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# Does Futures Price Volatility Differ Across Delivery Horizon?

We study the difference in the volatility dynamics of CBOT corn, soybeans, and oats futures prices across different delivery horizons via the smoothed Bayesian estimator of Karali, Dorfman, and Thurman (2010). We show that the futures price volatilities in these markets are affected by the inventories, time to delivery, and the crop progress period. Some of these effects vary across delivery horizons. Further, it is shown that the price volatility is higher before the harvest starts in most of the cases compared to the volatility during the planting period. These results have implications for hedging, options pricing, and the setting of margin requirements.

**Key words**: Bayesian econometrics, futures markets, seasonality, theory of storage, volatility

#### Introduction

Understanding and characterizing futures price volatility has been a key issue in futures market research. Understanding price volatility is important, in part, because initial margin requirements for futures contracts are computed based on estimates of price volatility. Further, the prices of options on futures contracts depend on the price volatility of the underlying contract. Previous research has explained futures price volatility by information variables (volume, time to delivery, seasonality), economic variables (demand and supply conditions, inventories), and by market structure variables (the ratio of speculators to hedgers). Samuelson (1965), Anderson (1985), Milonas (1986), Streeter and Tomek (1992), and Goodwin and Schnepf (2000) are among the many studies that analyze futures price volatility. Some of these studies use a single futures contract, like the December corn contract, and roll it over as maturity nears. Other approaches use multiple contracts and analyze them separately, or use single time series obtained by rolling over nearby contracts.

Which way to approach the construction of the data to be studied matters if volatility behavior and effects vary across delivery horizons, delivery months, or specific contracts. If volatility varies by delivery month, splicing together a series based on the nearest contract could lead to biased results and erroneous economic conclusions. Similarly if it is delivery horizon (nearest contract, second nearby, etc.) that determines behavior, that is the way the data need to be modeled as to do otherwise would again lead to poor and incorrect results. Additionally, to get the most accurate results, one should take into account the fact that multiple contracts are traded simultaneously. Streeter and Tomek (1992) modeled the volatility of March and November soybean contracts in a seemingly unrelated regressions framework and achieved improved results. Recently, Smith (2005) developed a partially overlapping time series (POTS) approach to model all simultaneous contracts together. Such approaches should yield improved statistical efficiency which should hopefully lead to better economic insights.

In the current study we analyze the determinants of volatility for specific delivery horizons using a smoothed Bayesian estimator developed in Karali, Dorfman, and Thurman (2010). This approach is in a similar spirit to Smith (2005) in treating the simultaneous nature of the overlapping contracts as something to be exploited for statistical gain. We model the evolution of volatility as a smooth function of inventories, time to delivery, calendar time, and stage of the crop production cycle, imposing smoothness through a prior density. This results in delivery-horizon-specific estimates that are smoothed through the use of a prior distribution that centers each delivery horizon's parameter estimates over a weighted average of the estimates for all delivery horizons. With this approach we model the differences in the volatilities of the first nearby contract, second nearby contract, third nearby contract, etc., following Colling and Irwin (1990) and Schaefer, Myers, and Koontz (2004) in positing that delivery horizon is the correct grouping of the data for modeling volatility of CBOT corn, soybean, and oats futures contracts.

The analysis shows there are, in fact, differences across the delivery horizons in the effect of a number of variables on volatility. We find the inventory effect for corn varies considerably across delivery horizons, being generally larger in magnitude for the nearer contracts. The time-to-delivery effect shows significant cross-horizon variation in the oats data. Calendar time has variable effects at different delivery horizons for all three commodities studied. Finally, we also find that the volatility of all three commodities varies by the stage of the production cycle (planting, pre-harvest, and post-harvest). These differences may be exploitable to construct more efficient hedges or better options pricing formulas.

The paper proceeds as follows. First, we discuss earlier work related to explaining futures volatility and how to construct and analyze data from simultaneous, overlapping futures contracts. Then we describe our data on CBOT corn, soybean, and oats contracts. Estimation methods are explained next, followed by results. Finally, the conclusions complete the paper.

#### Previous Related Work on Futures Volatility

Several previous studies on futures price volatility have used a single time series of futures prices obtained by rolling over the nearby contract. Yang and Brorsen (1993), for example, use a continuous price series constructed from the futures contract closest to delivery in their analysis of seasonality, day-of-the-week, and maturity effects in several futures markets. They find that the price volatility of corn, soybeans, and wheat exhibit seasonality, and maturity effect only exists for soybean and oats futures. Khoury and Yourougou (1993) use spliced nearby contract series while analyzing the determinants of agricultural futures price volatilities at the Winnipeg Commodity Exchange. They find that the volatility of barley, canola, feed wheat, oats, flaxseed, and rye are influenced by the year, calendar month, contract month, maturity, and trading session. In their analysis of soybean, corn, wheat, and cotton futures prices, Chatrath, Adrangi, and Dhanda (2002) use spliced price series obtained by rolling over the nearby

contracts. They show that daily returns on all four futures contracts are highly seasonal and persistent. Further, they show evidence for the Samuelson effect for soybean and corn futures contract.

Another common approach is to use a single delivery month contract, like the December corn futures or March soybean futures, and roll it over as the maturity approaches. Kenyon et al. (1987) show, using rolled over March corn, March soybean, and July wheat contracts, the season of the year, lagged volatility, and loan rates are important determinants of price volatility in these markets. Streeter and Tomek (1992) use rolled over November and March soybean contracts to show strong seasonality in price volatility, with volatility increasing in summer months. They also show the nonlinear effect of time to delivery on price volatility, with volatility decreasing in the months before maturity. Further, they go beyond the usual practice in the literature and model jointly the November and March soybean contracts using the method of Seemingly Unrelated Regressions (SUR) and show that the system estimation results in improvements of the results compared to single equation estimation. Hennessy and Wahl (1996), analyzing several delivery months separately, find that while seasonal effects on the price volatility of corn, soybeans, Chicago wheat, Kansas wheat, and Minneapolis wheat are significant the inventory and time-to-delivery effects are not. In their study of endogenous determinants of price risk, Goodwin and Schnepf (2000) use December corn and September wheat contracts and show the importance of inventories, growing conditions, seasonality, trading volume, and open interest on price volatility. They also find positive time-to-delivery effect for corn, contradicting the Samuelson hypothesis.

Finally, another approach used in the literature is to construct separate time series of the futures prices by the delivery horizon: first nearby, second nearby, third nearby etc. Colling and Irwin (1990) and Mann and Dowen (1996) study the effects of USDA Hogs and Pigs Reports on the near and distant live hog futures contracts. Similarly, Schaefer, Myers, and Koontz (2004) analyze the nearby, first deferred, and second deferred live cattle futures contracts to infer the efficiency of this market. All three studies analyze contracts as separate time series. In the recent study of Kalev and Duong (2008), a more elaborate approach is used. Instead of analyzing the time series of different delivery horizon contracts separately, they use five time series constructed by rolling over the first closest through the fifth closest maturity contracts in SUR framework. They find evidence for the Samuelson effect in corn, soybean, soybean oil, soybean meal, feeder cattle, lean hogs, live cattle, and pork bellies.

The novel model of Smith (2005), Partially Overlapping Time Series (POTS), allows one to use all futures contracts traded together. He studies corn futures market with this latent factor model of daily futures price changes and show that the corn futures price volatility is inversely related to inventories and it increases as maturity approaches. More recently, Suenaga, Smith, and Williams (2008) use the POTS model to analyze volatility dynamics of NYMEX natural gas futures prices. To show the effect of the strong seasonality in natural gas demand on the volatility of futures prices,

they consider all 12 delivery months and apply their analysis to optimal hedging strategy. They find that the December and summer (June through August) contracts are more effective than other natural gas contracts in minimizing the variance of portfolio returns.

We take a somewhat different approach and look at the effect of volatility determinants across different delivery horizons rather than specific delivery months. We construct time series of the first nearby, second nearby, etc. contracts but different than most of the previous work we use all concurrently traded contracts in a system of equations. To account for the contemporaneous correlations among the observations from the same day, we apply the Generalized Least Squares (GLS) method developed in Karali and Thurman (2009). Further, we allow the parameters of the volatility determinants to vary across delivery horizons through the use of the smoothed Bayesian estimator developed in Karali, Dorfman, and Thurman (2010). We impose smoothness on the delivery-horizon-specific estimates through a prior density, which centers the parameter estimates of each delivery horizon over a weighted average of the estimates for all delivery horizons.

# Data

We study corn, soybeans, and oat futures contracts traded at the Chicago Board of Trade, and employ daily settlement prices. For all of the commodities, prices are quoted in cents per bushel and contract size is 5,000 bushels. Delivery months are March, May, July, September, and December for corn, and oats; January, March, May, July, August, September, and November for soybeans. As in other futures markets, multiple contracts of these commodities are traded on a given day. Ten to 15 contracts are listed at any point in time, each with a delivery date up to three and a half years in the future. We trim the data set of each commodity to include an equal number of observations—the number of trading days of the shortest-lived contract for all contracts. This resulted in at most seven contracts on a given day for corn and soybeans, and five contracts for oats. However, great majority of trading days had six contracts for corn and four contracts for oats. In order to study volatility dynamics on a typical day, we consider only the first six delivery horizon contracts for corn, seven for soybeans, and four for oats. Sample characteristics are given in table 1(a). Total number of trading days is 4,202 for corn, 3,693 for soybeans, and 3,443 for oats. With multiple contracts on a given day, the total number of observations become 25,212 for corn, 25,851 for soybeans, and 13,772 for oats.

Grain Stocks report issued four times a year by the National Agricultural Statistics Service (NASS) is one of the closely-watched reports by market participants. These reports contain both on-farm stocks and off-farm stocks, which include stocks at mills, elevators, warehouses, terminals, and processors. Reported inventories are as of the first day of March, June, September, and December. We employ the total of on-farm and off-farm stocks and interpolate the quarterly time series by a linear spline method to obtain daily series. The resulting daily inventory series for all commodities are

presented in figure 1. As seen in the figure, inventories are highly seasonal. Corn and soybean inventories peak in December and reach a trough in September while oat inventories peak in September and reach a trough in June.

To capture this seasonality we partitioned a calendar year into three periods. We used Crop Progress reports published weekly during the growing season by the NASS as a reference to determine the planting, pre-harvest, and post-harvest periods. These reports are published from the first week of April through the last week of November each year and list planting, fruiting, and harvesting progress and overall condition of selected crops in major producing states. We determined in which months the data on planting have started and ceased, similarly for harvesting. We specified those months when there were information on planting progress as our "planting period." The gap between when the planting information is ceased and when the harvesting information is started is defined as "pre-harvest period." Finally, the months in which the harvesting progress is listed are specified as our "post-harvest period." It is well documented in the literature that price volatility of crops peak right before the harvest. At the end of planting period, market participants know how much crop is planted through these reports and the weather remains as the main uncertainty on the production. During the pre-harvest period then, new information other than the acreage planted causes fluctuations in price volatility. Partitioning a calendar year into three periods would allow one to see if uncertainties other than weather in the planting period results in higher volatility than in the pre-harvest period.

Time-to-delivery effect, or Samuelson effect, is one of the well-accepted determinants of the futures price volatility. The price volatility of a futures contract increases as the contract approaches delivery. We measure time to delivery as the number of trading days left to contract expiration. Futures contracts expire on the business day prior to the 15<sup>th</sup> calendar day of the contract month in all three markets studied.

As most financial assets, commodity futures markets also exhibit volatility persistence. High volatility days are followed by high volatility, whereas low volatility days are followed by low volatility. We include the lagged value of our volatility measure as an explanatory variable in our empirical analysis to capture the persistence in volatility.

Table 1(b) presents summary statistics for the daily variables used in the analysis. The average price volatility is 0.8 percentage points for corn, 0.9 percentage points for soybeans, and 1.2 percentage points for oats. Corn inventories are much larger than the inventories of other commodities, with a sample average of 4.4 billion bushels. In our sample, the longest time to delivery is 321 days for corn, followed by 246 days for soybeans, and 208 days for oats.

# Econometric Model and Smoothed Bayesian Estimation

We use the smoothed Bayesian estimator developed in Karali, Dorfman, and Thurman (2010) and model futures price volatility with separate coefficients for each delivery horizon group as follows:

$$|\%\Delta F_{it}| = a_i + b_{1i}S_t + b_{2i}S_t^2 + c_{1i}TTD_{it} + c_{2i}TTD_{it}^2 + d_{1i}t + d_{2i}t^2 + e_i|\%\Delta F_{i,t-1}| + h_{1i}D_{1t} + h_{2i}D_{2t} + \varepsilon_{it}, \quad (1)$$

where  $|\%\Delta F_{it}| \equiv |100 \times (\ln F_{it} - \ln F_{i,t-1})|$ , for  $i = 1, 2, \dots, k$  and  $t = 1, 2, \dots, T$ , and

 $\ln F_{it}$ : natural logarithm of the  $i^{\text{th}}$ -delivery contract's price on day t

k: the number of delivery horizons; six for corn, seven for soybeans, and four for oats

T: total number of trading days

 $S_t$ : inventory level on day t

 $TTD_{it}: i^{\mathrm{th}}$ -delivery contract's time to delivery on day t

 $D_{1t}$ : pre-harvest dummy variable, which takes the value of one if t is in July, August, and September for corn and soybeans, and in June and July for oats

 $D_{2t}$ : post-harvest dummy variable, which takes the value of one if t is in October, November, December, January, and February for corn and soybeans, and in August, September, October, November, December, January, and February for oats

The equation for each delivery horizon can be shown in matrix form as:

$$y_i = X_i \theta_i + \varepsilon_i, \qquad i = 1, 2, \cdots, k,$$
 (2)

where

$$oldsymbol{ heta_i} = egin{pmatrix} a_i & b_{1i} & b_{2i} & c_{1i} & c_{2i} & d_{1i} & d_{2i} & e_i & h_{1i} & h_{2i} \end{pmatrix}',$$
 $oldsymbol{X_i} = egin{bmatrix} oldsymbol{\iota} & S & S^2 & TTD_i & TTD_i^2 & t & t^2 & L(y_i) & D_1 & D_2 \end{bmatrix},$ 

and where  $\mathbf{y_i}$ ,  $\boldsymbol{\varepsilon_i}$ , and the elements of  $\mathbf{X_i}$  are the vertical concatenations of  $|\%\Delta F_{it}|$ ,  $\varepsilon_{it}$ , and the righthand-side variables in equation (1), respectively.  $\mathbf{L}(\mathbf{y_i})$  denotes the lagged dependent variable.

Bayesian estimation combines the prior distribution, which summarizes the prior beliefs about the unknown parameters, with the likelihood function, which is an objective measure of the information in the data, and results in a posterior distribution, which is the optimal combination of the two information sources (Zellner 1971).

The prior distributions on the regression parameters are specified as multivariate normal distribution:

$$p(\boldsymbol{\theta_i}) \sim N(\boldsymbol{\theta_i}, \sigma_i^2 \underline{\boldsymbol{V_i}}), \qquad i = 1, 2, \cdots, k,$$
 (3)

where  $\underline{\boldsymbol{\theta_i}}$  is the prior mean of the  $i^{\text{th}}$ -delivery contract's regression parameters, and  $\sigma_i^2 \underline{\boldsymbol{V_i}}$  is the prior variance-covariance matrix. The prior distribution of the inverse of  $\sigma_i^2$  is defined as gamma distribution:

$$p(\sigma_i^{-2}) \sim G(\underline{s_i}^{-2}, \underline{d_i}), \qquad i = 1, 2, \dots, k,$$
 (4)

where  $\underline{s_i}^{-2}$  is the prior mean for the inverse error variance, and  $\underline{d_i}$  is the prior degrees of freedom parameter.

The prior means of the parameters,  $\underline{\boldsymbol{\theta_i}}$ , for the  $i^{\text{th}}$ -delivery contract are computed as:

$$\underline{\boldsymbol{\theta}_i} = \frac{\sum_{\ell=1}^k w_{i\ell} \boldsymbol{\theta}_{\ell}}{\sum_{\ell=1}^k w_{i\ell}},\tag{5}$$

where  $\theta_{\ell}$  is a vector of parameter values for the  $\ell^{\text{th}}$ -delivery contract. We use a weighting scheme for prior means that forces the parameters of adjacent delivery contracts to be close. However, as the discrepancy between the delivery horizons increases so can the discrepancy between their parameters. More specifically, we use the following weight-

ing matrix:  $\mathbf{w}_i = [w_{i1} \ w_{i2} \ \dots \ w_{ik}]$  where  $w_{i\ell} = |\ell - i|^{-1}$ , for  $i, \ell = 1, 2, \dots, k$ , and  $w_{i\ell} = 0$  when  $i = \ell$ . For instance, with a total of seven delivery horizons, the weighting matrices for the 2<sup>nd</sup>-delivery and 5<sup>th</sup>-delivery contracts are defined as:

$$\mathbf{w_2} = \begin{bmatrix} 1 & 0 & 1 & \frac{1}{2} & \frac{1}{3} & \frac{1}{4} & \frac{1}{5} \end{bmatrix},$$

$$\mathbf{w_5} = \begin{bmatrix} \frac{1}{4} & \frac{1}{3} & \frac{1}{2} & 1 & 0 & 1 & \frac{1}{2} \end{bmatrix}.$$
(6)

When multiple contracts from the same day are used, one must recognize the correlation between the price observations. Even though futures contracts of a commodity have different delivery months, they are more or less subject to similar shocks on a given day. Therefore, contemporaneous correlation among the observations from different delivery horizons should be taken into account. We apply a GLS method similar to the one developed in Karali and Thurman (2009), which eliminates the contemporaneous correlation and allows one to pool all observations on a given day.

The likelihood function for each delivery horizon is assumed to follow a standard form after the GLS transformation, and is represented by

$$L_{i}(\boldsymbol{y_{i}^{*}}|\boldsymbol{\theta_{i}}, \sigma_{i}^{2}, \boldsymbol{X_{i}^{*}}) = (2\pi\sigma_{i}^{2})^{-n_{i}/2} \exp\{-0.5\sigma_{i}^{-2}(\boldsymbol{y_{i}^{*}} - \boldsymbol{X_{i}^{*}}\boldsymbol{\theta_{i}})'(\boldsymbol{y_{i}^{*}} - \boldsymbol{X_{i}^{*}}\boldsymbol{\theta_{i}})\},$$

$$i = 1, 2, \dots, k, \qquad (7)$$

where \* denotes the transformed data and  $n_i$  is the number of observations in the

 $i^{\text{th}}$ -delivery contract group. It can be shown (Poirier 1995) that the joint posterior is

$$p(\boldsymbol{\theta_i}, \sigma_i^2 | \boldsymbol{y_i^*}, \boldsymbol{X_i^*}) \sim NG(\overline{\boldsymbol{\theta_i}}, \overline{\boldsymbol{V_i}}, \overline{s_i}^2, \overline{d_i}), \qquad i = 1, 2, \cdots, k,$$
 (8)

where NG denotes the joint normal gamma distribution, and

$$\overline{\boldsymbol{\theta}_i} = \overline{\boldsymbol{V}_i} \left( \underline{\boldsymbol{V}_i}^{-1} \underline{\boldsymbol{\theta}_i} + (\boldsymbol{X}_i^{*\prime} \boldsymbol{X}_i^*) \hat{\boldsymbol{\theta}}_i \right), \tag{9}$$

$$\overline{\boldsymbol{V}_{i}} = \left(\underline{\boldsymbol{V}_{i}}^{-1} + \boldsymbol{X}_{i}^{*\prime} \boldsymbol{X}_{i}^{*}\right)^{-1}, \tag{10}$$

$$\overline{s}_i^2 = \overline{d}_i^{-1} \left[ \underline{d}_i \underline{s}_i^2 + (n_i - m_i) s_i^2 + (\hat{\boldsymbol{\theta}}_i - \underline{\boldsymbol{\theta}}_i)' \left( \underline{\boldsymbol{V}}_i + (\boldsymbol{X}_i^{*\prime} \boldsymbol{X}_i^*)^{-1} \right)^{-1} (\hat{\boldsymbol{\theta}}_i - \underline{\boldsymbol{\theta}}_i) \right], \quad (11)$$

$$\overline{d}_i = \underline{d}_i + n_i, \tag{12}$$

$$\hat{\boldsymbol{\theta}}_{i} = (\boldsymbol{X}_{i}^{*\prime} \boldsymbol{X}_{i}^{*})^{-1} \boldsymbol{X}_{i}^{*\prime} \boldsymbol{y}_{i}^{*}, \tag{13}$$

$$s_i^2 = \left(\frac{1}{n_i - m_i}\right) \boldsymbol{\varepsilon_i^{*\prime} \boldsymbol{\varepsilon_i^*}}. \tag{14}$$

We set  $\underline{\boldsymbol{V_i}} = 0.001 \times \boldsymbol{I_{m_i}}$ ,  $\underline{d_i} = 5$ ,  $\underline{s_i}^2 = 0.8\sigma_{\boldsymbol{y}}^2$ , where  $m_i$  is the number of regressors for the  $i^{\text{th}}$ -delivery contract and equal to ten for all i. The prior variance on the time-to-delivery terms are set to 0.1 for all commodities and the one for inventory terms are set to one for oats. The number of observations per delivery horizon,  $n_i$ , is the same for all i and equal to 4,202 for corn, 3,693 for soybeans, and 3,443 for oats.

# Results

Tables 2-4 present the posterior means, posterior standard errors, pseudo t-values, 95% highest posterior density lower and upper limits of the model parameters along with the marginal inventory, time-to-delivery, and calendar time effects and their posterior

probabilities of having negative sign. Marginal effects are evaluated at the mean values of variables. Figures 2-6 show posterior means and 95% highest posterior density regions (HPDR) as well as the probability density function for the selected variables.

# Inventory effect

As seen in table 2(a), the linear inventory term is negative for all six delivery horizons for corn with all 95% HPDRs excluding zero. The sign of the quadratic inventory term is less clear. However, one must compute the first derivative of the volatility measure with respect to inventories to see the overall inventory effect. Table 2(b) shows that the inventory effect is negative across delivery horizons. Only the sixth-delivery contract's 95% HPDR includes zero. However, the posterior probability of a negative inventory effect for this delivery group is considerably high with 0.88. This probability is unity for all other delivery groups. The second-nearby contract exhibits the largest inventory effect in magnitude followed by the third-nearby contract. Holding everything else constant, the price volatility of the second-nearby corn futures decreases by 0.64 percentage points when corn inventories increase by their sample range of 9.3 billion bushels (see table 2(c)). The same increase in inventories causes a 0.50 percentage point decrease in the volatility of the third-delivery contract, and a 0.32 percentage point decrease for the nearest-delivery contract. The volatility of the farthest-delivery contract decreases by only 0.05 percentage points. The posterior density regions and the probability density functions presented in figures 2(a) and 2(b) suggest that the inventory effect varies across delivery horizons, with the second-, third-, and sixthnearby contracts showing dissimilarities from each other as well as from the remaining contracts.

Both linear and quadratic inventory terms are reliably signed across seven delivery horizons for soybeans (table 3(a)). Table 3(b) shows that the inventory effect is negative with probability of one for all delivery horizons. The volatility of the sixth-delivery contract decreases by 0.6 percentage points while the volatility of the first-, third-, and fourth-delivery contracts decreases by about 0.5 percentage points when inventories increase by their sample range of 2.1 billion bushels. Contrary to corn, the second-delivery soybean contract has the smallest inventory effect. Figures 2(c) and 2(d) show that the inventory effect is somewhat different for the second- and sixth-delivery horizons.

Finally, tables 4(a)-4(c) show a strong negative inventory effect on oat futures price volatility. This holds for all four delivery horizons. An increase in inventories equal to their sample range of 0.2 billion bushels would cause the price volatility to drop by about 1.2 percentage points. The largest inventory effect is found for the farthest-delivery horizon. The three nearest delivery contracts exhibit very similar inventory effect (see figures 2(e) and 2(f)). However, the inventory effect is estimated less precisely for oats than it is for corn and soybeans as evidenced by the wider density regions.

To summarize, there is empirical evidence of the theory of storage for the three crops studied. The economic significance of the inventory effect for corn is slightly higher than it is for soybeans. The largest movement in corn futures volatility due to a change in inventories is 0.64 percentage points, three-fourths of the average volatility

of 0.85 percentage points on a typical day. For soybeans, it is two-thirds of the daily volatility on a typical day. On the other hand, price volatility of oats decreases by 1.1 times the average volatility when inventories increase by their sample range. The most noticeable difference among delivery horizons is observed in corn futures market. The inventory effect on the second-nearby contract is approximately twice the effect on the first-nearby contract. The inventory effect does not vary much across soybean and oat contracts.

# Time-to-delivery effect

Table 2(b) shows that for all but the third-delivery horizon, the marginal time-to-delivery effect is negative when evaluated at the mean value of TTD variable for each delivery group. The posterior probability of a negative time-to-delivery effect is 0.7 for the first-delivery horizon while it is considerably lower for the second- and third-delivery contracts. The probability of a negative time-to-delivery effect increases with the delivery horizon after the third-delivery group. The marginal time-to-delivery effect varies across delivery groups, with the second-nearby contract having the smallest effect and the farthest-delivery contract having the largest effect. As seen in table 2(c), estimated posterior means imply from 0.01 to 0.17 percentage point increase over the life of a contract. Figures 3(a) and 3(b) show that the time-to-delivery effect for the nearest-delivery contract is not estimated precisely.

For soybeans, the first-nearby contract has a positive time-to-delivery effect with probability of 0.6 at the half-life of the contract. The other delivery horizons, on the other

hand, have posterior probability higher than 0.7 for a negative time-to-delivery effect. Table 3(c) shows that the volatility of the nearest-delivery contract increases by 0.12 percentage points from the first to the last trading day. Like corn, the first-delivery contract's posterior density region and probability density function shown in figures 3(c) and 3(d) are considerably wide.

As seen in table 4(b), the marginal time-to-delivery effect is negative for the first two delivery horizons in oat futures market with posterior probabilities of one. On the other hand, the time-to-delivery effect evaluated at the mean value of TTD for the fourth-delivery contract is strongly positive. However, table 4(c) shows that over the life of a contract, oat futures price volatility increases by 0.07 percentage points for the first-delivery contract while it increases by 0.09 percentage points for the fourth-delivery contract. The posterior density regions and the probability density functions shown in figures 3(e) and 3(f) are very different than the ones shown for corn and soybeans. The time-to-delivery effect varies considerably across delivery horizons.

One must note the limited variation left in time to delivery variable after the data is separated by delivery horizon. This would make inference on the time-to-delivery effect difficult. In fact, the positive marginal effects seen in soybean and oat futures might be a consequence of this limited variation within a delivery horizon.

# Time trend and volatility persistence

We do not have a priori expectation on the time trend. The posterior mean of the linear time trend are negative and posterior mean of the quadratic term is positive for all delivery horizons in each market. However, the sign of the overall calendar time effect is not precisely estimated for most of the corn and soybean delivery horizons. Table 2(c) shows that except the first- and second-nearby corn contracts, the price volatility of all delivery horizons has decreased by about 0.1 percentage points from the beginning to the end of the sample period. The price volatility of the first-, third-, fourth-, and sixth-nearby soybean futures has increased while the volatility of other delivery horizons has decreased over the sample period as seen in table 3(c). For oats, the price volatility of all four delivery horizons declined towards the end of the sample period (table 4(c)).

Volatility persistence is prominent in all three markets. Days with high volatility are followed by days with high volatility, similarly for low volatility. For all crops, the price volatility of the nearest-delivery contract shows higher persistence compared to other delivery horizons.

# Harvest effect

In the estimation, planting period is used as the base category. Thus, a positive posterior mean for the pre-harvest dummy variable implies that futures prices are more volatile during the pre-harvest period than they are in the planting period. The same holds for the post-harvest dummy variable. Because of the uncertainty about weather, hence about the harvest output, one would expect to see higher volatility in the pre-harvest period than in the post-harvest period. Therefore, the posterior mean of the pre-harvest dummy variable is expected to be larger than that of the post-harvest

dummy variable if they are both positive, and smaller in magnitude if they are both negative. A positive posterior mean for the pre-harvest dummy and a negative one for the post-harvest dummy would also show that the price volatility is higher in the pre-harvest period compared to the planting and post-harvest periods, and further the post-harvest period is less volatile than the planting period.

Tables 2(a), 3(a), and 4(a) show that this effect varies to some extent both across crops and across delivery horizons for a given crop. Only the first- and sixth-delivery corn contracts' 95% HPDRs for the pre-harvest parameter do not include zero. The post-harvest parameter, on the other hand, is negative for all corn delivery horizons. The first- and second-delivery contracts exhibit higher volatility in the pre-harvest period compared to both the planting and post-harvest periods. Further, the volatility in the post-harvest period is lower than it is in the planting period. For the third- through sixth-delivery contracts, both pre- and post-harvest periods are found to be less volatile than the planting period. However, the volatility is more noticeably lower in the post-harvest period than in the planting period compared to the pre-harvest period. Figure 7(a) shows the predicted volatility for all three seasonal periods when holding all other variables constant at their mean values. It is seen that volatility in the pre-harvest period is about 0.2 percentage point higher than it is in the post-harvest period.

While the pre-harvest parameter estimation for soybeans is negative for the seconddelivery horizon, it is positive for all other delivery horizons. The post-harvest parameter is found to be negative for all contracts. Except the second-delivery contract, the price volatility is slightly higher in the pre-harvest period and lower in the post-harvest period compared to the planting period. Figure 7(b) shows that while the predicted volatility in the planting and pre-harvest periods is almost the same, it is consistently smaller in the post-harvest period for all delivery horizons.

For oats, the posterior means of both pre- and post-harvest dummy variables are positive for all delivery horizons and none of the 95% HPDRs include zero. Pre-harvest volatility is at least 0.34 percentage points higher than the planting period volatility. Also, the volatility in the post-harvest period is at least 0.06 percentage point higher than it is in the planting period. Thus, the pre-harvest volatility exceeds the post-harvest volatility by about 0.3 percentage points as seen in figure 7(c). The seasonal pattern of volatility is the same across delivery horizons. Further, the predicted volatility in any period declines as the delivery horizon becomes farther.

#### Conclusions

The Bayesian estimator developed in Karali, Dorfman, and Thurman (2010) again proved effective in modeling partially overlapping futures contracts. Results for the volatility of CBOT corn, soybeans, and oats contracts uncovered some significant differences in the effects of variables on volatility depending on the delivery horizon. We take these differences as a sign that grouping data by delivery horizon is a better approach for a study of this sort than to create a single series from the nearby contract. We also believe that future work with these models might show results similar to those in Suenaga, Smith, and Williams (2008) by identifying particular delivery horizons that yield the most effective hedges.

The inventory effect for corn varied considerably across delivery horizons, being largest in magnitude for the second nearby contract at which point it was roughly seven times the magnitude of the inventory effect on the most distant contract. Soybeans showed more muted differences although the probability density functions in figure 2(d) still reveal some distinction, and oats showed little effect of the delivery horizon on the inventory effect. The time-to-delivery effect showed highly significant cross-horizon variation in the oats data. The probability density functions for all four delivery horizons were clearly distinct in figure 3(f).

Calendar time has variable effects at different delivery horizons for all three commodities studied, as the various sections of figure 4 showed. Finally, we also find that the volatility of all three commodities varies by the stage of the production cycle (planting, pre-harvest, and post-harvest) when production stage is considered alone; however, once placed in the volatility model with our other explainers, the effects of production cycle stage are more muted.

The differences in effects of visible, exogenous variables on volatility of corn, soybeans, and oats may be exploitable to construct better options pricing formulas. This is a suggested direction for future research.

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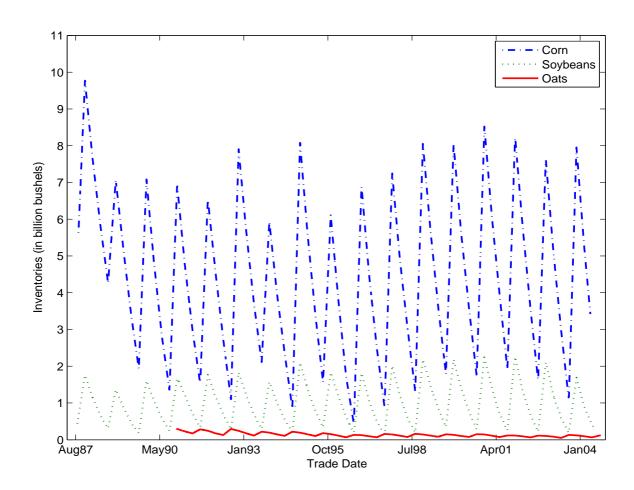


Figure 1: Interpolated Daily Inventories

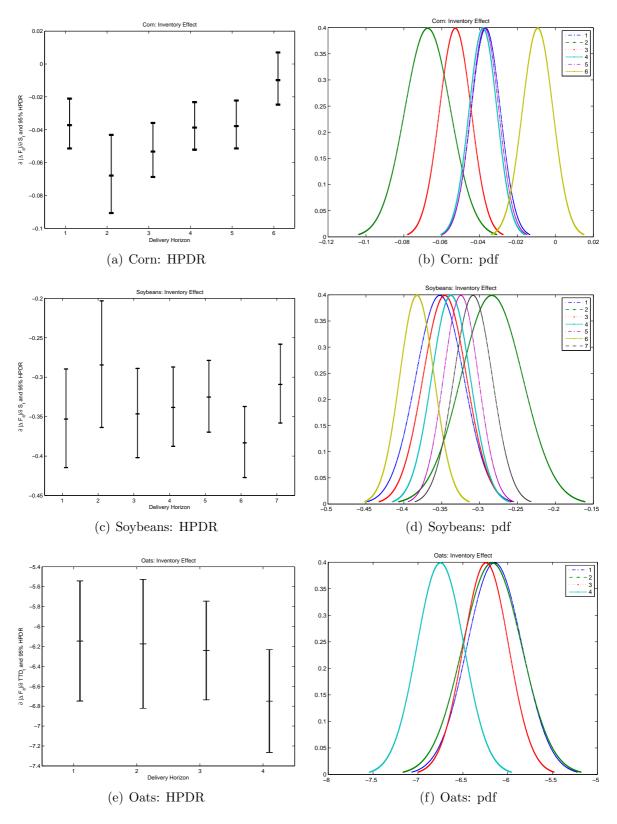


Figure 2: Posterior Means, 95% Highest Posterior Density Regions, and Probability Density Functions for the Inventory Effect

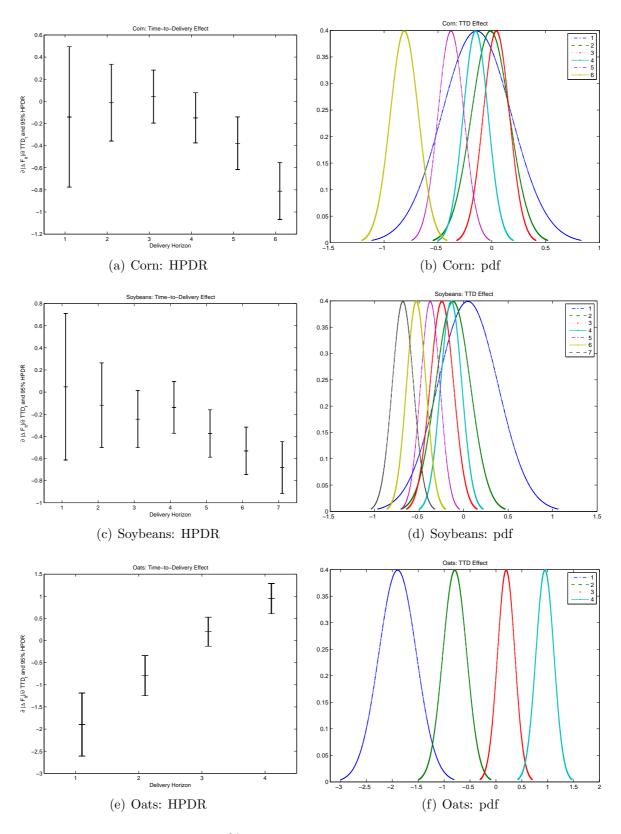


Figure 3: Posterior Means, 95% Highest Posterior Density Regions, and Probability Density Functions for the Time-to-Delivery Effect

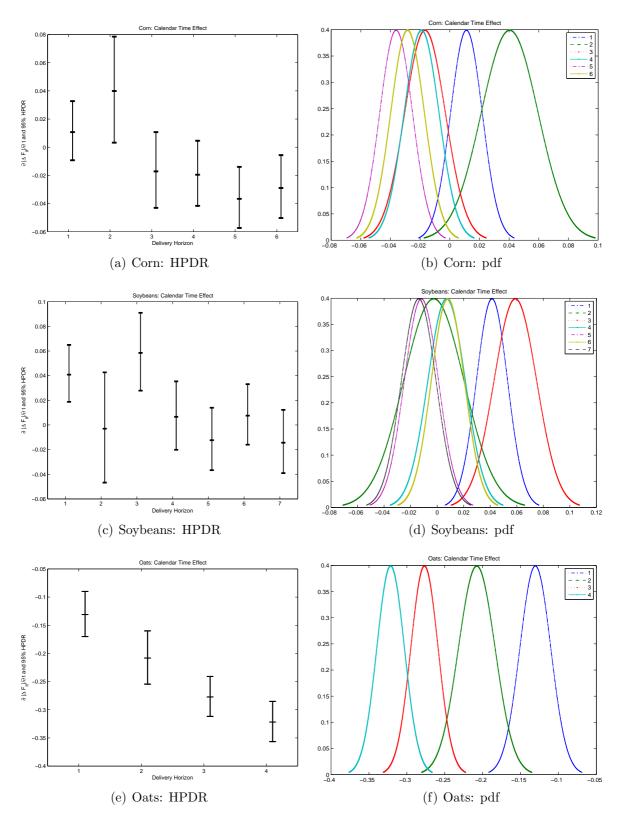


Figure 4: Posterior Means, 95% Highest Posterior Density Regions, and Porobability Density Functions for the Calendar Time Effect

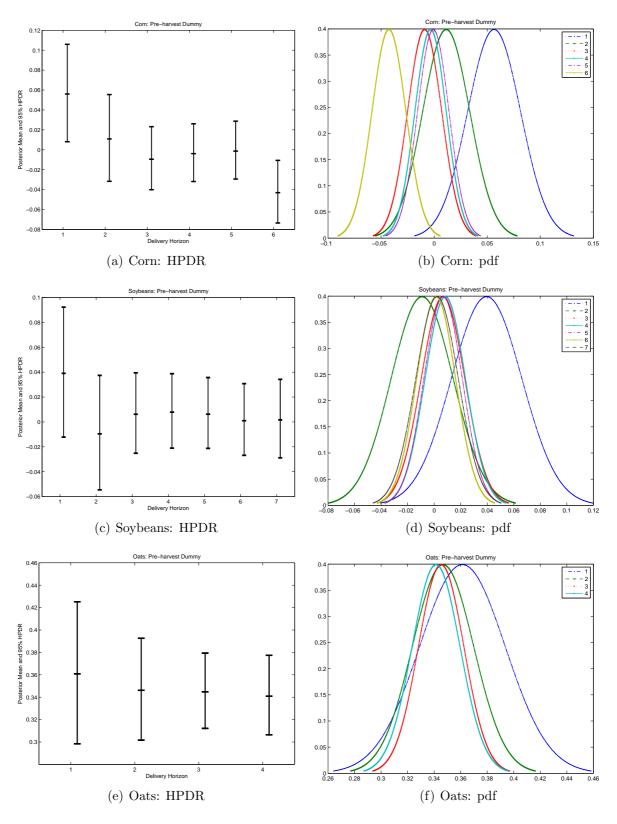


Figure 5: Posterior Means, 95% Highest Posterior Density Regions, and Probability Density Functions for the Pre-Harvest Dummy Variable

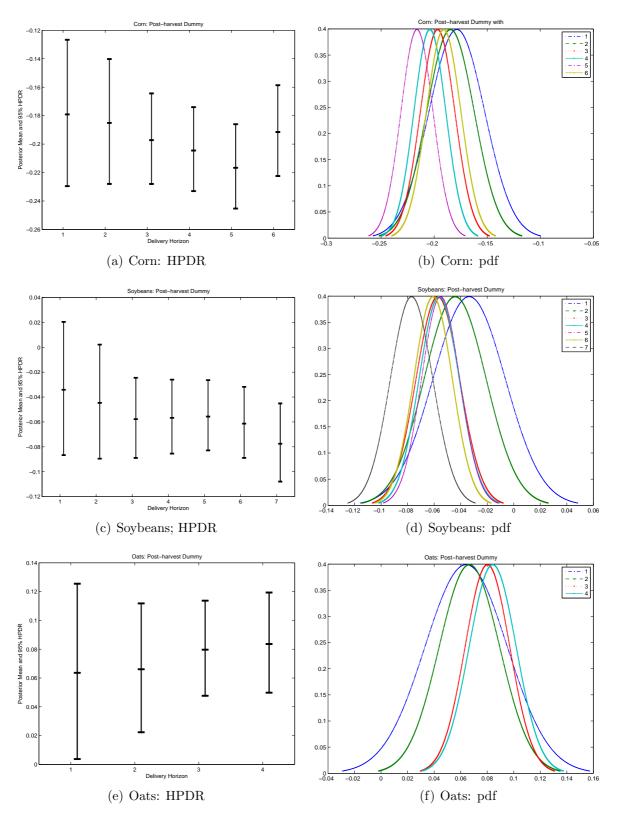


Figure 6: Posterior Means, 95% Highest Posterior Density Regions, and Probability Density Functions for the Post-Harvest Dummy Variable

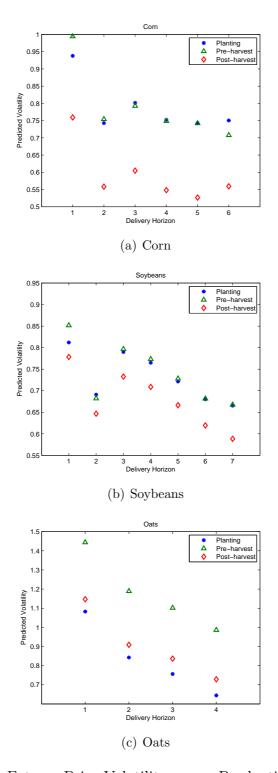


Figure 7: Futures Price Volatility across Production Seasons

Table 1: Summary Statistics

# (a) Sample Characteristics

|                        | Corn              | Soybeans  | Oats              |
|------------------------|-------------------|---|-------------------|
| Sample period          | 09/15/87-05/14/04 | $09/15/87 - 05/14/04 \qquad 09/01/87 - 07/14/04 \qquad 11/21/90 - 09/14/04$ | 11/21/90-09/14/04 |
| Contract months        | 3,5,7,9,12        | 1,3,5,7,8,9,11  | 3,5,7,9,12        |
| No. obs.               | 25,212            | 25,851  | 13,772            |
| Total no. trading days | 4,202             | 3,693   | 3,443             |
| Total no. contracts    | 88                | 124   | 72                |
| Max. contracts/day     | 9                 | 7   | 4                 |

(b) Daily Variables

|           |       |                      |        |       |             | ,                  | •     |                    |                           |       |                    |                       |       |                    |       |
|-----------|-------|----------------------|--------|-------|-------------|--------------------|-------|--------------------|---------------------------|-------|--------------------|-----------------------|-------|--------------------|-------|
|           |       | $\%  \Delta F_{it} $ |        |       | Inventories |                    | Tin   | Time to Delivery   | ry                        | Pre-  | Pre-harvest Dummy  | тту                   | Post- | Post-harvest Dummy | nmy   |
|           | Corn  | Soybeans             | Oats   | Corn  | Soybeans    | Oats               | Corn  | Corn Soybeans Oats | Oats                      | Corn  | Corn Soybeans Oats | Oats                  | Corn  | Corn Soybeans Oats | Oats  |
| Mean      | 0.854 | 0.917                | 1.195  | 4.432 | 1.003       | $0.138 \mid 0.151$ | 0.151 | 0.123              | $0.123  0.100 \mid 0.245$ | 0.245 | 0.221              | 0.221 $0.170$ $0.419$ | 0.419 | 0.432              | 0.577 |
| Median    | 0.626 | 0.689                | 0.870  | 4.385 | 0.961       | 0.126              | 0.150 | 0.123              | 0.100                     | 0     | 0                  | 0                     | 0     | 0                  | 1     |
| Min       | 0     | 0                    | 0      | 0.454 | 0.144       | 0.051              | 0     | 0                  | 0                         | 0     | 0                  | 0                     | 0     | 0                  | 0     |
| Max       | 9.426 | 11.665               | 20.294 | 9.771 | 2.255       | 0.300              | 0.321 | 0.246              | 0.208                     | 1     | 1                  | 1                     | 1     | П                  | 1     |
| Std. Dev. | 0.833 | 0.874                | 1.171  | 1.904 | 0.491       | 0.053              | 0.087 | 0.072              | 0.058                     | 0.430 | 0.415              | 0.376 0.493           | 0.493 | 0.495              | 0.494 |
|           |       |                      | ,      |       |             | 1                  |       |                    |                           |       |                    |                       |       |                    |       |

Notes:  $\%|\Delta F_{it}| = 100 \times (|\ln F_{it} - \ln F_{i,t-1}|)$ ,  $i = 1, 2, \cdots, k, \ t = 1, 2, \cdots, T$ , where k is the number of delivery horizon (six for corn, seven for soybeans, and four for oats), T is the number of trading days in the sample (4,202 for corn, 3,693 for soybeans, and 3,443 for oats), and  $\ln F_{it}$  is the natural logarithm of the i<sup>th</sup>-delivery contract's price on day t. Inventories are measured in billions of bushels. Time to delivery is measured as the number of trading days remaining to contract expiration divided by 1000. Pre-harvest dummy variable equals one if trading takes place in July, August, and September for corn and soybeans, and in June and July for oats. Post-harvest dummy variable equals one if trading takes place in October, November, December, January, and February for corn and soybeans, and in August, September, October, November, December, January, and February for oats.

Table 2: Determinants of Corn Futures Volatility

(a) Corn: Bayesian Estimation Results

|                               | 1      | 2              | 3       | 4       | 5              | (      |
|-------------------------------|--------|----------------|---------|---------|----------------|--------|
| Intercept                     | 1.223  | 1.222          | 1.220   | 1.212   | 1.215          | 1.209  |
| s.e.                          | 0.032  | 0.023          | 0.017   | 0.016   | 0.016          | 0.017  |
| pseudo t                      | 38.786 | 53.613         | 73.152  | 77.848  | 77.047         | 70.576 |
| 95% HPDL                      | 1.162  | 1.177          | 1.188   | 1.181   | 1.184          | 1.175  |
|                               |        |                |         |         |                |        |
| 95% HPDU                      | 1.285  | 1.267          | 1.253   | 1.242   | 1.246          | 1.242  |
| Inventories                   | -0.052 | -0.058         | -0.047  | -0.061  | -0.055         | -0.035 |
| s.e.                          | 0.018  | 0.019          | 0.014   | 0.013   | 0.013          | 0.014  |
| pseudo t                      | -2.863 | -3.029         | -3.338  | -4.635  | -4.091         | -2.436 |
| 95% HPDL                      | -0.087 | -0.095         | -0.075  | -0.087  | -0.081         | -0.064 |
| $95\%~\mathrm{HPDU}$          | -0.016 | -0.020         | -0.019  | -0.035  | -0.029         | -0.007 |
| Inventories <sup>2</sup>      | 0.002  | -0.001         | -0.001  | 0.003   | 0.002          | 0.003  |
|                               | !      |                |         |         |                |        |
| s.e.                          | 0.002  | 0.003          | 0.002   | 0.002   | 0.002          | 0.003  |
| pseudo t                      | 0.767  | -0.412         | -0.341  | 1.507   | 1.179          | 1.65   |
| 95% HPDL                      | -0.003 | -0.006         | -0.004  | -0.001  | -0.001         | -0.00  |
| 95% HPDU                      | 0.006  | 0.004          | 0.003   | 0.006   | 0.005          | 0.00   |
| Time to delivery              | -0.084 | 0.152          | 0.316   | 0.217   | 0.107          | -0.15  |
| s.e.                          | 0.323  | 0.176          | 0.121   | 0.113   | 0.116          | 0.125  |
| pseudo t                      | -0.261 | 0.864          | 2.615   | 1.922   | 0.927          | -1.28  |
| 95% HPDL                      | -0.718 | -0.193         | 0.079   | -0.004  | -0.120         | -0.39  |
| 95% HPDU                      | 0.549  |                |         |         |                |        |
| 95% HPDU                      | 0.549  | 0.497          | 0.552   | 0.439   | 0.335          | 0.083  |
| Time to delivery <sup>2</sup> | -1.100 | -1.074         | -1.091  | -1.052  | -1.080         | -1.179 |
| s.e.                          | 0.350  | 0.234          | 0.169   | 0.157   | 0.158          | 0.169  |
| pseudo t                      | -3.145 | -4.598         | -6.460  | -6.716  | -6.847         | -6.974 |
| 95% HPDL                      | -1.785 | -1.533         | -1.422  | -1.359  | -1.390         | -1.51  |
| $95\%~\mathrm{HPDU}$          | -0.414 | -0.616         | -0.760  | -0.745  | -0.771         | -0.848 |
| Calendar time                 | -0.246 | -0.239         | -0.247  | -0.264  | -0.255         | -0.25  |
| s.e.                          | 0.026  | 0.022          | 0.016   | 0.015   | 0.015          | 0.010  |
|                               |        |                |         |         |                |        |
| pseudo t                      | -9.344 | -11.012        | -15.566 | -17.974 | -17.212        | -15.61 |
| 95% HPDL                      | -0.298 | -0.281         | -0.278  | -0.293  | -0.285         | -0.28  |
| 95% HPDU                      | -0.195 | -0.196         | -0.215  | -0.235  | -0.226         | -0.22  |
| Calendar time <sup>2</sup>    | 0.055  | 0.059          | 0.049   | 0.052   | 0.047          | 0.04   |
| s.e.                          | 0.006  | 0.006          | 0.004   | 0.004   | 0.004          | 0.004  |
| pseudo t                      | 9.212  | 9.663          | 11.084  | 13.177  | 12.027         | 11.51  |
| 95% HPDL                      | 0.043  | 0.047          | 0.040   | 0.044   | 0.039          | 0.03   |
| 95% HPDU                      | 0.066  | 0.071          | 0.057   | 0.060   | 0.054          | 0.05   |
| I ammad svalatilitus          | 0.100  | 0.020          | 0.100   | 0.106   | 0.125          | 0.14   |
| Lagged volatility             | 0.199  | 0.029          | 0.102   | 0.106   | 0.135          | 0.14   |
| s.e.                          | 0.014  | 0.012          | 0.011   | 0.010   | 0.010          | 0.01   |
| pseudo t                      | 14.714 | 2.342          | 9.466   | 10.447  | 13.221         | 13.10  |
| 95% HPDL                      | 0.173  | 0.005          | 0.081   | 0.086   | 0.115          | 0.12   |
| 95% HPDU                      | 0.226  | 0.053          | 0.123   | 0.126   | 0.155          | 0.16   |
| Pre-harvest dummy             | 0.056  | 0.011          | -0.009  | -0.003  | -0.001         | -0.04  |
| s.e.                          | 0.025  | 0.022          | 0.016   | 0.015   | 0.015          | 0.01   |
| pseudo t                      | 2.264  | 0.513          | -0.554  | -0.229  | -0.056         | -2.65  |
| 95% HPDL                      | 0.008  |                | -0.041  | -0.229  |                | -0.07  |
| 95% HPDU                      | 0.008  | -0.032 $0.055$ | 0.023   | 0.032   | -0.030 $0.028$ | -0.07  |
|                               |        |                |         |         |                |        |
| Post-harvest dummy            | -0.179 | -0.185         | -0.197  | -0.204  | -0.216         | -0.19  |
| s.e.                          | 0.026  | 0.022          | 0.016   | 0.015   | 0.015          | 0.01   |
| pseudo t                      | -6.806 | -8.250         | -12.116 | -13.561 | -14.300        | -11.73 |
| 95% HPDL                      | -0.230 | -0.228         | -0.229  | -0.234  | -0.246         | -0.22  |
| 95% HPDU                      | -0.127 | -0.141         | -0.165  | -0.175  | -0.187         |        |

(b) Corn: Marginal Effects

|   | _      |        |        |        |        |        |
|---|--------|--------|--------|--------|--------|--------|
|   | 1      | 2      | 3      | 4      | 5      | 6      |
| $\partial  \% \Delta F_{it}  / \partial S_t = b_{1i} + 2b_{2i} \overline{S}$        | -0.037 | -0.067 | -0.053 | -0.038 | -0.037 | -0.009 |
| s.e.  | 0.008  | 0.012  | 0.008  | 0.007  | 0.007  | 0.008  |
| t-val   | -4.758 | -5.554 | -6.297 | -5.179 | -5.022 | -1.157 |
| 95% L   | -0.052 | -0.091 | -0.069 | -0.053 | -0.052 | -0.025 |
| 95% U   | -0.022 | -0.044 | -0.036 | -0.024 | -0.023 | 0.007  |
| $Prob(\partial  \% \Delta F_{it} /\partial S_t < 0)$                                | 1.000  | 1.000  | 1.000  | 1.000  | 1.000  | 0.876  |
| $\partial  \% \Delta F_{it}  / \partial TTD_{it} = c_{1i} + 2c_{2i} \overline{TTD}$ | -0.141 | -0.012 | 0.042  | -0.149 | -0.379 | -0.811 |
| s.e.  | 0.324  | 0.177  | 0.122  | 0.116  | 0.121  | 0.131  |
| t-val   | -0.436 | -0.069 | 0.347  | -1.290 | -3.124 | -6.194 |
| 95% L   | -0.775 | -0.359 | -0.197 | -0.376 | -0.617 | -1.068 |
| 95% U   | 0.493  | 0.335  | 0.282  | 0.078  | -0.141 | -0.555 |
| $Prob(\partial  \% \Delta F_{it} /\partial TTD_{it} < 0)$                           | 0.669  | 0.528  | 0.364  | 0.901  | 0.999  | 1.000  |
| $\partial  \% \Delta F_{it}  / \partial t = d_{1i} + 2d_{2i}\bar{t}$                | 0.011  | 0.040  | -0.017 | -0.019 | -0.036 | -0.028 |
| s.e.  | 0.011  | 0.019  | 0.014  | 0.012  | 0.011  | 0.011  |
| t-val   | 1.049  | 2.104  | -1.214 | -1.614 | -3.267 | -2.498 |
| 95% L   | -0.010 | 0.003  | -0.044 | -0.042 | -0.058 | -0.051 |
| 95% U   | 0.032  | 0.078  | 0.010  | 0.004  | -0.014 | -0.006 |
| $\text{Prob}(\partial  \% \Delta F_{it} /\partial t < 0)$                           | 0.147  | 0.018  | 0.888  | 0.947  | 0.999  | 0.994  |
|   |        |        |        |        |        |        |

(c) Corn: Implied Changes in Volatility

|  | 1      | 2      | 3      | 4      | 5      | 6      |
|--|--------|--------|--------|--------|--------|--------|
| Inventory effect:<br>$b_{1i}(S_{max} - S_{min}) + b_{2i}(S_{max}^2 - S_{min}^2)$ | -0.321 | -0.642 | -0.500 | -0.323 | -0.322 | -0.050 |
| TTD effect: $c_{1i}(TTD_{max} - TTD_{min}) + c_{2i}(TTD_{max}^2 - TTD_{min}^2)$  | -0.140 | -0.062 | -0.011 | -0.039 | -0.077 | -0.172 |
| CT effect:<br>$d_{1i}(t_{max} - t_{min}) + d_{2i}(t_{max}^2 - t_{min}^2)$        | 0.047  | 0.170  | -0.070 | -0.080 | -0.152 | -0.120 |

Notes: The model is  $|\%\Delta F_{it}| = a_i + b_{1i}S_t + b_{2i}S_t^2 + c_{1i}TTD_{it} + c_{2i}TTD_{it}^2 + d_{1i}t + d_{2i}t^2 + e_i|\%\Delta F_{i,t-1}| + h_{1i}D_{1t} + h_{2i}D_{2t} + \varepsilon_{it}$ , where  $|\%\Delta F_{it}| \equiv |100 \times (\ln F_{it} - \ln F_{i,t-1})|$  for  $i = 1, \dots, 6$  and  $t = 1, \dots, 4, 202$ . The subscript i denotes the i<sup>th</sup>-delivery contract. The variable  $\ln F_{it}$  is the natural logarithm of the price of the i<sup>th</sup>-delivery futures contract on day t,  $S_t$  is the inventory level measured in billions of bushels on day t,  $TTD_{it}$  is the number of remaining days to maturity of the i<sup>th</sup>-delivery contract t.  $D_{1t}$  is the pre-harvest dummy variable, which takes the value of one if t is in July, August, and September, zero otherwise.  $D_{2t}$  is the post-harvest dummy variable, which takes the value of one if t is in October, November, December, January, and February, zero otherwise. The remaining months of March, April, May, and June are defined as the planning period and used as the base category. In panel (a), for each parameter, its posterior mean, posterior standard error, pseudo t-value, 95% highest posterior density lower and upper limits are given, respectively. In panel (b), derivatives are evaluated at the mean value of the variables.

Table 3: Determinants of Soybean Futures Volatility

(a) Soybeans: Bayesian Estimation Results

| <u> </u>                      | í ·     |               | esian Esti       |                  |                  |               |               |
|-------------------------------|---------|---------------|------------------|------------------|------------------|---------------|---------------|
|                               | 1       | 2             | 3                | 4                | 5                | 6             | 7             |
| Intercept                     | 1.672   | 1.649         | 1.665            | 1.657            | 1.650            | 1.651         | 1.640         |
| s.e.                          | 0.028   | 0.023         | 0.016            | 0.015            | 0.015            | 0.015         | 0.017         |
| pseudo t                      | 59.411  | 70.334        | 100.945          | 108.140          | 112.017          | 109.757       | 98.800        |
| 95% HPDL                      | 1.616   | 1.603         | 1.633            | 1.627            | 1.622            | 1.621         | 1.607         |
| $95\%~\mathrm{HPDU}$          | 1.727   | 1.695         | 1.697            | 1.687            | 1.679            | 1.680         | 1.672         |
| Inventories                   | -0.912  | -0.898        | -0.908           | -0.911           | -0.911           | -0.918        | -0.914        |
| s.e.                          | 0.030   | 0.023         | 0.016            | 0.015            | 0.014            | 0.015         | 0.016         |
| pseudo t                      | -30.072 | -38.687       | -55.740          | -60.659          | -63.606          | -63.048       | -56.573       |
| 95% HPDL                      | -0.972  | -0.944        | -0.940           | -0.941           | -0.939           | -0.947        | -0.945        |
| 95% HPDU                      | -0.853  | -0.853        | -0.876           | -0.882           | -0.883           | -0.890        | -0.882        |
| Inventories <sup>2</sup>      | 0.279   | 0.306         | 0.280            | 0.286            | 0.292            | 0.267         | 0.302         |
| s.e.                          | 0.019   | 0.020         | 0.230            | 0.200            | 0.232            | 0.201         | 0.013         |
| pseudo t                      | 14.814  | 15.621        | 20.308           | 22.987           | 25.416           | 23.261        | 23.964        |
| 95% HPDL                      | 0.242   | 0.268         | 0.253            | 0.261            | 0.270            | 0.244         | 0.277         |
| 95% HPDU                      | 1       | 0.208 $0.345$ |                  |                  |                  |               | 0.277         |
| 95% HPDU                      | 0.316   | 0.545         | 0.307            | 0.310            | 0.315            | 0.289         | 0.520         |
| Time to delivery              | 0.134   | 0.157         | 0.218            | 0.497            | 0.435            | 0.451         | 0.467         |
| s.e.                          | 0.338   | 0.194         | 0.131            | 0.117            | 0.107            | 0.106         | 0.114         |
| pseudo t                      | 0.396   | 0.812         | 1.664            | 4.233            | 4.068            | 4.253         | 4.080         |
| 95% HPDL                      | -0.528  | -0.223        | -0.039           | 0.267            | 0.225            | 0.243         | 0.242         |
| $95\%~\mathrm{HPDU}$          | 0.795   | 0.538         | 0.474            | 0.727            | 0.645            | 0.659         | 0.691         |
| Time to delivery <sup>2</sup> | -2.589  | -2.590        | -2.625           | -2.584           | -2.561           | -2.548        | -2.503        |
| s.e.                          | 0.348   | 0.245         | 0.171            | 0.158            | 0.151            | 0.153         | 0.168         |
| pseudo t                      | -7.433  | -10.571       | -15.327          | -16.314          | -16.968          | -16.660       | -14.941       |
| 95% HPDL                      | -3.271  | -3.071        | -2.960           | -2.895           | -2.857           | -2.848        | -2.832        |
| $95\%~\mathrm{HPDU}$          | -1.906  | -2.110        | -2.289           | -2.274           | -2.265           | -2.248        | -2.175        |
| Calendar time                 | -0.307  | -0.308        | -0.297           | -0.304           | -0.306           | -0.311        | -0.318        |
| s.e.                          | 0.026   | 0.023         | 0.016            | 0.015            | 0.014            | 0.014         | 0.016         |
| pseudo t                      | -11.891 | -13.595       | -18.637          | -20.603          | -21.629          | -21.565       | -20.058       |
| 95% HPDL                      | -0.358  | -0.352        | -0.328           | -0.333           | -0.334           | -0.339        | -0.349        |
| 95% HPDU                      | -0.257  | -0.263        | -0.265           | -0.275           | -0.278           | -0.282        | -0.287        |
| Calendar time <sup>2</sup>    | 0.073   | 0.064         | 0.074            | 0.065            | 0.062            | 0.067         | 0.064         |
| s.e.                          | 0.006   | 0.007         | 0.005            | 0.004            | 0.004            | 0.004         | 0.004         |
| pseudo t                      | 12.114  | 9.205         | 15.317           | 14.926           | 15.238           | 16.686        | 14.792        |
| 95% HPDL                      | 0.061   | 0.050         | 0.065            | 0.057            | 0.054            | 0.059         | 0.055         |
| 95% HPDU                      | 0.001   | 0.030 $0.077$ | 0.084            | 0.037 $0.074$    | 0.054 $0.069$    | 0.039 $0.074$ | 0.033 $0.072$ |
| 95% HF DO                     | 0.065   | 0.077         | 0.064            | 0.074            | 0.009            | 0.074         | 0.072         |
| Lagged volatility             | 0.093   | 0.005         | 0.041            | 0.072            | 0.073            | 0.060         | 0.074         |
| s.e.                          | 0.014   | 0.014         | 0.012            | 0.011            | 0.011            | 0.011         | 0.012         |
| pseudo t                      | 6.569   | 0.347         | 3.566            | 6.452            | 6.659            | 5.382         | 6.248         |
| 95% HPDL                      | 0.066   | -0.023        | 0.019            | 0.050            | 0.052            | 0.038         | 0.050         |
| $95\%~\mathrm{HPDU}$          | 0.121   | 0.033         | 0.064            | 0.094            | 0.095            | 0.082         | 0.097         |
| Pre-harvest dummy             | 0.040   | -0.009        | 0.007            | 0.008            | 0.007            | 0.001         | 0.002         |
| s.e.                          | 0.027   | 0.023         | 0.016            | 0.015            | 0.015            | 0.015         | 0.016         |
| pseudo t                      | 1.485   | -0.386        | 0.405            | 0.549            | 0.464            | 0.102         | 0.136         |
| 95% HPDL                      | -0.013  | -0.055        | -0.026           | -0.022           | -0.022           | -0.027        | -0.029        |
| 95% HPDU                      | 0.092   | 0.037         | 0.039            | 0.038            | 0.035            | 0.030         | 0.034         |
| Post-harvest dummy            | -0.034  | -0.044        | -0.057           | -0.056           | -0.055           | -0.061        | -0.077        |
| s.e.                          | 0.027   | 0.023         | 0.016            | 0.015            | 0.014            | 0.015         | 0.016         |
| pseudo t                      | -1.233  | -1.890        | -3.485           | -3.711           | -3.831           | -4.181        | -4.817        |
| 95% HPDL                      | -0.087  | -0.090        | -0.089           | -0.086           | -0.083           |               | -0.108        |
| 95% HPDU<br>95% HPDU          | 0.020   | 0.090         | -0.089<br>-0.025 | -0.086<br>-0.027 | -0.083<br>-0.027 | -0.089        | -0.108        |
| 9970 III DU                   | 0.020   | 0.002         | -0.020           | -0.021           | -0.027           | -0.032        | -0.040        |

(b) Soybeans: Marginal Effects

|   | 1       | 2      | 3       | 4       | 5       | 6       | 7       |
|---|---------|--------|---------|---------|---------|---------|---------|
| $\partial  \% \Delta F_{it}  / \partial S_t = b_{1i} + 2b_{2i} \overline{S}$        | -0.353  | -0.284 | -0.346  | -0.338  | -0.325  | -0.383  | -0.309  |
| s.e.  | 0.032   | 0.041  | 0.029   | 0.026   | 0.023   | 0.023   | 0.025   |
| t-val   | -11.059 | -6.938 | -11.988 | -13.176 | -13.988 | -16.657 | -12.117 |
| 95% L   | -0.415  | -0.364 | -0.403  | -0.388  | -0.370  | -0.428  | -0.359  |
| 95% U   | -0.290  | -0.204 | -0.289  | -0.288  | -0.279  | -0.338  | -0.259  |
| $Prob(\partial  \% \Delta F_{it} /\partial S_t < 0)$                                | 1.000   | 1.000  | 1.000   | 1.000   | 1.000   | 1.000   | 1.000   |
| $\partial  \% \Delta F_{it}  / \partial TTD_{it} = c_{1i} + 2c_{2i} \overline{TTD}$ | 0.048   | -0.118 | -0.244  | -0.139  | -0.375  | -0.532  | -0.682  |
| s.e.  | 0.338   | 0.195  | 0.132   | 0.119   | 0.109   | 0.109   | 0.119   |
| t-val   | 0.142   | -0.608 | -1.849  | -1.166  | -3.429  | -4.859  | -5.726  |
| 95% L   | -0.614  | -0.500 | -0.503  | -0.373  | -0.589  | -0.746  | -0.916  |
| 95% U   | 0.710   | 0.264  | 0.015   | 0.095   | -0.160  | -0.317  | -0.449  |
| $Prob(\partial  \% \Delta F_{it} /\partial TTD_{it} < 0)$                           | 0.444   | 0.728  | 0.968   | 0.878   | 1.000   | 1.000   | 1.000   |
| $\partial  \% \Delta F_{it}  / \partial t = d_{1i} + 2d_{2i}\bar{t}$                | 0.041   | -0.003 | 0.059   | 0.007   | -0.012  | 0.008   | -0.014  |
| s.e.  | 0.012   | 0.023  | 0.016   | 0.014   | 0.013   | 0.013   | 0.013   |
| t-val   | 3.511   | -0.111 | 3.661   | 0.502   | -0.918  | 0.644   | -1.062  |
| 95% L   | 0.018   | -0.047 | 0.027   | -0.021  | -0.037  | -0.016  | -0.040  |
| 95% U   | 0.064   | 0.042  | 0.091   | 0.035   | 0.013   | 0.033   | 0.012   |
| $\operatorname{Prob}(\partial  \% \Delta F_{it} /\partial t < 0)$                   | 0.000   | 0.544  | 0.000   | 0.308   | 0.821   | 0.260   | 0.856   |

### (c) Soybeans: Implied Changes in Volatility

|  | 1      | 2      | 3      | 4      | 5      | 6      | 7      |
|--|--------|--------|--------|--------|--------|--------|--------|
| Inventory effect:<br>$b_{1i}(S_{max} - S_{min}) + b_{2i}(S_{max}^2 - S_{min}^2)$   | -0.514 | -0.346 | -0.499 | -0.477 | -0.444 | -0.587 | -0.402 |
| TTD effect:<br>$c_{1i}(TTD_{max} - TTD_{min}) + c_{2i}(TTD_{max}^2 - TTD_{min}^2)$ | -0.124 | -0.118 | -0.105 | -0.034 | -0.048 | -0.043 | -0.037 |
| CT effect:<br>$d_{1i}(t_{max} - t_{min}) + d_{2i}(t_{max}^2 - t_{min}^2)$          | 0.157  | -0.027 | 0.231  | 0.013  | -0.067 | 0.017  | -0.076 |

Notes: The model is  $|\%\Delta F_{it}| = a_i + b_{1i}S_t + b_{2i}S_t^2 + c_{1i}TTD_{it} + c_{2i}TTD_{it}^2 + d_{1i}t + d_{2i}t^2 + e_i|\%\Delta F_{i,t-1}| + h_{1i}D_{1t} + h_{2i}D_{2t} + \varepsilon_{it}$ , where  $|\%\Delta F_{it}| \equiv |100 \times (\ln F_{it} - \ln F_{i,t-1})|$  for  $i = 1, \dots, 7$  and  $t = 1, \dots, 3$ , 693. The subscript i denotes the i<sup>th</sup>-delivery contract. The variable  $\ln F_{it}$  is the natural logarithm of the price of the i<sup>th</sup>-delivery futures contract on day t,  $S_t$  is the inventory level measured in billions of bushels on day t,  $TTD_{it}$  is the number of remaining days to maturity of the i<sup>th</sup>-delivery contract t.  $D_{1t}$  is the pre-harvest dummy variable, which takes the value of one if t is in July, August, and September, zero otherwise.  $D_{2t}$  is the post-harvest dummy variable, which takes the value of one if t is in October, November, December, January, and February, zero otherwise. The remaining months of March, April, May, and June are defined as the planning period and used as the base category. In panel (a), for each parameter, its posterior mean, posterior standard error, pseudo t-value, 95% highest posterior density lower and upper limits are given, respectively. In panel (b), derivatives are evaluated at the mean value of the variables.

Table 4: Determinants of Oat Futures Volatility

(a) Oats: Estimation Results

| (a) Oats                         | s: Estima      | ation Res      | ults           |                |
|----------------------------------|----------------|----------------|----------------|----------------|
|                                  | 1              | 2              | 3              | 4              |
| Intercept                        | 2.911          | 2.910          | 2.910          | 2.905          |
| s.e.                             | 0.036          | 0.024          | 0.018          | 0.019          |
| pseudo t                         | 81.522         | 122.890        | 165.661        | 155.542        |
| 95% HPDL                         | 2.841          | 2.864          | 2.875          | 2.868          |
| 95% HPDU                         | 2.981          | 2.957          | 2.944          | 2.942          |
| Inventories                      | -12.090        | -12.047        | -12.093        | -12.582        |
| s.e.                             | 0.387          | 0.350          | 0.268          | 0.282          |
| pseudo t                         | -31.251        | -34.409        | -45.194        | -44.672        |
| 95% HPDL                         | -12.848        | -12.733        | -12.618        | -13.134        |
| 95% HPDU                         | -11.331        | -11.360        | -11.569        | -12.030        |
| Inventories <sup>2</sup>         | 21.488         | 21.230         | 21.156         | 21.087         |
| s.e.                             | 1.113          | 0.742          | 0.549          | 0.582          |
| pseudo t                         | 19.306         | 28.620         | 38.537         | 36.244         |
| 95% HPDL                         | 19.306         | 19.775         | 20.079         | 19.946         |
| 95% HPDU                         | 23.671         | 22.684         | 22.232         | 22.228         |
| Time to delivery                 | -2.411         | -2.318         | -2.309         | -2.527         |
| s.e.                             | 0.363          | 0.229          | 0.163          | 0.166          |
| pseudo t                         | -6.646         | -10.130        | -14.160        | -15.246        |
| 95% HPDL                         | -3.122         | -2.767         | -2.628         | -2.852         |
| 95% HPDU                         | -1.700         | -1.869         | -1.989         | -2.202         |
| Time to delivery <sup>2</sup>    | 10.019         | 10.029         | 10.037         | 9.981          |
| s.e.                             | 0.375          | 0.242          | 0.179          | 0.189          |
| pseudo t                         | 26.684         | 41.476         | 56.162         | 52.771         |
| 95% HPDL                         | 9.283          | 9.555          | 9.686          | 9.610          |
| 95% HPDU                         | 10.756         | 10.503         | 10.387         | 10.352         |
| Calendar time                    | -0.635         | -0.639         | -0.633         | -0.637         |
| s.e.                             | 0.032          | 0.023          | 0.017          | 0.018          |
| pseudo t                         | -19.731        | -28.415        | -37.922        | -35.967        |
| 95% HPDL                         | -0.698         | -0.684         | -0.666         | -0.671         |
| 95% HPDU                         | -0.572         | -0.595         | -0.600         | -0.602         |
| Calendar time <sup>2</sup>       | 0.135          | 0.115          | 0.095          | 0.084          |
|                                  |                |                |                |                |
| s.e.<br>pseudo t                 | 0.011 $12.829$ | 0.009 $12.924$ | 0.007 $14.450$ | 0.007 $12.425$ |
| 95% HPDL                         | 0.114          | 0.098          | 0.082          | 0.071          |
| 95% HPDU                         | 0.114          | 0.098 $0.133$  | 0.082 $0.108$  | 0.071          |
| T 1 1-4:1:4                      | 0.140          |                |                |                |
| Lagged volatility                | 0.149          | 0.084          | 0.086          | 0.107          |
| s.e.                             | 0.015          | 0.014          | 0.012          | 0.012          |
| pseudo t                         | 9.762          | 5.829          | 7.101          | 8.617          |
| 95% HPDL                         | 0.119          | 0.056          | 0.062          | 0.083          |
| 95% HPDU                         | 0.178          | 0.112          | 0.110          | 0.132          |
| Pre-harvest dummy                | 0.361          | 0.347          | 0.345          | 0.341          |
| s.e.                             | 0.032          | 0.023          | 0.017          | 0.018          |
| pseudo t                         | 11.171         | 14.952         | 20.110         | 18.855         |
| 95% HPDL                         | 0.298          | 0.301          | 0.312          | 0.306          |
| 95% HPDU                         | 0.425          | 0.392          | 0.379          | 0.377          |
| Post-harvest dummy               | 0.064          | 0.067          | 0.080          | 0.084          |
| s.e.                             | 0.031          | 0.023          | 0.017          | 0.018          |
|                                  | 1 0.004        | 2.919          | 4.761          | 4.742          |
| pseudo t                         | 2.064          | 2.313          | 101            |                |
| pseudo t<br>95% HPDL<br>95% HPDU | 0.003          | 0.022          | 0.047          | 0.049          |

(b) Oats: Marginal Effects

|  | 1       | 2       | 3       | 4       |
|--|---------|---------|---------|---------|
| $\partial  \% \Delta F_{it}  / \partial S_t = b_{1i} + 2b_{2i} \overline{S}$ | -6.146  | -6.174  | -6.241  | -6.749  |
| s.e.   | 0.308   | 0.330   | 0.253   | 0.263   |
| t-val  | -19.985 | -18.699 | -24.694 | -25.653 |
| 95% L  | -6.749  | -6.822  | -6.737  | -7.265  |
| 95% U  | -5.543  | -5.527  | -5.746  | -6.233  |
| $\text{Prob}(\partial  \% \Delta F_{it} /\partial S_t < 0)$                  | 1.000   | 1.000   | 1.000   | 1.000   |
|  |         |         |         |         |
| $\partial  \% \Delta F_{it}  / \partial TTD_{it} = c_{1i} + 2c_{2i}TTD$      | -1.898  | -0.794  | 0.199   | 0.946   |
| s.e.   | 0.363   | 0.231   | 0.168   | 0.174   |
| t-val  | -5.226  | -3.435  | 1.187   | 5.433   |
| 95% L  | -2.610  | -1.248  | -0.130  | 0.605   |
| 95% U  | -1.186  | -0.341  | 0.528   | 1.288   |
| $\text{Prob}(\partial  \% \Delta F_{it}  / \partial TTD_{it} < 0)$           | 1.000   | 1.000   | 0.118   | 0.000   |
|  |         |         |         |         |
| $\partial  \% \Delta F_{it}  / \partial t = d_{1i} + 2d_{2i}t$               | -0.131  | -0.208  | -0.277  | -0.321  |
| s.e.   | 0.020   | 0.024   | 0.018   | 0.018   |
| t-val  | -6.387  | -8.586  | -15.332 | -17.605 |
| 95% L  | -0.171  | -0.255  | -0.312  | -0.357  |
| 95% U  | -0.090  | -0.160  | -0.241  | -0.286  |
| $Prob(\partial  \% \Delta F_{it} /\partial t < 0)$                           | 1.000   | 1.000   | 1.000   | 1.000   |

(c) Oats: Implied Changes in Volatility

|  | 1      | 2      | 3      | 4      |
|--|--------|--------|--------|--------|
| Inventory effect:<br>$b_{1i}(S_{max} - S_{min}) + b_{2i}(S_{max}^2 - S_{min}^2)$ | -1.136 | -1.148 | -1.166 | -1.295 |
| TTD effect: $c_{1i}(TTD_{max} - TTD_{min}) + c_{2i}(TTD_{max}^2 - TTD_{min}^2)$  | -0.068 | -0.048 | -0.046 | -0.094 |
| CT effect:<br>$d_{1i}(t_{max} - t_{min}) + d_{2i}(t_{max}^2 - t_{min}^2)$        | -0.458 | -0.726 | -0.965 | -1.120 |

Notes: The model is  $|\%\Delta F_{it}| = a_i + b_{1i}S_t + b_{2i}S_t^2 + c_{1i}TTD_{it} + c_{2i}TTD_{it}^2 + d_{1i}t + d_{2i}t^2 + e_i|\%\Delta F_{i,t-1}| + h_{1i}D_{1t} + h_{2i}D_{2t} + \varepsilon_{it}$ , where  $|\%\Delta F_{it}| \equiv |100 \times (\ln F_{it} - \ln F_{i,t-1})|$  for  $i = 1, \dots, 4$  and  $t = 1, \dots, 3, 443$ . The subscript i denotes the i<sup>th</sup>-delivery contract. The variable  $\ln F_{it}$  is the natural logarithm of the price of the i<sup>th</sup>-delivery futures contract on day t,  $S_t$  is the inventory level measured in billions of bushels on day t,  $TTD_{it}$  is the number of remaining days to maturity of the i<sup>th</sup>-delivery contract t.  $D_{1t}$  is the pre-harvest dummy variable, which takes the value of one if t is in June and July, zero otherwise.  $D_{2t}$  is the post-harvest dummy variable, which takes the value of one if t is in August, September, October, November, December, January, and February, zero otherwise. The remaining months of March, April, and May are defined as the planning period and used as the base category. In panel (a), for each parameter, its posterior mean, posterior standard error, pseudo t-value, 95% highest posterior density lower and upper limits are given, respectively. In panel (b), derivatives are evaluated at the mean value of the variables.