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Single-Name Credit Risk, Portfolio Risk, and Credit Rationing

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Key Words: asymmetric information, credit rationing

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Abstract

This paper introduces non-diversifiable risk in the Stiglitz-Weiss adverse selection model, so that an increase in the average riskiness of the borrower pool causes higher portfolio risk. This opens up the possibility of equilibrium credit rationing. Comparative statics analysis shows that an increase in risk aversion turns a two-price equilibrium into a rationing equilibrium. A two-price equilibrium is more inefficient than a rationing equilibrium, and a usury law that rules out the higher of the two interest rates can be welfare-improving. Contrary to the common result, the equilibrium may be characterized by over-investment.

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1 Introduction

Models of adverse selection or moral hazard in the credit market show why lenders tend to finance investment projects which are too risky from their point of view. The reason why lenders dislike risky projects is that, when standard credit contracts are used, they generate lower expected repayment. Due to the common assumption of independence of the payoffs on different projects and the law of large numbers for large economies (Uhlig, 1996), the repayments on a well-diversified credit portfolio are safe, however (an implication that tends to irritate scholars with a background in portfolio theory): uncorrelated single-name credit risks cancel out and do not create any portfolio risk for the lender. A notable case in point is the seminal Stiglitz-Weiss (1981) (henceforth: “SW”) adverse selection model, in which an increase in the interest rate raises the average riskiness of the pool of active borrowers, but nonetheless the return on lending remains safe.

The present paper introduces non-diversifiable risk to the SW model (with two borrower types), so that an increase in the average riskiness of the borrower pool causes higher portfolio risk, which has to be borne by lenders. This has several interesting implications for equilibrium and welfare. First, it opens up the possibility of equilibrium credit rationing. As shown independently by Coco (1997) and Arnold and Riley (2009), non-monotonicity of the expected repayment lenders receive as a function of the interest rates potentially gives rise to a two-price equilibrium with market clearing at the higher rate in the SW model. There cannot be pure rationing at a single interest rate in the SW model, because it would be profitable to serve rationed risky borrowers at their maximum acceptable interest rate.¹ This strategy of “picking risky borrowers” might be unattractive to risk-averse lenders when the project payoffs are correlated, since it gives rise to a highly risky credit portfolio. Thus, a profitable deviation from the strategies that lead to rationing may not exist, restoring the possibility of equilibrium credit rationing.

A two-price equilibrium entails higher average project risks and, hence, higher portfolio risk than a rationing equilibrium, at a given level of investment. This is because market clearing at the higher interest rate implies that all risky projects are financed. As a consequence, whether a two-price equilibrium or a rationing equilibrium arises depends systematically on lenders’ degree of relative risk aversion. To address this issue, we use the constant relative risk aversion (CRRA) version of the Ordinal Certainty Equivalent (OCE) utility function proposed by Selden (1978), which allows to disentangle changes in risk aversion from changes in the preference for consumption smoothing over time. We show that starting from a two-price equilibrium, a rationing equilibrium arises when

¹Bester (1985, 1987), Riley (1987), Lensink and Sterken (2002), and De Meza and Webb (2006) provide alternative arguments against the relevance of equilibrium credit rationing.

(everything else equal) the degree of relative risk aversion grows sufficiently large. Thus, stronger risk aversion tends to make the emergence of a rationing equilibrium more likely.

A two-price equilibrium is, in a specific sense, more inefficient than a rationing equilibrium: consider a parameter change that leads to a switch from a two-price equilibrium to a rationing equilibrium. The level of investment is continuous in model parameters, so it changes only slightly. However, the riskiness of the pool of active borrowers deteriorates discontinuously, since all risky firms get finance in a two-price equilibrium. As a consequence, borrowers have to pay higher risk premia, so aggregate profit and total welfare jump downward. One way to avoid this inefficiency of a two-price equilibrium is to impose a usury law that prohibits interest rates above the equilibrium rate with pure rationing. Analogous results are derived by Coco (1997, Section 3) in a model with independent returns in which projects differ by both riskiness and mean return.²

Following De Meza and Webb (1987), next we address the question of whether there is too little or too much investment in equilibrium, relative to a first-best or a second-best optimum. Contrary to the common result that equilibrium is generally characterized by under-investment, we find that equilibrium over-investment may arise in a two-price equilibrium or in a rationing equilibrium, viz., when firms are endowed with a large amount of collateral and little weight is put on their expected utility in the optimum solution. Under these conditions, optimum saving is low, and may thus fall short of equilibrium investment, because households' consumption can be satisfied by reallocating the firms' collateral to them.³

The motivation for the analysis is the observation that returns on risky investment projects are correlated, so that someone has to bear the additional risk when firms carry out riskier investment projects. While direct evidence of correlation between individual firms' returns on investment is hard to come by, there are numerous country studies showing that economy-wide and industry factors explain much of the variance in firm earnings (e.g., Brealey, 1971, for the U.S.), that firm-level profitability is strongly affected by aggregate demand shocks (e.g., Machin and van Reenen, 1993, for the U.K.), and that macroeconomic shocks have a profound impact on business failures (e.g., Gaffeo and Santoro, 2009, for Italy). Generally, the fact that aggregate corporate profits are strongly procyclical means that individual firms' profits are positively correlated at business cycle frequency. These observations suggest that there is significant correlation between the returns

²See also De Meza and Webb (2006).

³This relates to variants of the SW model concerned with optimal risk sharing, such as Bester (1985, 1987) (see also the survey in Coco, 2000). These models usually assume risk-averse borrowers and independent project risks (i.e., no risk for lenders) and highlight the welfare loss due to the use of collateral as a sorting device: since a borrower's marginal utility is higher when his project fails, he should not put up any collateral in a first-best optimum.

on individual firms' risky endeavors. One might object that, even so, independence would be a convenient assumption if it reduced complexity without having a major impact on the results. However, the summary of the results above shows that the introduction of non-diversifiable risk leads to much richer model implications, as different types of non-market clearing equilibria emerge depending on consumers' risk attitudes, with interesting welfare implications.

Section 2 presents the model with CRRA preferences. In Section 3, we demonstrate that pure credit rationing can arise. Section 4 introduces OCE preferences and analyzes the impact of the degree of risk aversion on the type of equilibrium. Section 5 highlights the inefficiency of a two-price equilibrium. Section 6 addresses the question of equilibrium under-investment. Section 7 concludes.

2 Model

There are two time periods $t = 1, 2$. There is one homogeneous (perishable) good, which can be used for consumption or investment. There is a continuum of measure $M (> 0)$ of identical risk-averse consumers, each endowed with $y (> 0)$ units of the homogeneous good in the first period and nothing in the second period. So to consume in period 2, they have to save in period 1. The consumers' preferences are represented by the CRRA utility function

$$E[u(c_1, c_2)] = \frac{c_1^{1-\theta}}{1-\theta} + \delta E\left(\frac{c_2^{1-\theta}}{1-\theta}\right), \quad 0 < \delta, \theta < 1, \quad (1)$$

where c_t is period- t consumption of the homogenous good ($t = 1, 2$). θ measures the inverse of the intertemporal elasticity of substitution *and* the degree of relative risk aversion. The assumption that $1/\theta$ is greater than one ensures that capital supply is upward-sloping, so that the analysis is comparable to the existing literature. In the model with OCE preferences, we will maintain the assumption that the intertemporal elasticity of substitution exceeds unity but allow for arbitrary positive degrees of relative risk aversion.

There are a continuum of measure $N_S (> 0)$ of safe firms and a continuum of measure $N_R (> 0)$ of risky firms, each endowed with collateral $C (> 0)$ and the ability to turn $B (> C)$ units of the homogeneous good invested in the first period into a random second-period output of the homogeneous good. Safe firms' projects succeed with probability p_S and risky firms' projects with probability p_R ($0 < p_R < p_S < 1$). In case of success, they yield R_S or R_R , respectively. If a project fails, the payoff is zero. The projects are equally profitable on average: $p_S R_S = p_R R_R$ ($\equiv \bar{R} > B$). Standard debt is the only financial instrument.⁴ Firms' collateral C cannot be traded

⁴Standard debt is the optimal mode of finance in related models, e.g., when lenders can observe whether a project has succeeded or not but not its payoff in case of success (see Besanko and Thakor, 1987).

in the capital market.⁵ Firm owners are risk-neutral and apply for capital if the expected return on their investment (taking care of the possibility that they lose their collateral) is non-negative. There is asymmetric information: lenders are unable to observe whether a firm owns a safe or a risky project.

The novel assumption is that the projects' payoffs are not independent. There are three states of nature $s \in \{R, S, F\}$. In state R , which occurs with probability p_R , all projects succeed; in state S , which occurs with probability $p_S - p_R$, only the safe projects succeed; in state F (i.e., with probability $1 - p_S$), all projects fail. Thus, the returns of any two risky projects as well as of any two safe projects are perfectly correlated: if one risky project succeeds, all risky projects succeed; if one safe project succeeds, all safe projects succeed; and the risky projects never succeed unless the safe projects do.⁶ As a consequence of non-diversifiable risk, and contrary to the SW model, the single-name risks do not cancel out, so lenders face positive portfolio risk. For instance, in state F , all borrowers are unable to repay, so lenders merely receive the posted collateral.

We assume that the revenue from lending is passed through completely to the suppliers of capital. One interpretation is that loans are made and deposits are taken by intermediaries without operating costs and without equity. Alternatively, one may think of collateralized corporate bonds held by funds (so that the different types of firms are represented proportionally in each household's portfolio). A more complete model would introduce banks with positive equity, which serves as a buffer against losses on loans, so that deposits are safe and bank owners carry the non-diversifiable risk that does not remain in the firm sector. The main point, which the present model captures in the simplest possible fashion, is that the realization of investment projects creates non-diversifiable risks that someone has to bear and that these risks have a profound impact on the nature and efficiency of equilibrium in the credit market.⁷

⁵We maintain this assumption, even though it is not innocuous in the welfare analysis (cf. footnote 14), for the sake of comparability with the literature.

⁶We checked the robustness of our results by making different assumptions about the dependence of project returns. For instance, we analyzed the model under the alternative assumptions that there is positive but imperfect correlation and that the returns on safe projects are mutually independent. The sets of parameters for which, e.g., rationing or over-investment occur changes, but the essence of our results remains unaffected.

⁷It will be seen that for well-collateralized credit, firm owners bear significantly more risk than suppliers of credit (see footnotes 8 and 11).

3 Restoring credit rationing

This section demonstrates that, unlike in the SW model (cf. Coco, 1997, and Arnold and Riley, 2009), credit rationing may occur in the presence of portfolio risk.

Let r denote the interest rate (because of asymmetric information, lenders cannot set type-specific interest rates), $\lambda \equiv N_S/(N_S + N_R)$ the proportion of safe borrowers, and $p \equiv \lambda p_S + (1 - \lambda)p_R$ the average success probability among all firms. Firms of risk type $i \in \{S, R\}$ apply for capital if the expected profit $\pi_i(r) = p_i[R_i - (1 + r)B] + (1 - p_i)(-C)$ is non-negative, i.e., if $r \leq r_i$, where

$$r_i \equiv \frac{1}{B} \left(\frac{\bar{R} - C}{p_i} + C \right) - 1, \quad i \in \{S, R\}$$

($r_i > 0$ for $i \in \{S, R\}$).⁸ The demand for capital is

$$D(r) = \begin{cases} (N_S + N_R)B; & r \leq r_S \\ N_R B; & r_S < r \leq r_R \\ 0; & r_R < r \end{cases} . \quad (2)$$

The average success probability in the pool of credit applicants is p for $r \leq r_S$ and p_R for $r_S < r \leq r_R$. Denote the (random) return on lending at rate r in state s as $i(r)$. This function will be determined below. Since the revenue from lending is passed through completely, the consumers solve

$$\max_s : \frac{(y - s)^{1-\theta}}{1-\theta} + \delta E \left(\frac{\{[1 + i(r)]s\}^{1-\theta}}{1-\theta} \right)$$

by choosing an appropriate level of savings s . Let

$$\hat{R}(r) \equiv E \left\{ [1 + i(r)]^{1-\theta} \right\}. \quad (3)$$

Optimal saving is

$$s = \frac{y}{1 + \delta^{-\frac{1}{\theta}} \hat{R}(r)^{-\frac{1}{\theta}}} \equiv s^*(r) \quad (4)$$

⁸The variances of safe firms' profit and their repayment to lenders are $p_S(1 - p_S)[R_S - (1 + r)B + C]^2$ and $p_S(1 - p_S)[(1 + r)B - C]$, respectively. At $r = r_S$, the ratio of the variances is $[C/(\bar{R} - C)]^2$ and exceeds unity if $C > \bar{R}/2$, i.e., if collateral does not fall short of half of the expected project payoff. Similarly, the ratio of the variance of risky firms' profit to repayment at $r = r_S$

$$\left(\frac{\frac{p_S - p_R}{p_R} \bar{R} + C}{\bar{R} - C} \right)^2$$

exceeds unity under the weaker condition $C > [1 - p_S/(2p_R)]\bar{R}$. This confirms the assertion that for well-collateralized credit, firm owners bear the bulk of the risk created by their investments at an equilibrium with interest rate r_S , even though the return on lending is not safe either. If expected profit is a modest percentage of expected project payoff and, hence, of expected repayment, a comparison in terms of coefficients of variation reinforces this conclusion.

($0 < s^*(r) < y$), and the indirect utility function is

$$u(y - s^*(r)) + \delta E \{u([1 + i(r)]s^*(r))\} = \frac{y^{1-\theta}}{1-\theta} \left[1 + \delta^{\frac{1}{\theta}} \hat{R}(r)^{\frac{1}{\theta}}\right]^\theta \equiv v(r). \quad (5)$$

From (4), the total supply of capital by a measure M of consumers facing the stochastic return profile $i(r)$ (yet to be determined) is

$$S(r, M) = s^*(r)M. \quad (6)$$

For the sake of brevity, we focus attention on model specifications such that

$$N_R B < S(r_S, M) < (N_S + N_R)B. \quad (7)$$

That is, the supply of capital at interest rate r_S is sufficient to carry out the risky projects but not all projects. This assumption rules out single-price market clearing equilibria (with or without adverse selection) and thus allows us to focus on the two most interesting types of equilibria: pure credit rationing equilibrium and two-price equilibrium. A pure rationing equilibrium prevails when there is positive excess demand for capital but there is no interest rate that implies a more favorable return distribution for the consumers. Let X denote the quantity of capital channeled from lenders to borrowers.

Definition 1: (r_1, X) is a pure rationing equilibrium (PRE) if

- $\hookrightarrow X = S(r_1, M) < D(r_1),$
- \hookrightarrow there is no r such that $v(r) > v(r_1).$

A two-price equilibrium entails that credit is given at two different interest rates r_1 and r_2 ($> r_1$), with positive excess demand at the lower rate r_1 and equality of supply and residual demand at the higher rate r_2 . To qualify as an equilibrium, the two interest rates have to provide consumers with the same level of indirect utility. Moreover, interest rates at which there is positive residual demand (i.e., $r < r_2$) must be no more favorable to consumers than r_1 and r_2 . Let M_1 (> 0) and $M_2 = M - M_1$ (> 0) denote the measures of consumers giving credit and X_1 and X_2 the quantities of capital channelled from lenders to borrowers at r_1 and r_2 , respectively. Denote the residual demand at r_2 as \tilde{D}_2 .

Definition 2: $(r_1, r_2, M_1, M_2, X_1, X_2)$ with $r_2 > r_1$ is a two-price equilibrium (TPE) if

- $\hookrightarrow v(r_1) = v(r_2),$
- $\hookrightarrow X_1 = S(r_1, M_1) < D(r_1),$
- $\hookrightarrow X_2 = S(r_2, M_2) = \tilde{D}_2,$
- \hookrightarrow there is no $r < r_2$ such that $v(r) > v(r_2).$

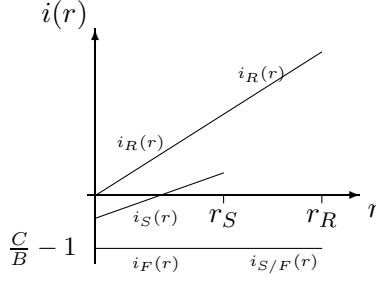


Figure 1: Return function

In the original SW model, pure credit rationing cannot arise in equilibrium; if there is excess demand at r_S but not at r_R , as stipulated in (7), the equilibrium is a TPE (see Coco, 1997, Arnold and Riley, 2009, and Appendix A.1). Our first main result states that, given the presence of aggregate risk and risk-averse suppliers of capital, a PRE *can* arise: a PRE exists whenever there is not a TPE (except in the measure-zero event $v(r_R) = v(r_S)$):

Proposition 1: *Let (7) hold. (a) If $v(r_R) < v(r_S)$, there is a PRE and not a TPE. (b) If $v(r_R) > v(r_S)$, there is a TPE and not a PRE. (c) If $v(r_R) = v(r_S)$, there are a TPE and a PRE.*

Proof: The crucial observation is that $S(r, M)$ (for given M) and $v(r)$ move in the same direction as r changes. This follows immediately from (4)-(6): r affects $S(r, M)$ and $v(r)$ only via $\hat{R}(r)$, and both functions increase when $\hat{R}(r)$ rises. From (3),

$$\hat{R}'(r) = (1 - \theta)E \left\{ [1 + i(r)]^{-\theta} i'(r) \right\} \quad (8)$$

whenever $i'(r)$ exists. Let $i_s(r)$ denote the return on lending in state s :⁹

$$\begin{pmatrix} i_R(r) \\ i_S(r) \\ i_F(r) \end{pmatrix} = \begin{pmatrix} r \\ \lambda r + (1 - \lambda) \left(\frac{C}{B} - 1 \right) \\ \frac{C}{B} - 1 \end{pmatrix}, \quad r \leq r_S, \quad (9)$$

and

$$\begin{pmatrix} i_R(r) \\ i_{S/F}(r) \end{pmatrix} = \begin{pmatrix} r \\ \frac{C}{B} - 1 \end{pmatrix}, \quad r_S < r \leq r_R \quad (10)$$

(see Figure 1). Both for $r < r_S$ and for $r_S < r < r_R$, we have $i'_s(r) \geq 0$ in all states and strict inequality for some state s . It follows from (8) that $\hat{R}'(r) > 0$ and, therefore, $(s^*)'(r) > 0$, $\partial S(r, M)/\partial r > 0$, and $v'(r) > 0$ for $r < r_S$ and for $r_S < r < r_R$. For $r > r_S$, let $\Delta(r)$ denote the difference between

⁹A subscript s/s' on a function or variable indicates that it applies in states s and s' .

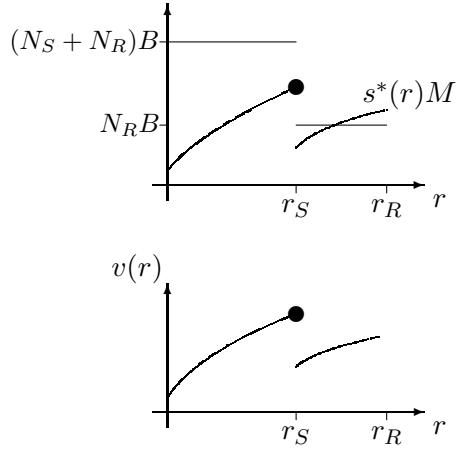


Figure 2: PRE

$\hat{R}(r)$ and $\hat{R}(r_S)$:

$$\Delta(r) \equiv \hat{R}(r) - \hat{R}(r_S), \quad r > r_S. \quad (11)$$

Using (3), (9), and (10), one obtains

$$\begin{aligned} \Delta(r) = & -(p_S - p_R) \left\{ \left[\lambda(1 + r_S) + (1 - \lambda) \frac{C}{B} \right]^{1-\theta} - \left(\frac{C}{B} \right)^{1-\theta} \right\} \\ & + p_R \left[(1 + r)^{1-\theta} - (1 + r_S)^{1-\theta} \right]. \end{aligned} \quad (12)$$

Let $\varepsilon > 0$ and $\varepsilon \rightarrow 0$. As the last term in square brackets goes to zero, while the term in braces does not (since $r_S > 0 > C/B - 1$), we have $\lim_{\varepsilon > 0, \varepsilon \rightarrow 0} \Delta(r_S + \varepsilon) < 0$. That is, $\hat{R}(r)$ and, hence, $s^*(r)$, $S(r, M)$, and $v(r)$ jump downward at r_S . Thus, both $S(r, M)$ (for given M) and $v(r)$ attain their respective global maxima on the interval $[0, r_R]$ either at r_S or at r_R . The two constellations consistent with (7) are illustrated in Figures 2 and 3 (evidently, parameter combinations giving rise to either case exist, as will be illustrated by means of example below).

(a) In Figure 2, $S(r, M)$ and $v(r)$ attain their (unique) respective maxima at r_S . $(r_S, S(r_S, M))$ is a PRE. Since $v(r_S) > v(r_1)$ and $r_S < r_2$ whenever $v(r_1) = v(r_2)$ for two interest rates r_1 and r_2 , a TPE does not exist.

(b) In Figure 3, $S(r, M)$ and $v(r)$ attain their (unique) maxima at r_R . In this case, a PRE does not exist. For whenever there is positive excess demand at r ($< r_R$), the second condition in the definition of a PRE is violated: $v(r_R) > v(r)$. Let $r_1 = r_S$. There exists an interest rate $r_2 > r_1$ such that $v(r_2) = v(r_1)$. Let X_1 and X_2 be determined by

$$X_1 = \frac{N_S + N_R}{N_S} [S(r_1, M) - N_R B] = S(r_1, M) - X_2 \quad (13)$$

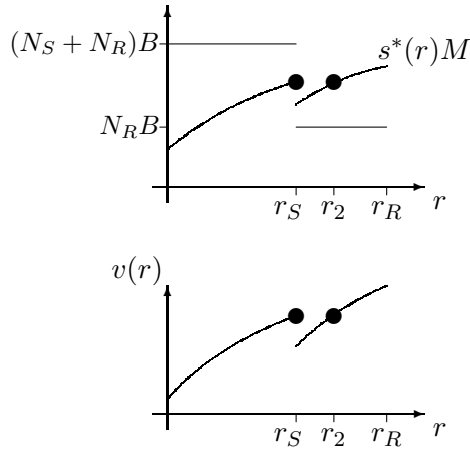


Figure 3: TPE

and $M_1 = X_1/s^*(r_1)$ (> 0) and $M_2 = X_2/s^*(r_2)$ (> 0). From (5), $v(r_2) = v(r_1)$ implies $\hat{R}(r_2) = \hat{R}(r_1)$. From (4), it follows that $s^*(r_2) = s^*(r_1)$. Using (6) and (13), it follows from the definitions of M_1 and M_2 that $M_1 + M_2 = M$. The residual demand at r_2 is

$$\tilde{D}_2 = \left[1 - \frac{X_1}{(N_S + N_R)B} \right] N_R B. \quad (14)$$

It is straightforward to check that $(r_1, r_2, M_1, M_2, X_1, X_2)$, thus defined, is a TPE: by construction, $v(r_1) = v(r_2)$; from (6) and $M_1 = X_1/s^*(r_1)$, $X_1 = S(r_1, M_1)$; from $M_1 < M$, $\partial S(r, M)/\partial M > 0$, $r_1 = r_S$, and (7), $S(r_1, M_1) < D(r_1)$; from (6) and $M_2 = X_2/s^*(r_2)$, $X_2 = S(r_2, M_2)$; from (13) and (14) together with (6) and $M_2 = X_2/s^*(r_2)$, $S(r_2, M_2) = \tilde{D}_2$ ($= -(N_R/N_S)S(r_1, M) + (N_S + N_R)(N_R/N_S)B$); by construction, $v(r) \leq v(r_2)$ for all $r < r_2$.

(c) The proofs that $(r_S, S(r_S, M))$ is a PRE in case (a) and $(r_1, r_2, M_1, M_2, X_1, X_2)$ is a TPE in case (b) also go through when $v(r_S) = v(r_R)$. ||

The multiplicity of equilibria in case (c) is not by itself remarkable, as $v(r_S) = v(r_R)$ is a measure-zero event.¹⁰ We will come back to this case, however, when we compare welfare levels in the two types of equilibria. For now, the important point is that aggregate risk makes the emergence of a PRE possible.¹¹

As a numerical example, let the model parameters be given by:

¹⁰One could avoid this multiplicity by making the inequality in the second condition of Definition 1 weak.

¹¹Since r_S is the single equilibrium interest rate in a PRE, the considerations of who bears how much risk in footnote 8 apply. In a TPE, the same considerations apply for safe firms and for risky firms which get credit at r_S . For risky firms that receive credit at r_2 , the ratio of the variances of profit and repayment is $([(1 - \lambda)(p_S - p_R)C - p_S \bar{R}]/\{[p_R + \lambda(p_S - p_R)]C\})^2$ and exceeds unity if $C > \{p_S/[(1 - 2\lambda)(p_S - p_R) - p_R]\} \bar{R}$.

y	M	θ	δ	B	p_S	p_R	\bar{R}	C	N_S	N_R
10	3,000	0.5	0.95	100	0.9	0.8	110	80	100	100

Safe and risky firms apply for credit up to interest rates $r_S = 13.33\%$ and $r_R = 17.5\%$, respectively. At r_S , each consumer supplies $s(r_S) = 4.9369$ units of capital, so the total supply of capital is 14,810.6541 ($= S(r_S, M)$). Since it is higher than 10,000 ($= N_R B$) and lower than 20,000 ($= (N_S + N_R)B$), condition (7) is satisfied. As $\Delta(r_R) = 0.0066 > 0$, the equilibrium is a TPE. The consumers' indirect utility is $v(r_S) = 8.8883$. A measure 96.2131 of randomly selected firms receive a loan at 13.33% interest, and the 51.8935 risky firms rationed at this interest receive a loan at 15.71% interest.

4 Risk aversion and rationing

In a PRE, since rationing is necessarily random, the proportion of realized projects which are risky is equal to the proportion of risky firms in the total population of firms (i.e., $1 - \lambda$). The presence of rationing means that some risky firms do not get funds. In a TPE, by contrast, all risky projects are carried out, which makes the return profile less attractive from the risk-averse lenders' point of view. This suggests that a PRE is more likely to emerge as the lenders' degree of relative risk aversion rises. The present section gives a precise statement of this proposition.

Within the framework used so far we encounter the familiar problem that both relative risk aversion and the preference for intertemporal consumption smoothing are parameterized by the same parameter θ . So in order to appropriately address the question of which type of equilibrium emerges under what degree of risk aversion, we generalize the analysis by following Selden's (1978) OCE approach. Let \hat{c}_2 be the certainty equivalent (CE) corresponding to the period utility function introduced in (1):

$$\frac{\hat{c}_2^{1-\theta}}{1-\theta} = E \left(\frac{c_2^{1-\theta}}{1-\theta} \right), \quad \theta > 0, \quad \theta \neq 1 \quad (15)$$

(we now allow for $\theta > 1$).¹² Households' utility is given by

$$u(c_1, \hat{c}_2) = \frac{c_1^{1-\eta}}{1-\eta} + \delta \frac{\hat{c}_2^{1-\eta}}{1-\eta}, \quad 0 < \eta < 1. \quad (16)$$

$1/\eta$ is the (intertemporal) elasticity of substitution between c_1 and \hat{c}_2 . The model analyzed above is the special case with $\eta = \theta$.

¹²For the sake of brevity, we omit the case of logarithmic utility (i.e., $\theta = 1$), which has to be treated separately.

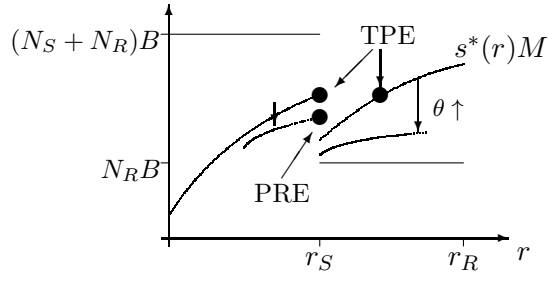


Figure 4: Switch from TPE to PRE

$\hat{R}(r)^{1/(1-\theta)}$ defined in (3) is the CE of $1 + i(r)$. Using this definition and $c_2 = [1 + i(r)]s$, the CE defined in (15) can be expressed as:

$$\hat{c}_2 = s \hat{R}(r)^{\frac{1}{1-\theta}}.$$

Households solve

$$\max_s : \frac{(y - s)^{1-\eta}}{1-\eta} + \delta \frac{[s \hat{R}(r)^{\frac{1}{1-\theta}}]^{1-\eta}}{1-\eta}.$$

The solution is

$$s = \frac{y}{1 + \delta^{-\frac{1}{\eta}} \hat{R}(r)^{-\frac{1}{1-\theta} \frac{1-\eta}{\eta}}} \equiv s^*(r), \quad (17)$$

and indirect utility is

$$v(r) \equiv \frac{y^{1-\eta}}{1-\eta} \left[1 + \delta^{\frac{1}{\eta}} \hat{R}(r)^{\frac{1}{1-\theta} \frac{1-\eta}{\eta}} \right]^\eta. \quad (18)$$

Given the novel definitions of $s^*(r)$ and $v(r)$, define capital supply (cf. (6)) and the two types of equilibria as before. Our first task is to generalize Proposition 1:

Proposition 2: *The assertion of Proposition 1 also holds true in the OCE framework.*

Proof: The crucial observation is that, as before, changes in the interest rate move $S(r, M)$ and $v(r)$ in the same direction, as is evident from (17) and (18). From (3),

$$\frac{d \left[\hat{R}(r)^{\frac{1}{1-\theta}} \right]}{dr} = \hat{R}(r)^{\frac{\theta}{1-\theta}} E \left\{ [1 + i(r)]^{-\theta} i'(r) \right\}.$$

Equations (9) and (10) hold true without modification. So for all $r < r_S$ and $r_S < r < r_R$, we have $d[\hat{R}(r)^{1/(1-\theta)}]/dr > 0$ and a fortiori $(s^*)'(r) > 0$ (from (17)), $\partial S(r, M)/\partial r > 0$ (from (6)), and $v'(r) > 0$ (from (18)). Since (3), (9), and (10) are unchanged, $\Delta(r)$ defined in (11) satisfies (12). For $\theta < 1$, the arguments put forward in the proof of Proposition 1 prove $\hat{R}(r_S + \varepsilon) < \hat{R}(r_S)$ for ε positive and small. Hence, $\hat{R}(r_S + \varepsilon)^{1/(1-\theta)} < \hat{R}(r_S)^{1/(1-\theta)}$. From (6), (17), and (18), it follows that $S(r, M)$ and $v(r)$ jump downward as r rises above r_S . For $\theta > 1$, the term in braces in (12) is

negative, so $\lim_{\varepsilon>0, \varepsilon \rightarrow 0} \Delta(r_S + \varepsilon) > 0$, i.e., $\hat{R}(r_S + \varepsilon) > \hat{R}(r_S)$ for ε positive and small. As before, this implies $\hat{R}(r_S + \varepsilon)^{1/(1-\theta)} < \hat{R}(r_S)^{1/(1-\theta)}$, so that, in this case also, $S(r, M)$ and $v(r)$ display downward discontinuities at r_S . This proves that both $S(r, M)$ and $v(r)$ attain their respective maxima either at r_S or at r_R . Under the maintained assumption (7), the remainder of the proof runs parallel to the proof of Proposition 1. ||

We are now in a position to give a precise statement of the proposition that higher risk aversion makes the emergence of a PRE more likely:

Proposition 3: *Starting from a TPE, as the degree of relative risk aversion θ rises and (7) remains satisfied, a PRE emerges.*

Proof: The proof consists of two steps. (a) Increases in θ reduce $s^*(r)$. (b) For θ large enough, $s^*(r)$ and $v(r)$ attain their maxima at r_S . Thus, starting from a TPE, as risk aversion becomes stronger, the capital supply schedule shifts downward, and at some point the maximum will occur at r_S and a PRE emerges, provided that (7) is still valid (see Figure 4).

(a) From (17), as noted by Basu und Ghosh (1993, p. 121), a decrease in the CE $\hat{R}(r)^{1/(1-\theta)}$ reduces $s^*(r)$. So we have to prove that an increase in θ reduces $\hat{R}(r)^{1/(1-\theta)}$ or, equivalently, $\ln[\hat{R}(r)^{1/(1-\theta)}]$. From (3) (suppressing the argument of the functions $\hat{R}(r)$ and $i(r)$),

$$\frac{\partial \left[\ln \left(\hat{R}^{\frac{1}{1-\theta}} \right) \right]}{\partial \theta} = \frac{1}{(1-\theta)^2} \frac{1}{E[(1+i)^{1-\theta}]} \cdot \left(E \left[(1+i)^{1-\theta} \right] \ln \{ E[(1+i)^{1-\theta}] \} - E \left\{ (1+i)^{1-\theta} \ln \left[(1+i)^{1-\theta} \right] \right\} \right).$$

Since $(1+i)^{1-\theta} \ln[(1+i)^{1-\theta}]$ is a strictly convex function of the random variable $(1+i)^{1-\theta}$, the derivative is negative by virtue of Jensen's inequality.

(b) From (12),

$$\Delta(r_R) = \left(\frac{C}{B} \right)^{1-\theta} \left\{ -(p_S - p_R) \left[\left(\lambda \frac{1+r_S}{\frac{C}{B}} + 1 - \lambda \right)^{1-\theta} - 1 \right] + p_R \left[\left(\frac{1+r_R}{\frac{C}{B}} \right)^{1-\theta} - \left(\frac{1+r_S}{\frac{C}{B}} \right)^{1-\theta} \right] \right\}.$$

Since $C/B < 1+r_S < 1+r_R$, as θ grows large (in particular larger than one), the power terms inside the braces become arbitrarily small, so $\Delta(r_R) > 0$. From (11), $\hat{R}(r_R) > \hat{R}(r_S)$ and $\hat{R}(r_R)^{1/(1-\theta)} < \hat{R}(r_S)^{1/(1-\theta)}$. From (17), it follows that $s^*(r)$ attains its maximum at r_S . ||

The example considered below shows that there exist model specifications such that a switch from a TPE to a PRE occurs. More generally, the following condition is sufficient to ensure that (7)

remains valid as θ rises:

$$N_{RB} < \frac{y}{1 + \delta^{-\frac{1}{\eta}} \left(\frac{C}{B}\right)^{-\frac{1-\eta}{\eta}}} M. \quad (19)$$

$i(r_S) \geq C/B - 1$ in all states of nature with strict inequality in some state of nature. Hence, $\hat{R}(r_S)^{1/(1-\theta)} > C/B$. From (6) and (17), $S(r_S, M)$ exceeds the term on the right-hand side of (19) for all θ , so $N_{RB} < S(r_S, M)$. The validity of the second inequality in (7) follows from the fact that it is satisfied in the TPE and $S(r_S, M)$ becomes smaller as θ increases (step (a) in the proof of Proposition 3).

In the numerical example introduced at the end of the preceding section (which implicitly assumes $\theta = \eta = 0.5$), the critical value for θ , above which the equilibrium is a PRE, is 2.5435. If, for instance, $\theta = 3$, we have $\Delta(r_R) = 0.0058$. The single equilibrium interest rate is $r_S = 13.33\%$, indirect utility is $v(r_S) = 8.8516$. The supply of capital is 14,684.1058, so 53.1589 (or 26.58%) of the borrowers are rationed.

Another way of analyzing the impact of portfolio risk on the nature of equilibrium is to compare the model with correlated payoffs to the (identically parameterized) model with independent payoffs. Since rationing cannot arise in the model with uncorrelated risks, it follows immediately from Propositions 1 and 2 that the introduction of correlated risks (holding everything else equal) potentially causes rationing. Details are in Appendix A.1.

5 Inefficiency of a two-price equilibrium

From a welfare point of view, a TPE is particularly unattractive. For even leaving aside the question of whether total investment is at its optimal level, it is precisely the risky projects, disliked by the risk-averse lenders, which have a one-hundred percent chance of being financed. In the present section, we show that, as a consequence, a change in a model parameters that gives rise to a switch from a TPE to a PRE has a discontinuous positive impact on welfare. Coco (1997, pp. 12-13) arrives at the same conclusion under the assumption that riskier projects have lower expected (uncorrelated) returns and concludes that a usury law that prevents lenders from attracting risky borrowers at high interest rates raises welfare. This conclusion naturally carries over to our set-up with non-diversifiable risk and risk-averse lenders.¹³

Proposition 4: *A parameter change that leads to a switch from a TPE to a PRE has a continuous effect on household utility but leads to a discontinuous increase in aggregate profit.*

¹³The effects of usury laws in the presence of moral hazard are analyzed in Coco and De Meza (2009).

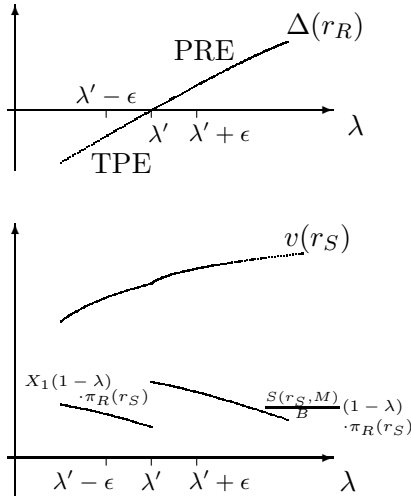


Figure 5: Welfare effect of a switch from TPE to PRE ($\theta > 1$)

Proof: The proof consists of two steps. We consider first (a) the knife-edge case $v(r_S) = v(r_R)$, in which both a PRE and a TPE exist, and then (b) a small change in a parameter that leads to a switch from a TPE to a PRE.

(a) Consider the model with OCE preferences and suppose parameters are such that $v(r_S) = v(r_R)$, i.e., $\Delta(r_R) = 0$. From part (c) of Proposition 1 and Proposition 2, $(r_S, S(r_S, M))$ is a PRE, and there is also a TPE with $r_2 = r_R$. Households' indirect utility is $v(r_S)$ in both equilibria. The mass of projects carried out in equilibrium $S(r_S, M)/B$ is also the same in both equilibria. But while in the PRE the loan rate is r_S for all borrowers who get funds, some borrowers have to pay the interest rate r_R in the TPE. This is the price firms have to pay in order to make households finance riskier projects. Total expected firm profit in the TPE is

$$\frac{X_1}{B}(1 - \lambda)\pi_R(r_S)$$

(where use is made of $\pi_S(r_S) = 0$ and $\pi_R(r_2) = \pi_R(r_R) = 0$). This compares with aggregate profit

$$\frac{S(r_S, M)}{B}(1 - \lambda)\pi_R(r_S)$$

in a PRE (where use is made of $\pi_S(r_S) = 0$). Since $X_1 < S(r_S, M)$, expected profit is strictly less in the TPE.

(b) If $\theta > 1$, a switch from a TPE to a PRE occurs when $\Delta(r_R)$ changes from negative to positive, and vice versa for $\theta < 1$. Consider model parameters $(B, C, p_S, p_R, \lambda) = (B', C', p'_S, p'_R, \lambda')$ such that $\Delta(r_R) = 0$. Consider a change in a parameter $a \in \{B, C, p_S, p_R, \lambda\}$ that has a first-order effect on

$\Delta(r_R)$ (i.e., $d\Delta(r_R)/da \neq 0$ for $(B, C, p_S, p_R, \lambda) = (B', C', p'_S, p'_R, \lambda')$). Without loss of generality, let $d\Delta(r_R)/da > 0$ (if the reverse inequality holds true, consider parameter $-a$). For instance, let $a = \lambda$ and $\theta > 1$: from (12), $d\Delta(r)/d\lambda > 0$ for $\theta > 1$ (see Figure 5). For $\theta > 1$, when the parameter rises from $a' - \varepsilon$ to $a' + \varepsilon$ (ε positive and small), $\Delta(r_R)$ turns from negative to positive and a PRE replaces a TPE, and vice versa for $\theta < 1$. From the analysis in the preceding section, it is clear that this is the only way a change in a parameter can bring about a switch from a TPE to PRE. From (3), (9), and (18), $v(r_S)$ is a continuous function of model parameters. From step (a), aggregate profit jumps upward. ||

As pointed out by Coco (1997), this special inefficiency of a TPE can be avoided by imposing an upper bound \bar{r} on the set of admissible interest rates:

Proposition 5: *If the equilibrium is a TPE, imposing an interest rate ceiling $\bar{r} \in (r_S, r_2)$ raises welfare.*

Proof: Define a PRE with an interest ceiling as in Definition 1 except that we add the condition $r_1 \leq \bar{r}$. The condition $r \leq \bar{r}$ rules out the existence of a TPE, for $v(r_S)$ exceeds $v(r)$ for any admissible pair of interest rates which yield the same expected utility. $(r_S, S(r_S, M))$ is a PRE with interest ceiling, since $v(r) < v(r_S)$ for all $r \leq \bar{r}$, $r \neq r_S$. ||

To illustrate the assertions of Propositions 4 and 5, we modify our running example such that θ is close to the critical value above which a PRE emerges:

y	M	η	θ	δ	B	p_S	p_R	\bar{R}	C	N_S	N_R
10	3,000	0.5	2.55	0.95	100	0.9	0.8	110	80	99	101

We also changed the numbers of safe and risky firms slightly, but will return to the original values in a second. At interest rate $r_S = 13.33\%$, each household saves $s^*(r_S) = 4.9029$, so the supply of capital is 14,708.7000. As demand drops from 20,000 to 10,100 at r_S , condition (7) is satisfied. The critical value for θ above which there is a PRE is 2.5720, so the equilibrium is a TPE with $r_2 = 17.47\%$ (which can also be inferred from $\Delta(r_R) = -0.0002$ and $\theta > 1$). Indirect utility and aggregate profit are $v(r_S) = v(r_2) = 8.8587$ and 157.8703, respectively. Now, let $N_S = N_R = 100$, holding all other parameters fixed, so that λ rises from 49.5% to 50%. Since θ is now above the critical value 2.5435 ($\Delta(r_R) = 6 \cdot 10^{-5}$ is positive), a PRE emerges. Indirect utility rises only slightly (to $v(r_S) = 8.8591$), but aggregate profit soars by more than 50% to 245.1701. In the former case, with 99 safe and 101 risky firms, consider the imposition of an interest rate ceiling \bar{r} between 13.34% and 15.70%. The equilibrium becomes a PRE with 13.33% interest. Borrower expected utility is

$(v(r_S) =) 8.8587$, as in the TPE without the interest ceiling. But aggregate firm profit rises from 157.8703 to $((X/B)(1 - \lambda)\pi_R(r_S) =) 247.5964$.

6 Optimum investment

A TPE is “more inefficient” than a PRE, but even a PRE entails inefficient risk sharing. This is because firms’ collateral is a potential hedge against households’ period-2 consumption risk, but is not used completely in order to insure consumers in equilibrium, as firms which do not invest or whose projects succeed keep their collateral.¹⁴ This section characterizes the optimal solution of the model, with efficient risk sharing, and addresses the question of whether the market brings forth too little or too much investment.

For the sake of simplicity, we return to the expected utility setup. In the main text, we maintain the assumption that information is asymmetric: when n projects are carried out, λn turn out to be safe and $(1 - \lambda)n$ risky. We assume that the firms’ collateral can only be consumed in the second period. The constrained-efficient solution maximizes household expected utility for a given level β of each firm owners’ expected utility by a suitable choice of investment and state-contingent consumption levels. We show that under-investment occurs, as one might expect, for high levels of β and for low values of collateral. However, since decreases in β reduce the need to invest in order to achieve the firm owners’ given level of expected utility and increases in collateral expand the pool of resources available for period-2 consumption, equilibrium over-investment can arise for low values of β and large values of collateral. (The first-best case, in which the planner can distinguish safe from risky projects, is analyzed in Appendix A.2.)

Suppose all firm owners receive the same levels of consumption in period 2, irrespective of whether their project is realized or not. Let α_s denote the firm owners’ consumption level in state $s \in \{R, S, F\}$. Since firm owners are risk-neutral, we can assume without loss of generality that their expected utility equals expected consumption, so the constraint that they receive a given level β (≥ 0) of expected utility becomes

$$p_R \alpha_R + (p_S - p_R) \alpha_S + (1 - p_S) \alpha_F = \beta. \quad (20)$$

In equilibrium, a firm owner’s expected consumption is $C + \pi_i(r)$ if he takes a loan at interest rate r and carries out his investment project (of type i) and C otherwise. So if, for instance, β equals the sum of collateral and expected equilibrium profit per firm owner, then the firm owners are as

¹⁴That is why the case in which firms’ collateral can be traded would require a separate analysis.

well-off in the CO as in the market equilibrium. Let $m \equiv (N_S + N_R)/M$ denote the “number” of firms per household and c_{2s} household consumption in state s . Then,

$$\begin{pmatrix} c_{2R} \\ c_{2S} \\ c_{2F} \end{pmatrix} = \begin{pmatrix} m(C - \alpha_R) + \frac{\lambda R_S + (1-\lambda)R_R}{B} s \\ m(C - \alpha_S) + \frac{\lambda R_S}{B} s \\ m(C - \alpha_F) \end{pmatrix}. \quad (21)$$

The level of investment per household s is bounded above by the minimum of mB and y , i.e., either by the amount of investment opportunities or by disposable income. For the sake of brevity, we restrict attention on the case in which My is sufficiently large to finance all investment projects:

$$mB \leq y.$$

The maximum attainable expected utility for firm owners is then $\beta = C + \bar{R}$ (achieved with $s = mB$ and $c_{2s} = 0$ for $s \in \{R, S, F\}$).

Definition 3: *Given $\beta \in [0, C + \bar{R}]$, $(c_1, s, c_{2R}, c_{2S}, c_{2F}, \alpha_R, \alpha_S, \alpha_F)$ is a constrained optimum (CO) if it maximizes (1) subject to the constraints $s \leq mB$, $c_1 = y - s$, (20), (21), and non-negativity of each component.*

Since (1) is continuous and the set of vectors which satisfy the constraints is non-empty and compact, a CO exists. Since households are risk-averse and firm owners are risk-neutral, household consumption is equalized across all states in which firm owners’ consumption is positive in a CO.¹⁵ For future reference, notice that period-2 household consumption is the same when all projects succeed or only the safe projects succeed (i.e., $c_{2R} = c_{2S}$) if

$$\alpha_R - \alpha_S = \frac{(1 - \lambda)R_R}{mB} s. \quad (22)$$

Similarly, $c_{2S} = c_{2F}$ if

$$\alpha_S - \alpha_F = \frac{\lambda R_S}{mB} s. \quad (23)$$

We have to distinguish three different cases, arising dependent on the choice of investment s .

(a) For

$$mB \frac{\beta - C}{\bar{R}} \leq s \leq mB \frac{\beta}{\bar{R}}, \quad (24)$$

s is sufficiently low and the share of investment returns in total wealth is sufficiently small so that it is possible to equalize households’ period-2 consumption across all three states. Solving (20), (22),

¹⁵If households consume different quantities in two states s and s' , say, their marginal rate of substitution differs from $p_s/p_{s'}$, which is the firm owners’ marginal rate of substitution. So if the firm owners’ consumption is positive in both states, there is scope for a mutually beneficial reallocation of resources.

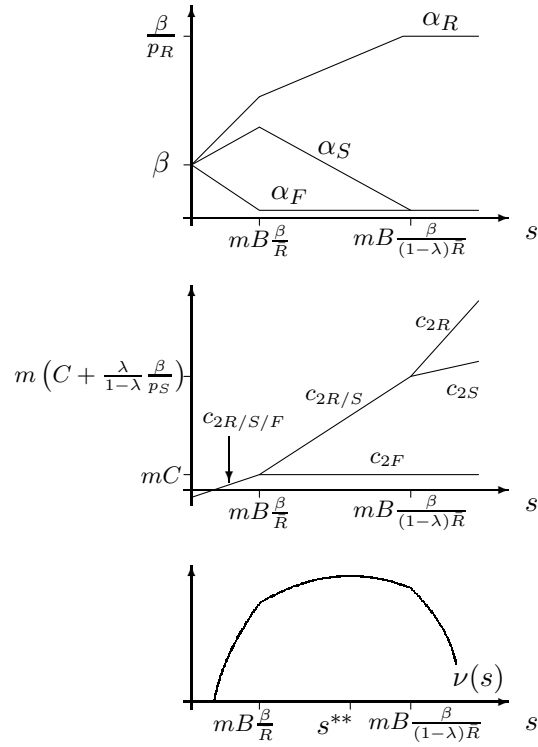


Figure 6: Consumption levels with optimal risk sharing under asymmetric information

and (23) for the α_s 's and substituting into (21) gives:

$$\begin{pmatrix} \alpha_R \\ \alpha_S \\ \alpha_F \end{pmatrix} = \begin{pmatrix} \beta + \frac{\lambda R_S + (1-\lambda)R_R - \bar{R}}{mB} s \\ \beta + \frac{\lambda R_S - \bar{R}}{mB} s \\ \beta - \frac{\bar{R}}{mB} s \end{pmatrix}$$

and

$$c_{2R/S/F} = m(C - \beta) + \frac{\bar{R}}{B} s \quad (25)$$

(consumers' and firm owners' state-contingent consumption levels in this and the following two cases are illustrated in the upper two panels of Figure 6). If the first inequality in (24) is violated, then the returns on investment are insufficient so as to satisfy (20). When the second inequality in (24) is violated, the non-negativity $\alpha_F \geq 0$ does not hold, i.e., the firm owners' risk bearing capacity in the state when all projects fail is exhausted.

(b) Second, consider investment levels

$$mB \frac{\beta}{\bar{R}} \leq s \leq mB \frac{\beta}{(1-\lambda)\bar{R}}. \quad (26)$$

In this case, it is possible to equalize consumption in states R and S , but not in F . From (20)-(22)

and $\alpha_F = 0$, we obtain:

$$\begin{pmatrix} \alpha_R \\ \alpha_S \\ \alpha_F \end{pmatrix} = \begin{pmatrix} \frac{1}{p_S} \left[\beta + \frac{(1-\lambda)p_S(R_R - R_S)}{mB} s \right] \\ \frac{1}{p_S} \left[\beta - \frac{(1-\lambda)\bar{R}}{mB} s \right] \\ 0 \end{pmatrix}$$

and

$$\begin{pmatrix} c_{2R/S} \\ c_{2F} \end{pmatrix} = \begin{pmatrix} m \left(C - \frac{\beta}{p_S} \right) + \frac{R_S}{B} s \\ mC \end{pmatrix}. \quad (27)$$

(c) Finally, for

$$mB \frac{\beta}{(1-\lambda)\bar{R}} \leq s, \quad (28)$$

c_{2R} exceeds c_{2S} and c_{2F} even if R is the only state in which firm owners consume:

$$\begin{pmatrix} \alpha_R \\ \alpha_{S/F} \end{pmatrix} = \begin{pmatrix} \frac{\beta}{p_R} \\ 0 \end{pmatrix}.$$

and

$$\begin{pmatrix} c_{2R} \\ c_{2S} \\ c_{2F} \end{pmatrix} = \begin{pmatrix} m \left(C - \frac{\beta}{p_R} \right) + \frac{\lambda R_S + (1-\lambda)R_R}{B} s \\ mC + \frac{\lambda R_S}{B} s \\ mC \end{pmatrix}. \quad (29)$$

Substituting $c_1 = y - s$ and c_{2s} from (25), (27), and (29) into (1) gives households' expected utility as a function $\nu(s)$ of s alone. The level of investment in a CO is $s^{**} = \arg \max_s \nu(s)$ s.t.: $0 \leq s \leq mB$. The function $\nu(s)$ is strictly concave (see the lower panel of Figure (6)). This follows from the fact that if two $(c_1, s, c_{2R}, c_{2S}, c_{2F}, \alpha_R, \alpha_S, \alpha_F)$ satisfy the constraints in Definition 3, then a convex combination also satisfies these constraints (because of linearity of the constraints) and yields higher expected utility (because of strict concavity of the function in (1) in $(c_1, c_{2R}, c_{2S}, c_{2F})$). $\nu(s)$ is continuous and it is differentiable in the interior of the intervals in (24), (26), and (28):

$$\nu'(s) + (y-s)^{-\theta} = \begin{cases} \frac{\delta \bar{R}}{B} \left[m(C - \beta) + \frac{\bar{R}}{B} s \right]^{-\theta}; & mB \frac{\beta - C}{R} < s < mB \frac{\beta}{R} \\ \frac{\delta \bar{R}}{B} \left[m \left(C - \frac{\beta}{p_S} \right) + \frac{R_S}{B} s \right]^{-\theta}; & mB \frac{\beta}{R} < s < mB \frac{\beta}{(1-\lambda)\bar{R}} \\ \frac{\delta p_R [\lambda R_S + (1-\lambda)R_R]}{B} \left[m \left(C - \frac{\beta}{p_R} \right) + \frac{\lambda R_S + (1-\lambda)R_R}{B} s \right]^{-\theta} \\ \quad + \frac{\delta (p_S - p_R) \lambda R_S}{B} \left(mC + \frac{\lambda R_S}{B} s \right)^{-\theta}; & mB \frac{\beta}{(1-\lambda)\bar{R}} < s \end{cases}. \quad (30)$$

It has kinks at $mB\beta/\bar{R}$ and at $mB\beta/[(1-\lambda)\bar{R}]$. There are four possible types of solutions to $s^{**} = \arg \max_s \nu(s)$ s.t.: $0 \leq s \leq mB$. For $s'(mB) \geq 0$, all projects are realized in the CO: $s^{**} = mB$. For $s'(0) \leq 0$, it is optimal not to invest at all: $s^{**} = 0$. If $\nu'(s^{**}) = 0$ for some s^{**} that satisfies one of the inequalities on the right-hand side of (30), then it is optimal to invest s^{**} . The final possibility

is that $\nu(s)$ attains its maximum at one of the kinks, i.e., at $s^{**} = mB\beta/\bar{R}$ or $mB\beta/[(1-\lambda)\bar{R}]$. By virtue of the theorem of the maximum, s^{**} is a continuous function of β and C . In a CO with $\nu'(s^{**}) = 0$, investment s^{**} increases when β rises or C falls. This follows from the strict concavity of $\nu(s)$ and the fact that an increase in β or a decrease in C raises $\nu'(s)$ (see (30)). Optimum investment s^{**} also increases with β when it occurs at a kink of $\nu(s)$.

Depending on the model parameters β , \bar{R} , and λ , some or all of cases (a)-(c) arise for admissible investment levels $0 \leq s \leq mB$.

$\bar{\mathbf{R}} \leq \beta$:

In this case, $s \leq mB \leq mB\beta/\bar{R}$. From (24), only case (a) can arise. It is optimal to carry out all projects (i.e., $s^{**} = mB$) if $(\nu'(mB) \geq 0$, i.e.)

$$\beta \geq \bar{R} + C - \left(\frac{y}{m} - B\right) \left(\frac{\delta\bar{R}}{B}\right)^{\frac{1}{\theta}}.$$

Otherwise optimum investment satisfies the first-order condition $\nu'(s^{**}) = 0$, i.e.,

$$s^{**} = \frac{y - \left(\frac{\delta\bar{R}}{B}\right)^{-\frac{1}{\theta}} m(C - \beta)}{1 + \delta^{-\frac{1}{\theta}} \left(\frac{\bar{R}}{B}\right)^{-\frac{1-\theta}{\theta}}}. \quad (31)$$

Since $\beta \geq \bar{R}$ and $\bar{R} > C$, we have $\beta > C$. So, as noted in the discussion of case (a) above, $s^{**} > 0$.

$(1 - \lambda)\bar{\mathbf{R}} \leq \beta < \bar{\mathbf{R}}$:

For s small enough, case (a) applies. For $s = mB$, (26) holds with strict inequalities, i.e., case (b) applies. From (30), it is optimal to finance all projects if

$$\beta \geq \bar{R} + p_S \left[C - \left(\frac{y}{m} - B\right) \left(\frac{\delta\bar{R}}{B}\right)^{\frac{1}{\theta}} \right].$$

If the right-hand side of this inequality is non-positive for $C = 0$, then it is optimal to realize all projects (i.e., $s^{**} = mB$) for C low enough, irrespective of β . If the right-hand side is positive for $C = 0$, $s^{**} = mB$ becomes optimal once β is large enough. If $\beta \geq C$, zero investment is ruled out on the same grounds as before. If, on the other hand, $\beta < C$, $s^{**} = 0$ is optimal (i.e., $\nu'(0) \leq 0$) if

$$\beta \leq C - \left(\frac{\delta\bar{R}}{B}\right)^{\frac{1}{\theta}} \frac{y}{m}. \quad (32)$$

$\beta < (1 - \lambda)\bar{\mathbf{R}}$:

Each of the cases (a)-(c) arises for some $s \leq mB$. It is optimal to realize all projects if $\nu'(mB) \geq 0$,

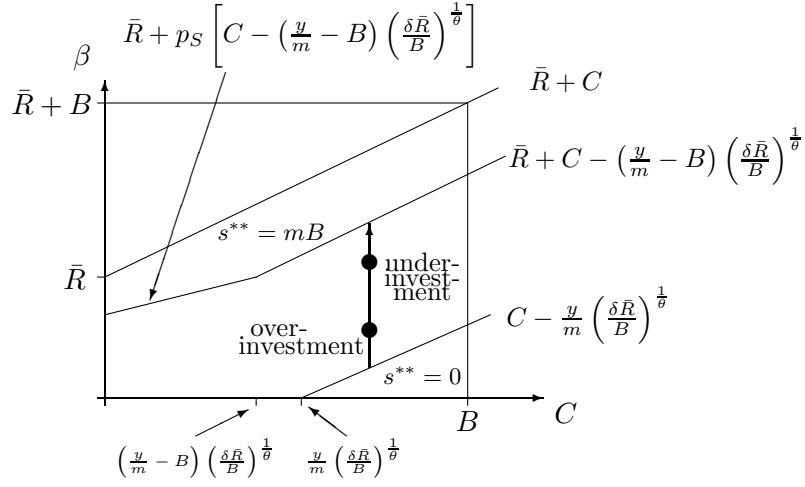


Figure 7: Optimum investment with asymmetric information

where now the last lines on the right-hand side of (30) are relevant. As in the preceding case, $s^{**} > 0$ for $\beta \geq C$, while $s^{**} = 0$ if (32) holds for $\beta < C$.

The results are summarized in Figure 7 (where $s^{**} < mB$ for $\beta = 0$) and in:

Proposition 6: For $\beta = 0$, $s^{**} > 0$ or $s^{**} = 0$, depending on whether $C < (y/m)(\delta\bar{R}/B)^{1/\theta}$ or $C \geq (y/m)(\delta\bar{R}/B)^{1/\theta}$, respectively. In the latter case, $s^{**} > 0$ for $\beta > C - (y/m)(\delta\bar{R}/B)^{1/\theta}$. s^{**} is non-decreasing in β . $s^{**} = mB$ for $\beta \geq \bar{R} + C - (y/m - B)(\delta\bar{R}/B)^{1/\theta}$.

In words, consumers respond to a reduction in their period-2 consumption possibilities brought about by an increase in the firm owners' expected utility β with higher investment s^{**} . Increases in collateral C relax the constraint that firm owners receive a given level of expected utility, so optimum investment s^{**} decreases.

Having characterized both the credit market equilibrium and the CO, we can now address the question of whether there is too little or too much investment in equilibrium.

Proposition 7: In a PRE or a TPE, (a) for C small enough, there is under-investment relative to the CO for all β ; (b) for C large enough, there is over-investment for sufficiently low values of β and under-investment for sufficiently high values of β .

Proof: (a) Suppose $\beta = C = 0$. From (28), case (c) applies for all investment levels. As noted above, $\beta \geq C$ implies that the CO entails $s^{**} > 0$. From (30), s^{**} satisfies

$$s^{**} = \frac{y}{1 + \delta^{-\frac{1}{\theta}} \left\{ p_R \left[\frac{\lambda R_S + (1-\lambda)R_R}{B} \right]^{1-\theta} + (p_S - p_R) \left(\frac{\lambda R_S}{B} \right)^{1-\theta} \right\}^{-\frac{1}{\theta}}}.$$

The term in braces exceeds

$$p_R \left(\frac{R_S}{B} \right)^{1-\theta} + (p_S - p_R) \left(\frac{\lambda R_S}{B} \right)^{1-\theta} = \hat{R}(r_S). \quad (33)$$

So, from (4), $s^{**} > s^*(r_S)$ for $\beta = C = 0$. $C = 0$ is not admissible in the equilibrium analysis (in the absence of collateral, firms would have nothing to lose from taking a credit). But both optimum investment s^{**} and $s^*(r_S)$ are continuous functions of C (for $s^*(r_S)$, this follows from the definition of r_S , (3), (4), and (9)). Therefore, there is under-investment (i.e., $s^{**} > s^*(r_S)$) for $\beta = 0$ and C sufficiently small. The fact that s^{**} is non-decreasing in β and $s^*(r_S) (< mB)$ is independent of β implies under-investment for all β .

(b) The fact that $s^{**} = 0$ for $\beta = 0$ and $C \geq (y/m)(\delta \bar{R}/B)^{1/\theta}$ (from Proposition 6) and $s^*(r_S) > (1 - \lambda)mB > 0$ (from (7)) implies that there is over-investment for $\beta = 0$ and C large enough. Under-investment occurs as β becomes sufficiently large so that $s^{**} = mB$. ||

From (4), for each C (and other model parameters except y), there exists y such that (7) is satisfied. So, from Proposition 1, for each $C > 0$ and $\beta \geq 0$, there exist parameterizations of the model such that a PRE or a TPE exists. This proves that parameter combinations exist such that over-investment arises in a PRE or a TPE.

The standard under-investment result holds true if firms have little collateral and a sufficiently high weight in the planning solution. However, if there is abundant collateral a large portion of which can be reallocated to the consumers in the CO, then there is equilibrium over-investment. To illustrate this, let us return to our running example:

y	M	θ	δ	B	p_S	p_R	\bar{R}	C	N_S	N_R	β
10	3,000	0.5	0.95	100	0.9	0.8	110	80	100	100	81.1737

As mentioned at the end of Section 3, there is a TPE with $s^*(r_S) = 4.9369$ and $v(r_S) = 8.8883$. Total expected firm profit $(X_1/B)(1 - \lambda)\pi_R(r_S)$ is 234.7463, so expected profit per firm owner is 1.1737. This motivates our choice of β , which equals the value of the collateral plus expected profit per capita. Optimal investment is $s^{**} = 5.0123$, which is the solution to $\nu'(s^{**}) = 0$ in case (b). There is under-investment: the “number” of projects financed in the credit market equilibrium (i.e., $3,000 \cdot 4.9369/100 = 148.1065$) falls short of the optimum “number” (i.e., $3,000 \cdot 5.0143/100 = 150.3701$). Indirect utility is $(\nu(s^{**})) = 8.8962$. The credit market equilibrium is doubly inefficient: there is too little investment, and too large a proportion of the investment capital is dedicated to risky projects. Proposition 7 suggests that over-investment tends to arise when β falls. In fact, when $\beta = 75$ (holding everything else equal), we have $s^{**} = 4.8147 < 4.9369 = s^*(r_S)$ (maximum utility rises to 9.0621).

7 Conclusion

The present paper deviates from the common assumption in models of the credit market with asymmetric information that single-name credit risks cancel out and do not create any portfolio risk. The introduction of portfolio risk raises several interesting questions and yields several interesting results. A credit market equilibrium with pure credit rationing becomes possible. Rationing tends to arise when lenders become more risk-averse. Changes in the parameters of the credit market that lead to a switch from a two-price equilibrium to pure rationing, such as the introduction of a usury law, raise welfare. Compared to the optimal allocation of resources, the credit market equilibrium is characterized by either under-investment or over-investment, where the latter tends to occur when collateral is high and firm owners get little consumption. The implications of the model are thus much richer than the standard SW model's, in which a non-market clearing equilibrium is generally a TPE and there is too little investment in equilibrium.

The most promising route for future research seems to be the introduction of bank capital. In the model considered here, there are no intermediaries with positive equity that could serve as a buffer against loan losses. Introducing bank capital would help make the model more complete, as one could distinguish the portions of the banks' credit risk borne by the bank owners (the major part or all) and by the depositors (little or none at all), respectively. Another possible extension of the model would introduce lenders with heterogeneous risk attitudes, which would raise the question of the optimal allocation of risks across differently risk-averse agents. Risky collateral would shed further light on optimal risk sharing. These, and possibly further, issues can only be meaningfully addressed in a model with portfolio risk, and we are confident that our results will hold true with a small amount of bank capital, a small second group of consumers with different risk attitude, or uncertain collateral with a small variance.

Appendix A.1: Uncorrelated payoffs

This appendix compares the model with non-diversifiable risk to the standard SW model, in which single-name risks cancel out completely. We show that the introduction of correlation between project payoffs possibly causes the emergence of credit rationing and that it tends to reduce equilibrium investment. We also prove the standard equilibrium under-investment result for the SW model.

Call the model with non-diversifiable risk described in Sections 2 and 4 model "C". Let "U" be the identically parameterized (SW) model that is obtained by replacing correlated with uncorrelated

payoffs. In model U, the (due to Uhlig's, 1996, law of large numbers) safe return on lending is $\check{i}(r) = pr + (1-p)(C/B - 1)$ for $r \leq r_S$ and $\check{i}(r) = p_R r + (1-p_R)(C/B - 1)$ for $r_S < r \leq r_R$. Since consumers do not face any risk, the expected utility and OCE approaches yield the same results for U. Let $\check{R}(r) = [1 + \check{i}(r)]^{1-\theta}$. Optimal saving $\check{s}^*(r)$ is given by

$$\check{s}^*(r) = \frac{y}{1 + \delta^{-\frac{1}{\eta}} \check{R}(r)^{-\frac{1}{1-\theta} \frac{1-\eta}{\eta}}} \quad (\text{A.1})$$

(cf. (17)). Let

$$N_R B < \check{s}^*(r_S) M < (N_S + N_R) B \quad (\text{A.2})$$

(cf. (7)).

The following proposition states that the introduction of portfolio risk possibly ‘‘causes’’ the emergence of credit rationing:

Proposition A.1: (a) *There exist parameters such that U does not have a PRE but C does. (b) The reverse is not true.*

Proof: The return function $\check{i}(r)$ attains its unique global maximum $\check{i}(r_R) = \bar{R}/B - 1$ at r_R . Given (A.2), similar arguments as in the proofs of Proposition 1 and 2 prove that there is a TPE and not a PRE. So U does not possess a PRE. (a) Jointly with Proposition 2, this proves the former part of the proposition. (b) The fact that U does not possess a PRE also contradicts the supposition that C does not have a PRE but U does. ||

Another way to compare models U and C is to ask which one brings forth higher investment. The following proposition states that the introduction of portfolio risk reduces equilibrium investment:

Proposition A.2: *Equilibrium investment is lower in C than in U.*

Proof: From (9) and (10), $E[\check{i}(r)] = \check{i}(r)$ for all $r \leq r_R$. Using $\check{R}(r) = [1 + \check{i}(r)]^{1-\theta}$, it follows that

$$\check{R}(r) = \{1 + E[\check{i}(r)]\}^{1-\theta}.$$

Jensen's inequality implies $\hat{R}(r) < \check{R}(r)$ if $\theta < 1$ and $\hat{R}(r) > \check{R}(r)$ if $\theta > 1$. In either case, from (17) and (A.1), we have $s^*(r) < \check{s}^*(r)$ (given $\eta < 1$). That is, the presence of risk reduces optimal saving (cf. Basu and Ghosh, 1993, Proposition 1, pp. 121-2). Since (7) holds, the equilibrium of C is a TPE or a PRE, and equilibrium investment is $S(r_S, M) = s^*(r_S)M$. Since (A.2) holds, the equilibrium of U is a TPE, and equilibrium investment is $\check{s}^*(r_S)M$. The validity of the assertion follows from $s^*(r) < \check{s}^*(r)$. ||

De Meza and Webb (1987, Proposition 5A, pp. 287-8) argue that the equilibrium of the SW model is characterized by under-investment (irrespective of whether the credit market clears or not). They

assume that, in the optimum, the proceeds of the investment projects accrue to the consumers, but the collateral remains with the firm owners, i.e., using the notation of Section 6, $\beta = C$. Since diversification eliminates project-specific risks, safe and risky projects are equally good under these assumptions, whether there is asymmetric information or not does not matter, the first-best and second-best optima coincide. The following proposition generalizes the De Meza-Webb (1987) result:

Proposition A.3: *In an equilibrium of U with $s^*(r) < mB$ and $r < r_R$, there is under-investment for $\beta \geq C$.*

Proof: Consider the optimal solution for model U. Given independent project risks, the safe rate of return is $\bar{R}/B - 1$. Since consumers do not face any risk, we can use the notation of the model with expected utility (i.e., $\eta = \theta$). Period-2 consumption c_2 is then given by the right-hand side of (25). If optimum investment is mB , the condition $s^*(r) < mB$ of the proposition implies that there is under-investment. So we can focus on an interior optimum. Optimum investment \check{s}^{**} is then given by the right-hand side of (31), so

$$\check{s}^{**} \geq \frac{y}{1 + \delta^{-\frac{1}{\theta}} \left(\frac{\bar{R}}{B}\right)^{-\frac{1-\theta}{\theta}}} \quad (\text{A.3})$$

for $\beta \geq C$. In an equilibrium of U, as shown in the proof of Proposition A.1, we have $\check{i}(r) < \bar{R}/B - 1$ for $r < r_R$. Hence, $\check{R}(r) < (\bar{R}/B)^{1-\theta}$. From (A.1) with $\eta = \theta$ and (A.3), $\check{s}^{**} > s^*(r)$. ||

If $s^*(r) = mB$ in equilibrium, under-investment evidently cannot arise. Optimum investment is then also equal to mB . The condition $r < r_R$ rules out the special case in which the supply of funds at the projects' expected rate of return $\bar{R}/B - 1$ is merely sufficient to finance N_R projects. Equilibrium and optimum again coincide in this case.

Appendix A.2: First-best optimum

This appendix analyzes the first-best optimum that can be achieved when, contrary to what has been assumed in Section 6, it is possible to distinguish safe and risky projects.

Clearly, for $M_s \leq N_S B$, the first-best solution entails that only safe projects are realized. For $M_s > N_S B$, all N_S safe projects as well as $M_s/B - N_S$ risky projects are realized. So

$$\begin{pmatrix} c_{2R/S} \\ c_{2F} \end{pmatrix} = \begin{pmatrix} m(C - \gamma_S) + \frac{R_S}{B} s \\ m(C - \gamma_F) \end{pmatrix},$$

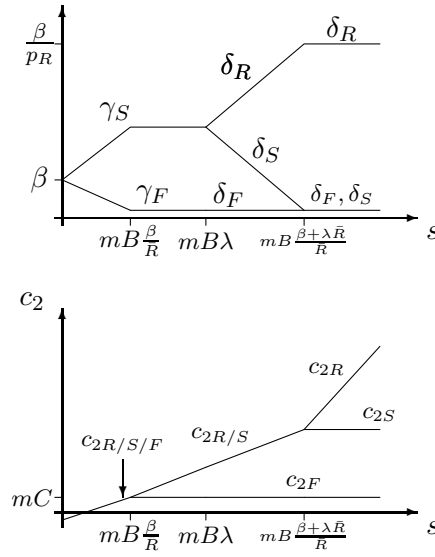


Figure A.1: Consumption levels with optimal risk sharing under symmetric information

for $s \leq mB\lambda$ and

$$\begin{pmatrix} c_{2R} \\ c_{2S} \\ c_{2F} \end{pmatrix} = \begin{pmatrix} m(C - \delta_R) - \lambda m(R_R - R_S) + \frac{R\bar{R}}{B}s \\ m(C - \delta_S) + \lambda mR_S \\ m(C - \delta_F) \end{pmatrix}$$

for $mB\lambda < s$, where the γ 's and δ 's are the firm owners' consumption levels in the respective cases and states of nature. The constraint that they receive expected utility β reads:

$$p_S\gamma_S + (1 - p_S)\gamma_F = \beta$$

for $s \leq mB\lambda$ and

$$p_R\delta_R + (p_S - p_R)\delta_S + (1 - p_S)\delta_F = \beta$$

for $mB\lambda < s$. For the sake of brevity, we confine attention to cases where β is not too large:¹⁶

$$\beta < \lambda\bar{R}. \quad (\text{A.4})$$

Similarly as in the preceding section, we obtain four different cases:

(a) First, let

$$mB\frac{\beta - C}{R} \leq s \leq mB\frac{\beta}{R}.$$

The second inequality and (A.4) jointly imply $s \leq mB\lambda$. c_2 is equalized across all states:

$$\begin{pmatrix} \gamma_S \\ \gamma_F \end{pmatrix} = \begin{pmatrix} \beta + \frac{1}{m}\frac{R_S - \bar{R}}{B}s \\ \beta - \frac{1}{m}\frac{\bar{R}}{B}s \end{pmatrix},$$

¹⁶This condition is inessential for the determination of optimum investment (cf. footnote 17 below).

and c_{2s} is given by (25).

(b) For

$$mB\frac{\beta}{\bar{R}} < s \leq mB\lambda, \quad (\text{A.5})$$

still only safe projects are realized, but the non-negativity constraint on γ is binding:

$$\begin{pmatrix} \gamma_S \\ \gamma_F \end{pmatrix} = \begin{pmatrix} \frac{\beta}{p_S} \\ 0 \end{pmatrix},$$

and c_{2s} is given by (27).

(c) For

$$mB\lambda < s \leq mB\frac{\beta + \lambda\bar{R}}{\bar{R}},$$

some risky projects are carried out:

$$\begin{pmatrix} \delta_R \\ \delta_S \\ \delta_F \end{pmatrix} = \begin{pmatrix} \frac{\beta}{p_S} - \lambda(R_R - R_S) + \frac{1}{m} \frac{R_R - R_S}{B} s \\ \frac{1}{p_S} \left(\beta + \lambda\bar{R} - \frac{1}{m} \frac{\bar{R}}{B} s \right) \\ 0 \end{pmatrix},$$

and (27) gives period-2 consumption.

(d) For

$$mB\frac{\beta + \lambda\bar{R}}{\bar{R}} < s,$$

firm owners consume only in the state when the risky projects succeed:

$$\begin{pmatrix} \delta_R \\ \delta_{S/F} \end{pmatrix} = \begin{pmatrix} \frac{\beta}{p_R} \\ 0 \end{pmatrix}.$$

Household consumption is given by

$$\begin{pmatrix} c_{2R} \\ c_{2S} \\ c_{2F} \end{pmatrix} = \begin{pmatrix} m \left(C - \frac{\beta}{p_R} \right) - \lambda m (R_R - R_S) + \frac{R_R}{B} s \\ mC + \lambda m R_S \\ mC \end{pmatrix}.$$

Consider the expected utility setup. Substituting the state-contingent consumption levels c_{2s} into (1) yields expected utility as a function of investment s alone.¹⁷ This allows us to conduct the welfare analysis parallel to the second-best case. One noteworthy implication is that the state-contingent consumption levels c_{2s} ($s \in \{R, S, F\}$) are identical in the second-best and first-best solutions for

¹⁷The reason why condition (A.4) is inessential is that the same function $\nu(s)$ also gives utility if the inequality sign in (A.4) is reversed.

investment levels s up to the minimum of $mB\beta/[(1-\lambda)\bar{R}]$ and $mB(\beta + \lambda\bar{R})/\bar{R}$. So if the second-best and first-best investment levels are small enough, the second-best and first-best household consumption and utility levels coincide. This is because the fact that the projects are more risky on average in the second-best solution does not matter for consumer welfare as long as the firm owners' risk bearing capacity is sufficient so as to absorb the additional risk. A simple sufficient condition is that $\beta \geq (1-\lambda)\bar{R}$, for in this case, $s \leq mB \leq \min\{mB\beta/[(1-\lambda)\bar{R}], mB(\beta + \lambda\bar{R})/\bar{R}\}$. For the sake of brevity, we merely prove that there is equilibrium under-investment for low levels of collateral and firm owner expected utility and illustrate the possibility of over-investment by means of the running example.

Proposition A.4: *In a PRE or TPE, for C and β small enough and $s^*(r_S) < mB\lambda$, there is under-investment.*

Proof: Let \tilde{s}^{**} denote optimum investment. We can focus on the case $\tilde{s}^{**} < mB\lambda$, since otherwise the assertion of the proposition trivially holds true. Given $\beta = 0$, case (b) applies (cf. (A.5)). Using (27) and $\beta = C = 0$, we obtain optimal investment:

$$\tilde{s}^{**} = \frac{y}{1 + \delta^{-\frac{1}{\theta}} p_S^{-\frac{1}{\theta}} \left(\frac{R_S}{B}\right)^{-\frac{1-\theta}{\theta}}}.$$

From (4) and (33), it follows that $\tilde{s}^{**} > s^*(r_S)$. $C = 0$ is not admissible in the equilibrium analysis. But both optimum investment s^{**} and $s^*(r_S)$ are continuous functions of β and C (this follows from the theorem of the maximum for \tilde{s}^{**} and from the definition of r_S , (3), (4), and (9) for $s^*(r_S)$). So $\tilde{s}^{**} > s^*(r_S)$ holds true for β and C sufficiently close to zero. ||

To illustrate the fact that the second-best solution does not necessarily imply a welfare loss compared to the first-best optimum and the possibility of over-investment relative to the first-best outcome, we return to the example at the end of Section 6, except that we now set $\beta = 50$, so that (A.4) is satisfied. The CO displays over-investment: $s^{**} = 4.0145 < 4.9369 = s^*(r_S)$. Indirect utility $\nu(s^{**}) = 9.7036$ exceeds the equilibrium value by 9.17%. The first-best optimum is characterized by the same amount of investment (and, therefore, over-investment) and the same level of indirect utility. This is because $s^{**} = 4.0145 < 6.06 = \min\{mB\beta/[(1-\lambda)\bar{R}], mB(\beta + \lambda\bar{R})/\bar{R}\}$.

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