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Abstract

This paper examines the segregative properties of endogenous processes of jurisdiction formation *à la* Tiebout in the presence of a central government who makes equalization transfers to jurisdictions in such a way as to maximize a welfarist objective. Choice of location by households, of local public good provision by jurisdictions, and of equalization grants and tax by the central government are assumed to be made simultaneously, taking the choices of others as given. Two welfarist objectives for the central government are considered in turn: Leximin and Utilitarianism. If the central government pursues a Leximin objective, it is easily shown that the only stable jurisdiction structures that can emerge are those in which the jurisdictions' poorest households have all the same wealth. A richer class of stable jurisdiction structures are compatible with a central utilitarian government. Yet, it so happens that, if individual preferences are additively separable, the class of households preferences that garantee the segregation of any stable jurisdiction structure remains unchanged by the presence of a central government.

1   Introduction

There is a wide presumption (see for instance Wooders (1999)) that decentralized processes of jurisdiction formation *à la* Tiebout (1956) lead individ-

uals to self-sort into homogenous communities. In a recent paper, Gravel and Thoron (2007) investigate the validity of this intuition in the classical model of jurisdictions formation developed by Westhoff (1977) (see also Greenberg and Weber (1986) and Demange (1994) among many other contributions). In this model, unequally wealthy households who have the same preference for a local public good and a private good choose simultaneously their place of residence in a finite set of locations. Households who choose the same location form a *jurisdiction* and produce a local public good by applying a democratically chosen tax rate to all residents' wealth. Any such simultaneous choice of residence by households is referred to as a *jurisdiction structure*. The analysis of Gravel and Thoron (2007) concerns *stable* jurisdiction structures, which satisfy the additional property of being *robust to individual deviations*. The question raised by Gravel and Thoron (2007) is whether stable jurisdiction structures lead households to self sort, or segregate, themselves according to their wealth. The notion of segregation used is that known under the heading of *consecutiveness* in the coalition formation literature (see e.g. Greenberg and Weber (1986)). A jurisdiction structure is segregated in this sense if, for any two jurisdictions with different *per capita* wealth, the richest individual in the poorer jurisdiction is (weakly) poorer than the poorest individual in the richer jurisdiction. This form of segregation is clearly extreme and is interesting precisely for the reason that it represents a pure and "idealized" notion of segregation. Gravel and Thoron (2007) identify a condition on households preferences for the private and the local public good that is necessary and sufficient for the segregation of any stable jurisdiction structure. The condition requires households to consider public good to be either *always* a gross complement, or *always* a gross substitute to the private good. While stringent, and violated by several preferences, including additively separable ones, the condition is not implausible. For this reason, the analysis of Gravel and Thoron (2007) seems to provide some theoretical ground on the widespread belief that decentralized processes of jurisdiction formation are driven by segregative forces.

In this paper, we investigate the extent to which the segregative feature of endogenous jurisdiction formation is affected by the introduction of a central government. Introducing a central government in models of endogenous jurisdiction formation strikes us as an important step toward improving the realism of these models. In many countries, one finds indeed a juxtaposition of several levels of governments: central and local. It is also commonly observed that the central government puts into place *equalization payment schemes* which aims at redistributing funds across jurisdictions so as to achieve specific normative objectives. It is therefore interesting to examine the consequence of central government's intervention on the segregative properties of the endogenous formation of local jurisdictions by freely mobile households.

Doing this requires one to specify:

1) the instruments available to the central government

2) the objective of the central government and

3) the nature of the interaction between central government, households and local governments.

As for the first point, we assume that the central government tax households at a fixed rate and redistributes tax revenues between jurisdictions in such a way as to maximize some objective function. Although stylized, this modeling of the redistribution performed by the central government does not provide a bad approximation of many existing systems of equalization payments observed in practice. It is consistent both with the so-called *horizontal* equalization payments scheme of the sort existing in Scandinavian countries and Germany - where the central government transfer money between jurisdictions directly and do not tax households - and the *vertical* schemes observed in several other countries (like Belgium, France, Canada, Australia, Switzerland and India, to mention just a few) where the funds used in the transfers to local jurisdictions come from households' tax. A normative analysis of equalization payment in federations can be found in Gravel and Poitevin (2006) while a broader recent discussion of equalization in structures with multiple levels of governance is provided by Boadway (2006). Yet our modelling of the central government is stylized in the sense that it does not allow the central government to redistribute wealth between households by using, for instance, a progressive wealth tax scheme. We further specify the objective of the central government by assuming it to be *welfarist*. Hence the central government is depicted as choosing equalization grants and households tax in such a way as to maximize an increasing function of the households' utilities (assumed to be interpersonally comparable and cardinally measurable, using Sen (1977a)'s terminology). Thanks to the classical work of Deschamps and Gevers (1978), there are only two "natural" types of such welfarist central governments: A Utilitarian one, which maximizes the sum of households utilities and a Leximin one, which maximizes the utility of the worst off household and, in case of indifference of the worst off household, switch to the second worst off and so on. We perform the analysis with the two objectives even though, as it turns out, the analysis is (much) more interesting when done with a Utilitarian one.

As for the interaction between households, central and local governments, we model it as taking place *simultaneously*, as done, without central government, in the conventional Westhoff (1977)'s setting. Specifically, we define a stable jurisdiction structure with a central government to be an assignment, to every location, of a local tax rate, a central government net transfer, and a set of households such that:

1) the local tax rate is, in every jurisdiction, the favorite one of some member of the jurisdiction (say the median), given central government transfers and wealth taxes and local tax base

2) the equalization transfers given to jurisdictions and the central gov-

ernment tax rate maximize the welfarist objective of the central government, given the partition of households into jurisdictions and local tax rates and

3) each household weakly prefers its jurisdiction to any other, given the central government transfers, local and central tax rates and local public good provision.

The question addressed is whether the condition on households' preferences identified in Gravel and Thoron (2007) remains necessary and sufficient for ensuring the segregation of any stable jurisdiction structure in presence of a central welfarist government. This condition, called the Gross Substitutability/Complementarity (GSC) condition, requires households preferences to be such that they consider the public good to be either always a gross substitute, or always a gross complement, to the private good. If the GSC condition remains necessary but ceases to be sufficient, we would interpret this as an indication that the presence of a central government *mitigates* the segregative tendencies of endogenous jurisdiction formation. Conversely, if the GSC condition is not any more necessary but remains sufficient, we would interpret this as an evidence that central government intervention increases the self-sorting forces underlying jurisdiction formation (as segregation of any stable jurisdiction structure would then be observed on a wider class of individual preferences). If the GSC condition remains necessary and sufficient, we would conclude that central government intervention has no impact on the segregative properties of jurisdiction formation. A last, and interpretatively difficult, logical possibility is for the GSC condition to become neither necessary and sufficient for guaranteeing the segregation of a stable jurisdiction structure. As it happens however, this eventuality will not arise.

We examine this question by considering in turn a Leximin and a Utilitarian central government, making, in the later case, the extra assumption that the utility function used by the social planner is additively separable between the public and the private good. There is not much analysis to be performed in the case of a Leximin government. For it is shown easily in that case that the only stable jurisdiction structures that can exist are those in which all jurisdictions' poorest households have the same wealth and where, as a result, it is optimal for the central government to perform no equalization (and to raise therefore no tax). Results are more interesting with a Utilitarian central government because the optimal redistribution by such a government is compatible with a much wider class of stable jurisdiction structures. Yet we show in this paper that, if the central Utilitarian government uses additively separable individual utility functions, the GSC condition is necessary and sufficient for the segregation of any stable jurisdiction structure in the presence of central government. Hence it appears that central government intervention, at least when performed in the way modelled in this paper, has no impact on the segregative properties of endogenous processes of jurisdiction formation. Stable jurisdictions structure will be

segregated in the presence of a welfarist redistributive central government if and only if households preferences are such that any stable jurisdictions structures without central government are segregated.

The rest of the paper is organized as follows. The next section introduces the basic notation and presents the result for Leximin government. Section 3 states and proves the main result for the Utilitarian government and section 5 concludes.

2 Endogenous jurisdictions formation with a central government

The model we consider is similar to that of Gravel and Thoron (2007) and Westhoff (1977), with the exception that, contrary to these authors, we assume a *finite number*, rather than a continuum, of households. As we hope to make clear, this change in the formal setting does not affect substantially the analysis done in Gravel and Thoron (2007). Yet it facilitates the writing of the central government's objective (the Leximin objective is not easy to define if there is a continuum of individuals). On the other hand, the continuum setting is probably more natural for modeling processes of jurisdiction formation in which households are "small" relative to jurisdictions in the sense that they behave as if their individual decisions had no impact on the jurisdiction aggregate wealth and/or tax rate.

We assume specifically that there are n households, indexed by h, taken from some finite set N. Household h has an exogenously given strictly positive wealth ω_h. We denote by $\omega = (\omega_1, ..., \omega_n)$ the distribution of wealth in the population written in such a way that $\omega_h \leq \omega_{h+1}$ for $h = 1, .., n-1$. Households have all the same preference for a public good (Z) and a private good (x) that are represented by a twice differentiable, strictly increasing and strictly concave[1] utility function $U : \mathbb{R}_+^2 \to \mathbb{R}$. For one of the result concerning the Utilitarian central government, we will also assume that households preferences are *additively separable* so that they can be represented by a utility function $U : \mathbb{R}_+^2 \to \mathbb{R}$ that can be written, for every $(\overline{Z}, \overline{x}) \in \mathbb{R}_+^2$, as:

$$U(\overline{Z}, \overline{x}) = f(\overline{Z}) + h(\overline{x}) \tag{1}$$

for some twice differentiable increasing and concave real valued functions f and g having both $\mathbb{R}_+$ as domain. Given any bundle of public and private good $(\overline{Z}, \overline{x}) \in \mathbb{R}_+^2$, we define $MRS(\overline{Z}, \overline{x})$, the *marginal rate of substitution*

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[1]Our terminology for concavity and quasi-concavity of a function $f : A \to \mathbb{R}$ $(A \subseteq \mathbb{R}^k)$ is as follows: f is strictly concave if, for every $\alpha \in]0, 1[$ and for every distinct a, $b \in A$, $f(\alpha a + (1 - \alpha)b) > \alpha f(a) + (1 - \alpha)f(b)$ and f is quasi-concave if, for every a, b, $x \in A$, and every $\alpha \in [0, 1]$, $f(a) \geq f(x)$ and $f(b) \geq f(x)$ imply $f(\alpha a + (1 - \alpha)b) \geq f(x)$.

of public good to private good evaluated at $(\overline{Z}, \overline{x})$, by:

$$MRS(\overline{Z}, \overline{x}) = \frac{\partial U(\overline{Z}, \overline{x})/\partial Z}{\partial U(\overline{Z}, \overline{x})/\partial x} \qquad (2)$$

As in Gravel and Thoron (2007), we find useful to express the condition on households preferences that is necessary and sufficient for the segregation of a stable jurisdiction in terms of the properties of the *Marshallian demands* associated with these preferences. For this purpose, we denote by $Z^M(p_Z, p_x, R)$ and $x^M(p_Z, p_x, R)$ the households' Marshallian demands for the public and private good (respectively) when the prices of these goods are p_Z and p_x and the household's income is R. Marshallian demand functions are the (unique under our assumptions) solution of the program:

$$\max_{Z,x} U(Z, x) \text{ subject to } p_Z Z + p_x x \le R$$

Given again our assumptions, Marshallian demands are differentiable functions of their arguments (except, possibly, at the boundary of $\mathbb{R}_+^2$). We emphasize that we view Marshallian demands as dual representations of preferences rather than description of behavior (after all households rarely if ever purchase local public good on competitive markets).

We further assume that Marshallian demand for public good satisfies the following additional *regularity* condition (see Gravel and Thoron (2007) for a discussion of the meaning of this condition).

Condition 1: If there exists a public good price $\overline{p}_Z$, an income level R and a non-degenerate interval I of strictly positive real numbers such that, $Z^M(\overline{p}_Z, p_x, R) = Z^M(\overline{p}_Z, p'_x, R)$ for all prices p'_x and p_x in I, then, for all $(p_Z, p_x, R) \in \mathbb{R}_+^3$, we must have $Z^M(p_Z, p_x, R) = h(p_Z, R)$ for some function $h : \mathbb{R}_{++}^2 \to \mathbb{R}_+$.

The problem considered in this paper is that of identifying the properties of the various *jurisdiction structures* that can emerge when households freely choose their location and share, when they locate at the same place, the benefit of a local public good produced by local tax revenues and a central government grant. This intervention of the central government is the main distinctive ingredient of our model as compared to what is done in the literature (e.g. Demange (1994), Gravel and Thoron (2007), Greenberg (1983), Westhoff (1977), Greenberg and Weber (1986), Wooders (1978) and Wooders (1999)).

From a formal point of view, we define a jurisdiction structure with a central government as follows.

Definition 1 *A jurisdiction structure with a central government is a triplet* $S = (\{N_j\}_{j=1}^l; g, c; t)$ *made of:*
1) a collection $\{N_j\}_{j=1}^l$ of l non-empty sets of households, to be referred to

as jurisdictions, satisfying $N_j \cap N_{j'} = \varnothing$ for all j and j' (with $j \neq j'$) in $\{1,...,l\}$ and $\bigcup_{j=1} N_j = N$,

2) a vector $g = (g_1,...,g_l,c\} \in \mathbb{R}^l \times [0,1]$ of l equalization grants and one central government tax rate satisfying $\sum_{j=1}^{l} g_j \leq c \sum_{j=1}^{l} g_j$, $g_j \geq -\overline{\omega}_j$ for every $j \in \{1,...,l\}$ where $\overline{\omega}_j = \sum_{i \in N_j} \omega_i$,

3) a vector $t = (t_1,...,t_l) \in \mathbb{R}^l$ of local tax rates satisfying, for every $j = 1,...,l$, $t_j \in [\frac{-g_j}{\overline{\omega}_j}, 1-c]$

In words, a jurisdiction structure with a central government is a partition of the set of households into l jurisdictions (condition 1), each of which receiving a (possibly negative) central government grant and being characterized by a (possibly negative) local tax rate. Condition 2) requires the central government to balance its budget so that the sum of the (possibly negative) grants given to jurisdictions can not exceed the revenues obtained from taxing all households at the rate c. Condition 2) also limits the fiscal power of the central government to raise taxes in a given jurisdiction to the extent of this jurisdiction's tax base. Condition 3) requires jurisdiction's local tax rates to be less than one and greater than (the negative of) the ratio of the central government grant over the jurisdiction tax base. Since in every jurisdiction j, household h in N_j has access to $t_j \overline{\omega}_j + g_j$ units of public good and has $(1-t_j)\omega_h$ units of wealth available for private consumption, the condition that $t_j \in [\frac{-g_j}{\overline{\omega}_j}, 1]$ guarantees that both $t_j \overline{\omega}_j + g_j \geq 0$ and $(1-t_j)\omega_h \geq 0$ are satisfied. We emphasize that negative local tax rates are possible in a world with a central government. A household living in a jurisdiction which receives a large grant may prefer local tax rate to be negative and, therefore, use part of the central government grant in private spending.

Given any jurisdiction structure, we denote by n_j the number of households in N_j (with of course $\sum_{j=1}^{l} n_j = n$), and we write the set N of households as:

$$N = \{1,...,n_1, n_1+1,...,n_1+n_2, n_1+n_2+1,...,n-n_l+1,...,n\}$$

where, for all $m = 0,...,l-1$, one has $\omega_h \leq \omega_{h+1}$ for $h \in \{\sum_{i=0}^{m} n_i + 1,...,\sum_{i=0}^{m+1} n_i - 1\}$ (under the convention that $n_0 = 0$).

As mentioned in the introduction, our modeling of the central government covers both the possibility that it transfers money between jurisdictions without taxing households (a form of equalization often referred to as

"horizontal" in the literature) and the possibility that it combines horizontal with vertical taxation. Yet our model, by limiting the taxation power of the central government to linear wealth tax scheme, rules out the possibility for the central government to use wealth tax for redistributive purposes (by making it progressive for instance). Of course providing the central government with the *full* power of redistributing wealth (by choosing the tax paid by each household for instance based on its characteristic) would devoid the problem examined in this paper of much of its interest. For any central government that is averse to wealth inequality would obviously choose, with such a power, to equalize wealth perfectly between households living in the same jurisdiction. Hence any stable jurisdiction structure with a central government endowed with the power of redistributing perfectly wealth between households would be trivially segregated in the sense that it will be characterized by a collection of jurisdictions, each of which populated by households with identical net of tax wealth. But between the full power given to a central government of taxing individually each household, and the extremely small one considered here of taxing all of them at the same rate, there is a large spectrum of possibilities that, undoubtedly, deserves further analysis.

Denote by $\Phi(\tau, \varpi, \omega_i, \gamma, c) = U(\tau\varpi + \gamma, (1 - \tau - c)\omega_i)$ the utility received by a household with wealth ω_i when the government chooses a tax rate of c and when the household live in a jurisdiction with local tax rate τ, aggregate wealth ϖ and central government grant γ. The function Φ so defined has several properties that we record in the following lemma (whose straightforward proof is omitted).

Lemma 1 *If U satisfies the conditions discussed above, Φ is a twice differentiable function of its five arguments, is strictly increasing and concave with respect to ω_i, ϖ and γ (taking τ and c as given) and is strictly concave and single peaked[2] with respect to τ (taking ω_i, ϖ, γ and c as given).*

The important property of Φ is its strict single peakedness. It implies that a household with wealth ω_i and who faces a rate c of central tax rate has a unique favorite local tax rate $\tau^*(\varpi, \omega_i, \gamma, c)$ in any jurisdiction with wealth ϖ and central government grant γ to which it may belong. This unique favorite local tax rate is the solution of the program:

$$\max_{\tau \in [\frac{-\gamma}{\varpi}, 1-c]} \Phi(\tau, \overline{\varpi}, \omega_i, \gamma, c) \tag{3}$$

and is, for this reason, a continuous function of all its three arguments.

We are interested in the properties of the likely outcome of a free choice of a location by households in the presence of a central government. This likely outcome must be such that:

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[2] A function $f : A \to \mathbb{R}$ ($A \subset \mathbb{R}$) is strictly single peaked if, for all a, b and $c \in A$ such that $a < b < c$, $f(c) > f(b) \Rightarrow f(b) > f(a)$ and $f(a) > f(b) \Rightarrow f(b) > f(c)$.

1) each household finds its location optimal given central government's equalization scheme and local tax rates, under the assumption that it can move freely between locations and that it has no effect on jurisdictions' choices of tax rates and aggregate wealth,

2) the central government finds optimal its vector of equalization grants and wealth tax rate, given the partition of individuals into jurisdictions and jurisdictions' choices of local tax rates

3) each jurisdiction finds its choice of tax rate and public good provision optimal, given its population, tax base and central government grant.

Concerning the last point, we adopt the view that each jurisdiction's choice of local tax rate is minimally democratic in the sense that it is contained *between* the *smallest* and the *largest* favorite tax rates of the jurisdiction members. The rule for selecting this tax rate is inconsequential for the results. In many models of endogenous jurisdiction formation with public good provision where voting is assumed, the jurisdiction tax rate would the one that occupies the *median position* in the jurisdiction's distribution of favorite taxes. While the analysis of this paper applies to this particular rule of selection of local tax rates, they are valid for other rules as well. Given a jurisdiction structure with a central government $(\{N_j\}_{j=1}^l; g, c; t)$, define, for every jurisdiction $j \in \{1, ..., l\}$ the smallest and largest favorite tax rates t_{j*} and t_j^* respectively by:

$$
\begin{aligned}
t_{j*} &= \min_{i \in N_j} \arg \max_{\tau \in [\frac{-g}{\overline{\omega}}, 1]} \Phi(\tau, \overline{\omega}_j, \omega_i, g_j, c) \text{ and} \\
t_j^* &= \max_{i \in N_j} \arg \max_{\tau \in [\frac{-g}{\overline{\omega}}, 1]} \Phi(\tau, \overline{\omega}_j, \omega_i, g_j, c)
\end{aligned}
$$

We shall therefore assume that, in any jurisdiction j, the local tax rate t_j satisfies $t_j \in [t_{j*}, t_j^*]$.

As for the objective of the central government, we represent it by means of a social ordering[3] R^S, depending upon the jurisdiction structure S, and defined on the set $\mathbb{G}(S)$ of all vectors of grants $g \in \mathbb{R}^l$ and tax rate $c \in [0, 1]$ such that $g_j + t_j \overline{\omega}_j \geq 0$ and $\sum_{j=1}^l g_j \leq c \sum_{i=1}^n \omega_i$. For any (g, c) and $(g', c') \in \mathbb{G}(S)$, we interpret the statement $(g, c) \ R^S \ (g', c')$ as meaning "equalization scheme (g, c) is socially weakly better than equalization scheme (g', c')". We further assume that R^S is welfarist (see e.g. Sen (1977b) or, for a recent account, Blackorby, Bossert, and Donaldson (2005)) in the sense that it compares alternative vectors of equalization grants and central tax rate on the sole basis of the *distribution of utilities* achieved by households in the various jurisdictions of S. Formally, R^S is welfarist if there exists an ordering R^U on the set $\mathbb{R}^n$ of utility vectors such that:

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[3]An ordering is a reflexive, complete and transitive binary relation.

$$(g,c) \ R^S \ (g',c') \Longleftrightarrow$$

$$\Phi(t_1, \overline{\omega}_1, \omega_1, g_1, c), ..., \Phi(t_l, \overline{\omega}_l, \omega_n, g_l, c) \ R^U \ \Phi(t_1, \overline{\omega}_1, \omega_1, g_1', c'), ..., \Phi(t_l, \overline{\omega}_l, \omega_n, g_l', c')$$

As is well-known, welfarism requires utilities to be interpersonally comparable and, sometimes, cardinally measurable as well. Following Deschamps and Gevers (1978), we adopt the view that utilities are cardinally measurable and interpersonally comparable. We also assume that the ordering R^U of utility vectors on which the central government ordering R^S is based is *Pareto inclusive* (increasing households utility levels *ceteris paribus* improves matters), *anonymous* (individual names don't matter) and satisfies the so-called principle of "*independence with respect to unconcerned*" households (see e.g. Blackorby, Bossert, and Donaldson (2005), (p. 115)) according to which the ranking of any two states should be independent from the utilities of the households who are indifferent between the states. We also assume that the central government has a minimal aversion to utility inequality as expressed in Hammond (1976)'s *minimal equity principle*.

With this specification of the central government's objective, we define formally our notion of stability as follows.

Definition 2 *A jurisdiction structure* $S = (\{N_j\}_{j=1}^l; g, c; t)$ *with a welfarist central government endowed with a social objective* R^S *is stable if*
1) for all $j \in \{1, ..., l\}$ *and all* $i \in N_j$, $\Phi(t_j, \overline{\omega}_j, \omega_i, g_j, c) \geq \Phi(t_{j'}, \overline{\omega}_{j'}, \omega_i, g_{j'}, c)$ *for all* $j' \in \{1, ..., l\}$
2) for all $j \in \{1, ..., l\}$, $t_j \in [t_{j*}, t_j^*]$
3) $(g,c) \ R_S \ (g',c')$ *for all* $(g',c') \in \mathbb{G}(S)$.

This definition of stability rides on the assumption that households as well as local and central governments take their decision simultaneously, considering as given the behavior of others. While this way of proceeding generalizes naturally the (atemporal) Westhoff framework, it may be viewed as limiting the power of the central government to shape the process of jurisdiction formation. An alternative could have been to assume a *two-stage* setting in which the central government, like a Stackelberg leader, would play before households and local governments and would choose its redistributive grants and households tax by anticipating the impact of its choice on the stable jurisdiction structure that would emerge in a second stage. While this alternative strategy would have given more power to the central government to influence jurisdiction structures, it would have raised delicate modeling issues. First, choosing a set of equalization grants and households wealth tax rate before knowing the jurisdiction structure that will prevail raises the problem of the financial viability of the equalizations grants. What if the central government chooses, in the first stage, an equalization scheme which imposes a tax burden to a jurisdiction which, in the second stage, will

be empty ? Second, and more importantly, there may be many partitions of individuals into jurisdictions that can give rise to a stable jurisdiction structure for a given equalization scheme. In that case, how is the central government going to predict which of the stable jurisdiction structures will emerge ? Third, everything else being the same, the central government, at least if it uses a Pareto inclusive social welfare function, would have a tendency to favour the "trivial" structure in which all households are put in the same jurisdiction and where, therefore, there is no role for a central government. The reason for this tendency is that, the local public good being non-rival, producing any quantity of it in a larger jurisdiction tend to be preferable from a social welfare view point because its cost can be shared by a larger number of tax payers.[4]

Accepting that households as well as central and local government take their decision simultaneously, we know from Deschamps and Gevers (1978) that there are *only two* welfarist social orderings for the central government that satisfy the properties mentioned above (Pareto, anonymity, independence with respect to unconcerned individuals, and Hammond's equity) and that use a cardinally meaningful and interpersonally comparable information on households' utilities. These are the *Leximin* and the *Utilitarian* orderings. The Leximin ordering compares social states by lexicographically comparing the households' *positions* in the distribution of utilities associated to the states, with utilities ordered from the smallest to the largest. Formally, for every utility vector $u \in \mathbb{R}^n$, let $u_{(.)}$ denote the (ordered) permutation of u such that $u_{(i)} \le u_{(i+1)}$ for $i = 1, ..., n-1$. The Leximin ordering R^U_{LMIN} of the set $\mathbb{R}^n$ of all possible utility vectors is defined as follows:

$$u \ P^U_{LMIN} \ u' \quad \Leftrightarrow \quad \exists \ i \in N \text{ such that } u_{(h)} = u'_{(h)} \text{ for all } h < i \text{ and } u_{(i)} > u'_{(i)}$$

$$u \ I^U_{LMIN} \ u' \quad \Leftrightarrow \quad u_{(h)} = u'_{(h)} \text{ for all } h \in \{1, ..., n\} \text{ and}$$

$$u \ R^U_{LMIN} \ u' \quad \Leftrightarrow \quad u \ P^U_{LMIN} \ u' \text{ or } u \ I^U_{LMIN} \ u'$$

As it turns out, there is not much analysis to be performed with a Leximin central government. For, as established in the following proposition, the only stable jurisdiction structures that can exist with a Leximin central government are those where the poorest households of each jurisdiction have all the same wealth. The trivial jurisdiction structure in which all households are in the same jurisdiction is of course a particular example of such jurisdiction structures.

Proposition 1 *A jurisdiction structure $S = (\{N_j\}^l_{j=1}; g, c; t)$ with a Leximin central government is stable iff $\omega_1 = \omega_{n_1+1} = \omega_{n_1+n_2+1} = ... =$*

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[4]This is true at least if the central government has full information on households' wealths. See Gravel and Poitevin (2005) for an analysis of the (difficult) problem of the optimal choice of a jurisdiction structure when the central government is imperfectly informed about households' wealth.

ω_{n-n_l+1}.

Proof. *By contraposition, assume that $S = (\{N_j\}_{j=1}^l; g, c, t)$ is a stable jurisdiction structure with a welfarist central government in which there are jurisdictions j and $j' \in \{1, ..., l\}$ such that $\omega_{n_1+...+n_{j'-1}+1} \neq \omega_{n_1+...+n_{j-1}+1}$. We wish to show that the welfarist central government can not be Leximin. Without loss of generality, assume that jurisdiction j contains a household who is the worst off in the whole population. Because this household must be the (not necessarily unique) poorest in its jurisdiction, one has:*

$$\Phi(t_j, \overline{\omega}_j, \omega_{n_1+...+n_{j-1}+1}, g_j, c) \leq \Phi(t_k, \overline{\omega}_k, \omega_i, g_k, c) \qquad (4)$$

for all $k \in \{1, ..., l\}$ and $i \in N_k$. By stability one has also:

$$\Phi(t_j, \overline{\omega}_j, \omega_{n_1+...+n_{j-1}+1}, g_j, c) \geq \Phi(t_{j'}, \overline{\omega}_{j'}, \omega_{n_1+...+n_{j-1}+1}, g_{j'}, c) \qquad (5)$$

and:

$$\Phi(t_{j'}, \overline{\omega}_{j'}, \omega_{n_1+...+n_{j'-1}+1}, g_{j'}, c) \geq \Phi(t_j, \overline{\omega}_j, \omega_{n_1+...+n_{j'-1}+1}, g_j, c) \qquad (6)$$

Either (i) $\omega_{n_1+...+n_{j'-1}+1} < \omega_{n_1+...+n_{j-1}+1}$ or:
(ii) $\omega_{n_1+...+n_{j'-1}+1} > \omega_{n_1+...+n_{j-1}+1}$. Yet assuming (i) would imply, using (5) and the fact that Φ is increasing with respect to private wealth, that:

$$\Phi(t_j, \overline{\omega}_j, \omega_{n_1+...+n_{j-1}+1}, g_j, c) > \Phi(t_{j'}, \overline{\omega}_{j'}, \omega_{n_1+...+n_{j'-1}+1}, g_{j'}, c)$$

in contradiction with (4). Hence (i) can not hold. If (ii) holds, then one has, thanks to (6) and the monotonicity of Φ with respect to private wealth, that:

$$\Phi(t_{j'}, \overline{\omega}_{j'}, \omega_{n_1+...+n_{j'-1}+1}, g_{j'}, c) > \Phi(t_j, \overline{\omega}_j, \omega_{n_1+...+n_{j-1}+1}, g_j, c)$$

so that household $n_1 + ... + n_{j-1} + 1$ is strictly worse off than household $n_1+...+n_{j'-1}+1$ who is the poorest (and therefore the worst off by monotonicity of Φ) in jurisdiction j'. Now, let W be defined by
$W = \{k \in \{1, ..., l\} : \Phi(t_k, \overline{\omega}_k, \omega_{n_1+....+n_{k-1}+1}, g_k, c) = \Phi(t_j, \overline{\omega}_j, \omega_{n_1+...+n_{j-1}+1}, g_j, c)\}$
and let, for all $k \in W$, $\min(k)$ be defined by:

$$\min(k) = \{i \in \{\sum_{h=1}^{k-1} n_h + 1, ..., \sum_{h=1}^{k} n_h\} : \omega_i = \omega_{n_1+....+n_{k-1}+1}\}.$$

W is clearly non-empty since $j \in W$. Moreover, if $k \in W$ and $k \neq j$, then $\omega_{n_1+....+n_{k-1}+1} = \omega_{n_1+...+n_{j-1}+1}$. Indeed assume by contradiction that $\omega_{n_1+....+n_{k-1}+1} \neq \omega_{n_1+...+n_{j-1}+1}$. Either (i) $\omega_{n_1+...+n_{k-1}+1} < \omega_{n_1+...+n_{j-1}+1}$ or (ii) $\omega_{n_1+...+n_{k-1}+1} > \omega_{n_1+...+n_{j-1}+1}$. As above, case (i) is incompatible, given (5), with (4) while case (ii) is incompatible, given (6), with the definition of W that $\Phi(t_k, \overline{\omega}_k, \omega_{n_1+....+n_{k-1}+1}, g_k, c) = \Phi(t_j, \overline{\omega}_j, \omega_{n_1+...+n_{j-1}+1}, g_j, c)$

*for all $k \in W$. Let $A = \bigcup_{k \in W} \min(k)$. Hence A is the set of all households
who are the worst off in the whole population in the jurisdiction structure
$S = (\{N_j\}_{j=1}^l; g, c; t)$. By the reasoning just made, all households in A have
the same wealth. Moreover A is a finite set. Now consider taking away from
jurisdiction j' some amount of grant Δ and dividing it up equally among
all jurisdictions in k so as to keep constant the central government budget
constraint. For a suitably small Δ, this change in the central government
transfer policy increases the well-being of all households in A. But this shows
that the original equalization grant vector g was not maximizing a Leximin
ordering, given c. ∎*

Hence, the only stable jurisdiction structures that can exist with a Lex-
imin central government are those where all households who are the poorest
of their jurisdictions have the same wealth. The intuition behind this result
is quite simple. A Leximin government wants to transfer money to the ju-
risdiction that contains one of the worst off households (who must clearly
be the poorest in its jurisdiction). By stability, any such worst off household
prefers staying in its jurisdiction than moving elsewhere while the poorest
households in other jurisdictions also prefer staying where they are than
moving to the jurisdiction containing the worst off households. Except if the
wealth of these poorest households in all jurisdictions is the same, these two
conditions for stability imply that worst off households are strictly worse off
than at least one household who is the poorest in its jurisdiction. But if this is
the case, then the transfers given by the central government to jurisdictions
are not optimal from a Leximin point of view. Notice that the reasoning
holds irrespective of the tax rate chosen by the central government.

Jurisdiction structures in which all jurisdictions contain some of the pop-
ulation poorest households are, admittedly rather peculiar. Hence, if one
wants to go beyond the nihilistic message that, except this for this pecu-
liar case, no analysis of stable jurisdiction structures can be performed in
presence of a central government, we must abandon the assumption that it
could be Leximin and focus on the other possibility that it is Utilitarian.

For a Utilitarian government, there are several stable jurisdiction struc-
tures that are not of the peculiar variety described in proposition 1, as
illustrated in the following simple example.

Example 1 *There are 7 households with utility function $U(Z, x) = \ln(1 +
Z) + x$. Households 1, 2 and 3 have a wealth of 2 and households 4 to 7 have
a wealth of $3/2$. Consider the 2-jurisdictions structure in which the central
government gives no equalization grants (and therefore levy no taxes) and
where the households 1, 2 and 3 are put in jurisdiction 1 while households 4
to 7 are put in jurisdiction 2. The local tax rates t_1 and t_2 that will prevail in
the two jurisdictions will be the favorite ones of the identical households who*

live there. Solving program (3), these optimal tax rates are easily found to be $t_1 = 1/3$ and $t_2 = 1/2$. We notice that with these tax rates, any household in jurisdiction 1 enjoys a utility level of $\ln 3 + 4/3 \approx 2.4319$, which is larger than the utility of $\ln 4 + 1 \approx 2.386$ the household would enjoy if it were to move to jurisdiction 2 and to get the tax and public good package available there. Analogously, a household in jurisdiction 2 enjoys a utility level of $\ln 4 + 3/4 \approx 2.1363$ by staying where it is while the move to jurisdiction 1 would provide this household with a (lower) utility of $\ln 3 + 1 \approx 2.0986$. Hence households have no incentive to move from their jurisdiction. To see that a Utilitarian central government finds optimal to gives zero equalization grants and raise no taxes, it is sufficient (given concavity of U) to show that:

$$(0,0) \in \arg \max_{\gamma, t} 3[\ln(3 + \gamma) + 2(2/3 - t) + 4[\ln(4 + 12t - \gamma) + \frac{3}{2}(1/2 - t)]$$

so that the (first order) conditions:

$$\frac{3}{(3 + \gamma^*)} = \frac{4}{(4 + 12t^* - \gamma^*)} \tag{7}$$

and:

$$-6 + \frac{48}{(4 + 12t^* - \gamma^*)} - 6 = 0 \tag{8}$$

hold for $\gamma^* = t^* = 0$, which is indeed the case. Hence we have a non trivial stable structure with a central government.

We want to find a condition on the household's utility function used by the Utilitarian central government that is necessary and sufficient for guaranteeing that any stable jurisdiction structure will be wealth-segregated. This requires one to define what is meant by a wealth-segregated jurisdiction structure. We take the definition to be that used in Gravel and Thoron (2007) (see also Westhoff (1977) and Greenberg and Weber (1986)).

Definition 3 *A jurisdiction structure with a central government $S = (\{N_j\}_{j=1}^l; g, c; t)$ is wealth-segregated if, for every jurisdictions j, $j' \in \{1, ..., l\}$, and every households h, i and $k \in N$, $(h, k \in \{\sum_{e=0}^{j-1} n_e + 1, ..., \sum_{e=0}^{j} n_e\}$, $\omega_h < \omega_i < \omega_k$ and $i \in \{\sum_{e=1}^{j'-1} n_e + 1, ..., \sum_{e=1}^{j'} n_e\}) \Rightarrow t_j = t_{j'} = \frac{g_{j'} - g_j}{\omega_j - \omega_{j'}}$.*

In words, a jurisdiction structure is wealth-stratified if, whenever a jurisdiction contains two households h and k with different levels of wealth, it also contains all households whose wealth levels are strictly between that of h and k or, if it does not contain those households, it is because they belong to some jurisdiction j' that offers the same tax rate and the same amount of public good than j (a logical, if not likely, possibility).

3 RESULTS

As in Gravel and Thoron (2007), the monotonicity of τ^* with respect to household's wealth (given jurisdiction's wealth and central government grant) will be a key element for guaranteeing the wealth segregation of stable jurisdiction structures. This property of monotonicity of the household's most preferred tax rate can be expressed conveniently in terms of standard consumer theory. In order to do this, we establish the following lemma.

Lemma 2 *Let $(\overline{\omega}, \omega_i, \gamma, c) \in \mathbb{R}^2_{++} \times \mathbb{R} \times [0,1]$. Then for all U satisfying the above properties, $\frac{1}{\overline{\omega}}[Z^M(\frac{1}{\overline{\omega}}, \frac{1}{\omega_i}, 1 - c + \frac{\gamma}{\overline{\omega}}) - \gamma)$ is the solution of (3).*

Proof. *Since $\frac{Z^M(\frac{1}{\overline{\omega}}, \frac{1}{\omega_i}, 1-c+\frac{\gamma}{\overline{\omega}})}{\overline{\omega}} \geq 0$ and $\frac{x^M(\frac{1}{\overline{\omega}}, \frac{1}{\omega_i}, 1-c+\frac{\gamma}{\overline{\omega}})}{\omega_i} \geq 0$, the fact that the bundle $(Z^M(\frac{1}{\overline{\omega}}, \frac{1}{\omega_i}, 1 - c + \frac{\gamma}{\overline{\omega}}), x^M(\frac{1}{\overline{\omega}}, \frac{1}{\omega_i}, 1 - c + \frac{\gamma}{\overline{\omega}}))$ satisfies the consumer's budget constraint $\frac{1}{\overline{\omega}}Z^M(\frac{1}{\overline{\omega}}, \frac{1}{\omega_i}, 1 - c + \frac{\gamma}{\overline{\omega}}) + \frac{x^M(\frac{1}{\overline{\omega}}, \frac{1}{\omega_i}, 1-c+\frac{\gamma}{\overline{\omega}})}{\omega_i} = 1 - c + \frac{\gamma}{\overline{\omega}}$*
$\iff \frac{1}{\overline{\omega}}[Z^M(\frac{1}{\overline{\omega}}, \frac{1}{\omega_i}, 1 - c + \frac{\gamma}{\overline{\omega}}) - \gamma] + \frac{x^M(\frac{1}{\overline{\omega}}, \frac{1}{\omega_i}, 1-c+\frac{\gamma}{\overline{\omega}})}{\omega_i} = 1 - c$ implies obviously that $\frac{1}{\overline{\omega}}[Z^M(\frac{1}{\overline{\omega}}, \frac{1}{\omega_i}, 1 - c + \frac{\gamma}{\overline{\omega}}) - \gamma] \in [-\frac{\gamma}{\overline{\omega}}, 1 - c]$. Suppose by contradiction that $\frac{1}{\overline{\omega}}[Z^M(\frac{1}{\overline{\omega}}, \frac{1}{\omega_i}, 1 - c + \frac{\gamma}{\overline{\omega}}) - \gamma]$ does not solve (3). That is, suppose that there exists $\widehat{\tau} \in [0,1]$ such that:

$$U(\widehat{\tau}\overline{\omega} + \gamma, (1 - c - \widehat{\tau})\omega_i) >$$

$$U(\frac{1}{\overline{\omega}}(Z^M(\frac{1}{\overline{\omega}}, \frac{1}{\omega_i}, 1 - c + \frac{\gamma}{\overline{\omega}}) - \gamma)\overline{\omega} + \gamma, (1 - c - \frac{1}{\overline{\omega}}[Z^M(\frac{1}{\overline{\omega}}, \frac{1}{\omega_i}, 1 - c + \frac{\gamma}{\overline{\omega}}) - \gamma])\omega_i))$$

$$= U(Z^M(\frac{1}{\overline{\omega}}, \frac{1}{\omega_i}, 1 - c + \frac{\gamma}{\overline{\omega}}), x^M(\frac{1}{\overline{\omega}}, \frac{1}{\omega_i}, 1 - c + \frac{\gamma}{\overline{\omega}})).$$

Since the bundle $(\widehat{\tau}\overline{\omega} + \gamma, (1 - c - \widehat{\tau})\omega_i)$ satisfies the (budget) constraint $\frac{\widehat{\tau}\overline{\omega}+\tau}{\overline{\omega}} + \frac{(1-\widehat{\tau})\omega_i}{\omega_i} = 1 - c + \frac{\gamma}{\overline{\omega}}$, this inequality is incompatible with the very definition of $Z^M(\frac{1}{\overline{\omega}}, \frac{1}{\omega_i}, 1 - c + \frac{\gamma}{\overline{\omega}})$ and $x^M(\frac{1}{\overline{\omega}}, \frac{1}{\omega_i}, 1 - c + \frac{\gamma}{\overline{\omega}})$. ∎

Lemma 2 states that, in a jurisdiction with aggregate wealth $\overline{\omega}$ and central government transfer γ, the favorite tax rate of a household with (net of central government tax) wealth $\omega_i(1-c)$ can be viewed as the *expenditure* that the household would like to devote to local public good in excess of the central government grant if the prices of public and the private goods were $\frac{1}{\overline{\omega}}$ and $\frac{1}{\omega_i}$, and if this household had an income of $1 - c + \frac{\gamma}{\overline{\omega}}$. Interpreted in this fashion, monotonicity of τ^* with respect to ω_i is *equivalent* to monotonicity of the Marshallian demand for public good with respect to the price of the private good. In the language of standard consumer theory, this is equivalent to requiring the public good to be, at any price of public good, either always a gross *complement* to (if Z^M is monotonically *decreasing* with respect to p_x) or always a gross *substitute* for (if Z^M is monotonically *increasing* with respect to p_x) the private good.

As discussed in Gravel and Thoron (2007), without further assumptions on households' preferences, it is possible for the public good to be always a gross substitute of the private good at some price of the public good while being always a gross complement to the private good at some other price of the public good. Yet this possibility is ruled out if condition 1 above is imposed on the utility function.

Lemma 3 *For every $U \in \mathbb{U}$, the function τ^* that solves (3) is monotonic with respect to ω_i for any given jurisdiction level $\overline{\omega}$ and per capita wealth if and only if the public good is always either a gross complement to, or a gross substitute for, the private good.*

Let us refer to this property according to which the substitutability/ complementarity relationship between the public and private good is independent from all possible prices as to the **Gross Substitutability/ Complementarity (GSC) condition**. Although not unreasonable, the GSC condition is nonetheless a significant restriction that, as discussed in Gravel and Thoron (2007), can be violated even by additively separable preferences.

An information used in Gravel and Thoron (2007) to show that this condition is necessary and sufficient for guaranteeing the segregation of any stable jurisdiction structure is the structure of households' indifference curves in the tax-jurisdiction's wealth space. While this information is also useful in the present context, we need to account for the fact that the relevant space of location characteristics of a household with (net of central government tax) wealth $\omega_i(1-c)$ is now three, rather than two, dimensional and must include central government grant as well as local tax rate and jurisdiction's aggregate wealth. Specifically, the indifference surface of a household with (net of central government) wealth $(1-c)\omega_i$ passing through some point $(\overline{\tau}, \overline{\omega}, \overline{\gamma})$ $\in \mathbb{R}^3$ such that $\Phi(\overline{\tau}, \overline{\omega}, \omega_i, \overline{\gamma}, c) = \overline{\Phi}$ is the graph of the implicit function $f^{\overline{\Phi}} : [\frac{-\overline{\gamma}}{\overline{\omega}}, 1] \times \mathbb{R} \times \mathbb{R}_+ \longrightarrow \mathbb{R}_+$ defined by $\Phi(\tau, f^{\overline{\Phi}}(\tau, \gamma, c; \omega_i), \omega_i, \gamma, c) \equiv \overline{\Phi}$. The assumption imposed on U guarantees that the function $f^{\overline{\Phi}}$ exists and is derivable everywhere. Its partial derivative $f_\tau^{\overline{\Phi}}(\overline{\tau}, \overline{\gamma}, c; \omega_i)$ with respect to τ evaluated at $(\overline{\tau}, \overline{\gamma}, c, \omega_i)$ is given by:

$$f_\tau^{\overline{\Phi}}(\overline{\tau}, \overline{\gamma}, c; \omega_i) = \frac{1}{\overline{\tau}} \left[\frac{\omega_i}{MRS(\overline{\tau}\overline{\omega} + \overline{\gamma}, (1 - c - \overline{\tau})\omega_i)} - \varpi \right] \qquad (9)$$

where $\varpi = f^{\Phi}(\overline{\tau}, \overline{\gamma}, c; \omega_i)$. Figure 1 below illustrates the shape of these indifference curves in the (τ, ϖ) plane for given values of $\overline{\gamma}$ and c. Specifically, indifference curves of a household with private net of central government wealth $(1 - c)\omega_i$ are U-shaped and reach a minimum at this household's most preferred tax rate for the corresponding jurisdiction wealth level. It can be seen indeed that, at the minimum of an indifference curve, the term within the bracket of (9) is zero thanks to the first order conditions of (3)). Despite what figure 1 suggests, indifference curves need *not* be globally

convex. The only property that indifference curves possess is that of being "single caved" (monotonically decreasing at the left of the minimum and monotonically increasing at the right).
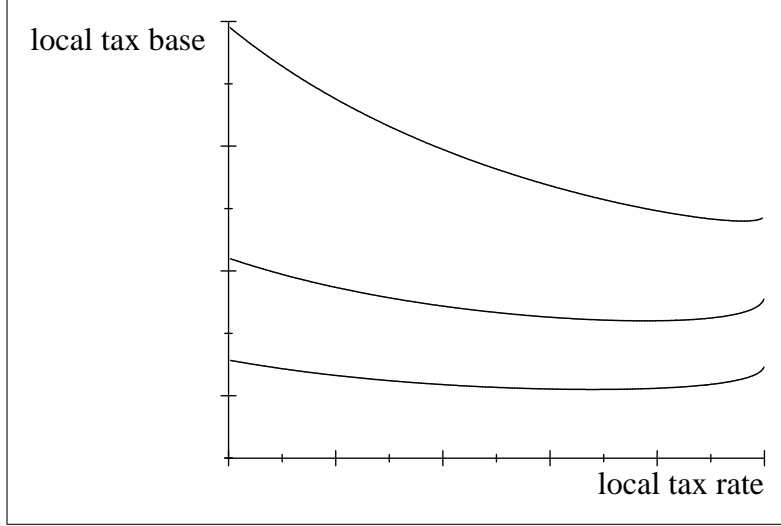

local tax base

local tax rate

Figure 1

Analogously, one can fix local tax rate at $\overline{\tau}$ and examine the property of the derivative of $f^{\overline{\Phi}}$ with respect to the central government's grant. This partial derivative $f_\gamma^{\overline{\Phi}}(\overline{\tau}, \overline{\gamma}, c; \omega_i,)$ with respect to γ evaluated at $(\overline{\tau}, \overline{\gamma}, c, \omega_i)$ is given by:

$$f_\gamma^{\overline{\Phi}}(\overline{\tau}, \overline{\gamma}, c; \omega_i) = \frac{-1}{\overline{\tau}} \tag{10}$$

Hence, when looked in the $(\gamma, \overline{\omega})$ space, indifference surfaces are straight line with negative slope (if at least $\overline{\tau}$ is positive). There is therefore a constant marginal trade off between tax base and central government grant as envisaged by a mobile household. This is of course not surprising since both central government grant and local tax base are perfectly substitutable ways of getting public expenditure in a given jurisdiction. The rate at which the household is willing to sacrifice local tax base in order to get more central government transfer depends obviously upon the local tax rate that converts tax base into public spending. The following picture shows a typical indifference surface in the tax rate, tax base and central government space in which location choice by households is made.
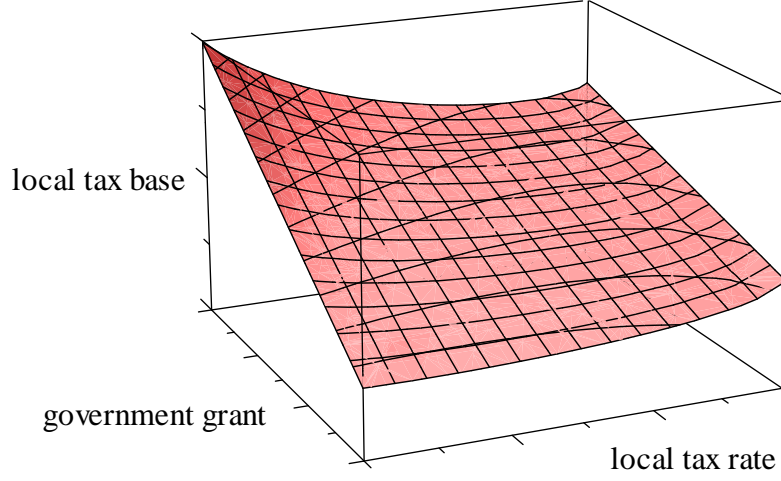
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Figure 2

We first establish, in the following lemma, that the ordering of the *slopes* of these indifference curves at every point in the tax- jurisdictions wealth space (for a given level of central government grant) coincides with the ordering of the households' wealth if and only if preferences for the public and the private good satisfy the GSC condition.

Lemma 4 *Assume that households preferences are represented by a utility function satisfying the properties mentioned above. Then, Z^M is everywhere a gross substitute (resp. complement) to the private good if and only if one has, at any $(\overline{\tau}, \varpi, \overline{\gamma}, c) \in \mathbb{R}^4$ satisfying $\overline{\gamma} \geq -\overline{\tau}\varpi$, $\overline{\tau} \leq 1 - c$, $\varpi \geq 0$ and $c \in [0,1]$, $f_\tau^{\overline{\Phi}_j}(\overline{\tau}, \overline{\gamma}; \omega_h) \leq$ (resp. $\geq$) $f_\tau^{\overline{\Phi}_k}(\overline{\tau}, \overline{\gamma}, \omega_k)$ for every h, k such that $\omega_h < \omega_k$ where, for every i, $\overline{\Phi}_i = \Phi(\overline{\tau}, \varpi, \overline{\gamma}, c; \omega_i)$.*
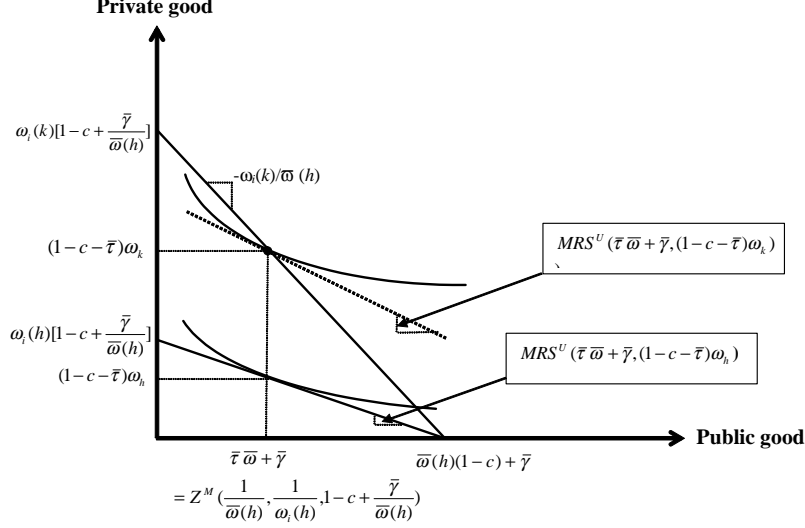
Figure 3.

Proof. *We provide the argument for the case of gross substitutability (the complementarity case being symmetric). For the first implication, assume that Z^M is everywhere increasing with respect to p_x and let $(\overline{\tau}, \varpi, \overline{\gamma}, c) \in \mathbb{R}^4$ be a combination of local tax rate $\overline{\tau}$, jurisdiction wealth ϖ, central government grant $\overline{\gamma}$ and central government tax rate c satisfying $\overline{\gamma} \geq -\overline{\tau}\varpi$, $\overline{\tau} \leq 1 - c$, $\varpi \geq 0$ and $c \in [0,1]$ and let h and k be two households such that $\omega_h < \omega_k$. Refer to figure 3 and define $\overline{\omega}(h)$ and $\omega_i(h)$ to be the numbers that generate public and private good prices $1/\overline{\omega}(h)$ and $1/\omega_i(h)$ which would lead a consumer with an income of $1 - c + \frac{\overline{\gamma}}{\overline{\omega}(h)}$ to choose the bundle $(\overline{\tau}\varpi + \overline{\gamma}, (1 - c - \overline{\tau})\omega_h)$ of public and private good. Hence $\overline{\omega}(h)$ and $\omega_i(h)$ satisfy the standard tangency and budget equality conditions:*

$$MRS^U(\overline{\tau}\varpi + \overline{\gamma}, (1 - c - \overline{\tau})\omega_j) = \frac{\omega_i(h)}{\overline{\omega}(h)}$$

and

$$\frac{\overline{\tau}\varpi + \overline{\gamma}}{\overline{\omega}(h)} + \frac{(1 - c - \overline{\tau})\omega_h}{\omega_i(h)} = 1 - c + \frac{\overline{\gamma}}{\overline{\omega}(h)}$$

Combining these two equations yields:

$$\frac{(1 - c - \overline{\tau})}{\overline{\omega}(h)(1 - c) - \overline{\tau}\varpi} = \frac{MRS^U(\overline{\tau}\varpi + \overline{\gamma}, (1 - c - \overline{\tau})\omega_h)}{\omega_h} \qquad (11)$$

Define now $\omega_i(k)$ to be a level of household wealth which would generate private good price $1/\omega_i(k))$ that is just sufficient to enable a consumer with the same income of $1 - c + \frac{\overline{\gamma}}{\overline{\omega}(h)}$ and facing public good price $1/\overline{\omega}(h)$ to afford

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the bundle $(\overline{\tau}\varpi + \overline{\gamma}, (1 - c - \overline{\tau})\omega_k)$. This $\omega_i(k)$ (which is clearly larger than $\omega_i(h)$ if $\omega_k > \omega_h$) is defined by the budget constraint equality:

$$\frac{\overline{\tau}\varpi + \overline{\gamma}}{\overline{\varpi}(h)} + \frac{(1 - c - \overline{\tau})\omega_k}{\omega_i(k)} = 1 - c + \frac{\overline{\gamma}}{\overline{\varpi}(h)}$$

$$\Longleftrightarrow$$

$$\omega_i(k) = \frac{\overline{\varpi}(h)(1 - c - \overline{\tau})\omega_k}{\overline{\varpi}(h)(1 - c) - \overline{\tau}\varpi} \quad (12)$$

Now, since Z^M is increasing with respect to p_x, we must have $Z^M(\frac{1}{\overline{\varpi}(h)}, \frac{1}{\omega_i(h)}, 1 - c + \frac{\overline{\gamma}}{\overline{\varpi}(h)}) \geq Z^M(\frac{1}{\overline{\varpi}(h)}, \frac{1}{\omega_i(k)}, 1 - c + \frac{\overline{\gamma}}{\overline{\varpi}(h)})$ and, therefore (see figure 3), the slope of the indifference curve passing through $(\overline{\tau}\varpi + \overline{\gamma}, (1 - c - \overline{\tau})\omega_k)$ must be, in absolute value, less than the price ratio $\omega_i(k)/\overline{\varpi}(h)$. Formally, this amounts to saying that:

$$MRS^U(\overline{\tau}\varpi + \overline{\gamma}, (1 - c - \overline{\gamma})\omega_k) \leq \frac{\omega_i(k)}{\overline{\varpi}(h)}$$

or, using (12):

$$\frac{MRS^U(\overline{\tau}\varpi + \overline{\gamma}, (1 - \overline{\tau})\omega_k)}{\omega_k} \leq \frac{(1 - c - \overline{\tau})}{\overline{\varpi}(h)(1 - c) - \overline{t}\varpi} \quad (13)$$

Combining inequality (13) and equality (11), we get:

$$\frac{MRS^U(\overline{\tau}\varpi + \overline{\gamma}, (1 - c - \overline{\tau})\omega_k)}{\omega_k} \leq \frac{MRS^U(\overline{\tau}\varpi + \overline{\gamma}, (1 - c - \overline{\tau})\omega_h)}{\omega_h}$$

$$\Longleftrightarrow$$

$$\frac{\omega_k}{MRS^U(\overline{\tau}\varpi + \overline{\gamma}, (1 - c - \overline{\tau})\omega_k)} \geq \frac{\omega_h}{MRS^U(\overline{\tau}\varpi + \overline{\gamma}, (1 - c - \overline{\tau})\omega_h)}$$

which, using the definition of $f_\tau^{\overline{\Phi}_j}$ provided by (9), establishes the result. For the second implication, assume that Z^M is not everywhere increasing with respect to p_x so that there are positive private good prices p_x^0, p_x^1 and p_x^2 satisfying $p_x^0 < p_x^1 < p_x^2$ as well as positive public good price p_Z and income level R for which one has:

$$Z^M(p_Z, p_x^0, R) = Z^M(p_Z, p_x^2, R) > Z^M(p_Z, p_x^1, R) \quad (14)$$

Let $c \in [0, 1]$ be any central government tax rate and let ϖ, ω_i and $\overline{\gamma}$ be defined by:

$$\varpi = \frac{1}{p_Z} \quad (15)$$

$$\omega_i = \frac{1}{p_x^i} \text{ and} \quad (16)$$

$$\overline{\gamma} = \varpi(R - 1 + c) \quad (17)$$

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for $i = 0, 1, 2$. By lemma 2 and definitions (15)-(17), one has $Z^M(p_Z, p_x^i, R) = \tau^*(\varpi, \omega_i, \overline{\gamma}, c)\varpi + \overline{\gamma}$ and $x^M(p_Z, p_x^i, R) = (1 - c - \tau^*(\varpi, \omega_i, \overline{\gamma}, c)\omega_i)$ for $i = 0, 1, 2$. Consider the tax rate $\overline{\tau}$ such that:

$$\overline{\tau} = \frac{Z^M(p_Z, p_x^0, R) - \overline{\gamma}}{\varpi} = \frac{Z^M(p_Z, p_x^2, R) - \overline{\gamma}}{\varpi} \qquad (18)$$

Using (9), we have:

$$
\begin{aligned}
f_\tau^{\overline{\Phi}}(\overline{\tau}, \overline{\gamma}, c; \omega_0) &= \frac{1}{\overline{\tau}}\left[\frac{\omega_0}{MRS(\overline{\tau}\varpi + \overline{\gamma}, (1 - c - \overline{\tau})\omega_0)} - \varpi\right] \\
&= 0 \text{ (by the FOCs of the consumer's program)} \\
&= \frac{1}{\overline{\tau}}\left[\frac{\omega_2}{MRS(\overline{\tau}\varpi + \overline{\gamma}, (1 - c - \overline{\tau})\omega_2)} - \varpi\right] \\
&= f_\tau^{\overline{\Phi}}(\overline{\tau}, \overline{\gamma}, c; \omega_2)
\end{aligned}
$$

Moreover, since $Z^M(p_Z, p_x^1, R) = \tau^*(\varpi, \omega_1, \gamma, c)\varpi + \overline{\gamma} < Z^M(p_Z, p_x^2, R) = \overline{\tau}(\varpi, \omega_2, \overline{\gamma}, c)\varpi + \overline{\gamma} = \overline{\tau}(\varpi, \omega_0, \overline{\gamma}, c)\varpi + \overline{\gamma}Z^M(p_Z, p_x^0, R)$, we must have that $MRS(\overline{\tau}\varpi + \overline{\gamma}, (1 - c - \overline{\tau})\omega_1) < \frac{\omega_1}{\varpi}$. Hence thanks to (9), we have

$$
\begin{aligned}
f_\tau^{\overline{\Phi}}(\overline{\tau}, \overline{\gamma}, c, \omega_1) &= \frac{1}{\overline{\tau}}\left[\frac{\omega_1}{MRS(\overline{\tau}\varpi + \overline{\gamma}, (1 - c - \overline{\tau})\omega_1)} - \varpi\right] \\
&> 0
\end{aligned}
$$

We therefore have $f_\tau^{\overline{\Phi}}(\overline{\tau}, \overline{\gamma}, c; \omega_0) = f_\tau^{\overline{\Phi}}(\overline{\tau}, \overline{\gamma}, c; \omega_2) < f_\tau^{\overline{\Phi}}(\overline{\tau}, \overline{\gamma}, c, \omega_1)$ for ω_0, ω_1 and ω_2 satisfying $\omega_0 > \omega_1 > \omega_2$ so that the slopes of indifference curves in the (τ, ϖ) plane are not monotonically decreasing with respect to ω_i for a certain $\overline{\gamma}$ and c. ∎

This lemma thus tells us that GSC condition is equivalent to the requirement that the projections of indifference surfaces of households with different wealth in the local tax rate and jurisdiction wealth space be *single crossing* at any point of that space for any exogenous central government grant and tax rate. In proposition 3 below, we shall show that this single crossing in the two-dimensional space of tax rate and aggregate wealth for a given government grant holds true as well in the three dimensional space of government grants, local tax and jurisdiction's wealth.

We now establish that, if the utility functions aggregated by the Utilitarian central government are additively separable, the GSC condition is *necessary* for the wealth segregation of any stable jurisdiction structure. That is to say, for any violation of the GSC condition obtained for additively separable preferences, it is possible to construct a stable jurisdiction structure with a utilitarian central government who uses an additively separable utility function that is not segregated as per definition 2.

Proposition 2 *Stable jurisdictions structures with a utilitarian government who use additively separable utility functions are segregated for every economy only if the preferences represented by the utility function satisfies the GSC condition.*

Proof. *Assume that the utility function used by the utilitarian planner is additively separable and that the GSC condition is violated. Hence there are private good prices p_x^0, p_x^1 and p_x^1 satisfying $0 < p_x^0 < p_x^1 < p_x^2$ and public good price $p_Z > 0$ such that:*

$$
\begin{aligned}
Z^M(p_Z, p_x^0, 1) &= Z^M(p_Z, p_x^2, 1) \\
&> Z^M(p_Z, p_x^1, 1) \text{ or} \\
&< Z^M(p_Z, p_x^1, 1)
\end{aligned}
$$

We provide the argument for the first of these two inequalities. Define the numbers θ, α, β and δ by:

$$
\begin{aligned}
\theta &= \frac{1}{p_Z} \\
\alpha &= \frac{1}{p_x^2} \\
\beta &= \frac{1}{p_x^1} \\
\delta &= \frac{1}{p_x^0}
\end{aligned}
$$

Consider now an economy with n_α households with wealth α, n_β households with wealth β and n_δ households with wealth δ. Let us construct a stable jurisdiction structure with a Utilitarian government that is not segregated. Specifically, we are going to put households with wealth α and δ in one jurisdiction - called it 1- and households with wealth β in another, 2 say. For this purpose we are going to choose number of households n_α, n_β and n_δ so that:

$$n_\alpha \alpha + n_\delta \delta = n_\beta \beta = \theta \tag{19}$$

Moreover, we want to choose these numbers n_α, n_β and n_δ in such a way that a Utilitarian central government would find optimal to give no equalization grants to any jurisdictions and to collect no taxes as a result. That is to say,

we would like

$$(0,0) \in \arg \max_{(\gamma,c)\in\mathbb{R}\times[0,1]} (n_\alpha + n_\delta)f(Z^M(\frac{1}{\theta}, \frac{1}{\alpha}, 1) + \gamma)$$

$$+ n_\beta f(Z^M(\frac{1}{\theta}, \frac{1}{\beta}, 1) + 2\theta c - \gamma)$$

$$+ n_\alpha h(Z^M(\frac{1}{\theta}, \frac{1}{\alpha}, 1) - c\alpha)$$

$$+ n_\beta h(x^M(\frac{1}{\theta}, \frac{1}{\beta}, 1) - c\beta)$$

$$+ n_\delta h(x^M(\frac{1}{\theta}, \frac{1}{\delta}, 1) - c\delta) \qquad (20)$$

Since the objective function of the program (20) is concave, any combination of values of jurisdiction 1's grant γ^* and wealth tax c^* that satisfy the first order conditions of this program is a solution to it. The first order conditions of this program for $\gamma^* = 0$ and $c^* = 0$ write:

$$(n_\alpha + n_\delta)\frac{\partial f(Z^M(\frac{1}{\theta}, \frac{1}{\alpha}, 1))}{\partial Z} = n_\beta \frac{\partial f(Z^M(\frac{1}{\theta}, \frac{1}{\beta}, 1))}{\partial Z} \qquad (21)$$

and:

$$n_\alpha \alpha \frac{\partial h(x^M(\frac{1}{\theta}, \frac{1}{\alpha}, 1))}{\partial x} + n_\beta \beta \frac{\partial h(x^M(\frac{1}{\theta}, \frac{1}{\beta}, 1))}{\partial x} + n_\delta \delta \frac{\partial h(x^M(\frac{1}{\theta}, \frac{1}{\delta}, 1))}{\partial x}$$

$$= 2\theta n_\beta \frac{\partial f(Z^M(\frac{1}{\theta}, \frac{1}{\beta}, 1)}{\partial Z} \qquad (22)$$

By definition of Marshallian demands, condition (22) writes:

$$n_\alpha \theta \frac{\partial f(Z^M(\frac{1}{\theta}, \frac{1}{\alpha}, 1))}{\partial Z} + n_\beta \theta \frac{\partial f(Z^M(\frac{1}{\theta}, \frac{1}{\beta}, 1))}{\partial Z} + n_\delta \theta \frac{\partial f(Z^M(\frac{1}{\theta}, \frac{1}{\delta}, 1))}{\partial Z}$$

$$= 2\theta n_\beta \frac{\partial f(Z^M(\frac{1}{\theta}, \frac{1}{\beta}, 1)}{\partial Z}$$

or (since $Z^M(\frac{1}{\theta}, \frac{1}{\delta}, 1) = Z^M(\frac{1}{\theta}, \frac{1}{\alpha}, 1)$):

$$(n_\alpha + n_\delta)\frac{\partial f(Z^M(\frac{1}{\theta}, \frac{1}{\alpha}, 1))}{\partial Z} = n_\beta \frac{\partial f(Z^M(\frac{1}{\theta}, \frac{1}{\beta}, 1))}{\partial Z}$$

which is nothing else than condition (21). Hence, the only other condition that the numbers n_α, n_β and n_δ must satisfy beside (19) is condition (21). Since α, β, δ and θ are given numbers, we have that $n_\beta = \theta/\beta$. Hence, in order for our jurisdiction structure to be stable with a utilitarian government who chooses (optimally) not to intervene, we only need to find numbers n_α and n_δ such that equalities:

$$n_\alpha = \frac{\theta}{\alpha} - \frac{\delta}{\alpha}n_\delta \qquad (23)$$

and:

$$n_\alpha = \frac{\theta \partial f(Z^M(\frac{1}{\theta}, \frac{1}{\beta}, 1))/\partial Z}{\beta \partial f(Z^M(\frac{1}{\theta}, \frac{1}{\alpha}, 1))/\partial Z} - n_\delta$$

hold. From the first order condition that defines Marshallian demands, we can write the later equality (using additive separability) as: :

$$n_\alpha = \frac{\theta \partial h(x^M(\frac{1}{\theta}, \frac{1}{\beta}, 1))/\partial x}{\alpha \partial h(x^M(\frac{1}{\theta}, \frac{1}{\alpha}, 1))/\partial x} - n_\delta \qquad (24)$$

Since the preferences represented by the utility function are additively separable, no good is inferior and, as a result, the private good is not a Giffen good. Hence x^M is decreasing with respect to its own price so that, since $\frac{1}{\beta} < \frac{1}{\alpha}$, $x^M(\frac{1}{\theta}, \frac{1}{\beta}, 1) > x^M(\frac{1}{\theta}, \frac{1}{\alpha}, 1)$ and, since h is concave, $\partial h(x^M(\frac{1}{\theta}, \frac{1}{\beta}, 1)/\partial x < \partial h(x^M(\frac{1}{\theta}, \frac{1}{\alpha}, 1)/\partial x$. Hence the intercept of the linear equation (24) is smaller than that of equation (23). Moreover, the abscissa at the origin $n_\delta^0(1)$ of equation (23) is given by:

$$n_\delta^0(1) = \frac{\theta}{\delta}$$

while the abscissa at the origin $n_\delta^0(2)$ of equation (24) is of course:

$$
\begin{aligned}
n_\delta^0(2) &= \frac{\theta \partial h(x^M(\frac{1}{\theta}, \frac{1}{\beta}, 1)/\partial x}{\alpha \partial h(x^M(\frac{1}{\theta}, \frac{1}{\alpha}, 1)/\partial x} \\[2mm]
&= \frac{\theta \partial f(Z^M(\frac{1}{\theta}, \frac{1}{\beta}, 1)/\partial Z}{\beta \partial f(Z^M(\frac{1}{\theta}, \frac{1}{\alpha}, 1)/\partial Z} \quad \text{\textit{(by separability and the definition of Marshallian demands)}} \\[2mm]
&= \frac{\theta \partial f(Z^M(\frac{1}{\theta}, \frac{1}{\beta}, 1)/\partial Z}{\beta \partial f(Z^M(\frac{1}{\theta}, \frac{1}{\delta}, 1)/\partial Z} \quad \text{\textit{(since } } Z^M(\frac{1}{\theta}, \frac{1}{\delta}, 1) = (Z^M(\frac{1}{\theta}, \frac{1}{\alpha}, 1)) \\[2mm]
&= \frac{\theta \partial h(x^M(\frac{1}{\theta}, \frac{1}{\beta}, 1)/\partial x}{\delta \partial h(x^M(\frac{1}{\theta}, \frac{1}{\delta}, 1)/\partial x} \quad \text{\textit{(by separability and the definition of Marshallian demands)}}
\end{aligned}
$$

$$\frac{\theta \partial h(x^M(\frac{1}{\theta},\frac{1}{\beta},1)/\partial x}{\alpha \partial h(x^M(\frac{1}{\theta},\frac{1}{\alpha},1)/\partial x}$$

equation (2)

$$\frac{\theta \partial h(x^M(\frac{1}{\theta},\frac{1}{\beta},1)/\partial x}{\delta \partial h(x^M(\frac{1}{\theta},\frac{1}{\delta},1)/\partial x}$$
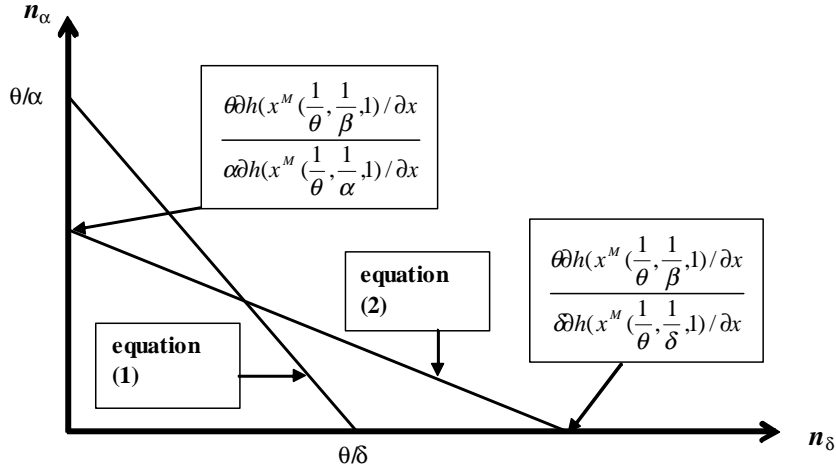
equation (1)

Figure 4.

Now, again, since the private good is not inferior (and therefore not Giffen) thanks to additive separability, we have that $x^M(\frac{1}{\theta},\frac{1}{\beta},1) < x^M(\frac{1}{\theta},\frac{1}{\delta},1)$ and, by concavity of h, that $n_\delta^0(2) > n_\delta^0(1) > 0$. Hence, the two straight lines represented by equations (23) and (24) are as in figure 4) and cross in the strictly positive orthant. There is therefore a set of positive numbers n_α and n_δ that satisfy equations (23) and (24) and this completes the proof. ∎

The additive separability condition under which proposition 2 is obtained is, admittedly, stronger than required. It is certainly possible, for *some* violation of the GSC condition obtained with non-additively separable preference, to obtain stable jurisdiction structure that are not segregated by proceeding in the same way than in the proof of proposition 2. Yet, without additive separability or some other condition that restricts the behavior of the marginal utility of private wealth in the neighborhood of the consumer's optimal price taking choice, it is not clear that we can construct an example of non-segregated and yet stable jurisdiction structure in the spirit of that exhibited in the proof of proposition 2. We emphasize, however, that proposition 2 proves in fact something slightly stronger than what is required. Indeed, what is established in proposition 1 is that, for any violation of the GSC condition obtained with additively separable preferences, one can find a non-segregated stable jurisdiction structure in which a utilitarian government finds optimal to perform zero equalization (and accordingly to levy no wealth taxes). The possibility of proving, less demandingly, that any violation of the GSC condition can give rise to a non-segregated stable

jurisdiction structure in which the utilitarian central government performs non-zero equalization without assuming additive separability of households utility function remains an open, if not difficult, question.

We now establish, without any further condition on household's preferences, the converse proposition that the GSC condition is sufficient for the wealth segregation of any stable jurisdiction structure.

Proposition 3 *Assume that households' preferences satisfy the GSC condition. Then, any stable jurisdiction structure with a Utilitarian central government is wealth segregated.*

Proof. *We sketch the argument for the case where public good is everywhere a gross complement to the private good. Assume therefore that Z^M is decreasing with respect to p_x and, by contradiction, let $S = (\{N_j\}_{j=1}^l, g, t)$ be a jurisdiction structure that is not wealth-stratified. Hence, there are jurisdictions j and $j' \in \{1, ..., l\}$ (with $j \neq j'$), and households h, i and $k \in N$ with $\omega_h < \omega_i < \omega_k$ for which one has h and $k \in N_j$, $i \in N_{j'}$ and either $t_j \neq t_{j'}$ or $t_j \overline{\omega}_j + g_j \neq t_{j'} \omega_j + g_{j'}$. It is clear that if only one of the two inequalities $t_j \neq t_{j'}$ and $t_j \overline{\omega}_j + g_j \neq t_{j'} \omega_j + g_{j'}$ holds, then the jurisdiction structure can not be stable because there would be unanimity of the members of one of the jurisdictions j and j' to go to the jurisdiction with the low tax rate (if $t_j \neq t_{j'}$ and $t_j \overline{\omega}_j + g_j = t_{j'} \omega_j + g_{j'}$) or to the jurisdiction with the largest public good provision (if $t_j = t_{j'}$ and $t_j \overline{\omega}_j + g_j \neq t_{j'} \omega_j + g_{j'}$). Hence we can assume that both $t_j \neq t_{j'}$ and $t_j \overline{\omega}_j + g_j \neq t_{j'} \omega_j + g$ hold. Define now $\overline{\omega}$ by:*

$$
\begin{aligned}
t_j \overline{\omega} + g_{j'} &= t_j \overline{\omega}_j + g_j \\
&\Leftrightarrow \\
\overline{\omega} &= \overline{\omega}_j + \frac{g_j - g_{j'}}{t_j}
\end{aligned}
$$

Clearly we have that:

$$
\begin{aligned}
U(t_j \overline{\omega} + g_{j'}, (1 - c - t_j)\omega_m) &= \Phi(t_j, \overline{\omega}, \omega_m, g_{j'}) \\
&= U(t_j \omega_j + g_j, (1 - c - t_j)\omega_m) \\
&= \Phi(t_j, \overline{\omega}_j, \omega_m, g_j, c) \quad (25)
\end{aligned}
$$

for every household m. For this non-stratified jurisdiction structure to be stable, we must have:

$$
\begin{aligned}
\Phi(t_j, \overline{\omega}_j, \omega_h, g_j, c) &\geq \Phi(t_j', \overline{\omega}_{j'}, \omega_h, g_{j'}, c) \\
\Phi(t_j, \overline{\omega}_j, \omega_i, g_j, c) &\leq \Phi(t_j', \overline{\omega}_{j'}, \omega_i, g_{j'}, c)
\end{aligned}
$$

and

$$
\Phi(t_j, \overline{\omega}_j, \omega_k, g_j, c) \geq \Phi(t_j', \overline{\omega}_{j'}, \omega_k, g_{j'}, c)
$$

or, using (25):

$$\Phi(t_j, \overline{\omega}, \omega_h, g_{j'}, c) \quad \geq \quad \Phi(t'_j, \overline{\omega}_{j'}, \omega_h, g_{j'}, c) \tag{26}$$

$$\Phi(t_j, \overline{\omega}, \omega_i, g_{j'}, c) \quad \leq \quad \Phi(t'_j, \overline{\omega}_{j'}, \omega_i, g_{j'}, c) \tag{27}$$

and

$$\Phi(t_j, \overline{\omega}, \omega_k, g_{j'}, c) \geq \Phi(t'_j, \overline{\omega}_{j'}, \omega_k, g_{j'}, c) \tag{28}$$

We have seen by lemma 4 that the slopes of indifference curves in the space of all combinations of local tax rate and jurisdiction aggregate wealth are ordered as per the individual wealth for any level of central government grant and, therefore, in particular for the level $g_{j'}$. Hence indifference curves of households h, i and k at the combination (t_j, ϖ) must be as they are depicted in figure 5. Clearly from this figure, unless the indifference curves of households k and i or h and i cross in the wrong order at some point (such as (τ, ϖ'')), the set of combinations of local tax rates and jurisdiction aggregate wealth that household i considers weakly worse (given a central government grant of $g_{j'}$) than
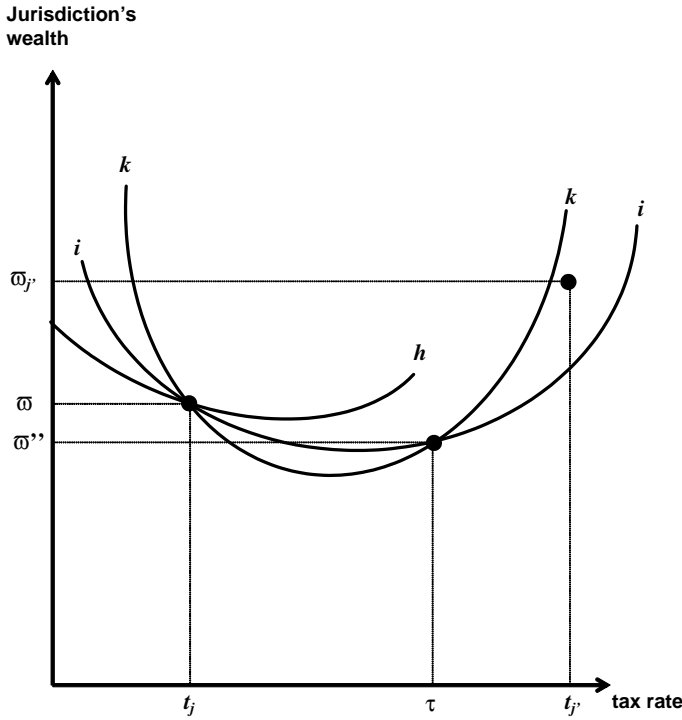

Figure 5

(t_j, ϖ) *is contained in the set of such combinations that either household h or household k considers strictly worse than (t_j, ϖ). Hence, unless indifference curves of two households cross in the "wrong" order at a point such as*

(τ, ϖ'') , *inequalities (26)-(28) can not simultaneously hold for distinct combinations* (t_j, ϖ) *and* $(t_{j'}, \varpi_{j'})$ *of local tax rates and jurisdiction tax rates. Hence the jurisdiction structure can not be stable.* ∎

4 Conclusion

The main conclusion of this paper is that the welfarist intervention of a central government does not alter substantially the segregative properties of endogenous jurisdiction formation, at least when this jurisdiction formation is modelled within a framework *à la* Westhoff. Specifically, the GSC condition that it is necessary and sufficient to impose on household preferences for guaranteeing the wealth segregation of any stable jurisdiction is not affected by the presence of a central government who gives equalization grants to jurisdictions and finance the grants by linear households taxation. This of course does not mean that central government intervention does not affect jurisdiction formation. As the Leximin government case dramatically reveals, the redistributive behavior of the central government tends to reduce the number of stable jurisdiction structure. Yet the stable jurisdictions structures that will remain under a Utilitarian central government will be segregated under exactly the same conditions on household's preferences than would be the case without central government.

While we believe that the message according to which central government intervention does not modify the segregative forces underlying Tiebout-like processes of jurisdiction formation is of some interest, it is worth recalling the limitations of the analysis on which it stands. For one thing, the result is obtained, at least for its necessity part, under the assumption that preferences are additively separable and that the utility function summed by the utilitarian central government is also additive. It would be nice to relax this assumption. Another limitation of the analysis lies, perhaps, in the simultaneous setting in which the decisions by households, central and local government are considered. As discussed in the paper, an alternative approach would be to assume a form of "leadership" of the central government in the process of jurisdiction formation. A third limitation of the analysis is the rather limited power given to the central government in our model to redistribute private wealth. Extending the analysis of this paper over these limitations, as well as many others, seems to us a worthy objective for future research.

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