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Multidimensional Poverty and its Discontents

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Abstract

More data on non-income dimensions of poverty are available than at any previous time in history. Alongside this, multidimensional measurement methodologies have advanced considerably in the past fifteen years. These advances together have created new possibilities to measure multidimensional poverty at the local, national and international level. Yet the fact that overall measures can be constructed does not mean that they will necessarily add value. This paper focuses on the question of when, how and why certain multidimensional poverty measures add value, sketches the limits of the contribution, and introduces a set of standing questions. The key value-added of a rigorously implemented multidimensional poverty index is that it conveys additional information not captured in single-dimensional measures (or in a dashboard) on the joint distribution of disadvantage and the composition of poverty. It also provides a consistent account of the overall change in multidimensional poverty across time and space. The paper discusses the joint distribution approach to multidimensional poverty measurement and presents one class of poverty measures within this approach. It then introduces one recently implemented measure within this family: the 104-country Multidimensional Poverty Index 2010 and uses concrete examples to explain its construction further. For example, without weights one can only identify the multidimensionally poor by the union or the intersection approaches; by these approaches the 2010 MPI would have identified an average of 58% or 0% of people across the 104 countries as poor. It also shows how to 'unfold' the MPI by sub-group or dimension, and also by intensity - because similar 'intensities' of poverty can conceal different distributions of intensity among the poor. Pointing out the added value of multidimensional poverty indexes is not to suggest that single-dimensional measures be abandoned but rather supplemented. Investing further in multidimensional measures has the potential to generate significant advances in understanding and useful policy tools.

Keywords: poverty measurement, multidimensional poverty, identification, poverty indices, FGT measures, joint distribution.

JEL classification: I3, I32, O1

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The Oxford Poverty and Human Development Initiative (OPHI) is a research centre within the Oxford Department of International Development, Queen Elizabeth House, at the University of Oxford. Led by Sabina Alkire, OPHI aspires to build and advance a more systematic methodological and economic framework for reducing multidimensional poverty, grounded in people's experiences and values.

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Introduction

The multidimensionality of poverty is not in dispute.¹ Poverty can mean poor health, inadequate education, low income, precarious housing, difficult or insecure work, political disempowerment, food insecurity, and the scorn of the better off. The components of poverty change across people, time, and context, but multiple domains are involved.

An emerging question is how multidimensionality should be reflected in measures of poverty.² The launch of a new 104 country multidimensional poverty index (MPI) in 2010 attracted attention and interest in many countries, and provoked lively discussion.³ This paper examines how one aggregate measure of multidimensional poverty adds value to an assemblage of deprivation and income poverty indicators. These issues have become vivid both due to an increasing body of studies on the interrelationships among indicators of disadvantage, as well as to the increased possibility of creating multidimensional poverty measures.⁴ More data on non-income dimensions of poverty are available than at any previous time in history. Alongside this, multidimensional measurement methodologies have advanced considerably in the past fifteen years. These advances together have created new possibilities to measure multidimensional poverty at the local, national and international level. Yet the fact that one can construct an overall measure does not mean that it will necessarily add value. As Sen writes, ‘The passion for aggregation makes good sense in many contexts, but it can be futile or pointless in others... The [overall] view does have its uses, but it has no monopoly of usefulness. When we hear of variety, we need not invariably reach for our aggregator.’⁵ This paper will focus on the question of when, how and why certain multidimensional poverty measures may add value, sketch the limits of the contribution, and introduce a set of standing questions.

To explore these issues, the paper discusses one general approach to multidimensional poverty measurement – that which reflects joint distribution. It then presents one class of poverty measures within this approach, namely an extension to the FGT class of measures (Foster, Greer and Thorbecke 1984) proposed by Alkire and Foster (2007; 2011a). It also introduces one recent implementation of one measure within this family: the Multidimensional Poverty Index. While this paper focuses on one broad approach to poverty measurement, it is important to acknowledge that measurement comprises only a subset of the broad range of techniques that have been developed to assess multidimensional poverty; other methods include qualitative and participatory techniques, dashboards and poverty profiles, dominance techniques, multivariate techniques, and multidimensional inequality indices. Among multidimensional poverty measures, this paper also covers a narrow terrain, and does not address relevant and interesting measures that use information theory,⁶ fuzzy set theory,⁷ latent variable

¹ Grusky and Kanbur 2006, Jenkins and Micklewright 2007, Ravallion 1996, Ravallion 2010, Sen 1992, Sen 1993, Thorbecke 2008. For example, Cappellari and Jenkins write (in Jenkins and Micklewright 2007), ‘It is widely agreed nowadays that being poor does not simply mean not having enough money’ (opening sentence, p 166).

² In Alkire and Foster 2011b, we clarify our measurement methodology and its basis in unidimensional poverty methods; this paper builds upon it, and highlights additional issues in empirical implementation.

³ After the launch of the Multidimensional Poverty Index (MPI) in July 2010, the Oxfam Blog, the World Bank Blog, VOXEU, and other *The Journal of Economic Inequality* carried substantive exchanges on the MPI.

⁴ Of course, there is also a parallel rise in multidimensional welfare assessments. See for example Becker, Philipson and Soares 2005, Brighthouse and Robeyns 2010, Fleurbaey 2009, Fleurbaey and Gaulier 2009, Jones and Klenow 2010, Kreitler and Kreitler 2006, McGillivray 2007, Robeyns and Van der Veen 2007, Stiglitz, Sen and Fitoussi 2009 among others.

⁵ Sen 1987b, p 33.

⁶ Deutsch and Silber 2005, Maasoumi and Lugo 2008.

⁷ Balestrino 1998, Cerioli and Zani 1990, Cheli and Lemmi 1995, Chiappero-Martinetti 1994, Chiappero-Martinetti 2006, Deutsch and Silber 2005, Lelli 2001, Lemmi and Betti 2006, Qizilbash 2002.

techniques,⁸ multiple correspondence analysis,⁹ alternative counting approaches,¹⁰ alternative axiomatic approaches,¹¹ or dominance.¹² While a number of the research questions are shared among approaches, in the limited space available we can only formulate the issues for one measurement approach. However, it seems potentially useful to set out a clear account of this particular approach, so that its strengths and limitations can be grasped, and areas for further research advanced efficiently.

In the absence of such an account, multidimensional measures of poverty may be viewed as a somewhat sweet distraction. In an eloquent criticism of the parsimony for which economics is known A.O. Hirschman (1984) proposed complicating economic discourse by, among other things, introducing a more adequate treatment of love. Love, Hirschman argued, is poorly handled in economics, being neither a scarce resource nor an augmentable skill. Lofty as Hirschman's suggestion might have been, it did not, in practice, take off. There could be many reasons that parsimony endured in this respect; perhaps it was not sufficiently clear when and how such a complication would add value, or perhaps it has yet to find its time. While multidimensional poverty measurement might seem more familiar to economists than Hirschman's favoured topic, it runs the risk of seeming to threaten legitimate parsimony if its potential contribution – and the limits of its contribution – are not sketched more precisely.

I. Multidimensional Poverty

One well-known normative motivation to measure multidimensional poverty arises because poor people's lives can be battered by multiple deprivations that are each of independent importance (Sen 1992). The other key motivation arises from the empirical mismatch between poverty measured in any single space such as income, and additional important single and multidimensional measures of disadvantage. If it were the case that income (or any other unidimensional measure) were a sufficiently good proxy of other disadvantages for practical purposes (such as targeting or tracking change over time or guiding policy) then, in the interests of parsimony, one might not need to go further.¹³

But empirically, many studies note that the extent of mis-match between key social and income indicators, and even between income and key material deprivations, can be considerable across countries and across groups. For example, Brandolini and D'Alessio (2009) used Italy's Survey on Household Income and Wealth (SHIW) 1995 data for six dimensions and found that the correlation coefficients 'show low degrees of association', and that the cross-classifications show 'low redundancy'.¹⁴ They argue that 'the implied shift towards multidimensionality may certainly originate purely empirical grounds as being driven by the necessity to enrich the information set and to overcome the deficiencies of monetary indicators.' Similarly Franco et al. cross-tabulated data in India and Peru on child and adult deprivations

⁸ Kakwani and Silber 2008b, Krishnakumar 2004, Krishnakumar and Ballon 2008, Schokkaert and Van Ootegem 1990.

⁹ Asselin 2009.

¹⁰ Atkinson 2003, Erikson 1993, Gordon et al. 2003, Nolan and Whelan 1996, Subramanian 2007.

¹¹ Bossert, D'Ambrosio and Peragine 2007, Bourguignon and Chakravarty 2002, Chakravarty 1998, Chakravarty and D'Ambrosio 2006, Chakravarty and Silber 2008, Deutsch and Silber 2005, Tsui 2002.

¹² Duclos, Sahn and Younger 2006.

¹³ This issue is discussed in Foster and Sen's Appendix 7 of Sen (1997), which discusses various forms of income poverty measures as well as indicators of other functionings.

¹⁴ The need to look beyond correlations is well known and empirically important. To give just one example of many, Jones and Klenow (2010) find a correlation of 0.95 between GDP and their welfare index, but also find that 'across 134 countries, the typical deviation [between the two indices] is around 46%.'

in health and education with income poverty, and found that the percentage of people who were capability poor but not income poor, or vice versa, ranged from 21 per cent to 93 per cent.¹⁵

But even if there are discrepancies between individual indicators, it could be that income, being a general purpose means, is an accurate representative of *multiple* deprivations. Again, empirical studies have not necessarily substantiated this. Klasen (2000) found that while correlations between expenditure and levels of deprivation in South Africa were strong overall, they were weaker for the most deprived and for certain population groups (Africans, rural, female-headed households etc). In that study, 17 per cent of those identified as functionings deprived were not expenditure poor. Other studies focus on certain population groups such as the disabled, and argue that income poverty measures need to be supplemented by information on additional disadvantages. In a 16 country study Mitra et al. (2010) find that disability is not significantly associated with consumption poverty in most countries, but is significantly associated with multidimensional poverty (using different functional forms and thresholds for multidimensional poverty measures) (see also Kuklys 2005, Zaidi and Burchardt 2005). And in the European context, Nolan and Marx (2009) conclude that the multidimensionality of poverty generally requires multiple variables:

Both national and cross-country studies suggest that the numbers experiencing high levels of deprivation across a number of dimensions are often quite modest and that low income alone is not enough to predict who is experiencing different types of deprivation: poor housing, neighborhood deprivation, poor health and access to health services, and low education are clearly related to low income but are distinct aspects of social exclusion.¹⁶

	Neither persistently income poor nor deprived	Persistently income poor only	Persistently deprived only	Persistently income poor and deprived
Denmark	82.8	6.9	8.9	1.4
The Netherlands	78.8	7.1	7.3	6.8
Belgium	73.0	9.3	8.8	8.9
France	70.8	11.6	8.5	9.0
Ireland	64.8	11.4	9.7	14.0
Italy	68.8	9.2	11.3	10.7
Greece	68.8	11.2	9.9	10.1
Spain	72.7	9.2	8.7	9.4
Portugal	64.5	12.0	11.3	12.2
All	70.7	10.4	9.2	9.7

Figure 1: Distribution across combined income poverty and deprivation persistence variable by country
Source: Whelan Layte and Maitre (2004).

Other analyses explore the relationships between income poverty and other deprivations across time. For example, Whelan et al. 2004 study material asset deprivation and income poverty across five waves of the European Community Household Panel (ECHP) data in nine European countries and find no strong direct or lagged relationship between them. Figure 1 above, from their paper, shows that on average, 70.7 per cent of people were neither persistently income poor nor persistently deprived, and 9.7 per cent of people were persistently both income poor and deprived. However the measures disagreed for 19.6 per cent of people. These people were either persistently income poor but not materially deprived (10.4 per cent) or persistently materially deprived but not income poor (9.2 per cent). To use

¹⁵ Franco in Stewart et al. 2004. See also Klasen 2000, Qizilbash 2002, Ruggeri-Laderchi, Saith and Stewart 2003, Ruggeri-Laderchi 1997, Ruggieri-Laderchi 2008.

¹⁶ Nolan and Marx 2009. See also Balestrino 1996, Balestrino and Sciclone 2001, Brandolini and D'Alessio 1998, Chiappero-Martinetti 2000.

either measure alone would be to overlook half of those deprived in the other. To compare persistent deprivation (18.9 per cent) and persistent income poverty (19.6 per cent) measures individually, one would not know this divergence and might even presume that the reference populations coincided. Also, if each measure were used singly, one would lose information on which households were deprived in *both* ways and which in only one. Such studies of the empirical mismatches between income poverty and other deprivations motivate the by now well-established practice of considering multiple dimensions of poverty.¹⁷

Considering multiple dimensions does not, however, require a multidimensional poverty *index*. The following section considers why an index might add value. In breve, the key value-added of a rigorously implemented multidimensional poverty index is that it conveys additional information not captured in single-dimensional measures on the joint distribution of disadvantage and the composition of poverty among different multiply deprived groups. It also provides a consistent account of the overall change in multidimensional poverty across time and space. To argue this is not to suggest that single-dimensional measures be abandoned; it is to suggest that they be supplemented.

Measurement Approaches to Multidimensional Poverty

As has been often cited, Sen's (1976) paper 'Poverty: An Ordinal Approach to Measurement' opens with the following sentence:

In the measurement of poverty two distinct problems must be faced, viz., (i) identifying the poor among the total population, and (ii) constructing an index of poverty using the available information on the poor.¹⁸

Based on that paper, most poverty measurement methodologies include the two components of identification and aggregation. Whereas in income poverty measures, a person is identified as poor if their income falls beneath a poverty line, identification in multidimensional space is more complex because it may involve the identification of deprivations with respect to each dimension as well as across dimensions.¹⁹

Poverty measures that employ data on multiple dimensions can be broadly distinguished according to which of the following operations they include, and the order in which these are conducted. While the details vary, broadly speaking four steps can be identified (Figure 2):

- (i) apply dimensional cutoff(s) to identify whether a person is deprived in a dimension²⁰
- (ii) aggregate across dimensions
- (iii) identify whether each person is multidimensionally poor
- (iv) aggregate across people

The first methodology in Figure 7 engages the same component steps as income or consumption poverty measures. When the component variables can be meaningfully aggregated, a cutoff can be set

¹⁷ Building upon this, recently a multidimensional index was adopted at the European level, which combines income with material deprivation and unemployment data to provide a more accurate assessment of economic deprivations. See for example <http://ec.europa.eu/eu2020/pdf/115346.pdf> at page 12.

¹⁸ Sen 1976. In that paper, Sen focuses on aggregation, because the recent literature at that time had focused on identification.

¹⁹ These issues are discussed extensively in Alkire and Foster (2011b); this paper draws upon that account.

²⁰ This discussion refers to the 'person' as a unit of analysis for ease of presentation. Similar measures could be constructed for distinct units of analysis such as the household or some population subgroup like youth or women, or an institution such as a school or health clinic.

across the aggregate attainments to identify who is poor, and a poverty index constructed in the same way as for unidimensional poverty. This is depicted in the first column below.

Figure 2: Order of Operations

	1	2	3
	Unidimensional	Multidimensional (Marginal)	Multidimensional (Joint)
Apply Deprivation Cutoffs	n/a	1	1
Aggregate Across Dimensions	1	3	(2)
Identify Who is Poor	2	n/a	3
Aggregate Across People	3	2	4

Bourguignon and Chakravarty (2003) confine the term ‘multidimensional poverty’ to measures that use cutoffs for each dimension or attribute: *‘the issue of multidimensionality of poverty arises because individuals, social observers or policy makers want to define a poverty limit on each individual attribute: income, health, education, etc...’*²¹ Hence the other two methods in Figure 2 might be categorized as multidimensional because they apply deprivation cutoffs to multiple dimensions; however only the last approach necessarily identifies whether each person is multidimensionally poor.

The marginal approach (column 2) uses deprivation cutoffs to identify who is deprived in a particular dimension. It then aggregates information across a population to generate a deprivation *measure* for each dimension. The vector of marginal deprivation measures are then aggregated. Note that people are identified as deprived or non-deprived with respect to each dimension individually; the measure does not identify people as ‘multidimensionally’ poor or non-poor. Nor does it reflect the joint distribution of deprivations. We refer to such indices as marginal (Alkire and Foster 2011b; see also Anand and Sen 1997, Atkinson 2003, Jenkins and Micklewright 2007).

Marginal indices, being insensitive to joint deprivation, do not require all variables to come from the same survey. Also, they can directly aggregate deprivations that pertain to different reference populations – such as children and adults, or rural and urban populations. They can also bring together deprivations that occur with different frequencies or orders of magnitude in the population. For example, a marginal measure might combine an indicator on the percentage of people living in households without access to sanitation (a relatively frequent event let us suppose, with data for all households) with an indicator on maternal mortality per 100,000 women, and an indicator on child malnutrition for children under the age of three. Yet consider a man whose house has sanitation but whose wife perished in childbirth and whose young child is malnourished. Is he poor? Marginal measures do not identify each *person* in the society as multidimensionally poor or non-poor so could not answer this question. They might not fulfill the identification criterion of Sen (1976), which might require that identification clarify whether each person in the population was poor or non-poor.

The third column above provides the general order of aggregation for multidimensional poverty measures that reflect joint distribution. As the empirical examples in the previous section demonstrated,

²¹ 2003:25; emphasis in the original. Other approaches such as the ‘counting’ approaches widely implemented in Europe and the Unsatisfied Basic Needs approaches in Latin America also use this approach (Atkinson 2003, Feres and Mancero 2001).

in many situations, information regarding simultaneous deprivations might contribute independent value to an overall assessment of poverty. Measures reflecting joint distribution first apply a set of deprivation cutoffs in order to identify the dimensions in which that person is deprived. These measures then identify whether each person is multidimensionally poor.

For example, a person is identified as multidimensionally poor by the *union* approach if the person is deprived in *any* dimension (Atkinson 2003; see also Duclos et al. 2006). A person is identified as multidimensionally poor by the *intersection* approach if and only if she or he is deprived in *all* dimensions. In both of these cases, identification is accomplished by considering the vector of deprivations, but aggregation across dimensions is not required. Alternative identification methods – such as our dual cutoff approach²² – may require aggregation across dimensions. The multidimensional poverty measure aggregates across poor people to construct an overall measure of multidimensional poverty for the society.

A key point to note is that the joint approach alone identifies people as being multidimensionally poor on the basis of their joint or simultaneous deprivations. This methodology requires all data to be available for each person, which in many cases means that the data must originate from the same household survey. This methodology also requires a common unit of analysis. If the unit of analysis is the person, then each person may be identified as poor based on their own direct deprivations. But the unit of analysis might also be the household, or youth aged 15–24, or it might be a school or health clinic.

The distinctions between these broad approaches to multidimensional poverty measurement are vital and are often overlooked, creating considerable confusion. The remainder of this paper focuses on multidimensional measures that reflect the joint distribution of disadvantage, in order to probe more completely their characteristics and value-added.

The relevance of understanding interconnections among multiple deprivations was highlighted in the 2009 Report of the Commission on the Measurement of Economic Performance and Social Progress, which argues that: ‘Some of the most important policy questions involved relate to how developments in one area (e.g. education) affect developments in others (e.g. health status, political voice and social connections), and how developments in all fields are related to those in income.’ The report also highlights the particular relevance of joint distribution when studying disadvantage: ‘For example, the loss of quality of life due to being both poor and sick far exceeds the sum of the two separate effects, implying that governments may need to target their interventions more specifically at those who cumulate these disadvantages’ (Stiglitz et al. 2009: p 55). The conclusion affects both survey design as well as the development of summary measures. ‘Developing measures of these cumulative effects requires information on the “joint distribution” of the most salient features of quality of life across everyone in a country through dedicated surveys.’²³

The identification and aggregation steps of one multidimensional measure reflecting joint distribution will now be illustrated in the following sections, using our most basic and applicable index M_0 drawn from the class M_a introduced in Alkire and Foster (2007, 2011a).

²² James Foster and I have adopted an intermediary approach, in which a person can be identified as multidimensionally poor if they are poor in some (weighted) sum or ‘count’ of dimensions that can include union and intersection as well as intermediary cutoffs (Alkire and Foster 2007).

²³ Consideration of these issues can be traced to Atkinson and Bourguignon (1982).

One Multidimensional Poverty Measure: the M_0

This section briefly introduces the Alkire-Foster (AF) class of M_α measures that build on the FGT index. We describe our general measurement approach thus:

‘A methodology \mathcal{M} for measuring multidimensional poverty is made up of an identification method and an aggregate measure (Sen 1976). Following Bourguignon and Chakravarty (2003) we represent the former using an *identification function* $\varrho: \mathbf{R}_+^d \times \mathbf{R}_{++}^d \rightarrow \{0,1\}$, which maps from person i ’s achievement vector $y_i \in \mathbf{R}_+^d$ and cutoff vector z in \mathbf{R}_{++}^d to an indicator variable in such a way that $\varrho(y_i; z)=1$ if person i is poor and $\varrho(y_i; z)=0$ if person i is not poor. Applying ϱ to each individual achievement vector in y yields the set $Z \subseteq \{1, \dots, n\}$ of persons who are poor in y given z . The aggregation step then takes ϱ as given and associates with the matrix y and the cutoff vector z an overall level $M(y; z)$ of multidimensional poverty. The resulting functional relationship $M: Y \times \mathbf{R}_{++}^d \rightarrow \mathbf{R}$ is called an *index*, or *measure*, of multidimensional poverty.’ A methodology is then given by $\mathcal{M} = (\varrho, M)$. (Alkire and Foster 2011a: p 477).

Let us consider poverty in d dimensions across a population of n individuals. Let $y = [y_{ij}]$ denote the $n \times d$ matrix of achievements for i persons across j dimensions. The typical entry in the achievement $y_{ij} \geq 0$ represents individual i ’s achievement in dimension j . Each row vector $y_i = (y_{i1}, y_{i2}, \dots, y_{id})$ gives individual i ’s achievements in each dimension, whereas each column vector $y_{.j} = (y_{1j}, y_{2j}, \dots, y_{nj})$ gives the distribution of achievements in dimension j across individuals. To weight the dimensions, define a weighting vector w whose j^{th} element w_j represents the weight that is applied to dimension j . We set $\sum_{j=1}^d w_j = d$, that is, the dimensional weights sum to the total number of dimensions.

The M_0 measurement methodology can be summarized as follows. Let $z_j > 0$ be the deprivation cutoff in dimension j , and z be the vector of deprivation cutoffs. Define a matrix of deprivations $g^0 = [g_{ij}^0]$, whose typical element is defined by $g_{ij}^0 = w_j$ when $y_{ij} < z_j$, and $g_{ij}^0 = 0$ when $y_{ij} \geq z_j$. From the g^0 matrix construct a column vector c of *deprivation intensity*, whose i^{th} entry $c_i = \sum_{j=1}^d g_{ij}^0$ represents the sum of the entries in a given row, and represents the weighted deprivations suffered by person i .

Next, identify who is multidimensionally poor. Select a poverty cutoff k , such that $0 < k \leq d$ and apply it across this column vector c . A person is identified as poor if their weighted deprivation count $c \geq k$. This can be called a *dual cutoff* identification method, because it uses the *deprivation* cutoffs z_j to determine whether a person is deprived or not in each dimension, and the *poverty* cutoff k to determine who is to be considered multidimensionally poor.

Construct a second matrix $g^0(k)$, obtained from g^0 by replacing its i^{th} row g_i^0 with a vector of zeros whenever $c_i < k$. This matrix contains the weighted deprivations of exactly those persons who have been identified as poor and *excludes* deprivations of the non-poor. M_0 is the mean of the matrix $g^0(k)$. That is $M_0 = \mu(g^0(k))$, where μ denotes the arithmetic mean operator.

M_0 can also be expressed as the product of the (multidimensional) headcount ratio (H) and the average deprivation share among the poor (A). H is simply the proportion of people that are poor, or q/n where q is the number of poor people. A is the average of fraction of deprivations poor people experience – $A = \sum_{i=1}^n c_i(k) / dq$ – and reflects the average *intensity* of multidimensional poverty.

M_0 satisfies dimensional monotonicity: if a poor person becomes deprived in an additional dimension, the M_0 will increase. M_0 is also decomposable by population subgroups. Additionally, after identification, M_0 can be broken down by dimension. The intuitiveness of M_0 – that it is the product of headcount (H) and intensity (A) – together with these three properties in particular, enable M_0 to be taken apart in various ways to generate multiple insights, as the next section details.²⁴

If data are cardinal and satisfy additional assumptions, we identify other measures within the M_x family that can be computed to reflect the depth and severity of multidimensional poverty. These replace the binary g^0 matrix with a matrix of normalized gaps, or with normalized gaps raised to a positive power α ; apply the identification function; censor deprivations of the non-poor; and take the mean of the respective matrices to generate other measures.

Unfolding M_0

A relevant feature of the Alkire-Foster (AF) methodology is that M_0 (and other measures in that class) can be unfolded directly into multiple meaningful indices, which clarify the extent and composition of poverty in a coherent way. This ‘dashboard’ of internally consistent measures provides a fuller account of the information summarized in the overall poverty measure. The key partial and intuitive indices for M_0 are the multidimensional poverty headcount (H) and the measure of intensity (A). M_0 can also be broken down by dimension, to generate censored headcounts for each dimension, and dimensional contributions to poverty. M_0 , H and A can be decomposed by population subgroups such as regions or ethnic groups. Intensity (A) can be broken down by levels to explore who is poorest or least poor. These indicators are used alongside the summary measure to provide a depth of understanding and policy insight that is not possible from the overall measure alone. All of these numbers are generated from the censored matrix $g^0(k)$, and are briefly introduced below.

Partial Indices

1. **Headcount (H)** The percentage of people who are identified as multidimensionally poor. In multidimensional as in unidimensional poverty, the headcount is familiar, intuitive and easy to communicate. It can be compared directly with an income poverty headcount, or with the incidence of deprivations in another indicator, and also compared across time.
2. **Intensity (A)** The percentage of weighted dimensions in which the average poor person is deprived. Intensity reflects the extent of simultaneous deprivations poor people experience. Its lower bound is the percentage k/d (the poverty cutoff as a percentage of the total number of dimensions just below the poverty cutoff) and its upper bound is 100 per cent.

Break Down by Dimension (post-identification)

3. **Censored Dimensional Headcounts H_j** , From the censored matrix, the mean of each column generates the percentage of people who are both multidimensionally poor and deprived in each dimension.
4. **Percentage Contribution of each Dimension to Multidimensional Poverty.** The intensity figure A can be broken down by dimension to show the percentage that each

²⁴ M_0 also satisfies other properties: replication invariance, symmetry, poverty focus, deprivation focus, weak monotonicity, non-triviality, normalisation, and weak re-arrangement (Alkire and Foster 2011a). These axioms are joint restrictions on the identification and aggregation methodologies.

(weighted) dimension contributes to poverty. This is similar to the property of factor decomposability (Chakravarty et al. 1998), but uses the censored matrix $g_0(k)$.

Decomposition by Subgroup

5. One can decompose the M_0 , H , and A by population sub-group, to show how each of these varies by region, by ethnicity, by rural and urban areas, or other subgroups for which the sample is representative. These could be used to create poverty maps, for example.

Break Down by Intensity

6. The intensity (A) is constructed as the mean of each person or household's deprivation count c_i (with appropriate sampling weights applied). The average hides inequality in intensities across people. So the intensity can be broken into different bands, to show the percentage of poor people who experience different levels of intensity or to target the poorest of the poor.

Related Indices (from g^0 rather than $g^0(k)$ matrix)

7. **Raw Dimensional Headcounts.** From the raw matrix g_0 the mean of the column vector generates the 'raw' headcounts of persons who are deprived – whether or not they are multidimensionally poor. This can be compared with the censored headcounts to see, for example, which deprivations are most common among the non-poor, or the percentage of deprived persons who are also multidimensionally poor. This can also be useful to prioritise service delivery and to decide between universal versus targeted delivery mechanisms.

II. One Particular Application of M_0 : the MPI 2010

In 2010, the UNDP Human Development Report Office and the Oxford Poverty and Human Development Initiative (OPHI) released an acute Multidimensional Poverty Index (MPI) for 104 developing countries (Alkire and Santos 2010; UNDP 2010); in 2011 the MPI covers 109 countries. This section sets out briefly how the MPI was constructed and provides an overview of the MPI 2010 results.

Parameters

The MPI generates multidimensional poverty measures by analyzing existing publicly available data sources. In particular, the 2010 MPI is based on Demographic and Health Surveys (DHS) for 48 countries, on Multiple Indicator Cluster Surveys (MICS) for 35 countries, and on the World Health Survey (WHS) for 19 countries. Distinct surveys were used for Mexico and urban Argentina. All surveys used were between 2000 and 2008. All questions for each country were drawn from the same household survey for that country.

The MPI implements the first measure in the dual-cutoff approach of Alkire and Foster (2011a) introduced above: M_0 . This methodology was chosen because it can be used for ordinal and categorical data; it builds upon the FGT income poverty measures, is straightforward and intuitive in construction, and satisfies a number of desirable axioms. In terms of policy relevance, the resulting measure can be decomposed by population group and broken down by factor to show the composition of poverty, hence can describe how the extent and composition of multidimensional poverty varies across states or ethnic communities, or across time.

The MPI is constructed using ten indicators covering three dimensions.²⁵ The three dimensions are health, education, and standard of living. The indicators are nutrition (anthropometric measures) and child mortality for health; years of schooling and school attendance for education; and electricity, water, sanitation, cooking fuel, flooring, and asset ownership for living standard. Each dimension is equally weighted at one-third. Each indicator within a dimension is also equally weighted. Thus the health and education indicators are weighted at one-sixth each, and standard of living at one-eighteenth.

Deprivation cutoffs: The MPI first identifies who is deprived in each of the 10 indicators. The indicators, cutoffs and weights are summarized in the figure below. Note at this point that we take the household as the unit of analysis. For standard of living indicators, a person is deprived if their household is deprived in that particular indicator. However for health and education indicators, a person's deprivations depend on the achievements of other household members. We will return to this issue and call for further research on this combination of individual and household level data.

Figure 3: Dimensions, indicators, cutoffs and weights of the MPI

Dimension	Indicator	Deprived if...	Weight
Education	Years of Schooling	No household member has completed 5 years of schooling	16.67%
	School Attendance	At least one school-aged child is not attending school years 1 to 8	16.67%
Health	Child Mortality	A child has died within the household	16.67%
	Nutrition	Any adult or child for whom there is nutritional information is malnourished	16.67%
Standard of Living	Electricity	The household has no electricity	5.56%
	Cooking Fuel	The household cooks on wood, dung or charcoal	5.56%
	Floor	The household's floor is dirt, sand or dung	5.56%
	Sanitation	The household does not have adequate sanitation (according to the MDG guidelines) or it is shared	5.56%
	Water	The household does not have clean drinking water (according to MDG guidelines) or it is more than a 30 minute walk away	5.56%
	Assets	The household does not own more than one of: radio, television, telephone, bicycle, motorbike, or refrigerator; and does not own a car or truck.	5.56%

Adapted from: Alkire and Santos (2010)

²⁵ It is important at this point to draw attention to an important change in terms: in the methodological discussion each entry in the matrix constituted a 'dimension'. In describing the MPI, the terminology changes, and each entry in the matrix is termed an 'indicator'; the term dimension is used in the MPI to reflect conceptual categories ('health') that do not appear in the g^0 matrix directly.

Poverty Cutoff: Once it has been identified who is deprived in each indicator, the next step is to determine who is multidimensionally poor. The second cutoff, called the ‘poverty cutoff’ k , is set across the weighted sum of a person’s deprivations. In the case of the MPI, every person is identified as multidimensionally poor if and only if they are deprived in at least one-third of the weighted indicators.²⁶ That is, a person is poor if they are deprived in any two health or education indicators, in all six standard of living indicators, or in three standard of living and one health or education indicator.

Whenever the poverty cutoff k requires deprivation in more than one indicator there will be people who, despite experiencing some deprivation will not be considered multidimensionally poor, simply because their total weighted deprivations is less than the k poverty cutoff. A person might cook with wood, but have a separate kitchen and ventilation system, so that it does not indicate poverty in their case. An uneducated person may nonetheless be a self-made millionaire blossoming with good health. Furthermore, deprivations may also be caused by inaccuracies in the data themselves or by inappropriate indicators for that context. In some climates and cultures a natural floor may not indicate deprivation, for example. Finally, in some cases the data may be inaccurate or people may voluntarily abstain from some dimension due to personal values: for example they may have a low body mass due to fasting or fashion.

In the AF methodology, those deprivations are censored since they are assumed to correspond to people who are not multidimensionally poor. Their values are replaced by zeros in the $g^0(k)$ matrix (which differs from the g^0 matrix precisely in the censoring of these deprivations). Subsequent MPI analyses are not based on the original raw data (that would appear in a poverty profile or dashboard for example and that it is contained in the g^0 matrix) but rather on the deprivations of multidimensionally poor people. This censoring of any deprivations of non-poor people is a novel step, so is easily overlooked. It influences all subsequent analysis, at times considerably as is mentioned below.

The MPI, as the more general M_0 measure, is the mean of the censored matrix of weighted deprivations. It can equivalently be calculated as the product of the headcount or *incidence* of poverty – the percentage of people who are multidimensionally poor – and the *intensity* or average proportion of weighted deprivations a poor person experiences. For example, if a person is deprived in nutrition, years of schooling, and three standard of living indicators, then the intensity of their poverty is 50 per cent ($1/6 + 1/6 + 3/18$). If – on average – every person in a country is deprived in 50 per cent of the weighted indicators, and 40 per cent of the population are poor in that country, then the MPI for that country is 0.20.

Data Considerations

The indicators which could be compared across the DHS, MICS, and WHS datasets were limited in several ways; indeed data limitations proved to be a binding constraint for the MPI. Alkire and Santos (2010) provide detailed description of the limitations, which include the use of surveys from different years, the fact that not all indicators were present for all countries, that some households did not have eligible populations, and that some subgroups are systematically excluded from the household surveys. These data limitations affect national accuracy as well as cross-country comparability. The MPI values cannot be used to compare the 104 countries’ acute poverty in a definitive way, as they are drawn from different years, vary in the definition of certain variables, and some countries lack indicators. The study does claim to provide a more comprehensive and accurate baseline of acute multidimensional poverty that reflects joint deprivations than is possible using a dashboard of the same indicators, provide an estimate of acute multidimensional poverty in each of the 104 countries using available information

²⁶ When there are ten indicators present, the persons who are identified as poor by $k=3$ are the same persons who would be identified as poor by $k=3.33$.

about three core dimensions of human development, and demonstrate the AF methodology for measuring multidimensional poverty – which can be adapted to national or regional settings having different objectives or more and better data.

The 2010 MPI results thus form a baseline for each country, drawing on the most recent publicly available dataset containing the MPI indicators. When it is updated using comparable surveys and sampling frameworks, changes over time can be measured and analysed.

Illustrative Results²⁷

About 1.7 billion people in the 104 countries covered – 32 per cent of the entire population – are poor according to the MPI.²⁸ As the aim of the MPI is to complement income poverty measures with a direct measure of deprivation, Alkire and Santos (2010) compare the MPI headcount to the income poverty headcounts in those countries that have data for both measures (93 of the 104 countries), and find that it lies between the US \$1.25 and US \$2/day poverty lines. Across these countries, 26 per cent of the population are estimated to live on US \$1.25 a day or less and 49 per cent live on less than US \$2 a day. At the national level, they find a clear overall relationship between income and multidimensional poverty, but considerable differences for particular countries. The MPI captures deprivations in health and educational outcomes and key services such as water, sanitation and electricity – hence the literature would predict some mis-match. The MPI and income values will also differ due to different survey years as well as measurement error and data inaccuracies. Further analysis is required to understand the differences and potential complementarities more fully.

In terms of regional distribution of acute multidimensional poverty, the study finds that 51 per cent of the world's poor as measured by the MPI live in South Asia (844 million people) and 28 per cent in Africa (458 million). In Sub-Saharan Africa 64.5 per cent of people are MPI poor; in South Asia it is 55 per cent. The intensity of poverty – the average number of deprivations experienced by each household – is also greatest in Sub-Saharan Africa and South Asia. Multidimensional poverty in both continents is troubling both in regards to the number of people who are multiply deprived and the intensity of their poverty.

The study compares the headcount (H) and its intensity (A) across the 104 countries and finds a disconcerting relationship: countries with a higher incidence of multidimensional poverty tend to have a higher average intensity. The study shows that decompositions reveal considerable disparity in MPI among population subgroups – a finding that stimulated decompositions of over 50 countries' MPI values, and analyses of the spatial disparities that emerge (Alkire, Roche et al. 2011).

The study also breaks down the MPI by indicators. This is a post-identification decomposition, hence results still exclude the deprivations experienced by those not identified as poor. This decomposition reveals the structure of poverty among the poor. For example, among the Kikuyu ethnic group in Kenya, deprivation in child mortality and malnutrition (both health indicators) contribute most to the poverty. Deprivations in electricity, sanitation and cooking fuel, contribute most to the poverty of the Embu, another ethnic group. Decomposition of poverty in India and Bolivia also reveals interesting differences among ethnic, caste, and religious groups.

²⁷ This section very briefly mentions some results from Alkire and Santos (2010); for a fuller discussion the reader is referred to that paper.

²⁸ In all of these figures 2007 population data is used; it would also be possible to apply the population data from the year in which the survey was conducted in each country.

Is the MPI robust to a range of weights?²⁹ As an initial exploration, Alkire and Santos (2010) estimate the MPI using three additional weighting structures: (i) giving 50 per cent weight to health and 25 per cent weight each to education and standard of living, (ii) giving 50 per cent weight to education and 25 per cent weight each to health and standard of living, and finally (iii) giving 50 per cent weight to standard of living and 25 per cent weight each to health and education. The pairwise country comparisons show that 88 per cent of pairwise comparisons of country rankings are robust for all weighting structures. They also find the Kendal Tau-b correlation coefficients between the MPI rankings and each of the three new methods to be above 0.90. Alkire et al. (2010) estimate the concordance between all four rankings using several methods. The concordance is high (0.975 and above) and as expected the null hypothesis of rank independence across the four rankings is rejected with 99 per cent confidence by the Friedman test. In terms of large changes in ranking, among the 60 countries whose MPI scores range from 0.05 to 0.64, five countries exhibit rank changes of 10 or more places.³⁰ On the basis of this we conclude that the MPI country rankings are quite robust to weights.

Analyses are also underway to explore trends in MPI over time for a number of countries. For example, in Bangladesh, 68 per cent of people were MPI poor in 2004; by 2007 multidimensional poverty had fallen to 58 per cent. Although progress was made in a number of indicators, an improvement in school attendance was the most striking aspect of poverty reduction in Bangladesh. In contrast, Ethiopia reduced poverty by improving nutrition and water, whereas Ghana improved several indicators at once.³¹

Operationality

The MPI is a very elementary international baseline multidimensional poverty measure that is operational. It is deeply constrained by data, but it does implement a recognizable methodology, perform in robustness tests, and invite improvements. In the 2006 Grusky and Kanbur volume, Bourguignon argues that:

the key challenge in the field of poverty analysis is clear. It consists of building a set of instruments, starting with a satisfactory definition of poverty, that would meet part or all of the critiques of the [income poverty] paradigm described above, *while retaining at least part of its 'operationality'*. Current economic analysis of poverty clearly falls short of this objective... The poverty income paradigm is presently often used in situations calling for alternative definitions of poverty, essentially because instruments to handle these definitions are not available. The challenge is to create those instruments, rather than trying to make the initial paradigm artificially fit a different conceptual basis (p 78-79).

Essentially, the most basic claim of the MPI is that it is an operational instrument, whose strengths and limitations have been made quite clear, and which can be developed and strengthened in the future. The remainder of this paper seeks to identify further issues and catalyze research by which to advance.

III. Research Questions and Debates

The above summarizes the MPI and the methodology that underlies it. The MPI is one possible implementation of M_0 ; which itself is one measure within the AF family of M_α measures. The M_α family is itself one possible approach to multidimensional poverty measurement that reflects joint distribution. The MPI has come under scrutiny and criticism; it could be useful to categorize the 'discontents' in

²⁹ The results in this paragraph draw on Alkire, Santos et al. (2010).

³¹ Apablaza et al. 2010.

terms of their generality. Many of the issues raised with respect to the MPI would be shared by any multidimensional poverty measure that reflects the joint deprivations that poor people experience – for example, Bourguignon and Chakravarty (2003), Bossert et al. (2007), or a counting headcount. The latter include *methodological* issues, such as aggregation, weights and cutoffs; *data* issues, such as whether it is possible to obtain sufficiently accurate data on relevant dimensions from one survey; *political* issues, such as updating and manipulation; and *economic* issues, in particular the link between multidimensional poverty measures and welfare economics. Some issues pertain to the AF methodology directly – such as its neutrality with respect to compensation among dimensions, and the focus axiom. Others pertain to the implementation of AF methodology in the Multidimensional Poverty Index (MPI).

This final section discusses four sets of issues: weights, unit of analysis, data, and aggregation into a single index. They are issues in two senses: first, they may be ‘critiques’ that have been offered; and second, they may be questions for future research and innovation. I articulate most issues with respect to the MPI and the M_0 methodology; however many questions would be answered differently by different measurement approaches. The debate thus far has also passed over some issues within these categories that may be of equal or greater importance to those already articulated, so I take the liberty of proposing these as well.

A. Joint Distribution Measures

As was signaled above, poverty measures that reflect the joint distribution of deprivations for one person or household proceed by aggregating [weighted] information on deprivations across all dimensions for each person, identifying multidimensionally poor persons on that basis, and subsequently aggregating across poor people to construct a poverty measure. This section identifies some areas for further work, both on general-purpose methodologies and on how these methodologies are implemented in practice.

Weights

Shortly after the release of the MPI, Ravallion (2010) drew attention, among other things, to questions regarding the robustness of the MPI to a plausible range of weights as well as to the space in which weights were articulated.³² Recall that the MPI sets weights between the incidence of deprivations, with health, education and standard of living being equally weighted, and the weights entering in a linear form.

Amartya Sen, among others, sees the need to set weights in multidimensional measures as a strength not an embarrassment: ‘There is indeed great merit... in having public discussions on the kind of weights that may be used’ (1997). After all, any national budget implicitly sets weights on many dimensions of welfare, often with little debate. The weights on the MPI are explicit: equal weights on each dimension, and on each indicator within a dimension. Yet given the legitimate diversity of human values, Sen also argues that it may not be necessary to agree on a precise set of weights: ideally, measures would be developed that are robust to a range of weights. However it is pertinent to ask why weights are required, and in particular to note that in the absence of weights, in many situations it will not be possible to identify a plausible group of people who are multidimensionally poor. This section outlines the reason weights are needed – both methodologically and also empirically – and summarizes the MPI robustness results, then identifies issues for research.

³² It will be interesting and fruitful to relate this discussion to the distinct but related discussion of the ‘indexing problem’. See Arneson 1990, Arrow 1973, Fleurbaey 2007, Rawls 1971, Sen 1991a and references therein.

Weights can enter at the identification and or aggregation steps; in the AF methodology they often enter in both steps. Observe that if either the union or the intersection identification approaches are used, *no particular weights on dimensions are required in order to identify who is poor*. Recall that the *union* approach identifies someone who is deprived in any dimension as poor. The *intersection* approach requires a person to be deprived in all dimensions in order to be identified as multidimensionally poor. The appeal of union or intersection approaches is simplicity: neither require specific weights to be set across the dimensions. Note however that both do impose some non-zero weight on each dimension. Indeed the selection of dimensions and cutoffs plays a very significant role in shaping the results, but weights are not required. Why then does the AF methodology introduce a different approach?

A key reason to develop an intermediary approach to identification is practical. In empirical applications that have greater than two dimensions, often both union and intersection approaches give quite extreme values. Of course this depends upon the dimensions, indicators, and cutoffs selected in each context. The intersection approach can be rather stringent. For example, if we were to identify who was poor using the intersection approach instead, using the indicators, cutoffs and weights used in the Multidimensional Poverty Index (MPI) launched in 2010 (Alkire and Santos 2010), we find that the average percentage of poor people across 104 countries is 0 per cent (Table 1), and in only one country would more than 2 per cent of people be identified as multidimensionally poor, Burundi (5.1 per cent).

Table 1: Union and Intersection and MPI Identification using the MPI deprivation matrix, 2010

Identification Method	Average Headcount (H) ³³	Number of the 104 countries	
		with H > 90%	with H < 5%
Union	58%	32	0
Intersection	0%	0	103
MPI (k=3)	32%	1	30

On the other hand, if we use the union approach, the average (population-weighted) headcount of multidimensionally poor people would be 58.4 per cent, with a range from 4.6 per cent in Hungary to 99.6 per cent in the Central African Republic. In 32 countries more than 90 per cent of people would be identified as multidimensionally poor by the union approach (Table 1). Furthermore, the variation can be quite wide: the difference between the union and the intersection headcounts of multidimensionally poor people is more than 90 per cent in 32 of the 104 countries. For example, in Ethiopia, 98 per cent of people would be identified as poor according to the union approach, and 0 per cent of persons were identified as poor by the intersection approach. So in some contexts, both approaches to identification are rather unwieldy for policy purposes because they identify no one or almost everyone as poor.

If a poverty cutoff to identify the multidimensionally poor is set at any intermediary level between union and intersection, then weights are required. In the AF methodology, general weights are applied to the 0-1 deprivation vectors, where the sum of the weights is equal to the number of dimensions d . A person is identified as poor depending upon the sum of the weights of the dimensions in which they are deprived. However other approaches could be implemented. The point to note at the moment is that without

³³ Population-weighted average of multidimensional poverty headcount using the 2007 population data, the union approach to identification and MPI indicators/cutoffs/weights across all 104 countries. The simple average headcount, with each country equally weighted, is also 58 per cent.

weights we cannot identify who is multidimensionally poor *except* by using the union or the intersection method. Weights are needed merely to identify multidimensionally poor persons, even if no summary or aggregate measure is constructed. As is widely understood, all aggregate measures that reflect joint distribution will also impose weights at the aggregation stage on measures. If weights are to be set with transparency and without embarrassment, further research is required on three issues:

Standards and Kinds of Robustness to Weights: First additional work on robustness to a range of weights is required. The 2010 MPI values might be thought of as a baseline and are not directly comparable for the data reasons already outlined. This limits the power of studies regarding the robustness of country MPI rankings. Furthermore, standards have not yet been established regarding the kinds of robustness to weights a multidimensional poverty measure *should* satisfy. Also, the AF measures are designed to inform poverty analysis, not just rankings by country or subgroup; and the level of multidimensional poverty, as well as its break-down by indicator, is affected by the weighting structure. So methodologies are required which explore the robustness of different relevant descriptive analyses to a range of plausible weights.

Source of Weights: A second key question is how to generate weights, and what the conceptual as well as practical and empirical considerations are in the choice of method. Approaches to setting weights that have been implemented include participatory consultations, survey questions (on time trade-offs, gambling, socially perceived necessities, and subjective well-being), statistical techniques, expert opinion, axiomatic approaches,³⁴ and, most commonly, normative weights applied by the author.³⁵ Quantitative as well as conceptual studies have implemented, compared and scrutinized a range of approaches to setting weights, for example in health economics and social policy (Dibben et al. 2007); such studies are required for multidimensional poverty measures.

Space of Weights: Third, even if the weights on indicators were brilliantly clear, there could be questions about the *space* in which to set normative weights. In the MPI, the weighting vector w applies to the incidence of deprivation. However one might transform the vector such that the normative weights apply in a different space. For example, if 10 per cent of people are deprived in clean water and 40 per cent are deprived in sanitation, then there might be arguments for an 80-20 or 20-80 weighting structure. Alternatively, the weights might be fixed, if possible, in the space of public expenditure. The appropriate space will depend on the purpose of the measure as well as the accuracy of potential transformations. It is necessary to set out the alternatives, their implications for the MPI and also the decompositions and further analyses, as well as their potential value-added and limitations.

Unit of Analysis

One of the features of multidimensional poverty measures is that they must construct a row of deprivations for the same unit of analysis. However in many cases household survey data contains information about the household as well as diverse individual members of the household. It would be

³⁴ In the case of Mexico, the legal guidelines governing the development of their national multidimensional poverty measure include principles – for example, that economic and social progress had to balance each other, and that the achievement of a certain level in each social dimension should be seen as a human right. James Foster and I then developed an axiomatic approach showing that these principles, together with assumptions regarding the accuracy of data, were sufficient to set the weights across dimensions and to uniquely identify people as multidimensionally poor. Alkire, S and Foster, JE. 2009b.

³⁵ See for example Dibben et al. 2007, and Decanq and Lugo forthcoming.

useful to consider further how to combine individual and household level data when the unit of analysis is the person, and when it is the household.³⁶

There are three kinds of combinations that need to be examined in each circumstance: a) how to combine information that is available for each household member (as years of schooling) and how to address ‘missing values’ in some responses or attribute scores to ineligible respondents; b) how to attribute household level data to individuals (taking into account literature and empirical studies on equivalence scales for income and on intra-household inequalities in distribution), and c) when and how it is justified to use a variable from a single respondent or from a subset of household members to represent all household members. These may require detailed expertise on each indicator.

The combination methods must also consider biases due to differently sized households, as well as households with different compositions. In the 2010 MPI, larger households have a greater probability of being deprived in the health and school attendance indicators, and less probability of being deprived in ‘years of schooling’ and at least the ‘asset’ indicator among the standard of living indicators. The overall effect is not clear. Household composition – the age and gender of household members, as well as their relationships to one another – also varies. A household of male migrant workers will have a relatively low probability of being deprived in nutrition (most surveys lack male malnutrition data), as well as child school attendance and child mortality, whereas a household with a great number of children will have a relatively larger probability.

Studies are needed to enumerate alternative methods of combining the data, what errors may be introduced by different methodologies, and how to check the robustness of results to choices made. Empirical studies also are needed to explore the magnitude of differences introduced by different methodologies and to generate examples of careful and rigorously verified methods of combining individual and household data. Alongside quantitative work, qualitative and ethnographic studies can be used to explore the assumptions underlying different alternatives, and consider which equivalence scales and intra-household aggregation methods are most accurate in a given context.³⁷

Data

The data restrictions on the MPI or any other global measure which requires internationally comparable indicators are considerable, as was detailed earlier. The data constraints at a country level are less binding, but it can be useful to itemize common constraints. Many of these are well recognised. For example, many household surveys omit institutionalized populations such as the imprisoned, the homeless, and the hospitalized; further, certain surveys exclude key groups such as the elderly or a gender group. The sampling frame, periodicity, and quality of household surveys are also regularly criticized. Multidimensional measures raise a distinctive set of questions in addition to these for two reasons.

Data on each variable must be available for the same person. If a multidimensional poverty measure follows Sen’s approach, and identifies *who* is multidimensionally poor first, then information on joint deprivations is required. Surveys reflecting joint distribution were advocated by the 2009 Stiglitz, Sen Fitoussi Commission on the Measurement of Economic Performance and Social Progress. The need for data on different dimensions for the same person or family is a fundamental issue for this class of measures.

³⁶ Of course in other feasible applications of AF measures, the unit of analysis might be an institution (school, community health clinic), a community (datazone), a business or cooperative, or even a state or country (for governance indicators). However in the case of multidimensional poverty the unit is likely to be a person or household.

³⁷ For example, identifying a household as non-deprived if any member has 5 years of schooling, as the MPI, presumes that education is shared across household members; in some cultural contexts or in some kinds of households, that assumption may not be accurate.

In developing countries all questions may need to come from the same survey (or else be generated for the same household through matching or mapping³⁸). Increasingly, multi-topic household surveys have specialized to explore multidimensional health *or* the quality of education *or* empowerment *or* water management, etc. Such surveys treat one or a few topics in some depth precisely because no single indicator has been identified as a sufficient single proxy for that dimension. A legitimate concern is whether it is possible to construct brief modules on each dimension such that the data generated are sufficiently accurate.³⁹ This requires the input of professionals from different disciplines and areas of expertise. It also requires participatory and qualitative work to explore the accuracy of indicators and measures after implementation.

Data must be accurate at the individual level. Second, because the data are aggregated first across each person or household, each variable must reflect deprivation at the level of the person or household itself – not merely when averaged. This affects survey design. For example, a question on ‘morbidity within the past two weeks’ may provide useful data on average, but is unlikely to be a good indicator of the respondent’s general health status. Related issues arise for indicators of maternal mortality where incidence is very rare. Also, the component indicators of an index must be scrutinized conceptually to identify whether they are stock or flow variables, whether subjective or objective, and whether they refer to resources, inputs, outputs or outcomes. For example the MPI includes child mortality, which could be a stock variable; it was included in part because no better health variables were available, and in part because empirical studies showed that it does change within relevant time periods. Research on each dimension-area is required to propose the shortest and most high quality questions that reflect different dimensions of poverty accurately for the relevant unit of analysis, type of indicator, and time period of interest.

B. The AF Methodology

The AF methodology has certain desirable properties as sketched above, and certainly M_0 is quite applicable due to its ability to use ordinal variables. But there are also a number of issues or questions that arise within the context of that methodology, of which one is discussed below.

Summary Index

A clear preliminary question is why to measure multidimensional poverty – in the sense of providing a summary – at all? It is already possible to consider a vector of deprivations. A vector of deprivations can be constructed from different data sources; data from the same survey are not required. Also, weights are not required, so a dashboard seems less controversial.

The first part of this question is why to provide an overall summary measure.⁴⁰ One foundational reason is that a summary measure defines each person as multidimensionally poor or non-poor, and so is a poverty measure in the sense of Sen 1976; whereas a dashboard or vector of deprivations may define deprivations one by one for different groups but does not look across dimensions at their joint distribution. Also, an overall measure – for example at the national level – allows for comparisons across time (has poverty gone up), and also across regions (which have higher poverty) using a consistent metric.

³⁸ When all relevant indicators are not present on the same survey, where feasible, surveys could be matched to provide data for the same individual or household from different surveys. Poverty mapping techniques might also be used, when they provide sufficient accuracy at the individual/household level, and are appropriate for policy formulation.

³⁹ For example, Browning 2003.

⁴⁰ See also Alkire and Foster 2011b.

A further question is why to provide a summary measure and alongside it a vector of deprivations (e.g. the censored headcounts). The censored headcounts we provide reflect joint distribution and cannot be generated without the imposition of weights unless union or intersection identification approaches are used. Depending upon the value of the poverty cutoff k and the shape of the distribution, the impact of censoring on single-dimensional headcounts can be quite significant, and noting the discrepancies themselves can be of interest.

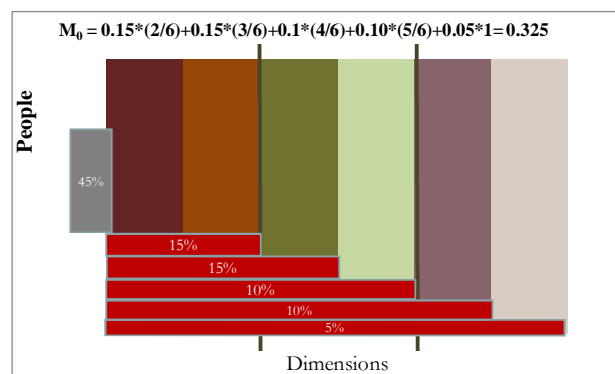
To take a first example from the MPI, Table 2 compares ‘raw’ headcounts – that are directly available from the data – and censored headcounts – which reflect our identification step. In the international MPI, deprivation in adequate sanitation in Ghana was 88 per cent in the raw matrix, but only one-third of these households were MPI poor, hence in the censored matrix, the percentage of people who are MPI poor and live in households lacking sanitation falls to 30 per cent. In Iraq, 29 per cent of households had school aged children not attending school, but only 41 per cent of these households were MPI poor, so the censored headcount of people living in MPI poor households where a child is not attending school is 12 per cent. And in India, 48 per cent of people lived in households with at least one malnourished member, but only 80 per cent of these people were MPI poor, so the censored matrix shows 38 per cent of people are MPI poor and live in households with a malnourished member. In general, across the MPI indicators, the discrepancy between raw and censored matrices was highest in sanitation and cooking fuel, and lowest among years of education.

Table 2. Selected Country Differences Between Raw and Censored Headcount Ratios (Percentage decrease)

Country	School Attend	Years of Education	Child Mortality	Nutrition	Electricity	Sanitation	Water	Floor	Cooking Fuel	Assets
Ghana	15.3%	7.4%	22.8%	17.1%	44.8%	65.9%	42.3%	34.0%	65.0%	47.9%
Haiti	5.7%	1.4%	9.9%	13.6%	24.6%	36.7%	25.6%	11.4%	41.0%	22.4%
India	7.7%	3.8%	12.4%	19.7%	13.5%	30.8%	24.6%	18.6%	31.0%	22.9%
Iraq	58.8%	37.9%	52.6%	50.9%	54.5%	77.3%	70.9%	48.2%	45.4%	55.1%
Lesotho	7.9%	1.7%	11.4%	7.9%	50.4%	47.4%	31.8%	26.0%	40.0%	42.6%
Niger	0.7%	0.3%	1.9%	1.5%	2.6%	5.2%	0.7%	2.2%	7.1%	2.4%
Somalia	3.3%	1.0%	8.3%	1.7%	7.3%	9.5%	7.1%	3.2%	18.7%	7.1%

As the example above illustrates, it is possible and interesting to compare the raw headcounts in each indicator – which includes all deprivations – and the censored headcounts – which focus on people who are deprived in k or more weighted indicators.

For a second example, we explore further insights from intensity. A distinctive feature of the M_0 measures is the partial index we call intensity. This is constructed, recall, by taking the average proportion of dimensions in which poor people are deprived. For example, in the figure on the right, there are six dimensions and $k = 2$. We can see that 5 per cent of people are deprived in 6/6 dimensions (100 per cent), 10 per cent each in 4/6 and 5/6 dimensions, and 15 per cent each in 2/6 and 3/6 dimensions, so

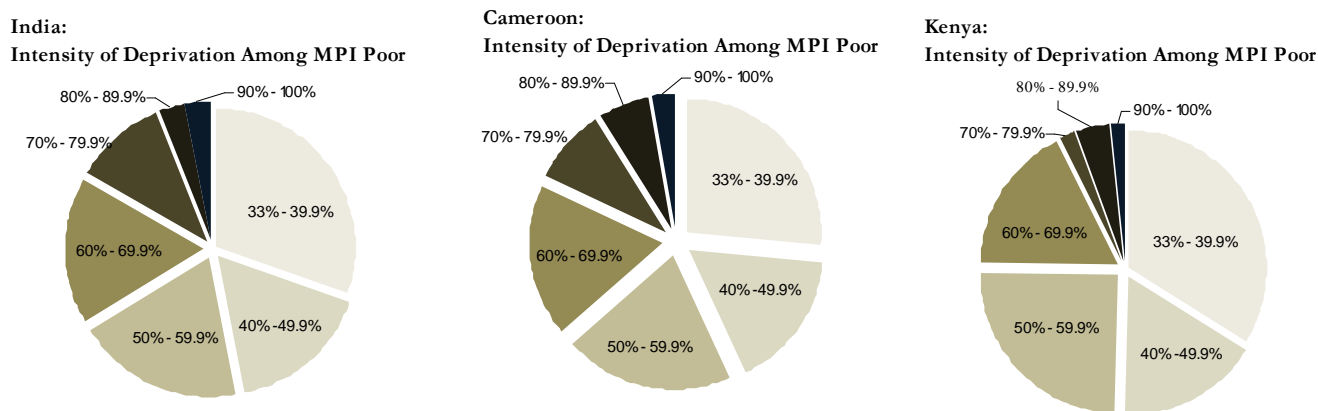


55 per cent of people are deprived in 59 per cent of the dimensions on average, where 59 per cent is the weighted sum of the above proportions. The area of the red bars is equivalent to M_0 , as would be a single rectangle, 55 per cent x 59 per cent.

Applying the intensity to the headcount creates a measure that can be broken down by dimension: the headcount cannot. Also because the poverty cutoff k can vary, one can choose to focus on only a proportion of the population at a time. For example, if we increased k to 4, we can see that 25 per cent of people would be identified as multidimensionally poor, if to 5, 15 per cent, and when $k=6$, 5 per cent of people would be identified as poor. In each successive increment of the value of k , the people considered poor are deprived in more dimensions simultaneously.

Because of its construction, analyses of M_0 for any *given* value of k can still describe the composition of *intensity* among poor people – the average proportion of dimensions in which multidimensionally poor people are deprived in a population. In the MPI, a person must be deprived in one-third of the weighted indicators in order to be identified as poor. We could break up the intensity in different ways. In each country briefing for the MPI, we present a pie diagram depicting the percentage of MPI poor people who are deprived by category of intensity. That is, the darkest slice shows the percentage of MPI poor people who are deprived in 90-100 per cent of the dimensions (e.g. $90 \leq c_i/d \leq 100$). The next lightest represent the percentage of people deprived in 80-89 per cent, and so on down to 30-39 per cent.⁴¹ As is visually evident the configuration varies between countries. Consider for example India, Cameroon, and Kenya, which are adjacent in the MPI rankings. The average intensity of MPI poverty ranges from 50 to 54.7 per cent. However the *composition* of average intensity varies: in Kenya – which has the highest MPI, the percentage of people who are deprived in 30-50 per cent of dimensions is just under 70 per cent, and about 25 per cent are deprived in 60 per cent or more dimensions. Cameroon has the highest intensity overall, and the highest percentage of people deprived in 60 to 90 per cent of dimensions – about a third. However India, which is quite similar in intensity, has a lower percentage of poor people who are deprived in 30 to 50 per cent of dimensions than Cameroon, and a higher percentage deprived in 50-60 per cent of dimensions.

Figure 4. Breakdown of Intensity of MPI for India, Cameroon and Kenya



⁴¹ As the MPI weights are 0.167 and .056 on different dimensions (with 10 indicators), there are additional values of k at which the intensity actually changes; we showed decile bands for clarity only.

The intensity of deprivations among the MPI poor

India	Cameroon	Kenya
MPI: 0.283	MPI: 0.287	MPI: 0.296
Intensity: 52.7%	Intensity: 53.9%	Intensity: 49.3%

Source: 'Country Briefings' on www.ophi.org.uk

This ability to break apart a multidimensional poverty index by strata of intensity can be useful, for example for targeting. If an institution were able to provide services for 18 per cent of the population, and if the multidimensional poverty measure were appropriately constructed for that country, then one could increase the value of the k cutoff until it identified about 18 per cent of people as poor.⁴² These would be the 18 per cent of people having the highest intensity of poverty. It can also be analysed further to inform policy interventions: in Cameroon, more people are deprived in 80 to 100 per cent of indicators; thus a question for further analysis is who these people are (spatially and in terms of social groups), and whether targeting the poorest poor and providing an integrated spectrum of services might reduce poverty most quickly. Recall that if one brings a high intensity person out of poverty, the reduction in poverty is greater than if one brings a just-poor person out of poverty,⁴³ because in the former case the average intensity, as well as the headcount, would decrease. It could also be possible to combine an analysis of intensity bands with a subgroup decomposition to show commonalities in the structure of deprivation: for example, if 80 per cent of the persons who are deprived in 40-50 per cent are deprived in the *same* two indicators, this suggests different policies than if there was a great dispersion of deprivation combinations. Finally, it is possible and could be useful to undertake these analyses not only for a population as a whole but also for various subgroups – states, ethnic groups, or other subgroups for which the data are representative and the measure is valid.

Hence the summary measure is not a stand-alone number. Once one has created the censored matrix $g^0(k)$, the mean of which is the MPI, it is natural to generate a range of related descriptions from it: comparisons, subgroup decompositions, analysis by indicator, and by intensity. The data source for these various tools is the post-identification censored matrix $g^0(k)$, created using the achievement matrix, the deprivation cutoffs vector, the weighting vector, and the poverty cutoff. In any identification other than the union approach (in which k takes the value of the lowest-weighted indicator), $g^0(k)$ will differ from the deprivation matrix. Analyses based on this matrix are consistent with the summary measure, and serve to unfold its insights. Such analyses can also be compared with the original (raw) data, and with figures from other data sources.

Concluding remarks

This paper has introduced one approach to multidimensional poverty measurement, one particular methodology (AF), one implementation of it (MPI), and four research topics that are either being investigated or are issues for future research. The key strengths of the M_0 methodology are that it is a poverty measure, fulfilling the steps of identification and aggregation that Amartya Sen set out for poverty measures; that it is intuitive and easy to interpret; that it satisfies a set of desirability properties such as subgroup consistency; that it makes explicit the weights set upon dimensions; that it identifies joint deprivations and has multiple ways of presenting joint deprivation through the measurements such

⁴² Alkire and Foster 2009. The degree of precision in adjusting the headcount by increasing k depends upon the weighting structure and the number of variables as well as the distribution of deprivations. Note that the poverty cutoff k is, in the example given, itself a policy tool (see Alkire and Foster 2009, section 8).

⁴³ For example, by identifying exactly the deprivation(s) lacked by those deprived in 30-40% in India and Kenya.

as intensity. Finally, the AF methodology is flexible: the dimensions, cutoffs, and weights can all be chosen to reflect the purpose of the measure and its context; the MPI is only one example out of many possible applications of the underlying methodology.

We used the results from the 2010 104-country Multidimensional Poverty Index which implemented the M_0 methodology to illustrate some analyses that the measure can generate. We noted that MPI analyses differ from analyses using the original data and indicators because the basic matrix used by all MPI-related figures is ‘censored’ to focus *only* on the disadvantages of people who are jointly deprived in 33 per cent (in this case) of dimensions.

The last section briefly introduced an incomplete yet substantive set of research topics, progress in which would take this work to the next stage. These include a set of issues related to many multidimensional measurement approaches – such as work on weights, cutoffs, income, combining individual and household data, policy analysis, linkages to preferences and welfare economics, treatment of ordinal and categorical data, and so on. There are also issues specific to the AF methodology – such as incorporating complementarity and substitutability, and relaxing the focus axiom at identification.

The fundamental question is whether undertaking the further field-building research and collecting missing data is likely to significantly advance various agents’ abilities to reduce the incidence, intensity, depth and duration of human poverty. I have argued that investing further in multidimensional poverty measures has the potential to generate significant advances in understanding and to create useful policy tools. To develop this potential, it is vital to establish and convey good practices for the implementation of multidimensional poverty measures, such that measures are implemented with rigour and transparency in the upcoming phase. The paper also identified a few of the many methodological advances that are required. If the methodologies of multidimensional poverty measurement adequately navigate these challenges, they may be seen not as a threat to economics’ legitimate parsimony, but as an extension of its core strengths.

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