta > 0$. To keep computations as simple as possible the effort function will be quadratic, $g(x) = \zeta x^2$ with $\zeta > 0$, as discussed in Remark 4. Whenever parameter $\zeta$ is chosen sufficiently large so that condition (47) holds, *i.e.*, $\zeta \geq \beta^2/[2(1-\alpha)]$, Proposition 6 guarantees that all the constraints in the maximization problems described in Subsections 2.1 to 2.3 are convex. While when only the principal or only the agent is fully liable (Subsections 2.1 and 2.2) the objective functions in the principal's maximization problems are concave, in all other scenarios characterized by different degrees of liability borne by the agent, $0 < \lambda < 1$, (quasi)concavity of the objective function in (14) jointly in the three variables $x$, $r$ and $E$ cannot be guaranteed (see, e.g., Sévi and Yafil, 2005, and the literature quoted therein). Hence, uniqueness of the optimal solution $(x^*_\lambda, r^*_\lambda, E^*_\lambda)$ cannot in general be assured.[19] However, we have been successful in establishing uniqueness of the solution for the model's specification just described; details are reported in Appendix 6.C.

## 4.1 Numeric examples

We shall fix all the parameters' values except for the sanction rate $s$ and the coefficient of absolute risk-aversion $\rho$. Specifically, $Y = 100$, $u_0 = -0.9$, $t = 0.4$, $\alpha = 0.1$ and $\beta = 1$, while the effort parameter $\zeta$ will be kept at its minimum compatible with concavity of all problems considered, that is, condition (47) will be binding: $\zeta = \beta^2 / [2(1-\alpha)] = 1/(1.8) \simeq 0.556$ (see Example 1). For the model in which liability for sanctions is shared (Subsection 2.3) the case in which $\lambda = 0.5$ will be considered.

We shall investigate how the optimal solutions $(x^*, r^*, E^*)$ characterized by the K.T. conditions described in Subsections 2.1 to 2.3, together with the principal's net benefit $\pi$, change for different values of the absolute risk-aversion coefficient $\rho$ and the sanction rate $s$. The results, as discussed in Appendix 6.C, are computed with the support of Maple 13. Table 1 reports the results for $s = 2.1$ while either $\rho = 10$ or $\rho = 0.5$. Table 2 reports the results for $s = 1$ while again either $\rho = 10$ or $\rho = 0.5$.

| Risk-aversion coefficient | Principal fully liable | Agent fully liable | Shared liability: $\lambda = 0.5$ |
|---|---|---|---|
| $\rho = 10$ | $x^*_P = 1.233$<br>$r^*_P = 0.290$<br>$E^*_P = 100$<br>$\pi_P = 34.158$ | $x^*_A = 0.175$<br>$r^*_A = 0.043$<br>$E^*_A = 0.132$<br>$\pi_A = 0.010$ | $x^*_\lambda = 0.562$<br>$r^*_\lambda = 0.201$<br>$E^*_\lambda = 1.012$<br>$\pi_\lambda = 0.150$ |
| $\rho = 0.5$ | $x^*_P = 0.839$<br>$r^*_P = 1.352$<br>$E^*_P = 100$<br>$\pi_P = 31.906$ | $x^*_A = 0.175$<br>$r^*_A = 0.850$<br>$E^*_A = 2.635$<br>$\pi_A = 0.204$ | $x^*_\lambda = 0.562$<br>$r^*_\lambda = 4.026$<br>$E^*_\lambda = 20.237$<br>$\pi_\lambda = 3.007$ |

TABLE 1: optimal solutions and principal's net benefit when the sanction rate is $s = 2.1$ in the three regimes – when the principal is fully liable, denoted by $(x^*_P, r^*_P, E^*_P, \pi_P)$, when the agent is, denoted by $(x^*_A, r^*_A, E^*_A, \pi_A)$, and when sanctions are equally shared between the principal and the agent, denoted by $(x^*_\lambda, r^*_\lambda, E^*_\lambda, \pi_\lambda)$ – for risk-aversion coefficients $\rho = 10$ and $\rho = 0.5$.

Note that when the agent is liable and the solutions are interior – *i.e.*, both $0 < E^*_A < 0$ and $0 < E^*_\lambda < Y$ hold, the optimal efforts $x^*_A$ and $x^*_\lambda$ turn out to be independent of the absolute risk-aversion coefficient $\rho$, as explained at the end of Appendix 6.C. Intuition on this purpose suggests the following explanation: since with CARA the desired amount of tax evasion does not depend on income, the latter can always be adjusted to accommodate the preferred effort while also securing the reservation utility. Hence the salary $r^*_{A,\lambda}$ and the evasion $E^*_{A,\lambda}$ change in response to changes in risk aversion, while effort does not. This property does not hold for corner solutions, *i.e.*, when $E^*_A = E^*_\lambda = Y$.

Numerical examples confirm that a liability shift from the principal onto the agent can actually entail a reduction of the agent's remuneration, which in turn implies that both evasion and effort are smaller. This effect is more likely at high sanction and/or high risk aversion

[19]See Appendix 6.B for a discussion.

| Risk-aversion coefficient | Principal fully liable | Agent fully liable | Shared liability: $\lambda = 0.5$ |
|---|---|---|---|
| $\rho = 10$ | $x_P^* = 1.213$<br>$r_P^* = 0.249$<br>$E_P^* = 100$<br>$\pi_P = 36.136$ | $x_A^* = 0.277$<br>$r_A^* = 0.077$<br>$E_A^* = 0.308$<br>$\pi_A = 0.047$ | $x_\lambda^* = 1.198$<br>$r_\lambda^* = 19.918$<br>$E_\lambda^* = 100$<br>$\pi_\lambda = 17.352$ |
| $\rho = 0.5$ | $x_P^* = 0.729$<br>$r_P^* = 1.005$<br>$E_P^* = 100$<br>$\pi_P = 34.367$ | $x_A^* = 0.277$<br>$r_A^* = 1.531$<br>$E_A^* = 6.163$<br>$\pi_A = 0.935$ | $x_\lambda^* = 0.774$<br>$r_\lambda^* = 15.384$<br>$E_\lambda^* = 100$<br>$\pi_\lambda = 21.234$ |

TABLE 2: optimal solutions and principal's net benefit when the sanction rate is $s = 1$ in the three regimes again for risk-aversion coefficients $\rho = 10$ and $\rho = 0.5$.

levels. The relevant sanction severity depends on both the rate $s$ and the amount of risk shifted upon the agent. On the other hand, an increase in the agent's compensation may entail either a decrease in effort and evasion, as in the second column of the second row in table 2, or an increase in effort, as in the case of full evasion (corner solution) reported in column three of the second row of Table 2. The latter case is more likely when the sanction is lower and/or the agent is less risk-averse.

An interesting feature of the framework under consideration, characterized by an agent with a CARA utility function, is that – when the agent is partially or fully liable – effort does not depend on the tax rate $t$, while it is increasing in $t$ when the principal is.[20] This property suggests that a shift of liability onto the agent increases the degrees of freedom of the tax policy, since even if the tax rate is increased there is no danger of inducing a larger effort in covering up tax evasion.

## 4.2 Comparative statics analysis

We end this Section with some comparative statics analysis specific to the regime in which the agent is fully liable and when the optimal solution is interior so that $0 < E_A^* < Y$.

**Proposition 5** *Consider the regime in which the agent is fully liable for the functional forms* $u(w) = -e^{-\rho w}$, $p(x) = \alpha/(\beta x + 1)$ *and* $g(x) = \zeta x^2$ *chosen in this section such that all parameters* $Y$, $u_0$, $t$, $\alpha$, $\beta$, $\zeta$, $\rho$ *and* $s$ *satisfy Assumption A.1 plus condition (47) in Appendix 6.A. If the optimal solution* $(x_A^*, r_A^*, E_A^*)$ *to problem (7) is interior in the sense that* $0 < E_A^* < Y$, *then the optimal effort in concealing misdemeanor,* $x_A^*$, *is decreasing in the sanction rate* $s$:

$$\frac{\partial x_A^*}{\partial s} < 0.$$

*Moreover, the optimal evasion* $E_A^*$ *turns out to be decreasing in both the sanction rate* $s$ *and the tax rate* $t$:

$$\frac{\partial E_A^*}{\partial s} < 0 \qquad \text{and} \qquad \frac{\partial E_A^*}{\partial t} < 0.$$

[20] After substituting the exponential terms from the second equations in systems (51) and (52) in Appendix 6.C, the single equations in $x$ thus obtained do not depend on $t$, while, by means of the implicit function theorem, it is easily seen that the optimal effort in equation (50) is increasing in $t$.

The proof is reported at the end of Appendix 6.C.

For corner solutions of the type $E_A^* = Y$ it is readily seen that the signs of $\partial x_A^*/\partial s$ and $\partial x_A^*/\partial t$ depend on the magnitude of $x_A^*$, and thus a clear-cut comparative statics result cannot be provided.

Summing up, with reference to internal solutions our results confirm the predictions of the standard individual's tax evasion model when a risk-averse agent is considered. This is in line with the fact, already noted in Subsection 2.2, that when all the risk is shifted onto the agent her optimal choices coincide with those that would arise if the activity was "sold" to the agent and the surplus extracted by the principal through a lump-sum fee.

While the computations needed for performing a comparative static analysis for the shared liability case are excessively demanding, one expects that similar results would arise, since also in that case one can interpret the result as a decision about tax evasion taken by an agent (see Subsection 2.3). Since, however, the slope of the budget constraint faced by the agent would be smaller in absolute value, the size of the reaction to the parameters' changes is likely to be smaller than in the case in which the whole risk is transferred onto the agent.

# 5 Conclusion

Tax evasion by firms is a very complicated phenomenon that can be understood only by tracing back its roots to the firm's internal organization, which describes how the responsibilities and the incentives are allocated and how they interact in producing the firm's behavior. The principal-agent model offers a natural framework for studying this problem. It also represents a very flexible framework, in which different scenarios can be studied, according to the assumptions made about the parties' attitudes toward risk, the information they possess, and the type of activity that the agent is expected to do within the firm.

In this paper we have studied the case in which the cooperation of a risk-averse agent (such as an employee or a consultant) is needed in order to evade taxes. Moreover, the agent can exert effort in order to reduce the ensuing risk. In this framework policies aimed at curbing tax evasion can target either the principal or the agent or both of them.

The immediate policy aspects of this paper are as follows. When liability is shifted from the principal to the agent, a problem of excessive loyalty might arise, and the agent might be pushed to help the principal in concealing tax evasion. However, we are also able to show that there are win-win scenarios, in which both tax evasion and effort in covering decrease. A tough enforcement policy and a high agent's risk-aversion render these scenarios more likely. An indicator that the win-win case has arisen is a decrease in the agent's remuneration. Thus, a signal of success of policies that crack on tax officers, consultants and tax preparers, in order to moralize their conduct and to curb tax evasion, would be a shrinking of their remunerations and income share, *ceteris paribus*. Indirect signals might be represented by the shrinking of the share of the population involved in activities pertaining to tax preparation and in the share of students demanding education in this field, of course once again controlling for other possible explanatory factors.

One might in principle also consider going the other way round, *i.e.*, setting caps on the compensations for assistance to the taxpayers, in order to discourage risky cooperation with evaders. However, since the ceilings should not fall short of the remuneration level adequate for legally permissible assistance, the information needed to design them seem beyond the reach of the regulator. What is sometimes done,[21] and might be advisable to do to a larger extent, is

[21] In USA under Code Sec. 6694 a tax preparer who prepares a tax return or claim for refund with an

setting penalties as an increasing function of the gatekeepers illegitimate gains, in order to cap at least the expected benefits that gatekeepers receive if they contribute to illegal activities.

# 6 Appendix

## 6.A Conditions for constraint's convexity

We aim at establishing sufficient conditions under which the first inequalities in the constraints (7) and (14) define a convex set in $\mathbb{R}^3_+$. Specifically, we shall provide conditions under which the following function,

$$v(x, r, E) = [1 - p(x)]\, u(r) + p(x)\, u(r - \phi E) - g(x), \tag{28}$$

where the constant $\phi$ is defined as

$$\phi = \begin{cases} (1+s)\, t \text{ when the shift of risk is full (Subsection 2.2)} \\ (1-\lambda)\, st \text{ when the risk is shared (Subsection 2.3),} \end{cases}$$

turns out to be quasiconcave in $x$, $r$ and $E$, so that its upper contours are convex sets.

**Lemma 2** *If, under Assumption A.1, the following conditions hold:*

**i)**

$$p(x)\, p''(x) \geq 2\, [p'(x)]^2 \text{ for all } x \geq 0 \text{ and}$$

**ii)**

$$-\frac{u''(w)}{[u'(w)]^2} \geq \frac{[p'(x)]^2}{[g''(x)\, p(x) - 2g'(x)\, p'(x)]\, p(x)\, [1 - p(x)]} \text{ for all } w > 0 \text{ and } x \geq 0,$$

*then $v(x, r, E)$ in (28) is quasiconcave, and thus its upper contours are convex sets.*

**Proof.** To prove quasiconcavity of $v(x, r, E)$ we directly tackle the upper contour sets:

$$\left\{(x, r, E) \in \mathbb{R}^3_+ : v(x, r, E) \geq \mu\right\}, \tag{29}$$

and show that under Assumption A.1 plus conditions (i) and (ii) of Lemma 2 they are convex sets for all $\mu \in \mathbb{R}$.

The sets (29) are equivalently defined as the sets of points $(x, r, E)$ satisfying

$$E \leq G_\mu(x, r),$$

where

$$G_\mu(x, r) = \frac{r}{\phi} - \frac{1}{\phi} u^{-1} \left\{ \frac{\mu - [1 - p(x)]\, u(r) + g(x)}{p(x)} \right\} \tag{30}$$

is a function of the only two variables $x$ and $r$ for each given $\mu$, in which $u^{-1}(\cdot)$ denotes the inverse of the utility $u(w)$. Note that the set (29) corresponds to the hypograph of the function $G_\mu$ in (30); therefore, the sets (29) are convex if and only if the hypographs of the functions $G_\mu$s are convex sets for any $\mu \in \mathbb{R}$. Moreover, it is well known that a function has convex

understatement of liability due to a "unreasonable position" might be subject to a penalty equal to or larger than 50% of the income derived from the tax return preparation.

hypograph if and only if it is a concave function; hence, our plan is to show that $G_\mu(x,r)$ is a concave function in its two variables $x$ and $r$.

Note that, since $u^{-1}(\cdot)$ is strictly increasing and strictly convex, $G_\mu(x,r)$ in (30) is the sum of a linear function, $r/\phi$, and the opposite of a strictly increasing and strictly convex transformation of the function

$$L(x,r)=\frac{\mu-[1-p(x)]\,u(r)+g(x)}{p(x)}=\frac{\mu-u(r)+g(x)}{p(x)}+u(r)\,. \tag{31}$$

Thus, in order to establish concavity of $G_\mu(x,r)$ in (30), it is sufficient to show that $L(x,r)$ in (31) is convex in $x$ and $r$. To achieve such goal we study the Hessian matrix of $L(x,r)$, whose partial, second-order derivatives are:

$$L_{xx}=\frac{g''(x)\,p(x)-2g'(x)\,p'(x)-[\mu-u(r)+g(x)]\left\{p''(x)-2\frac{[p'(x)]^2}{p(x)}\right\}}{[p(x)]^2}, \tag{32}$$

$$L_{rr}=-\frac{1-p(x)}{p(x)}u''(r)\,, \tag{33}$$

$$L_{xr}=\frac{p'(x)}{[p(x)]^2}u'(r)\,. \tag{34}$$

Since, by Assumption A.1 (i) and (ii), $L_{rr}$ in (33) is clearly positive, we must show that both $L_{xx}$ in (32) and the determinant of the Hessian matrix, $L_{xx}L_{rr}-(L_{xr})^2$, are non-negative.

By Assumption A.1, the first two terms on the numerator of the RHS in (32) are both positive; moreover, by definition of upper contour set in (29) the following holds:

$$\begin{aligned}\mu &\le [1-p(x)]\,u(r)+p(x)\,u(r-\phi E)-g(x)\\ &= u(r)-p(x)\,[u(r)-u(r-\phi E)]-g(x)\\ &\le u(r)-g(x)\,,\end{aligned}$$

where the last inequality holds as $p(x)\,[u(r)-u(r-\phi E)]\ge 0$. Therefore $[\mu-u(r)+g(x)]\le 0$ in the last term on the numerator of the RHS in (32); moreover, by condition (i), the last factor, $\left\{p''(x)-2\,[p'(x)]^2/p(x)\right\}$, is non-negative, thus establishing that $L_{xx}>0$.

Since the last term on the numerator of the RHS in (32) is non-negative, condition (ii) is sufficient for the determinant of the Hessian matrix, $L_{xx}L_{rr}-(L_{xr})^2$, with $L_{xx}$, $L_{rr}$ and $L_{xr}$ given by (32), (33) and (34) respectively, to be non-negative, and the proof is complete. ■

Condition (ii) in Lemma 2 can be rewritten in a clearer way by separating properties related to each function $u(w)$, $p(x)$ and $g(x)$, as the next Proposition shows.

**Proposition 6** *Let* $B=\inf_{w\ge 0}\left\{-u''(w)/[u'(w)]^2\right\}$ *and assume that* $B>0$. *Then, a sufficient condition for (ii) in Lemma 2 is the following:*

$$-\frac{p'(x)}{p(x)}\le B\,[1-p(0)]\left\{g'(x)+\sqrt{[g'(x)]^2+\frac{g''(x)}{B\,[1-p(0)]}}\right\}\quad \text{for all } x>0. \tag{35}$$

**Proof.** By using the definition of $B$, condition (ii) in Lemma 2 can be rewritten as

$$\left[\frac{p'(x)}{p(x)}\right]^2\le B\,[1-p(x)]\left[g''(x)-2g'(x)\,\frac{p'(x)}{p(x)}\right]. \tag{36}$$

Since $p(x)$ is a decreasing function of $x$, a sufficient condition for (36) is

$$\left[\frac{p'(x)}{p(x)}\right]^2 \leq B[1-p(0)]\left[g''(x) - 2g'(x)\frac{p'(x)}{p(x)}\right],$$

which is equivalent to

$$\left[\frac{p'(x)}{p(x)}\right]^2 + 2B[1-p(0)]\,g'(x)\left[\frac{p'(x)}{p(x)}\right] - B[1-p(0)]\,g''(x) \leq 0.$$

Let $\tau = -p'(x)/p(x)$, so that the last inequality can be rewritten as

$$\tau^2 - 2B[1-p(0)]\,g'(x)\,\tau - B[1-p(0)]\,g''(x) \leq 0, \tag{37}$$

where, for fixed $x$, the LHS represents an upward parabola in $\tau$ with roots given by

$$\tau = B[1-p(0)]\left\{g'(x) \pm \sqrt{[g'(x)]^2 + \frac{g''(x)}{B[1-p(0)]}}\right\}.$$

Since, by Assumption A.1 (iii), $g''(x) \geq 0$, one root is non-positive and thus it must be ruled out [recall that, by Assumption A.1 (ii), $\tau = -p'(x)/p(x) > 0$ must hold]; therefore the solution of (37) is

$$\tau \leq B[1-p(0)]\left\{g'(x) + \sqrt{[g'(x)]^2 + \frac{g''(x)}{B[1-p(0)]}}\right\},$$

which is (35). ■

Condition (35) states that the rate of decrease of the probability $p(x)$ must be bounded from above by a term involving the first and second derivatives of $u(w)$ and $g(x)$. The most striking feature of inequality (35) is that it provides the bound on $-p'(x)/p(x)$ – which involves only the function $p(x)$ – entirely in terms of properties of $g(x)$.

Conditions (i) and (ii) in Lemma 2 as well as (35) in Proposition 6 may appear a bit abstract at a first sight. As a matter of fact, the family of functions $u(w)$, $p(x)$ and $g(x)$ satisfying such conditions, besides being nonempty, includes some meaningful cases.

Specifically, condition (i) is satisfied (with equality) by the following family of hyperbolic probability functions:

$$p(x) = \frac{\alpha}{\beta x + 1} \tag{38}$$

for parameters $\alpha$ and $\beta$ such that $0 < \alpha < 1$ and $\beta > 0$, as can be easily checked. Moreover, condition (35) is satisfied by all HARA utilities of the form

$$u(w) = \frac{\gamma}{1-\gamma}\left(\theta + \frac{\rho}{\gamma}w\right)^{1-\gamma}, \tag{39}$$

with $\theta > 0$, $\rho > 0$, and satisfying $\gamma \geq 1$, and by the set of exponential effort functions of the type[22]

$$g(x) = \eta\left(e^{\delta x} - \delta x - 1\right), \tag{40}$$

with $\eta > 0$ and $\delta > 0$, provided that some conditions involving all parameters $\alpha$, $\beta$, $\theta$, $\gamma$, $\eta$ and $\delta$ are met, as stated in the next Proposition.

[22]Note that the effort function in (40) satisfies Assumption A.1 (iii) and (iv) for all $\eta > 0$ and $\delta > 0$.

**Proposition 7** *Assume that utility is of the family (39), probability is of the type (38) and the effort function is given by (40). Then condition (35) in Proposition 6 holds whenever*

$$\frac{\beta}{\sqrt{1-\alpha}} \leq \delta\sqrt{\eta\theta^{\gamma-1}}. \tag{41}$$

**Proof.** From (39) it is immediately seen that $-u''(w)/[u'(w)]^2 = [\theta + (\rho/\gamma)w]^{\gamma-1}$, and thus, since $\gamma \geq 1$, $B = \min_{w\geq 0}[\theta + (\rho/\gamma)w]^{\gamma-1} = \theta^{\gamma-1} > 0$. From (38) we have $p(0) = \alpha$ and $-p'(x)/p(x) = \beta/(\beta x + 1) \leq -p'(0)/p(0) = \beta$, so that an upper bound for the LHS of (35) is $\beta$; while from (40) we get $g'(x) = \eta\delta\left(e^{\delta x} - 1\right) \geq g'(0) = 0$ and $g''(x) = \eta\delta^2 e^{\delta x} \geq g''(0) = \eta\delta^2$, so that a lower bound for the RHS of (35) is

$$\theta^{\gamma-1}(1-\alpha)\sqrt{\frac{\eta\delta^2}{\theta^{\gamma-1}(1-\alpha)}} = \delta\sqrt{(1-\alpha)\eta\theta^{\gamma-1}}.$$

Rearranging terms in $\beta \leq \delta\sqrt{(1-\alpha)\eta\theta^{\gamma-1}}$ condition (41) is obtained. ∎

**Remark 1** *Condition (41) does not depend on parameter $\rho$ in (39).*

**Remark 2** *For $1 \leq \gamma < +\infty$ and $\theta > 0$ HARA utilities defined in (39) exhibit* decreasing absolute risk-aversion *but* increasing relative risk-aversion*. However, for large $w$ the relative risk-aversion of all utilities in (39) tend to become constant, approaching the value $\gamma$ from below. Moreover, if $\theta = 1$, for $\gamma \to +\infty$ (39) becomes the CARA 'negative exponential' utility:*

$$u(w) = -e^{-\rho w}, \tag{42}$$

*where $\rho > 0$ is the (constant) coefficient of absolute risk-aversion. Note that, by Remark 1, in this case condition (41) is independent of the absolute risk-aversion.*

*Finally, note that by applying l'Hôpital's rule to the equivalent version of (39) given by*

$$u(w) = \frac{\gamma}{1-\gamma}\left[\left(\theta + \frac{\rho}{\gamma}w\right)^{1-\gamma} - 1\right],$$

*for $\gamma \to 1$ we obtain* log *utility:*

$$u(w) = \ln(\theta + \rho w), \tag{43}$$

*and the lower bound $B$ in (35) boils down to $B = 1$, as in this case $-u''(w)/[u'(w)]^2 \equiv 1$ for all $w \geq 0$.*

**Remark 3** *Letting $\theta = 0$ in (43) we obtain*

$$u(w) = \ln(\rho w), \tag{44}$$

*which exhibits* constant relative risk-aversion *equals to 1. As for the general* log *utility defined in (43), the lower bound $B$ in (35) for $u(w)$ defined in (44) equals 1, and thus condition (41) in Proposition 7 becomes*

$$\frac{\beta}{\sqrt{1-\alpha}} \leq \delta\sqrt{\eta}, \tag{45}$$

*for both the constant relative risk-aversion utility defined in (44) and the increasing relative risk-aversion utility defined in (43). It turns out that (44) is the only member of the CRRA utility functions family that, when associated with probabilities defined as in (38) and effort functions defined as in (40), is capable of providing a sufficient condition for the function $v(x, r, E)$ in (28) to be quasiconcave along the arguments developed in this appendix, i.e., whenever (45) is satisfied.*

As the LHS is increasing in both $\beta$ and $\alpha$, the likelihood of holding condition (41) increases for smaller $\alpha$ and $\beta$; hence, Proposition 7 states that, for any choice of parameters $\gamma \geq 1$, $\theta > 0$ (or $\gamma = 1$ if $\theta = 0$), $\rho > 0$, $\eta > 0$ and $\delta > 0$, all related to the functions $u(w)$ and $g(x)$, there always exist a probability function $p(x)$ of the type (38), with parameters $\alpha$ and/or $\beta$ sufficiently small, that renders the function $v(x, r, E)$ in (28) quasiconcave. For fixed $\beta$, smaller $\alpha$s let the graph of $p(x)$ to be lower and flatter, while, for fixed $\alpha$, smaller $\beta$s let the graph of $p(x)$ to be higher and less convex.

When $\theta = \eta = 1$ [a situation encompassing all CARA utilities of the type (42), as clarified in Remark 2] condition (41) boils down to

$$\frac{\beta}{\sqrt{1-\alpha}} \leq \delta. \tag{46}$$

Inequality (46) says that the effort in concealing misdemeanor may cost less – *i.e.*, smaller $\delta$ in (40) – when the probability of detection is either uniformly smaller – *i.e.*, $\alpha$ in (38) is smaller – or it decreases more rapidly – *i.e.*, $\beta$ in (38) is smaller.

**Remark 4** *Condition (35) in Proposition 6 actually holds also for one single power effort function, the quadratic function $g(x) = \zeta x^2$, with $\zeta > 0$. In this case a lower bound for the square root in the RHS of (35) is given by $g''(x) \equiv 2\zeta$, so that, by keeping the probability and the utility functions as in (38) and (39) respectively, if $\theta = 1$ (or $\theta = 0$ and $\gamma = 1$) our condition becomes*

$$\frac{\beta^2}{2(1-\alpha)} \leq \zeta, \tag{47}$$

*which looks similar to (46), and hence the same comments apply with reference to parameter $\zeta$ in place of $\delta$ as a measure of effort cost.*

**Example 1** *If we take any* log *utility, either with increasing relative risk-aversion as in (43) or of the CRRA type as in (44), or any CARA utility as in (42), and a probability function given by (38) with $\alpha = 0.1$ and $\beta = 1$, then, by condition (46), an effort function as in (40) with $\eta = 1$ must have a parameter $\delta$ not smaller than $1/\sqrt{0.9} \simeq 1.054$, while by condition (47) a quadratic effort function of the type $g(x) = \zeta x^2$ requires its parameter $\zeta$ to be not smaller than $1/(1.8) \simeq 0.556$.*

## 6.B On uniqueness of the solution when liability is shared

Uniqueness of the optimal solution $(x_\lambda^*, r_\lambda^*, E_\lambda^*)$ in problem (14) of Subsection 2.3 cannot in general be guaranteed, as the three variables $x$, $r$ and $E$ appear both as a product and as a sum in the objective function, which thus, in general, turns out to be neither concave nor quasiconcave. However, in this appendix we briefly argue why it seems not unreasonable to expect problem (14) to be actually concave. We shall see in Appendix 6.C that this is the case for the numeric examples discussed in Section 4.

The function $v(x, r, E)$ defined in (28) is differentiable and strictly increasing in $r$, *i.e.*, $(\partial/\partial r)\, v(x, r, E) > 0$ for all feasible $x, r, E$; thus, by the implicit function theorem, the contour set $v(x, r, E) = \mu$ coincides with the graph of a function of the only two variables $x$ and $E$, $r^\mu(x, E)$. As the participation constraint in problem (14) is binding, after setting $\mu = u_0$ the function $r^{u_0}(x, E)$ can be substituted in the objective function and the whole problem simplifies to

$$\max \left\{ \pi(x, E) = [1 - p(x)(1 + \lambda s)]\, tE - r^{u_0}(x, E) \right\} \tag{48}$$

over the (convex) rectangle $\{(x, E) : x \geq 0,\ 0 \leq E \leq Y\}$.

Under the assumptions of Lemma 2 the upper contour set $\left\{(x, r, E) \in \mathbb{R}^3_+ : v(x, r, E) \geq u_0\right\}$ is a convex set. Because $(\partial/\partial r)\, v(x, r, E) > 0$, such contour set corresponds to the epigraph of function $r^{u_0}(x, E)$;[23] therefore, $r^{u_0}(x, E)$ is a $C^2$ convex function, that is, its Hessian matrix has second-order partial derivatives such that $r^{u_0}_{xx} \geq 0$, $r^{u_0}_{EE} \geq 0$ and $r^{u_0}_{xx} r^{u_0}_{EE} - \left(r^{u_0}_{xE}\right)^2 \geq 0$. Hence, the second-order partial derivatives in the Hessian matrix, $H_\pi$, of function $\pi(x, E)$ in (48) are:

$$\begin{aligned} \pi_{xx} &= -p_{xx}(1 + \lambda s)\, tE - r^{u_0}_{xx} < 0, \\ \pi_{EE} &= -r^{u_0}_{EE} \leq 0, \\ \pi_{xE} &= -p_x(1 + \lambda s)\, t - r^{u_0}_{xE}. \end{aligned}$$

While the signs of $\pi_{xx}$ and $\pi_{EE}$ are clearly negative and nonpositive respectively, the sign of $\pi_{xE}$ depends on the cross-effect of effort and evasion on the agent's compensation. The determinant of $H_\pi$ is

$$(1 + \lambda s)\, tEp_{xx} r^{u_0}_{EE} + r^{u_0}_{xx} r^{u_0}_{EE} - \left[(1 + \lambda s)\, tp_x\right]^2 - \left(r^{u_0}_{xE}\right)^2 - 2\left[(1 + \lambda s)\, tp_x r^{u_0}_{xE}\right]. \tag{49}$$

The first term in (49) is nonnegative, while, by convexity of $r^{u_0}(x, E)$, the sum of the second and fourth terms is nonnegative as well, $r^{u_0}_{xx} r^{u_0}_{EE} - \left(r^{u_0}_{xE}\right)^2 \geq 0$. Hence, the sign of (49) can be positive if $r^{u_0}(x, E)$ is strictly convex, $|(1 + \lambda s)\, tp_x|$ is not too large, and either $r^{u_0}_{xE} \geq 0$ or $|2\left[(1 + \lambda s)\, tp_x r^{u_0}_{xE}\right]|$ is not too large when $r^{u_0}_{xE} < 0$. Therefore, when $r^{u_0}(x, E)$ is strictly convex and the probability function $p(x)$ is sufficiently flat and/or the cross-effect between effort and evasion is not too large, the Hessian matrix, $H_\pi$, turns out to be positive definite, establishing concavity of function $\pi(x, E)$ in (48), so that problem (14) has a unique solution $(x^*_\lambda, r^*_\lambda, E^*_\lambda)$ fully characterized by conditions (17) to (19) of Proposition 2.

Note that at any rate the concavity of the whole problem and the uniqueness of the solution $(x^*_\lambda, r^*_\lambda, E^*_\lambda)$ is not needed in both Propositions 3 and 4, where the unique solution $(x^*_P, r^*_P, Y)$ under principal's full liability is compared to any solution $\left(x^*_{A,\lambda}, r^*_{A,\lambda}, E^*_{A,\lambda}\right)$ under different liability regimes – either shared or fully borne by the agent.

## 6.C Building the examples of Section 4

For the functions' specification chosen in Section 4 conditions (5) and (6) for the case in which the principal is fully liable become

$$\begin{cases} -e^{-\rho r} = \zeta x^2 + u_0 \\ \dfrac{\alpha\beta}{(\beta x + 1)^2}(1 + s)\, tY = \dfrac{2\zeta x}{\rho e^{-\rho r}}, \end{cases}$$

which, after substituting the common term $e^{-r}$, boils down to the following cubic equation in the only variable $x$:

$$\frac{2\zeta}{\rho\alpha\beta(1 + s)\, tY} x(\beta x + 1)^2 + \zeta x^2 + u_0 = 0, \tag{50}$$

which, in turn, is easily solved through Maple 13 yielding a unique positive value[24] $x^*_P$; $r^*_P$ is then immediately calculated.

[23] $r^{u_0}(x, E)$ represents the minimum compensation needed for satisfying the agent's participation constraint.
[24] The Maple 13 code for the solutions in all three liability scenarios is available from the authors upon request.

Similarly, when the agent bears all the risks conditions (11) to (13) can be written as

$$\begin{cases} -\dfrac{e^{-\rho r}}{\beta x+1}\left[\beta x+1-\alpha+\alpha e^{\rho(1+s)tE}\right]=\zeta x^2+u_0 \\ -\dfrac{e^{-\rho r}}{\beta x+1}\left[e^{\rho(1+s)tE}-1\right]=-\dfrac{2\zeta}{\alpha\beta}x\left(\beta x+1\right) \\ e^{\rho(1+s)tE}\leq\left(\beta x+1-\alpha\right)/\alpha s, \end{cases}$$

which, again, by substituting the common terms $-e^{-\rho r}/\left(\beta x+1\right)$ in the first two equations, boils down to the following system in the only two variables $x$ and $E$:

$$\begin{cases} \alpha\beta\left(\zeta x^2+u_0\right)\left[e^{\rho(1+s)tE}-1\right]+2\zeta x\left(\beta x+1\right)\left[\beta x+1-\alpha+\alpha e^{\rho(1+s)tE}\right]=0 \\ e^{\rho(1+s)tE}\leq\left(\beta x+1-\alpha\right)/\alpha s, \end{cases} \tag{51}$$

where, as before, the first equation is cubic in $x$, even after substituting from the last one. Such system can either have a unique interior solution – when $0<E_A^*<Y$ and the last inequality holds with equality – which is obtained by substituting $e^{\rho(1+s)tE}$ in the first equation from the second one and then solved through Maple 13 with respect to $x$, or a unique corner solution – when $E_A^*=Y$ and the last inequality is strict – which is again computed by solving the first equation through Maple 13 in $x$ for $E=Y$.

Finally, when liability for sanctions is shared between the principal and the agent conditions (17) to (19) can be reduced to

$$\begin{cases} -\dfrac{e^{-\rho r}}{\beta x+1}\left[\beta x+1-\alpha+\alpha e^{\rho(1-\lambda)stE}\right]=\zeta x^2+u_0 \\ -\dfrac{e^{-\rho r}}{\beta x+1}\left[\left(1+\lambda s\right)t\rho E\dfrac{\beta x+1-\alpha+\alpha e^{\rho(1-\lambda)stE}}{\beta x+1}+e^{\rho(1-\lambda)stE}-1\right]=-\dfrac{2\zeta}{\alpha\beta}x\left(\beta x+1\right) \\ e^{\rho(1-\lambda)stE}\leq\dfrac{\left(\beta x+1-\alpha\right)\left[\beta x+1-\alpha\left(1+\lambda s\right)\right]}{\alpha\left[\left(1-\lambda\right)s-1\right]\left(\beta x+1\right)+\alpha^2\left(1+\lambda s\right)}, \end{cases}$$

which, as before, by substituting the common terms $-e^{-\rho r}/\left(\beta x+1\right)$ in the first two equations, boils down to the following system in the only two variables $x$ and $E$:

$$\begin{cases} \left[2\zeta x\left(\beta x+1\right)^2+\alpha\beta\left(\zeta x^2+u_0\right)\left(1+\lambda s\right)t\rho E\right]\left[\beta x+1-\alpha+\alpha e^{\rho(1-\lambda)stE}\right] \\ \qquad\qquad +\alpha\beta\left(\zeta x^2+u_0\right)\left(\beta x+1\right)\left[e^{\rho(1-\lambda)stE}-1\right]=0 \\ e^{\rho(1-\lambda)stE}\leq\dfrac{\left(\beta x+1-\alpha\right)\left[\beta x+1-\alpha\left(1+\lambda s\right)\right]}{\alpha\left[\left(1-\lambda\right)s-1\right]\left(\beta x+1\right)+\alpha^2\left(1+\lambda s\right)}. \end{cases} \tag{52}$$

When $0<E_\lambda^*<Y$ and the last inequality holds with equality we can substitute from the second equation both terms $e^{\rho(1-\lambda)stE}$ and $E$ defined by

$$E=\frac{1}{\rho\left(1-\lambda\right)st}\ln\left\{\frac{\left(\beta x+1-\alpha\right)\left[\beta x+1-\alpha\left(1+\lambda s\right)\right]}{\alpha\left[\left(1-\lambda\right)s-1\right]\left(\beta x+1\right)+\alpha^2\left(1+\lambda s\right)}\right\} \tag{53}$$

(which is the second equation suitably rewritten) in the first equation and then solve through Maple 13 with respect to $x$; in this case, however, we need to rely on the Maple's numeric procedure 'fsolve', rather than on the symbolic one 'solve', which works well in the former cases. After substituting $e^{\rho(1-\lambda)stE}$ and $E$ from the second equation, uniqueness of the solution just found can be checked through graphic inspection by plotting the LHS of the first equation in (52) as a function of $x$: it is clearly seen that such function intersects the horizontal axis only at one point.[25] To make our uniqueness test more robust we also checked that the second order condition holds on the unique point found.[26] Again, a unique corner solution – when $E^*_\lambda = Y$ and the last inequality is strict, so that the objective function becomes concave in the two variables $x$ and $r$ – is computed by solving the first equation through Maple 13 (by means of 'solve') in $x$ for $E = Y$.

Note that when the solutions are interior, *i.e.*, both $0 < E^*_A < 0$ and $0 < E^*_\lambda < Y$ hold, after substituting for the exponential terms $e^{\rho(1+s)tE}$ in systems (51) and $e^{\rho(1-\lambda)stE}$ – as well as for $E$ given by (53) – in system (52), the optimal efforts $x^*_A$ and $x^*_\lambda$ turn out to be independent of the absolute risk-aversion coefficient $\rho$, as shown in Tables 1 and 2. Of course, this property does not hold for corner solutions, *i.e.*, when $E^*_A = E^*_\lambda = Y$.

**Proof of Proposition 5.** Assume that $0 < E^*_A < Y$. Then, after substituting the second equation in system (51) and rearranging terms we get the following unique equation in $x$:

$$\beta\left(\zeta x^2 + u_0\right)[\beta x + 1 - \alpha(1+s)] + 2\zeta x(\beta x + 1)(\beta x + 1 - \alpha)(1+s) = 0.$$

By labelling the LHS as $f(x, s)$ and differentiating it with respect to $x$ and $s$ respectively we get:

$$\begin{aligned} \frac{\partial f}{\partial x} &= \beta^2\left(\zeta x^2 + u_0\right) + 2\zeta\left[\beta x(\beta x + 1) + (3\beta x + 1)(\beta x + 1 - \alpha)(1+s)\right], \\ \frac{\partial f}{\partial s} &= -\alpha\beta\left(\zeta x^2 + u_0\right) + 2\zeta x(\beta x + 1)(\beta x + 1 - \alpha). \end{aligned}$$

Since Assumption A.1(iv) requires $u_0 \geq u(0) = -1$, while (47) in Appendix 6.A states that $\zeta \geq \beta^2/[2(1-\alpha)]$, the following holds:

$$\begin{aligned} \frac{\partial f}{\partial x} &\geq \beta^2\left[\frac{\beta^2}{2(1-\alpha)}x^2 - 1\right] + 2\frac{\beta^2}{2(1-\alpha)}\left[\beta x(\beta x + 1) + (3\beta x + 1)(\beta x + 1 - \alpha)(1+s)\right] \\ &> -\beta^2 + \beta^2(1+s) \\ &= \beta^2 s \\ &> 0, \end{aligned}$$

[25] Details and plots are available from the authors upon request. Note that, in order to check uniqueness of the zero for the LHS of the first equation in (52) as a function of the only variable $x$, we need to bound the range for such variable to some interval $[0, x_U]$. A useful upper bound $x_U$ can be calculated using the monotonicity of the probability $p(x)$ and the participation constraint in (14), obtaining $g(x) \leq [1 - p(0)]u(r) - u_0$; then, under condition (15), from the objective function get $r < [1 - p(x)(1 + \lambda s)]tE < tE \leq tY$, so that $x < g^{-1}\{[1 - p(0)]u(tY) - u_0\}$. Under our model's specification this condition becomes $x < \sqrt{\{[1 - p(0)]u(tY) - u_0\}/\zeta}$; however, as the term $[1 - p(0)]u(tY)$ turns out to be negligible under our parameters' values, we set the upper bound $x_U = \sqrt{-u_0/\zeta}$.

[26] As a matter of fact, we followed the approach outlined in Appendix 6.B and substituted the participation constraint, rewritten as $r^{u_0}(x, E) = (1/\rho)\ln\left\{\left[p(x)\left(1 - e^{e^{\rho(1-\lambda)stE}}\right) - 1\right]/\left[u_0 + \zeta x^2\right]\right\}$, into the objective function as in (48); then we checked graphically that the objective function $\pi(x, E)$ is concave jointly in $x$ and $E$ over the rectangle $[0, x_U] \times [0, Y]$, with $x_U$ defined in footnote 25. Moreover, we also plotted the determinant (49) of its Hessian matrix as a function of $x$ and $E$ to check that it is positive on the whole feasible set, so that the Hessian turns out to be everywhere definite negative. Details are available upon request.

where the second inequality uses the fact that $x > 0$. As for $\partial f / \partial s$,

$$\begin{aligned}\frac{\partial f}{\partial s} &= -\alpha\beta\zeta x^2 - \alpha\beta u_0 + 2\zeta x\left(\beta x + 1\right)\left(\beta x + 1 - \alpha\right) \\ &> \zeta x\left[2\left(\beta x + 1\right)\left(\beta x + 1 - \alpha\right) - \alpha\beta x\right] \\ &= \zeta x\left\{\left[2\beta x + 1 + 3\left(1 - \alpha\right)\right]\beta x + 2\left(1 - \alpha\right)\right\} \\ &> 0,\end{aligned}$$

where the first inequality holds since $u_0 < 0$, while the last inequality holds as $x > 0$ and the term in curly brackets is clearly positive. Therefore, by the implicit function theorem $\partial x_A^* / \partial s = -\left(\partial f / \partial s\right)\left(x_A^*, s\right) / \left(\partial f / \partial x\right)\left(x_A^*, s\right) < 0$ and the first statement is established.

By rewriting the second inequality in system (51) – which holds as equality – as

$$E = \frac{1}{\rho\left(1 + s\right)t} \ln\left(\frac{\beta x + 1 - \alpha}{\alpha s}\right)$$

and differentiating the RHS with respect to $s$ and $t$ respectively we get:

$$\begin{aligned}\frac{\partial E}{\partial s} &= -\frac{1}{\rho\left(1 + s\right)t}\left[\frac{1}{1 + s}\ln\left(\frac{\beta x + 1 - \alpha}{\alpha s}\right) - \frac{\beta\left(\partial x\right) / \left(\partial s\right)}{\beta x + 1 - \alpha} + \frac{1}{s}\right], \\ \frac{\partial E}{\partial t} &= -\frac{1}{\rho\left(1 + s\right)t^2}\ln\left(\frac{\beta x + 1 - \alpha}{\alpha s}\right).\end{aligned}$$

Since from the first part of the proof we know that at the optimum $\left(\partial x\right) / \left(\partial s\right) < 0$, a sufficient condition for $\left(\partial E\right) / \left(\partial s\right)$ to be negative is

$$\frac{\beta x + 1 - \alpha}{\alpha s} > 1 \qquad \text{for all } x > 0,$$

which is equivalent to (10) for $p\left(x\right)$ as in (38). Clearly, the same condition is sufficient for $\left(\partial E\right) / \left(\partial t\right) < 0$ as well, and the proof is complete. ∎

# References


[1] Allingham, M. G., and Sandmo, A. (1972), Income tax evasion: a theoretical analysis, *Journal of Public Economics* **1**: 323-38.

[2] Briys, E., and Schlesinger, H. (1990), Risk-aversion and the propensities for self-insurance and self-protection, *Southern Economic Journal* **57**: 458-467.

[3] Cowell, F. A. (2004), Carrots and sticks in enforcement, in: Aaron H.J. and Slemrod, J. (Eds.), *The Crisis in Tax Administration*, The Brookings Institution, Washington D. C.: 230-275.

[4] Chen, K., and Chu, C. Y. C. (2005), Internal control versus external manipulation: a model of corporate income tax evasion, *Rand Journal of Economics* **36**: 151-64.

[5] Chiu, W. H. (2000), On the propensity to self-protect, *Journal of Risk and Insurance* **67**: 555-78.

[6] Crocker, K. J., and Slemrod, J. (2005), Corporate tax evasion with agency costs, *Journal of Public Economics* **89**: 1593-1610.

[7] Ehrlich, I., and Becker, G. S. (1972), Market insurance, self-insurance and self-protection, *Journal of Political Economy* **80**: 623-48.

[8] Hamdani, A. (2003), Gatekeeper liability, *Southern California Law Review* **77**: 53-121.

[9] Lee, K. (2001), Tax evasion and self-insurance, *Journal of Public Economics* **81:** 73-81.

[10] Kraakman, R. (1986), Gatekeepers: The Anatomy of a third-party enforcement strategy, *Journal of Law, Economics & Organization* **2**: 53-104.

[11] Morck, R. (2009), Generalized agency problems, *NBER Working Paper series* **15051**.

[12] Privileggi, F., Marchese, C., and Cassone, A. (2001), Agent's liability versus principal's liability when attitudes toward risk differ, *International Review of Law and Economics* **21**: 181-95.

[13] Sandmo, A. (2005), The theory of tax Evasion: a retrospective view, *National Tax Journal* **58**: 643-63.

[14] Sévi, B., and Yafil, F. (2005), A special case of self-protection: the choice of a lawyer, *Economics Bulletin* **4**: 1-8.

[15] Sweeney, G., and Beard, T. R. (1992), The comparative statics of self-protection, *The Journal of Risk and Insurance* **59**: 301-309.

[16] Yitzhaki, S. (1987), On the excess burden of tax evasion, *Public Finance Quarterly* **15**: 123-37.

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