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Multidimensional Poverty Measures from an Information Theory Perspective[1]

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Abstract

This paper proposes to use an information theory approach to the design of multidimensional poverty indices. Traditional monetary approaches to poverty rely on the strong assumption that all relevant attributes of well-being are perfectly substitutable. Based on the idea of the essentiality of some attributes, scholars have recently suggested multidimensional poverty indices where the existence of a trade-off between attributes is relevant only for individuals who are below a poverty threshold in all of them (Bourguignon & Chakravarty 2003, Tsui 2002). The present paper proposes a method which encompasses both approaches and, moreover, it opens the door to an intermediate position which allows, to a certain extent, for substitution of attributes even in the case in which one or more (but not all) dimensions are above the set threshold. An application using individual well-being data from Indonesian households in 2000 is presented in order to compare the results under the different approaches.

1 Introduction

Evaluation of household or individual well being is now widely accepted as a multiattribute exercise. Far less agreement exists on such matters as which attributes to include, how such attributes are related or contribute to overall well being, and what criteria to employ for complete –index based– ranking of well-being situations.[1] A useful starting point, both for the believers and non-believers in the multidimensional approach, is to see the traditional univariate assessments in the multiattribute setting: it is as though a weight of one is attached to a single attribute, typically income or consumption, and zero weights are given to all other real and potential factors. Univariate approaches do not avoid but, rather, impose very strong a priori values.

The purpose of this paper is to propose multidimensional poverty measures adopting an information theory perspective. A brief description of the information theory (IT) approach is as follows: the distance between two distribution functions can be efficiently measured using the relative entropy measure, as proposed by Shannon (1948). From the relative entropy measure one can derive an individual level aggregate function whose distribution is the least divergent from the distribution of the constituent welfare attributes. The second step is to define the set of poor and select an appropriate method for aggregation across individuals. Based on this approach, this paper presents two alternative families of measures depending on the underlying definition of the poverty line. The resulting measures encompass the indices proposed by others (Bourguignon & Chakravarty 2003, Tsui 2002), while opening the way to more general measures of poverty, including more complex moments than the averaging function ($\frac{1}{n}\sum_{i=1}^{n}$). Additionally, the measures proposed in the paper allow for substitution from an attribute that exceeds its poverty level to another that

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[1]Some degree of robustness may be sought through weak uniform rankings of states, as by stochastic dominance and related criteria (Duclos, Sahn & Younger 2006).

falls short of it, a concept that we will refer to as the *weak poverty focus* property. We think that weak poverty focus is a very attractive feature of the multidimensional approach which deserves to be examined in many real life situations.[2]

The paper is structured as follows: Section 2 introduces the notation that will be used throughout the article. Section 3 presents a brief description of information theory and its application to the area of inequality and multidimensional well-being indices. Section 4 uses the IT approach to derive two alternative families of multidimensional poverty indices. Measures presented elsewhere are shown to be included in the proposed set. Section 5 illustrates the use of these measures utilizing household data from Indonesia in 2000 and makes remarks concerning implementation and practical issues. One issue concerns the identification of truly distinct attributes, highlighting the statistical role played by any chosen index and its ability to utilize information in different dimensions.[3] Section 6 concludes.

2   Notation

Let $N = \{1, ..., n\}$ be the set of individuals i and $Q = \{1, ..., q\}$ the set of dimensions j. The population is of size $n \geq 2$. A distribution matrix X is an $n \times q$ strictly positive real-valued matrix whose element x_i^j represents the attainments of individual i in dimension j. When $q = 1$ matrix X is a one-dimensional vector. The domain of the distribution matrices is denoted $\mathcal{D}$ and is restricted to the space of strictly positive real-valued matrices of size $n \times q$. Consider the $1 \times q$ vector of poverty lines $z = \{z^1, z^2, ..., z^q\} \in Z$, where z^j is the poverty line for attribute j. Define a *multidimensional poverty index* as a mapping from the matrix X and the vector z to a real number in $\mathbb{R}$.

$$P(X, z) = G[f(x_i^1, ..., x_i^q); z] : \mathcal{D} \to \mathbb{R}^+ \tag{1}$$

3   Information Theory and Welfare

The issue of the aggregation of attributes for welfare assessments using IT has been addressed in the context of the measurement of unidimensional and multidimensional inequality (Theil 1967, Maasoumi 1986). This section describes the main principles of information

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[2]Other approaches to derive multidimensional poverty measures of poverty include fuzzy set theory (Cerioli & Zani 1990, Cheli & Lemmi 1995, Vero & Werquin 1997) and efficiency analysis (Deutsch & Silber 2005). For a comparative study of these approaches, see Deutsch and Silber (2005)

[3]Since we consider only three dimensions – income, education and health – we do not deal with the clustering techniques that also use a consistent IT method for dimension reduction based on the similarity of the attribute distributions (Hirschberg, Maasoumi & Slottje 1991). We merely report several robust measures of dependence between our chosen attributes to shed light on their relations.

theory, its uses in the analysis of inequality, and the results of the multidimensional inequality literature. The section concludes by setting out the challenges that arises when poverty, rather than inequality, is the object of study. Multidimensional poverty measures derived using the IT approach are presented in the next section.

Information theory was developed in the 1940s by Claude Shannon as a discipline within the mathematical theory of communication. The goal was to determine how much data can be transmitted through a channel without significant losses or errors (Shannon 1948). The measure of data (information) transmitted is known as *entropy*, in reference to the concept used in thermodynamics. Shannon proposes to measure the information using the expected information content or entropy index:

$$H(X) = -\sum_{i=1}^{n} p(x)log_i p(x) = \sum_{i=1}^{n} p(x)log\frac{1}{p(x)}, \tag{2}$$

where X is a random variable with a probability function $p(x) = Pr\{X = x\}$. The more likely the event is –the higher the $p(x)$– the smaller the reduction in entropy caused by the event occurring. The entropy index is a measure of the average uncertainty of the random variable, in other words, a measure of the amount of information required on average to describe the random variable (Cover & Thomas 2003). Values of $H(X)$ lie between 0 and $logN$, where minimum entropy is achieved when the probability of one event i is 1 and $p(x_j) = 0, \forall j \neq i$, and maximum entropy is reached when all events are equally likely. $H(X)$ is a concave function of $p(x)$ and satisfies the properties of continuity, normalization, and grouping – akin to decomposability (Shannon 1948).[4]

When comparing two probability distributions $p(x)$ and $q(x)$, the *relative* entropy measure is used to measure the distance between them. The relative entropy measure, also referred to as *Kullback-Leibler divergence*, is defined as

$$D(p\|q) = \sum_{i=1}^{n} p(x)log\frac{p(x)}{q(x)}. \tag{3}$$

The relative entropy measure $D(p\|q)$ gives the minimum additional information that $q(x)$ provides over $p(x)$. It is shown that $D(p\|q) \geq 0$, $D(p\|q) = 0$ if and only if $p(x) = q(x)$ and, $D(p\|q)$ is convex in the pair (p, q) (Shannon 1948).

Henri Theil proposed to use the relative entropy measure to construct indices of economic inequality (Theil 1967). Income can be seen as a random variable with each person having a probability $p(x_i) = \frac{s_i x_i}{\sum_i x_i}$ (income share) of receiving income x_i where s_i is the proportion of people with income x_i. The income distribution is compared to an 'ideal'

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[4] Shannon used an axiomatic derivation for $H(X)$ where these three axioms were invoked.

distribution where everyone receives the same income, with probability $q(x) = \frac{1}{n}$. The first Theil Index is defined as

$$T_1 = \frac{1}{n} \sum_{i=1}^{N} \frac{x_i}{\overline{x}} \ln \left(\frac{x_i}{\overline{x}} \right), \tag{4}$$

where $\overline{x} = \frac{1}{n} \sum_{i=1}^{n} x_i$ is the mean income, and n is the number of people. If everyone has the same income ($x_i = \overline{x}$), the case of 'perfect equality', then the index $T = 0$ since $ln\overline{x}/\overline{x} = \ln 1 = 0$. At the other extreme, if one person has all the income ($x_i = n\overline{x}$) while the rest have nothing ($x_j = 0, \forall j \neq i$) - 'perfect inequality' - then the index achieves its maximum level $T = \ln n$.[5]

An alternative measure of inequality based on entropy can be defined in the analogous way interchanging the probability so that the population share is $p(x)$ and incomes share is $q(x)$.[6] The second Theil index is hence

$$T_2 = \frac{1}{n} \sum_{i=1}^{n} \ln \left(\frac{\overline{x}}{x_i} \right). \tag{5}$$

T_2 is equal to zero in the perfect equality scenario, and positive otherwise. Both measures proposed by Theil satisfy a set of desirable properties for measuring inequality (Anand 1983). In particular, they are ratio-scale invariant (if all incomes change proportionally the measurement does not change, hence the measure is independent of the units of measurement of income), they satisfy the principle of transfer (a transfer from a richer to a poorer person, without reversing the ranking, decreases inequality), and are decomposable by population subgroups (into between- and within-group inequality components).[7] It is this last property that, Theil argues, makes his measures preferable to other well-known inequality measures such as the Gini Coefficient, where its decomposition is not perfect (Theil 1967).[8]

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[5]With a little manipulation it can be shown that T_1 is directly related to the entropy index $H(X)$ as follows:
$$T_1 = \ln n - H(X)$$
where $\ln n$ is the maximum level of inequality possible, and $H(x)$ can be interpreted as a measure of equality.

[6]In Theil's own words the second index is the "the expected information content of the indirect message which transforms the population shares as prior probabilities into the income shares as posterior probabilities" (Theil 1967, p. 125).

[7]On the decomposability property see, for instance, Foster and Shneyerov (1999)

[8]On the other hand, as Sen pointed out "it is an arbitrary formula, and the average of the logarithms of the reciprocals of income shares weighted by income shares is not a measure that is exactly overflowing with intuitive sense." (Sen & Foster 1997, p. 36). However, if one were to think of the GE index in terms of its meaning -rather than its mathematical definition- as the measure that represents the minimum possible distance between an 'ideal' distribution (perfectly equal) and the one under study, it can then be understood as a lower bound on inequality. On the decomposition of the Gini coefficient see Mussard *et al.* (2003), Dagum (1997), Shorrocks (1980), and Pyatt (1976).

The two inequality measures proposed by Theil can be seen as belonging to a more general evaluation function of income shares. Cowell extended the family of IT-based inequality indices to the *Generalized Entropy measures* (Cowell 1977, Cowell & Kuga 1981a, Cowell & Kuga 1981b).

$$GE\alpha = \frac{1}{\alpha(1-\alpha)} \frac{1}{n} \sum_{i=1}^{n} \left[1 - \left(\frac{x_i}{\bar{x}} \right)^{\alpha} \right], \tag{6}$$

where $\alpha \in (-\infty, +\infty)$ is a parameter capturing the sensitivity of a particular GE index to different parts of the distribution. The smaller the α, the higher the measure's sensitivity to the lower tail, that is, the poor. For $\alpha = 1$, $GE = T_1$ and for $\alpha = 0, GE = T_2$. The GE measures are also ordinally equivalent to the inequality measure proposed by Atkinson (1970).

In the context of the multidimensional measurement of inequality, Maasoumi uses information theory both in the aggregation across attributes –to obtain a well-being index for each individual– and in the aggregation across individuals to obtain the inequality measure (Maasoumi 1986). In the first step, a function $S_i(.)$ ($f(.)$ in (1) above) would summarize the information on all attributes for each individual in an efficient manner. Every attribute j has a distribution $x^j = (x_1^j, x_2^j, \ldots, x_n^j)$ containing all the information about the variable that can be accessed and inferred objectively. The aim is to select a functional form for the aggregator function S_i that would have a distribution as close as possible to the distributions of its constituent members, x^js. The 'optimal' function $S_i(.)$ can be achieved by solving an information theory inverse problem, based on distributional distances, where the divergences represent the difference between their entropies – their *relative entropy.*

Let S_i denote the summary or aggregate function for individual i, based on her q attributes $(x_i^1, x_i^2, \ldots, x_i^q)$. The distance function $D_\beta(.)$ is the weighted average of the relative entropy divergences between $(S_1, S_2, \ldots, S_n)$ and each $x^j = (x_1^j, x_2^j, \ldots, x_n^q)$, defined as follows:

$$D_\beta(S\|X; w) = \sum_{j=1}^{q} w_j \frac{1}{\beta(1-\beta)} \sum_{i=1}^{n} S_i \left[1 - \left(\frac{S_i}{x_i^j} \right)^{\beta} \right], \tag{7}$$

where w_j is the weight attached to the generalized entropy distance from each attribute. Minimizing $D_\beta(S\|X; w)$ with respect to S_i subject to $\sum_i S_i = 1$ produces the following optimal aggregation functions:[9]

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[9]The solution functions would be the same if we considered normalized attributes, such as x_i^j/μ^j where $\mu^j = E(x^j)$, or $x_i^j / \sum_{i=1}^{n} x_i^j$ which are the attribute shares (Maasoumi 1986).

$$S_i \quad \propto \quad \Big(\sum_j^q w_j (x_i^j)^\beta \Big)^{1/\beta} \text{ when } \beta \neq 0, \tag{8}$$

$$S_i \quad \propto \quad \prod_j^q (x_i^j)^{w_j} \text{ when } \beta = 0, \tag{9}$$

where β is related to the trade-off between attributes in their contribution to the aggregator function.[10]

In the second stage, Maasoumi proposes to use a GE measure on the resulting well-being indices:

$$I^M = \frac{1}{\alpha(1-\alpha)} \frac{1}{n} \sum_{i=1}^N \Big[1 - \big(\frac{S_i}{\overline{S_i}} \big)^\alpha \Big]. \tag{10}$$

This measure has been criticized on theoretical grounds, most importantly for its failure to satisfy the multidimensional version of the Pigou-Dalton principle of transfer known as *Uniform Majorization* (Dardanoni 1996, Bourguignon 1999, Decancq, Decoster & Schokkaert 2007). As shown elsewhere (Lugo 2007) the key source of weakness is the fact that the distribution of achievements is normalized by the average level of well-being, rather than by the well-being of the average levels of attributes.[11] In the next section we propose to use a similar approach to Maasoumi's to derive a multidimensional poverty index that is free from this criticism.

Constructing a poverty measure also presents additional challenges. Notably, we need to (a) define a poverty threshold that accounts for the multidimensionality of deprivations, and (b) choose the degree to which the measure complies with specific distributional properties.

4   Measuring Poverty

4.1   Traditional approach

Poverty analysis is concerned with the lower part of the distribution of well-being. The measurement of poverty involves three steps: selecting an appropriate indicator to represent individuals' well-being; choosing a poverty line which identifies the lower part of the

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[10]Note that the standard consumer theory requirement of convexity of indifference curves in attribute space will demand that β is less than or equal to one. As we show in the next section, in the context of poverty indices, one might consider the relative deprivation functions, $q_i^j = 1 - \frac{x_i^j}{z^j}$, in place of x_i^j. In this case, the convexity requirement is the opposite $\beta \geq 1$. See the next section for this alternative.

[11]This index has been applied in various empirical studies of multidimensional inequality (Justino, Litchfield & Nimi 2004).

distribution; and finally, selecting some function of the level of well-being of 'poor' individuals relative to the poverty line (Sen 1976).

The monetary approach to poverty utilizes income or consumption expenditure (Y_i) as the indicator of well-being, identifies the poor as those with insufficient income to attain minimum basic needs (z), and aggregates their shortfall from the poverty line into a poverty index (Deaton 1997). The poverty headcount, poverty gap, and severity of poverty are the most common indices used in the literature, all belonging to the family of Foster-Greer-Thorbecke (FGT) poverty measures (Foster *et al.*, 1984).

In the monetary approach to poverty, where individual i consumes q goods x_i^j, $j = 1$, $2, \ldots, q$,

1. The well-being indicator is $Y_i = \sum_{j=1}^q r_j x_i^j$, where r_j is the market price for good j.

2. The poverty line is determined as $z = \sum_{j=1}^{q^0} r_j x_i^{j0}$, where x_{ij0} belongs to the set of basic needs and $q^0 \in Q$.

3. The individual poverty function is

$$p_i = \max \left[\frac{z - Y_i}{z}; 0 \right], \tag{11}$$

and the aggregator function is the FGT index

$$P(X; z) = FGT_\alpha \quad = \quad \frac{1}{n} \sum_{i=1}^n (p_i)^\alpha \tag{12}$$

$$= \quad \frac{1}{n} \sum_{i=1}^n \left[\max \left\{ \frac{z - Y_i}{z}; 0 \right\} \right]^\alpha, \tag{13}$$

where α is a parameter indicating the sensitivity of the index to the distribution among the poor - the higher its value, the more sensitive the index is to the poorest persons in the economy. For $\alpha = 0$, FGT is the headcount, for $\alpha = 1$, it is the poverty gap, and for $\alpha = 2$, it represents the severity of poverty.

For decades now, many scholars have favoured a multidimensional perspective to poverty where 'human deprivation is visualized not through income as an intermediary of basic needs but in terms of shortfalls from the minimum levels of basic needs themselves' (Tsui 2002, p. 70). The latter voices a common argument against the traditional income method on two main grounds. Firstly, it questions the assumption of the *existence* of known prices and markets for all relevant attributes determining deprivation. Moreover, even when market prices exist, it challenges the view that these are somehow 'right' in themselves. Instead, market prices can be thought of as no less arbitrary than any other weights chosen (Tsui 2002). In truth, explicitly chosen weights have the advantage that they allow for a clear understanding of the effects of the weighting scheme.

Secondly, the monetary approach relies on the assumption of perfect substitutability between attributes, probably too strong an assumption to make if the previous critique is accepted. In effect, for poverty or deprivation analysis, some authors argue that each attribute is to be considered *essential* in the sense that a person who does not reach the minimum threshold in any one dimension should be considered poor, irrespective of his attainment in all other attributes (Tsui 2002, Bourguignon & Chakravarty 2003). On this view, the existence of a substitution or trade-off between two attributes is relevant only for persons who are below the minimum level in all dimensions. The idea of essentiality of attributes is consistent with the *union* approach of poverty (Atkinson 2003, Duclos et al. 2006) and is expressed through the **strong poverty focus property**, defined as follows (Bourguignon & Chakravarty 2003):

Strong poverty focus (SF). For any $n \in N, (X, Y) \in \mathcal{D}, z \in Z, j \in \{1, 2, ..., q\}$, if for
(i) any i such that $x_i^j \geq z^j, y_i^j = x_i^j + \gamma$, where $\gamma > 0$,
(ii) $y_t^j = x_t^j$ for all $t \neq i$, and
(iii) $y_i^s = x_i^s$ for all $s \neq j$ and for all i, then $P(Y; z) = P(X; z)$.

In other words, if any attribute x_i^j changes so that $x_i^j \geq z^j$ before and after the change, then $P(X; z)$ does not change. This property leads us to ignore not only *individuals above* the poverty threshold in all relevant attributes, but also *attributes above* the poverty threshold for individuals who do not achieve the threshold in other attributes. Figure 4-1 illustrates the case of two attributes, health and education. Their respective poverty thresholds are z^h and z^e. The shaded area represents the set of people considered poor, and the lines the isopoverty contours. The monetary approach (on the left) assumes perfect substitutability between both attributes, so that the isopoverty contours are straight lines and the poverty set is the shaded triangle. Instead, the union approach (on the right) includes not only these individuals but also those that fall short in only one of the dimension, even if they possess a large amount of the second attribute. Note that the isopoverty curves are strictly convex up to the level of the threshold, while beyond z^j, the curve is either vertical or horizontal. In other words, beyond the threshold substitution between the abundant good and the scarce good is not permitted.[12]

In the next section we present a family of measures that incorporate the idea of the union approach, while also suggesting the use of an *intermediate* position which allows for substitution between attributes –up to a certain extent– that are above and below their corresponding thresholds (see figure 2). We reflect this intermediate view using the **weak version of the poverty focus property**, formally defined as follows:

Weak poverty focus (WF). For any $n \in N, (X, Y) \in \mathcal{D}, z \in Z$, if for some i $x_i^k \geq z_k$ for

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[12]The *Intersection Approach* to poverty considers only the set of individual who are poor in *all* relevant dimensions, that is, those within the rectangle $Oz_e P z_h$.
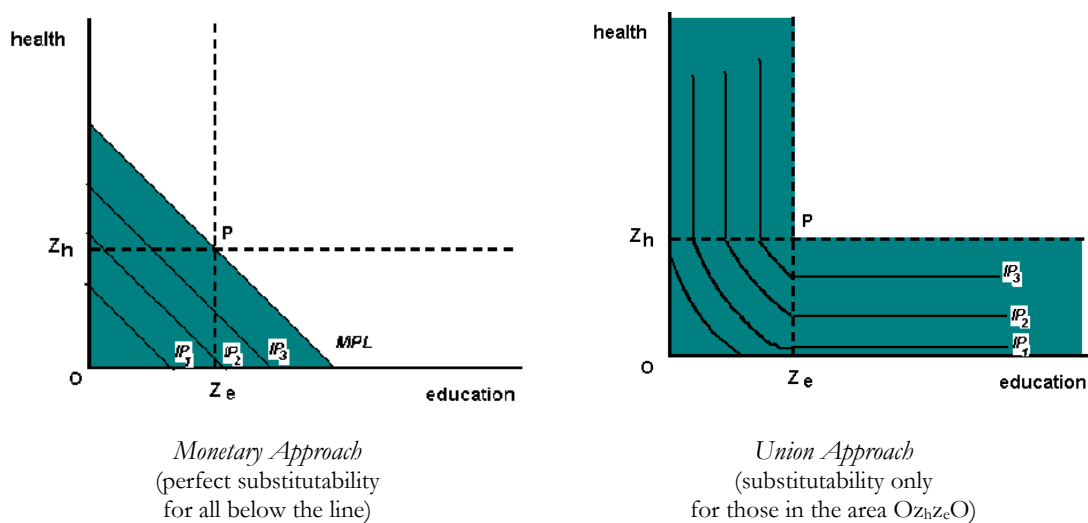
Figure 1: Monetary and Union Approaches to Poverty
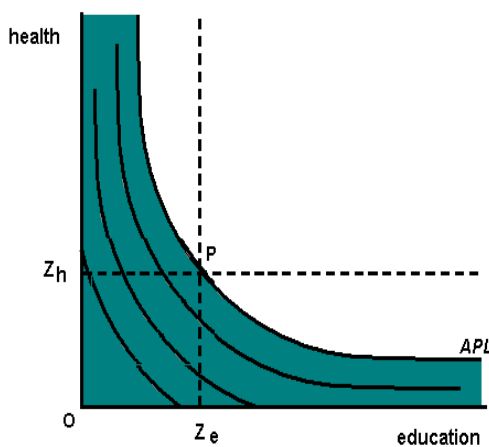
Figure 2: Intermediate Approach

all k and for

(i) any $j \in \{1, 2, ..., q\}$, , $y_i^j = x_i^j + \gamma$, where $\gamma > 0$,

(ii) $y_t^j = x_t^j$ for all $t \neq i$, and

(iii) $y_r^s = x_r^s$ for all $r \neq i$ and for all s, then $P(Y; z) = P(X; z)$.

WF makes the poverty index independent of the attribute levels of non-poor individuals only. In other words, some interplay between attributes above and below the poverty threshold is allowed (Bourguignon and Chakravarty, 2003).

4.2   IT Multidimensional Indices of Poverty

This section presents alternative approaches to the derivation of multidimensional poverty indices using instruments from information theory. Each route employs three steps as described above, and makes use of the IT perspective in defining both the well-being index and the poverty line. Two decisions will have to be made sequentially, each of them associated with two possible outcomes. First, we need to opt between a poverty line that represents the 'shortfall of well-being' or one based on the 'well-being of the shortfalls'. We refer to these two procedures as *aggregate poverty line* and *component poverty line* approaches, respectively. Second, we should choose between the strong poverty focus property or its weaker version. Thus we present four alternative measures of multidimensional poverty, though one will be dropped for having an undesirable property.

4.2.1   Aggregate Poverty Line Approach

From an IT perspective, the most efficient way to represent the information of the distribution of the attributes is through the composite index $S(x_i^j)$ derived in section 3, which minimizes the entropy divergence $D(S\|z)$ with respect to $S(.)$.

1. The well-being indicator is

$$S_i \;\; = \;\; \left(\sum_j^q w_j (x_i^j)^\beta \right)^{1/\beta} \qquad \text{for } \beta \neq 0, \tag{14}$$

$$S_i \;\; = \;\; \prod_j^q (x_i^j)^{w_j} \qquad \text{for } \beta = 0. \tag{15}$$

2. The poverty line is the *aggregate poverty line* S_z, consistent with the IT aggregator functions S_i derived above.

$$S_z \;\; = \;\; \left(\sum_j^q w_j (z^j)^\beta \right)^{1/\beta} \qquad \text{for } \beta \neq 0, \tag{16}$$

$$S_z \;\; = \;\; \prod_j^q (z^j)^{w_j} \qquad \text{for } \beta = 0. \tag{17}$$

Each attribute's poverty line, z^j, plays a role in defining a multi-attribute poverty line, S_z, which incorporates the same weights for, and relationship between, the attributes as considered for each individual. All of the axioms which support FGT are applied to individual summary functions of well being, S_i.

3. The individual multi-attribute poverty function is

$$p_i = \max\left\{\frac{S_z - S_i}{S_z}; 0\right\}, \tag{18}$$

and the aggregator function (across individuals) is

$$
\begin{aligned}
P(S;z) &= \frac{1}{n}\sum_{i=1}^{n} p_i^{\alpha} \\
&= \frac{1}{n}\sum_{i=1}^{n}\left[\max\left\{\frac{S_z - S_i}{S_z}; 0\right\}\right]^{\alpha},
\end{aligned}
\tag{19}
$$

which is the αth moment FGT poverty index based on the distribution of $S = (S_1, S_2, \ldots, S_n)$.

This leads to a general formulation which allows for the possibility of some substitution between attributes above and below the poverty thresholds provided the individual is poor in at least one dimension, which is consistent with the **weak poverty focus property**. The Aggregate Poverty Line (APL) multidimensional poverty measures is thus defined as

$$
P(APL_{weak}) = \frac{1}{n}\sum_{i=1}^{n}\left[\max\left\{\frac{\left(\sum_j^q w_j(z^j)^{\beta}\right)^{1/\beta} - \left(\sum_j^q w_j(x_i^j)^{\beta}\right)^{1/\beta}}{\left(\sum_j^q w_j(z^j)^{\beta}\right)^{1/\beta}}; 0\right\}\right]^{\alpha}
$$
$$
\text{for } \beta \neq 0, \tag{20}
$$

$$
P(APL_{weak}) = \frac{1}{n}\sum_{i=1}^{n}\left[\max\left\{\frac{\prod_j^q (z^j)^{w_j} - \prod_j^q (x_i^j)^{w_j}}{\prod_j^q (z^j)^{w_j}}; 0\right\}\right]^{\alpha} \qquad \text{for } \beta = 0. \tag{21}
$$

When $\beta = 1$ we are back with a weighted sum of consumption. In this case, if the attributes chosen are those included in the basic needs basket, and weights are market prices, the poverty index is consistent with the monetary approach to poverty. In other words, the proposed measure includes the standard unidimensional poverty index as a special case.

If one prefers to highlight the essentiality of each component and entertains a **strong** version of the focus axiom (the *union approach*), one has only to replace x_i^j by the expression $min\{x_i^j, z^j\}$.[13]

Thus the well-being index used is

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[13] A similar but somewhat different version of this approach may also be considered. Consider following the procedure described above but without the consistent derivation of the S_z. Suppose a multidimensional poverty line is chosen directly from the distribution $S = (S_1, S_2, \ldots, S_m)$, as though it were a target univariate distribution. Suitable candidates for this line would be the so called relative poverty lines, such as the lower quantiles, or a percentage of the median of the distribution. Indeed, this has been suggested by D'Ambrosio, Deutsch and Silber (2004) and by Miceli (1997) who appear to have been the first to apply the IT approach to poverty, exploiting Swiss data.
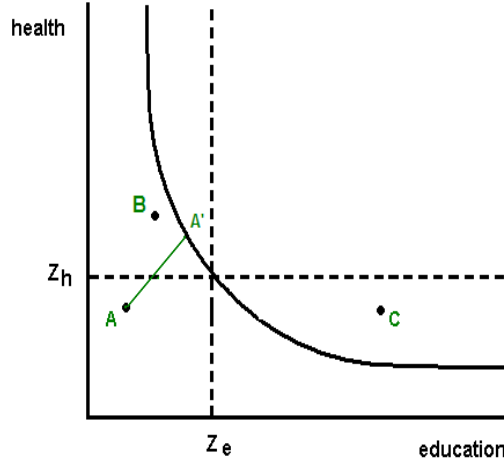
Figure 3: Aggregate Poverty Line. Three persons below thresholds

$$S_i(strong) = \left[\sum_j^q w_j (min\{x_i^j, z^j\})^\beta \right]^{1/\beta} \qquad \text{for } \beta \neq 0, \tag{22}$$

$$S_i(strong) = \prod_j^q \left(min\{x_i^j, z^j\} \right)^{w_j} \qquad \text{for } \beta = 0. \tag{23}$$

The poverty line and the aggregation of individuals is the same as before, so the APL_{strong} multidimensional poverty measure is

$$P(APL_{strong}) = \frac{1}{n} \sum_{i=1}^n \left[\max \left\{ \frac{\left(\sum_j^q w_j (z^j)^\beta \right)^{1/\beta} - \left(\sum_j^q w_j (min\{x_i^j, z^j\})^\beta \right)^{1/\beta}}{\left(\sum_j^q w_j (z^j)^\beta \right)^{1/\beta}}; 0 \right\} \right]^\alpha$$
$$\text{for } \beta \neq 0, \tag{24}$$

$$P(APL_{strong}) = \frac{1}{n} \sum_{i=1}^n \left[\max \left\{ \frac{\prod_j^q (z^j)^{w_j} - \prod_j^q (min\{x_i^j, z^j\})^{w_j}}{\prod_j^q (z^j)^{w_j}}; 0 \right\} \right]^\alpha \quad \text{for } \beta = 0. \tag{25}$$

To clarify the difference between weak and strong versions consider the individual poverty functions when only two attributes are included, health and education, as illustrated in figure 3.

For simplicity, we consider $\beta = 0$. For individuals who are poor in both dimensions, such as individual A, both the weak and the strong version would lead to

$$p_i(weak) = p_i(strong) = \max \left\{ \frac{(z^e)^{w_e} (z^h)^{w_h} - (x_i^e)^{w_e} (x_i^h)^{w_h}}{(z^e)^{w_e} (z^h)^{w_h}}; 0 \right\}, \tag{26}$$

which for person A is

$$p_A(weak) = p_A(strong) = \frac{(z^e)^{w_e}(z^h)^{w_h} - (x_A^e)^{w_e}(x_A^h)^{w_h}}{(z^e)^{w_e}(z^h)^{w_h}}.$$

The individual deprivation function measures the distance, in terms of 'well-being units', between point A and the closest point in the APL, represented by the line $\overline{AA'}$.

For persons who are poor only in one dimension, for instance health such as the person in the figure, the weak version would be

$$p_i(weak) = \max\left\{\frac{(z^e)^{w_e}(z^h)^{w_h} - (x_i^e)^{w_e}(x_i^h)^{w_h}}{(z^e)^{w_e}(z^h)^{w_h}}; 0\right\}, \tag{27}$$

which is different from the strong version

$$p_i(strong) = \max\left\{\frac{(z^e)^{w_e}(z^h)^{w_h} - (z^e)^{w_e}(x_i^h)^{w_h}}{(z^e)^{w_e}(z^h)^{w_h}}; 0\right\}. \tag{28}$$

Notice that because B and C are above the threshold in one of the dimensions, the relative deprivation value in the strong focus case will be larger than in the weak formula. This is the consequence of not allowing substitution between attributes below and above the poverty threshold. In figure 4 the lines represent the measurement of the distance of each individual from the APL. Weak poverty is the distance between the actual location of the individual and the APL (lighter solid line), which in the case of person B is the line between B and B'. The strong poverty measurement for person B, on the other hand, is the distance between b' and b'' where b' is defined according to the quantity of the good in which the person is short of, and the poverty threshold for the good in which she is not short $(x_B^e; z^h)$ (darker solid line).

Note that whether B or C will be counted as 'aggregate poor' will depend on the difference between the S_z and their respective S_i. While B is poor in both versions of the focus property, C is not counted as poor according to the weak definition (27). That is,

$$p_B(weak) = \frac{(z^e)^{w_e}(z^h)^{w_h} - (x_i^e)^{w_e}(x_i^h)^{w_h}}{(z^e)^{w_e}(z^h)^{w_h}}$$
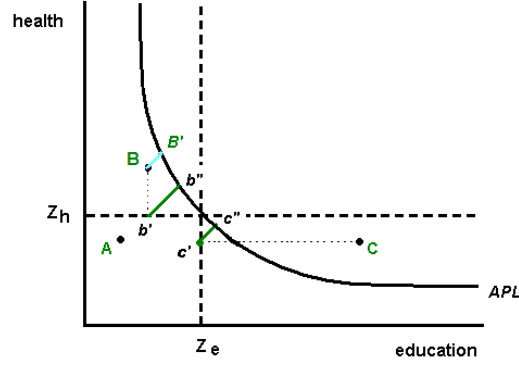
while

$$p_C(weak) = 0.$$

Figure 4: Aggregate Poverty Line. Weak and strong poverty focus versions

4.2.2   Component Poverty Line Approach

A second approach to setting a poverty threshold consistent with a multidimensional view is as follows. Consider obtaining summary functions of

$$d_i^j = \frac{z^j - x_i^j}{z^j} \tag{29}$$

in place of x_i^j. The d_i^j can be interpreted as *shortfalls to threshold* where for poor persons $0 \le d_i^j \le 1$ and for the non-poor $d_i^j \le 0$ (Bourguignon & Chakravarty 2003). The optimal IT functionals will be the same as given above, as a result of minimising the distance between the ideal distribution (with no person poor) and the actual distribution of shortfalls d_i^j. Notice that shortfall is a negative characteristic of the individual in that it is decreasing in well-being level. Note further that no explicit 'aggregate poverty line' is invoked. Rather, the poverty line is embedded in the first step.

1. Shortfall indicator is

$$
\begin{aligned}
S_{d_i} &= \left(\sum_j^q w_j \left(\frac{z^j - x_i^j}{z^j} \right)^\beta \right)^{1/\beta} && \text{for } \beta \ne 0, & (30) \\
S_{d_i} &= \prod_j^q \left(\frac{z^j - x_i^j}{z^j} \right)^{w_j} && \text{for } \beta = 0. & (31)
\end{aligned}
$$

The w_j are positive weights attached to each j shortfall, whereas β sets the level of substitutability between shortfalls; the higher the β, the lower the degree of substitutability between them. There are two interesting special cases. When β tends to infinity, relative deprivations are non-substitutes; and when $\beta = 1$ shortfalls are perfect substitutes. In both situations, poverty will be defined unidimensionally, in the first case by the attribute deprivation with the highest level of deprivation, and in the second, as a simple weighted sum of attributes. The second option resembles

the standard monetary approach to poverty if the weights are equal to market prices. Convexity of attributes – that is concavity in the space of deprivations – requires that $\beta \geq 1$.

2. The individual poverty function follows directly as

$$p_i = \left[\sum_{j}^{q} w_j \max \left\{ d_i^j; 0 \right\}^{\beta} \right]^{1/\beta}, \tag{32}$$

and the multidimensional poverty measure is

$$P_{\alpha}(S_d; z) = \frac{1}{n} \sum_{i=1}^{n} p_i^{\alpha}, \tag{33}$$

which is the αth moment of the distribution of $S_d = (s_{d_1}, s_{d_2}, ...s_{d_n})$.

The component poverty line (CPL) multidimensional poverty measure is thus

$$P(CPLstrong) = \frac{1}{n} \sum_{i=1}^{n} \left[\sum_{j}^{q} w_j \max \left\{ \frac{z^j - x_i^j}{z^j}; 0 \right\}^{\beta} \right]^{\alpha/\beta}, \tag{34}$$

and

$$P(CPLstrong) = \frac{1}{n} \sum_{i=1}^{n} \left[\prod_{j}^{q} \max \left\{ \frac{z^j - x_i^j}{z^j}; 0 \right\}^{w_j} \right]^{\alpha}. \tag{35}$$

In this second procedure, the strong focus axiom and the union definition of poverty are imposed. This step produces an aggregate of relative deprivations that allocates weights to each deprivation, and allows trade-offs between these relative deprivations in various attributes. Again, substitution is allowed only among attributes that are below the poverty threshold. The weak focus poverty axiom is not invoked in the second IT approach because $d_i^j < 0$ when the individual possesses more than the poverty threshold level of that attribute. Thus, when β takes an even value the farther away (richer) a person is the greater the value of his shortfall d_i^j. Clearly, this is counterintuitive and hence an undesirable property of the measure.

Figures 5 and 6 illustrate the measurement of individual poverty using the strong aggregate poverty line approach (in the left panel) and the composite poverty line procedure (in the right panel). Person A is poor in both health and education and has an individual CPL poverty index of

$$p_A(CPL) = \left[w_e \left(\frac{z^e - x_A^e}{z^e} \right)^{\beta} + w_h \left(\frac{z^h - x_A^h}{z^h} \right)^{\beta} \right]^{\frac{1}{\beta}} \tag{36}$$
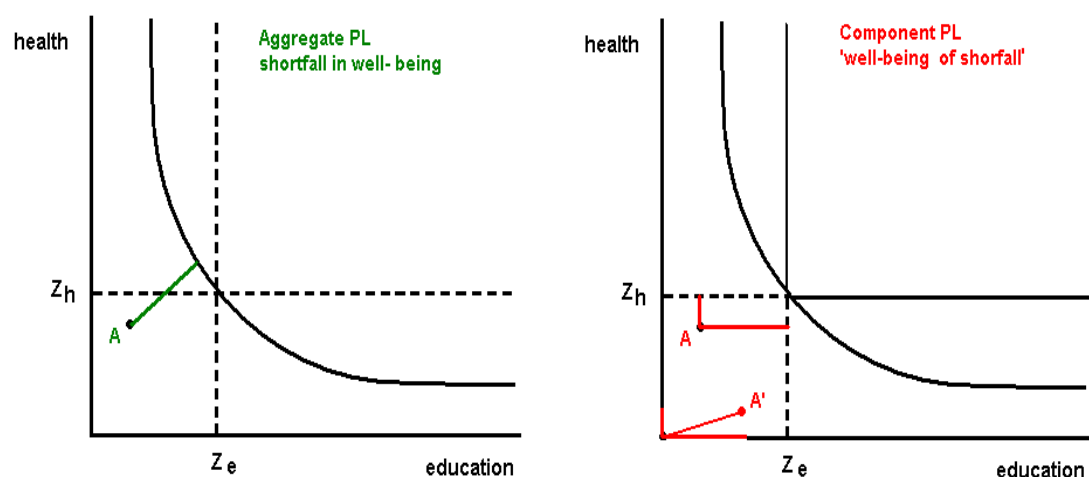
Figure 5: Aggregate Poverty Line and Component Poverty Line IT approaches
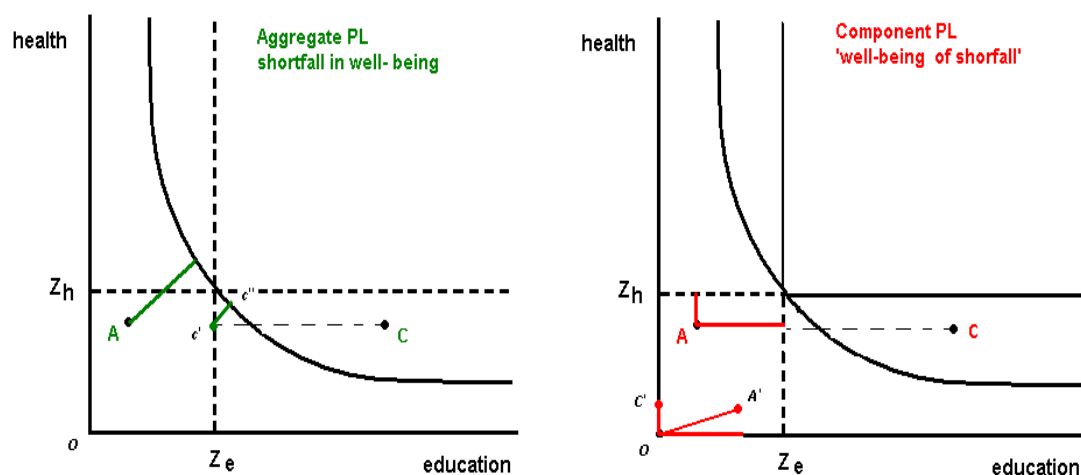
Figure 6: Aggregate Poverty Line and Component Poverty Line IT approaches

Graphically, the composite approach to poverty will evaluate the distance between the origin O and the 'well-being of the shortfall' A' which represents the reflected version of A. It is thus clear that any individual that falls below the threshold in any one dimension will be included in the set of poor, irrespective of her attainment in other dimensions. In the case of person C, the distance measured is $\bar{Oc'}$ (figure 6), where c' represents the difference between the education threshold z^e and x_C^e.

4.3   Other measures in the literature

This subsection presents the two most well-known multidimensional poverty measures in the literature. We show that both are special cases of either one of the measures derived above. Based on a critical appraisal of the 'market price approach', Tsui (2002) derives

a set of multidimensional poverty measures following an axiomatic approach, similar in spirit to his work on multidimensional inequality (Tsui 1995, Tsui 1999) discussed in Lugo (2007). Specifically, Tsui extends standard univariate axioms of unidimensional poverty indices, and presents new axioms tailored to the multivariate poverty context. Axioms are imposed on the poverty index $P(X; z)$ in (1) directly, rather than applied to some social evaluation function (as in Tsui 1999) but these properties will constrain the family of both individual functions $f(x)$ and the aggregate function $G(.)$. A standard basic set of axioms includes: continuity; symmetry; replication invariance; monotonicity; subgroup consistency; and ratio-scale invariance (Tsui 2002). These axioms are complemented with poverty specific properties including strong poverty focus, poverty criteria invariance, poverty non-increasing minimal transfer, and poverty non-decreasing rearrangement.[14]

The resulting multidimensional poverty measures are

$$P_1^{Tsui} = \frac{1}{n} \sum_{i=1}^{n} \left[\prod_{j=1}^{m} \left(\frac{z^j}{\min\left\{x_i^j; z^j\right\}} \right)^{\delta_j} - 1 \right],$$ (37)

with $\delta_j \geq 0, j = 1, 2, ..., m$, and δ_j chosen to maintain convexity of the functions, and

$$P_2^{Tsui} = \frac{1}{n} \sum_{i=1}^{n} \sum_{j=1}^{m} \delta_j \ln\left[\frac{z^j}{\min\left\{x_i^j; z^j\right\})} \right],$$ (38)

with $\delta_j \geq 0, j = 1, 2, ..., m$.

We can disentangle the elements of the measures in order to bring to the fore the underlying components. The individual poverty functions implicit in Tsui's measures are defined as

$$p_i^{T1} = \prod_{j=1}^{m} \left[\frac{z^j}{\min\left\{x_i^j; z^j\right\}} \right]^{\delta_j} - 1,$$ (39)

---

[14] Poverty axioms as defined by Tsui (2002)

- **Poverty criteria invariance**. If $z \neq z'$ then $P(X; z) \leq P(Y; z)$ and $P(X; z') \leq P(Y; z')$ whenever $X(z) = X(z')$ and $Y(z) = Y(z')$. There is no dramatic change in the evaluation of poverty for changes in the poverty threshold not affecting the number of poor;

- **Poverty non-increasing minimal transfer** with respect to a majorization criterion. $P(Y; z) \leq P(X; z)$ where $Y = BX$ and B is a bistochastic matrix and the transfer is among the poor. The poverty index must be sensitive to the dispersion of attributes among the poor;

- **Poverty non-decreasing rearrangement**. If Y is derived from X by a finite sequence of basic-rearrangements increasing transfers among the poor with no one becoming non-poor due to the transfer, then $P(X; z) \leq P(Y; z)$, where 'basic-rearrangements increasing transfer' is defined as a transfer between individuals p and q such that the resulting matrix has the same marginal distributions of attributes but higher correlation between them. More correlation between attributes among the poor increases (or leaves unchanged) the measurement of poverty.

and

$$p_i^{T2} = \sum_{j=1}^{m} \delta_j \ln \left[\frac{z^j}{\min \left\{ x_i^j; z^j \right\}} \right]. \tag{40}$$

Note that $p_i = 0$ only for those above the poverty line in *all* dimensions. δ_j represents the contribution that the relative shortfall in attribute j makes to the individual's poverty.

The implicit poverty index is:

$$P(X; z) = \frac{1}{n} \sum_{i=1}^{n} p_i, \tag{41}$$

which is the poverty gap, that is, the first moment of the discrete (empirical) distribution of p_i.

It is easy to show that for $\beta = 0$ and $w_j = -\delta_j$, the P_1^{Tsui} is ordinally equivalent to the strong version of the APL procedure (25), the main difference being that while $P(APL_{strong})$ is non-negative and normalized to be less than one, the P_1^{Tsui} index is non-negative but unbounded. This has the disadvantage that the upper bound is dependent on the units chosen for each poverty line z^j. One interpretation is that our IT measures include a normalized version of Tsui's when $\alpha = 1$.

In a closely related paper, Bourguignon and Chakravarty (2003) impose similar axioms, except for two. They replace subgroup consistency with the separability axiom, and allow for correlation increasing transfer to have either an increasing or decreasing effect on the evaluation of poverty, depending on the nature of the attributes involved. In other words, they accept both 'Poverty-non-decreasing rearrangement' and 'Poverty non-increasing rearrangement'. The resulting poverty index is of the following general CES-like form:

$$P^{BC}(X; z) = \frac{1}{n} \sum_{i=1}^{n} \left[\sum_{j=1}^{m} w_j \left[\max \left\{ 1 - \frac{x_i^j}{z^j}; 0 \right\} \right]^{\beta} \right]^{\alpha/\beta}, \tag{42}$$

which is essentially the CPL multidimensional index (34) proposed in the previous section.[15]

5 Empirical Application

This section applies the proposed poverty measures to data from Indonesia. The exercise highlights the inevitability of making value judgments when comparing any two multivariate distributions.

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[15]Interestingly, the effect of increasing correlation on the poverty index is dependent on the specific relation between the parameters β and α. Bourguignon and Chakravarty also present an interesting case where β depends on the poverty level, so that the substitution between shortfalls changes according to how far the individual is from the poverty line (Bourguignon & Chakravarty 1999, Bourguignon & Chakravarty 2003).

We compare three-dimensional distributions of Indonesians' expenditure, health status, and level of education across the three largest ethnic groups. These are Jawa, Sunda, and Betawi, which contain 52, 18 and 5 per cent of the total Indonesian population, respectively.[16] The exercise is meant to be merely illustrative and, for this reason, we choose to represent well-being by only three attributes. Naturally, results can be extended to more dimensions. The choice of dimensions was made given the wide agreement on their fundamental role as both means and ends - particularly in the case of education and health (Anand & Sen 2000).

Data come from the 2000 Indonesian Family Life Survey (IFLS) conducted by RAND, UCLA and the Demographic Institute of the University of Indonesia. The IFLS is an ongoing longitudinal socioeconomic and health survey, representing 83 per cent of the Indonesian population living in 13 (out of 26) provinces. It collects data on individual respondents, their families, their households, the communities in which they live, and the health and education facilities they use (Strauss, Beegle, Sikoki, Dwiyanto, Herawati & Witoelar 2004). The IFLS was previously conducted in 1993, 1997, and 1998, but data on health status is publicly available only for 2000.

Approximately 10,400 households and 39,000 individuals were interviewed in 2000. We will restrict the study to individuals for whom we have complete information on all relevant variables, omitting just over one per cent of the sample.

The indicators used are *real per capita expenditure, level of hemoglobin, and years of education achieved by the head of household.* Nominal per capita expenditure data are adjusted using a temporal deflator (Tornquist CPI, base year Dec 2000) and a spatial deflator (regional poverty lines) (Strauss et al. 2004). Individuals' hemoglobin levels are expressed in grams per decilitre (g/dl). Low levels of hemoglobin indicate deficiency of iron in the blood where '...[i]ron deficiency is thought to be the most common nutritional deficiency in the world today' (Thomas, Frankenberg & Friedman 2003, p.4).[17] Given that normal values of hemoglobin depend on sex and age, we adjusted individual values to transform them into equivalent adult levels.[18] Tables 2 and 3 in the statistical appendix present basic summary statistics for these variables, including correlation coefficients between them.

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[16]We assign to each individual the ethnic group as declared by his head of household. The question strictly refers to the influence of ethnicity on daily activities ('Which ethnic group is primarily influential in daily activities of your household?' Answers are classified in twenty-five ethnic groups, including 'Others').

[17]Low levels of hemoglobin are linked to susceptibility to diseases, fatigue, and lower levels of productivity. It reflects the combination of a diet that is low in animal proteins (primary source of iron) and lower absorption capacity (which is caused by disease insults, the presence of worms, loss of blood and diets high in rice). More generally, low levels are related to iron deficiency (World Health Organisation, 2001, Thomas & Frankenberg 2002).

[18]We use threshold values from the WHO 2001 report to compute the table of equivalence (World Health Organisation, 2001, table 6, chapter 7). Normal levels of hemoglobin also vary with long exposure to altitudes - which we ignore for our calculations but given our sample of Indonesia in this survey it should not be problematic. Additionally, studies show that in US individuals from African extraction tend to have normally lower values, suggesting that is possible to have normal ranges that varies according to ethnic groups. A thorough assessment of anemia for Indonesian population should consider both issues.

Computing poverty involves choosing a cut-off point for each indicator. To allow for sensitivity to different poverty lines we use two values representing reasonable boundaries for alternative thresholds. These can also be identified with extreme poverty and poverty lines as in the traditional poverty literature. In particular, for (monthly) per capita expenditure we utilize Strauss *et al.*'s (2004) values of Rp. 100,000 and Rp. 150,000, respectively;[19] for hemoglobin 12 g/dl and 13 g/dl;[20] and for education 4 and 6 years of schooling.[21]

Table 1 presents measurements of poverty for each attribute, using the FGT index for values of $\alpha = [0, 1, 2]$. Interestingly enough, the ordering of groups differs for each dimension. In particular, the poorest group in expenditure (Jawa) is in the second position in terms of health and education, whereas the poorest in terms of education outcomes (Betawi) is the richest both in expenditure and health outcomes. The Sunda group, on the other hand, has the highest poverty measurement in health, the second highest in expenditure, and the lowest in education.[22] In this context, the decision of how to aggregate the different dimensions across ethnic groups takes on a particular importance.[23]

Employing multidimensional poverty indices necessarily involves a significant loss of information. Depending on how the aggregation is done – in terms of functional form, indicator variables, and parameter values – the results will vary in terms of cardinal values and, in some cases, the ordinal rankings of the distributions. Figure 7 shows the resulting measurements (y-axis) using the two approaches presented in the previous section, as the parameter of substitutability between attributes (β) varies (x-axis). We utilize an equal weighting scheme (that is $w_j = 1/3$ for all j) and set α to correspond to the poverty headcount formula in the left panel and the severity of poverty in the right panel (Figure 8 presents a magnified version of the right-hand graphs). Tables 4 and 5 in the statistical appendix includes these poverty measurements and, for sensitivity analysis, reports alternative poverty lines, weighting schemes and poverty gap measures. We also include CDFs for each combination of aggregate well-being functions computed for the measurements (Figures 10 to 12 in the Statistical Appendix).

We first compare the results with those obtained from the univariate poverty analysis. The Sunda group, which is ranked first, second and third respectively in the distinct dimensions in the univariate analysis, unambiguously becomes the best-off ethnic group. This is true for all combinations of approaches and parameter values used here.

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[19]See chapter three in Strauss (2004). In December 2000, the exchange rate for the Rupiah was Rp. 9,480 per US dollar. Thus, the lines used in the text are US0.3 and US0.5 dollar a day, which are a fairly low thresholds.

[20]From the WHO report, a male adult is considered anemic, possibly suffering from iron deficiency, if his hemoglobin level is below 13 grams per decilitre.

[21]To avoid later computational problems, we assigned to individuals with no education a value of 0.5, instead of 0.

[22]The previous results should be evaluated in the light of the statistical significance of differences as presented in the table

[23]In the Statistical Appendix we include a table with basic statistics for variables employed.

Table 1: Univariate poverty measurement by ethnic groups. Indonesia, 2000

	Ethnic groups								
	Jawa			Sunda			Betawi		
Extreme poverty	$\alpha = 0$	$\alpha = 1$	$\alpha = 2$	$\alpha = 0$	$\alpha = 1$	$\alpha = 2$	$\alpha = 0$	$\alpha = 1$	$\alpha = 2$
Expenditure	0.141	0.032	0.011	0.135*	0.029*	0.011*	0.072	0.018	0.007
Hemoglobin	0.109	0.011*	0.003*	0.127	0.012	0.002	0.101	0.009*	0.002*
Education	0.320*	0.209	0.156	0.234	0.149	0.109	0.344	0.234	0.178
Poverty	$\alpha = 0$	$\alpha = 1$	$\alpha = 2$	$\alpha = 0$	$\alpha = 1$	$\alpha = 2$	$\alpha = 0$	$\alpha = 1$	$\alpha = 2$
Expenditure	0.344	0.102	0.042	0.323*	0.097*	0.040*	0.262	0.068	0.026
Hemoglobin	0.247*	0.024*	0.005*	0.273	0.026	0.005	0.261*	0.022*	0.004*
Education	0.404	0.269	0.204	0.318	0.201	0.148	0.455	0.301	0.23

* Not significantly different from the higher value at the 95% confidence level

Source: authors' calculations

Figure 7: Multidimensional Poverty Measurements, by ethnic groups. Indonesia, 2000
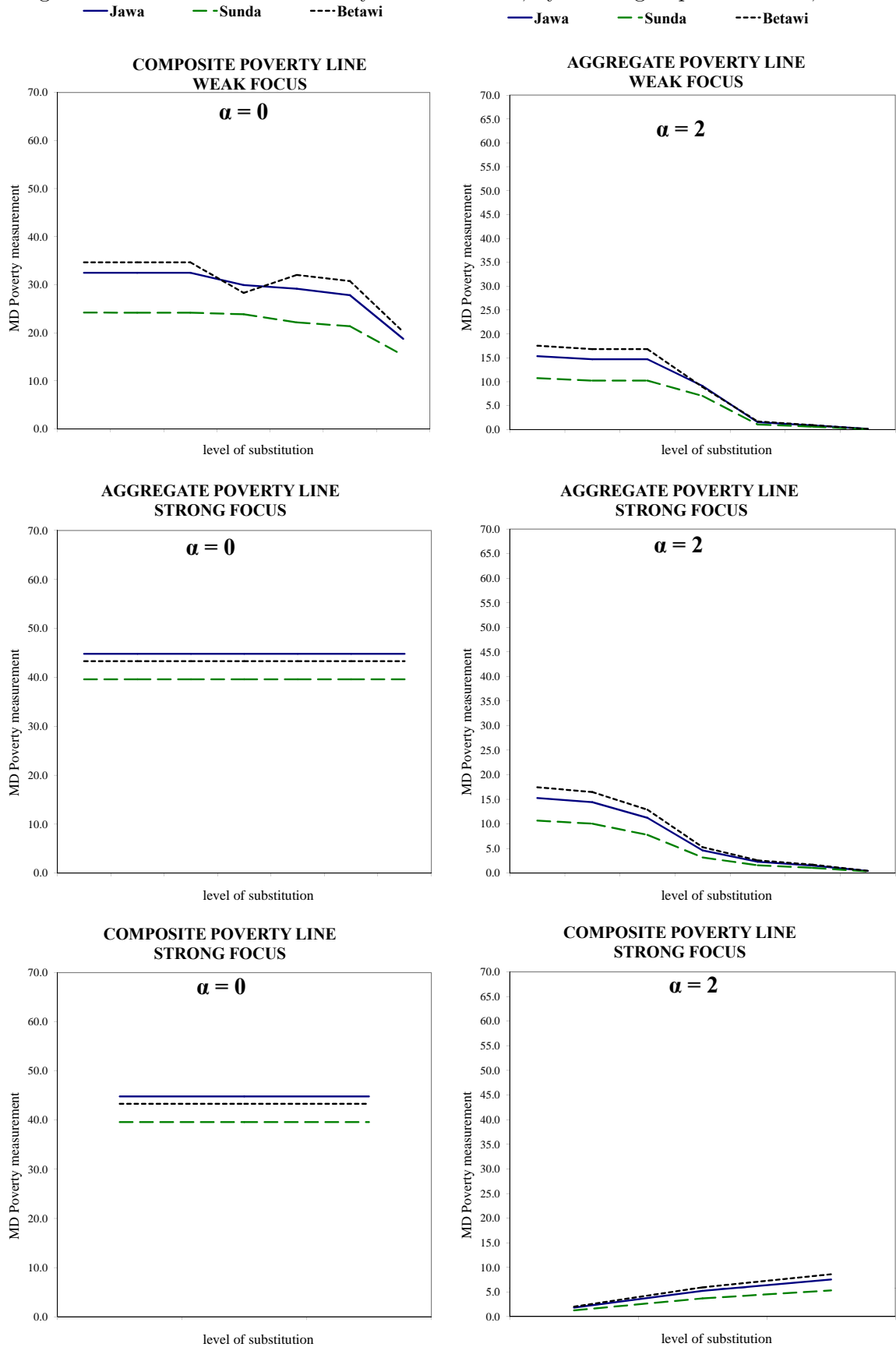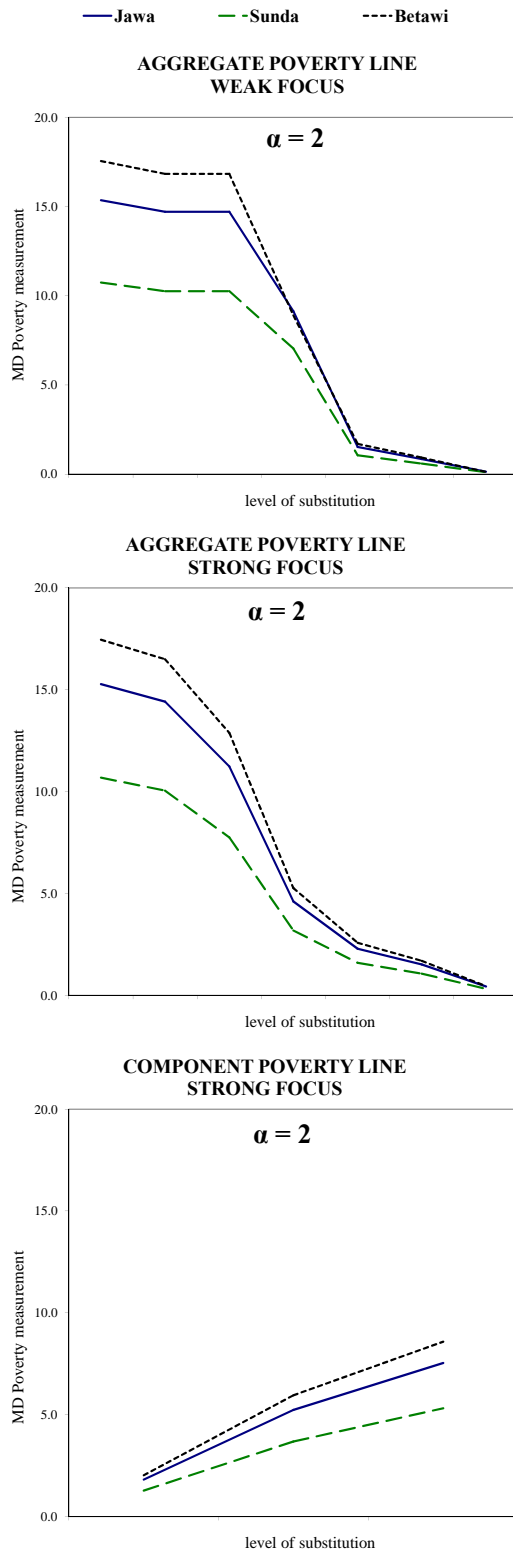
Figure 8: Multidimensional Poverty Measurements, by ethnic groups. Indonesia, 2000 (magnified)

The comparison between the Jawa and Betawi population is less straightforward. When $\alpha = 0$ (left-hand side of figure 7), the degree of substitution makes almost no difference to rankings for any measure, with the exception of just one point (a substitution of zero) in the APL approach with the weak focus axiom (top left hand panel, figure 7). But Betawi has higher poverty values when using the APL approach and weak focus while Jawa is placed first when strong focus is assumed. This might reflect the fact that among the Jawa population, low levels of expenditure are accompanied by relatively high education outcomes – relative to the Betawi group.

When the distribution among the poor is considered, with $\alpha = 2$ (right-hand side of figure 7 and figure 8), the ordering of groups depends on the level of substitutability between attributes. In particular, we find unambiguous rankings across measures for lower β, but the distinction between groups vanishes as β rises above zero and approaches unity.[24] All these results are robust to the two sets of weights employed here (see tables 4 and 5 in the statistical appendix). We expect that only very extreme a priori weighting assumptions will produce results that are closer to the unidimensional poverty values.

Note that, as expected, the measured poverty rates decreases as the substitutability between attributes increases. At the extreme, when there is no substitution, multidimensional poverty rates will equal the unidimensional poverty rate for the component of the index with the highest poverty. For all three ethnic groups this is education. Recall that higher substitution between attributes corresponds to *high* values of β in the first IT approach and to *low* values of β in the component poverty line approach (based on shortfalls).

Finally, within the Aggregate Poverty Line approach we can observe the implications of using the weak versus the strong poverty focus axiom. For each combination of (w_j, α, β) the measures that are consistent with weak poverty focus yield lower poverty levels than those consistent with strong focus (see tables 4 and 5). This is due to the fact that the former allows for some degree of substitution (compensation) between attributes for those who are poor in one dimension and not in some other dimension such that they end up being above the multidimensional poverty threshold. This example shows that employing the weak poverty focus axiom can be seen as an intermediate case between the union and intersection approaches.

6   Conclusion

This paper has presented the information theory (IT) approach to multidimensional poverty measurement in a way that allows a deeper interpretation of the existing methods and leads to the development of new measures, primarily based on axiomatic approaches. The IT

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[24]A proper assessment of this conclusion requires the computation of standard errors of multidimensional measurements. We intend to include these in future versions

approach emphasizes clarity in aggregation choices that, it is argued, are inevitable in any multidimensional setting. By making aggregation issues explicit, IT procedures reveal the meaning and the working of multidimensional measures when one allows attributes that are above the poverty threshold to 'compensate' for attributes that are below the threshold. We feel it is essential to have an accommodation for this possibility since, otherwise, the case for a multidimensional approach to poverty and welfare may not exceed far beyond adding up over many dimensions. Future work will consider differential elasticities substitution between attributes.

The paper has also shown that the families of measures proposed earlier in the literature are special cases of IT poverty measures derived under particular conditions, and that the latter allow for new indices when some conditions are relaxed. The Indonesian case study brings out some of these issues, but not all. The figures are merely indicative (but not statistically definitive) of a great degree of robustness in our ranking of the poverty status of different ethnic groups of the country at a particular point in time. Nevertheless, numerical conclusions were found to vary with the degree of substitution between attributes, the degree of inequality aversion within the group classified as poor, and the degree to which compensation was allowed between attributes above and below poverty thresholds. The size of the group which is not poor in all dimensions deserves closer scrutiny and may itself characterize economies and societies in meaningful ways.

Table 2: Summary Statistics, by ethnic groups. Indonesia, 2000

Variable	Obs	Mean	Std. Dev.	Min	Max
Jawa					
Real per capita expenditure (Rp.)	17,097	271,347	287,322	20,348	5,236,150
Hemoglobin (g/dl)	17,097	13.95	1.71	3.6	25.8
Education of head of hh	17,089	6.25	4.46	0.5	19.0
Sudan					
Real per capita expenditure (Rp.)	5,932	294,857	338,738	24,391	6,066,339
Hemoglobin (g/dl)	5,932	13.86	1.71	3.5	19.4
Education of head of hh	5,927	6.78	4.28	0.5	19.0
Betawi					
Real per capita expenditure (Rp.)	1,576	306,096	316,578	42,577	3,901,813
Hemoglobin (g/dl)	1,576	13.94	1.67	3.1	20.1
Education of head of hh	1,576	6.20	4.57	0.5	17.0

Source: authors' calculation for ILFS 2000.

Statistical Appendix

Table 3: Correlation coefficients. Indonesia, 2000

Pearson Correlation Coefficients (sign 0.05)

Jawa

	expenditure	hemoglobin	education
expenditure	1.0000		
hemoglobin	0.0675*	1.0000	
education	0.3354*	0.0688*	1.0000

Sunda

	expenditure	hemoglobin	education
expenditure	1.0000		
hemoglobin	0.0989*	1.0000	
education	0.3456*	0.1252*	1.0000

Betawi

	expenditure	hemoglobin	education
expenditure	1.0000		
hemoglobin	0.0703*	1.0000	
education	0.3104*	0.0635*	1.0000

Spearman Correlation Coefficients (sign 0.05)

Jawa

	expenditure	hemoglobin	education
expenditure	1.0000		
hemoglobin	0.0869*	1.0000	
education	0.3889*	0.0789*	1.0000

Sunda

	expenditure	hemoglobin	education
expenditure	1.0000		
hemoglobin	0.1277*	1.0000	
education	0.4327*	0.1119*	1.0000

Betawi

	expenditure	hemoglobin	education
expenditure	1.0000		
hemoglobin	0.1170*	1.0000	
education	0.3905*	0.0711*	1.0000

Kendall Correlation Coefficients (sign 0.05)

Jawa

	expenditure	hemoglobin	education
expenditure	0.9997		
hemoglobin	0.0577*	0.9935	
education	0.2632*	0.0520*	0.8713

Sunda

	expenditure	hemoglobin	education
expenditure	0.9991		
hemoglobin	0.0854*	0.9941	
education	0.2858*	0.0729*	0.8400

Betawi

	expenditure	hemoglobin	education
expenditure	0.9976		
hemoglobin	0.0780*	0.9935	
education	0.2616*	0.0470*	0.8832

Source: authors' calculation from IFL3 2000.

Figure 9: CDFs of univariate distributions, by ethnic groups. Indonesia, 2000
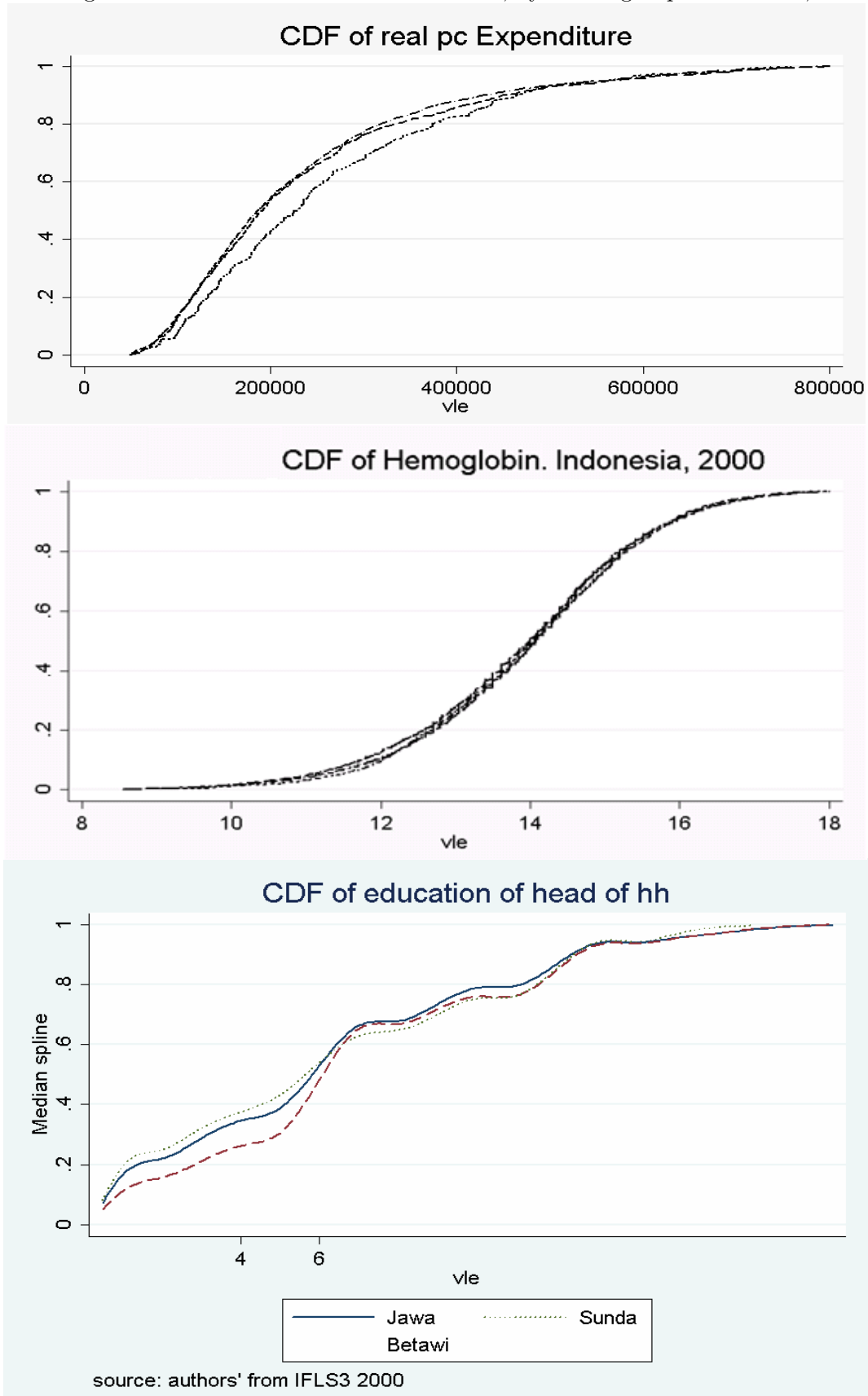
Table 4: Multivariate Poverty Measurements, by ethnic groups. Indonesia, 2000

	Ethnic groups								
	Jawa			Sunda			Betawi		

Panel (a) EXTREME POVERTY

IT - Aggregate Poverty Line Approach WEAK FOCUS

	$\alpha = 0$	$\alpha = 1$	$\alpha = 2$	$\alpha = 0$	$\alpha = 1$	$\alpha = 2$	$\alpha = 0$	$\alpha = 1$	$\alpha = 2$
weights equal									
$\beta = -3$	32.48	20.65	15.36	24.20	14.77	10.74	34.65	23.20	17.55
$\beta = -2$	32.49	20.05	14.71	24.17	14.31	10.26	34.65	22.56	16.84
$\beta = -1$	32.49	20.05	14.71	24.17	14.31	10.26	34.65	22.56	16.84
$\beta = 0$	29.92	31.32	9.13	23.84	23.37	7.03	28.31	33.68	8.90
$\beta = 1/3$	29.14	5.85	1.52	22.17	4.21	1.06	32.02	6.55	1.70
$\beta = 1/2$	27.82	4.25	0.84	21.37	3.09	0.59	30.76	4.71	0.92
$\beta = 1$	18.74	1.23	0.13	15.29	0.97	0.10	20.17	1.23	0.13
weights {1/2, 1/4, 1/4}									
$\beta = -3$	32.53	20.54	15.24	24.13	14.68	10.65	34.61	23.08	17.43
$\beta = -2$	32.50	19.67	14.32	24.13	14.03	9.97	34.61	22.17	16.41
$\beta = -1$	32.52	16.63	10.95	24.14	11.78	7.54	34.61	18.85	12.59
$\beta = 0$	29.07	30.39	7.14	23.35	22.94	5.49	27.53	32.89	6.89
$\beta = 1/3$	28.01	4.15	0.80	21.49	2.98	0.55	30.73	4.56	0.86
$\beta = 1/2$	26.43	2.88	0.42	20.50	2.08	0.28	29.51	3.09	0.43
$\beta = 1$	15.99	0.76	0.06	13.24	0.59	0.04	15.47	0.69	0.05

IT -Aggregate Poverty Line Approach STRONG FOCUS

	$\alpha = 0$	$\alpha = 1$	$\alpha = 2$	$\alpha = 0$	$\alpha = 1$	$\alpha = 2$	$\alpha = 0$	$\alpha = 1$	$\alpha = 2$
weights equal									
$\beta = -3$	44.81	20.67	15.27	39.59	14.81	10.68	43.30	23.18	17.46
$\beta = -2$	44.81	19.97	14.42	39.59	14.32	10.05	43.30	22.42	16.50
$\beta = -1$	44.81	17.35	11.23	39.59	12.42	7.76	43.30	19.50	12.88
$\beta = 0$	44.81	11.19	4.61	39.59	8.07	3.18	43.30	12.51	5.26
$\beta = 1/3$	44.81	8.05	2.29	39.59	5.89	1.59	43.30	8.91	2.58
$\beta = 1/2$	44.81	6.65	1.52	39.59	4.91	1.06	43.30	7.30	1.70
$\beta = 1$	44.81	3.68	0.43	39.59	2.83	0.32	43.30	3.90	0.46
weights {1/2, 1/4, 1/4}									
$\beta = -3$	44.81	20.80	15.48	39.59	14.89	10.83	43.30	23.34	17.68
$\beta = -2$	44.81	20.54	15.15	39.59	14.70	10.59	43.30	23.06	17.32
$\beta = -1$	44.81	19.45	13.77	39.59	13.91	9.58	43.30	21.85	15.76
$\beta = 0$	44.81	9.09	3.06	39.59	6.55	2.10	43.30	10.13	3.48
$\beta = 1/3$	44.81	6.15	1.33	39.59	4.51	0.92	43.30	6.76	1.48
$\beta = 1/2$	44.81	4.99	0.85	39.59	3.70	0.59	43.30	5.43	0.93
$\beta = 1$	44.81	2.70	0.23	39.59	2.09	0.17	43.30	2.81	0.23

IT - Component Poverty Line Approach STRONG FOCUS

	$\alpha = 0$	$\alpha = 1$	$\alpha = 2$	$\alpha = 0$	$\alpha = 1$	$\alpha = 2$	$\alpha = 0$	$\alpha = 1$	$\alpha = 2$
weights equal									
$\beta = 1$	44.81	7.38	1.82	39.59	5.43	1.28	43.30	8.11	2.03
$\beta = 2$	44.81	12.53	5.23	39.59	9.20	3.69	43.30	13.88	5.95
$\beta = 3$	44.81	15.04	7.54	39.59	11.04	5.31	43.30	16.66	8.59
weights {1/2, 1/4, 1/4}									
$\beta = 1$	44.81	5.68	1.07	39.59	4.20	0.75	43.30	6.19	1.17
$\beta = 2$	44.81	10.94	3.97	39.59	8.05	2.80	43.30	12.09	4.51
$\beta = 3$	44.81	13.73	6.27	39.59	10.09	4.42	43.30	15.20	7.14

Source: authors' calculation from IFL3 2000.
Note: the shading of cells indicates the ranking of the distributions, with the darkest being the highest poverty level in each combination of index and parameters.

Table 5: Multivariate Poverty Measurements, by ethnic groups. Indonesia, 2000 (*cont.*)
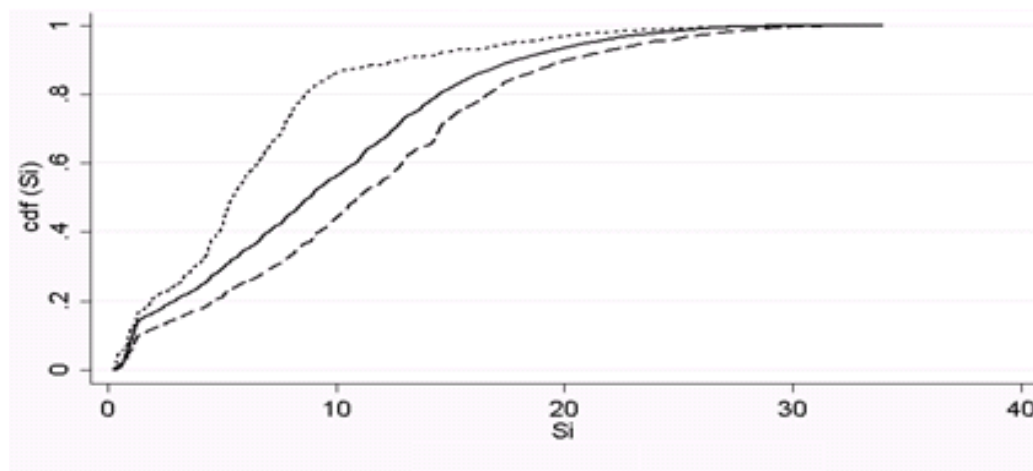
	Ethnic groups							
	Jawa			Sunda			Betawi		

Panel (a) EXTREME POVERTY

IT - Aggregate Poverty Line Approach WEAK FOCUS

	$\alpha = 0$	$\alpha = 1$	$\alpha = 2$	$\alpha = 0$	$\alpha = 1$	$\alpha = 2$	$\alpha = 0$	$\alpha = 1$	$\alpha = 2$
weights equal									
$\beta = -3$	32.48	20.65	15.36	24.20	14.77	10.74	34.65	23.20	17.55
$\beta = -2$	32.49	20.05	14.71	24.17	14.31	10.26	34.65	22.56	16.84
$\beta = -1$	32.49	20.05	14.71	24.17	14.31	10.26	34.65	22.56	16.84
$\beta = 0$	29.92	31.32	9.13	23.84	23.37	7.03	28.31	33.68	8.90
$\beta = 1/3$	29.14	5.85	1.52	22.17	4.21	1.06	32.02	6.55	1.70
$\beta = 1/2$	27.82	4.25	0.84	21.37	3.09	0.59	30.76	4.71	0.92
$\beta = 1$	18.74	1.23	0.13	15.29	0.97	0.10	20.17	1.23	0.13
weights {1/2, 1/4, 1/4}									
$\beta = -3$	32.53	20.54	15.24	24.13	14.68	10.65	34.61	23.08	17.43
$\beta = -2$	32.50	19.67	14.32	24.13	14.03	9.97	34.61	22.17	16.41
$\beta = -1$	32.52	16.63	10.95	24.14	11.78	7.54	34.61	18.85	12.59
$\beta = 0$	29.07	30.39	7.14	23.35	22.94	5.49	27.53	32.89	6.89
$\beta = 1/3$	28.01	4.15	0.80	21.49	2.98	0.55	30.73	4.56	0.86
$\beta = 1/2$	26.43	2.88	0.42	20.50	2.08	0.28	29.51	3.09	0.43
$\beta = 1$	15.99	0.76	0.06	13.24	0.59	0.04	15.47	0.69	0.05

IT -Aggregate Poverty Line Approach STRONG FOCUS

	$\alpha = 0$	$\alpha = 1$	$\alpha = 2$	$\alpha = 0$	$\alpha = 1$	$\alpha = 2$	$\alpha = 0$	$\alpha = 1$	$\alpha = 2$
weights equal									
$\beta = -3$	44.81	20.67	15.27	39.59	14.81	10.68	43.30	23.18	17.46
$\beta = -2$	44.81	19.97	14.42	39.59	14.32	10.05	43.30	22.42	16.50
$\beta = -1$	44.81	17.35	11.23	39.59	12.42	7.76	43.30	19.50	12.88
$\beta = 0$	44.81	11.19	4.61	39.59	8.07	3.18	43.30	12.51	5.26
$\beta = 1/3$	44.81	8.05	2.29	39.59	5.89	1.59	43.30	8.91	2.58
$\beta = 1/2$	44.81	6.65	1.52	39.59	4.91	1.06	43.30	7.30	1.70
$\beta = 1$	44.81	3.68	0.43	39.59	2.83	0.32	43.30	3.90	0.46
weights {1/2, 1/4, 1/4}									
$\beta = -3$	44.81	20.80	15.48	39.59	14.89	10.83	43.30	23.34	17.68
$\beta = -2$	44.81	20.54	15.15	39.59	14.70	10.59	43.30	23.06	17.32
$\beta = -1$	44.81	19.45	13.77	39.59	13.91	9.58	43.30	21.85	15.76
$\beta = 0$	44.81	9.09	3.06	39.59	6.55	2.10	43.30	10.13	3.48
$\beta = 1/3$	44.81	6.15	1.33	39.59	4.51	0.92	43.30	6.76	1.48
$\beta = 1/2$	44.81	4.99	0.85	39.59	3.70	0.59	43.30	5.43	0.93
$\beta = 1$	44.81	2.70	0.23	39.59	2.09	0.17	43.30	2.81	0.23

IT - Component Poverty Line Approach STRONG FOCUS

|	$\alpha = 0$	$\alpha = 1$	$\alpha = 2$	$\alpha = 0$	$\alpha = 1$	$\alpha = 2$	$\alpha = 0$	$\alpha = 1$	$\alpha = 2$
weights equal									
$\beta = 1$	44.81	7.38	1.82	39.59	5.43	1.28	43.30	8.11	2.03
$\beta = 2$	44.81	12.53	5.23	39.59	9.20	3.69	43.30	13.88	5.95
$\beta = 3$	44.81	15.04	7.54	39.59	11.04	5.31	43.30	16.66	8.59
weights {1/2, 1/4, 1/4}									
$\beta = 1$	44.81	5.68	1.07	39.59	4.20	0.75	43.30	6.19	1.17
$\beta = 2$	44.81	10.94	3.97	39.59	8.05	2.80	43.30	12.09	4.51
$\beta = 3$	44.81	13.73	6.27	39.59	10.09	4.42	43.30	15.20	7.14

Source: authors' calculation from IFL3 2000.

Note: the shading of cells indicates the ranking of the distributions, with the darkest being the highest poverty level in each combination of index and parameters.

Figure 10: CDFs of aggregated well-being, by ethnic groups. Indonesia, 2000
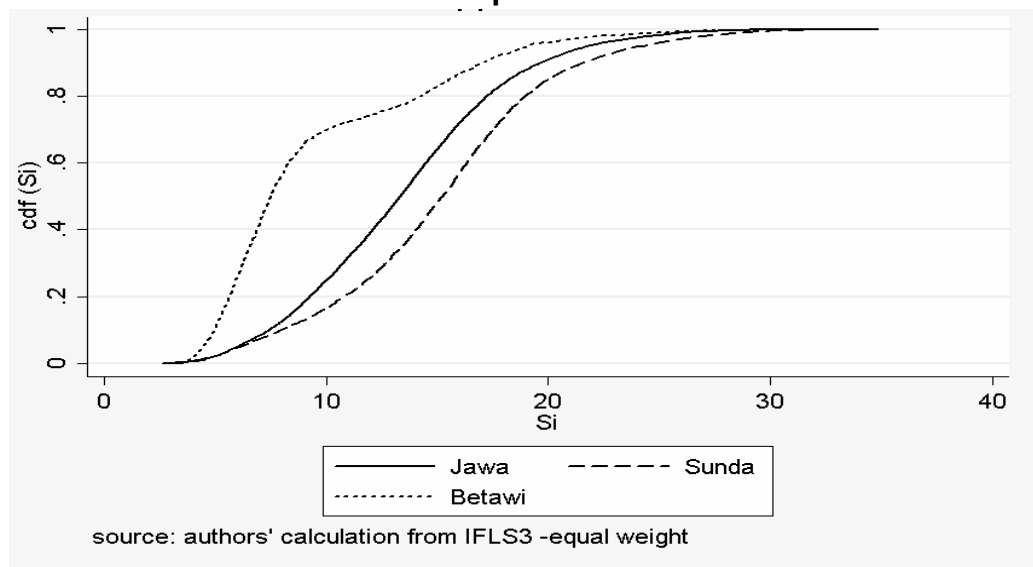Aggregate Poverty Line Approach (Weak Focus)
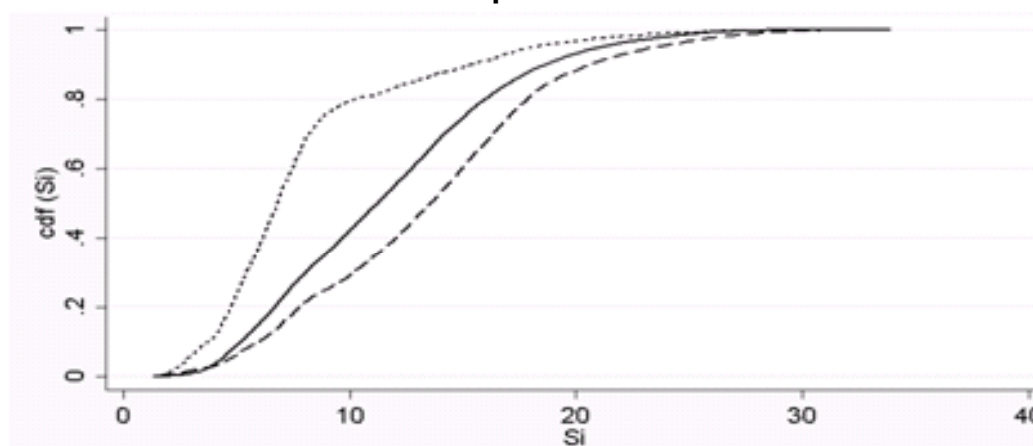
Figure 11: CDFs of aggregated well-being, by ethnic groups. Indonesia, 2000
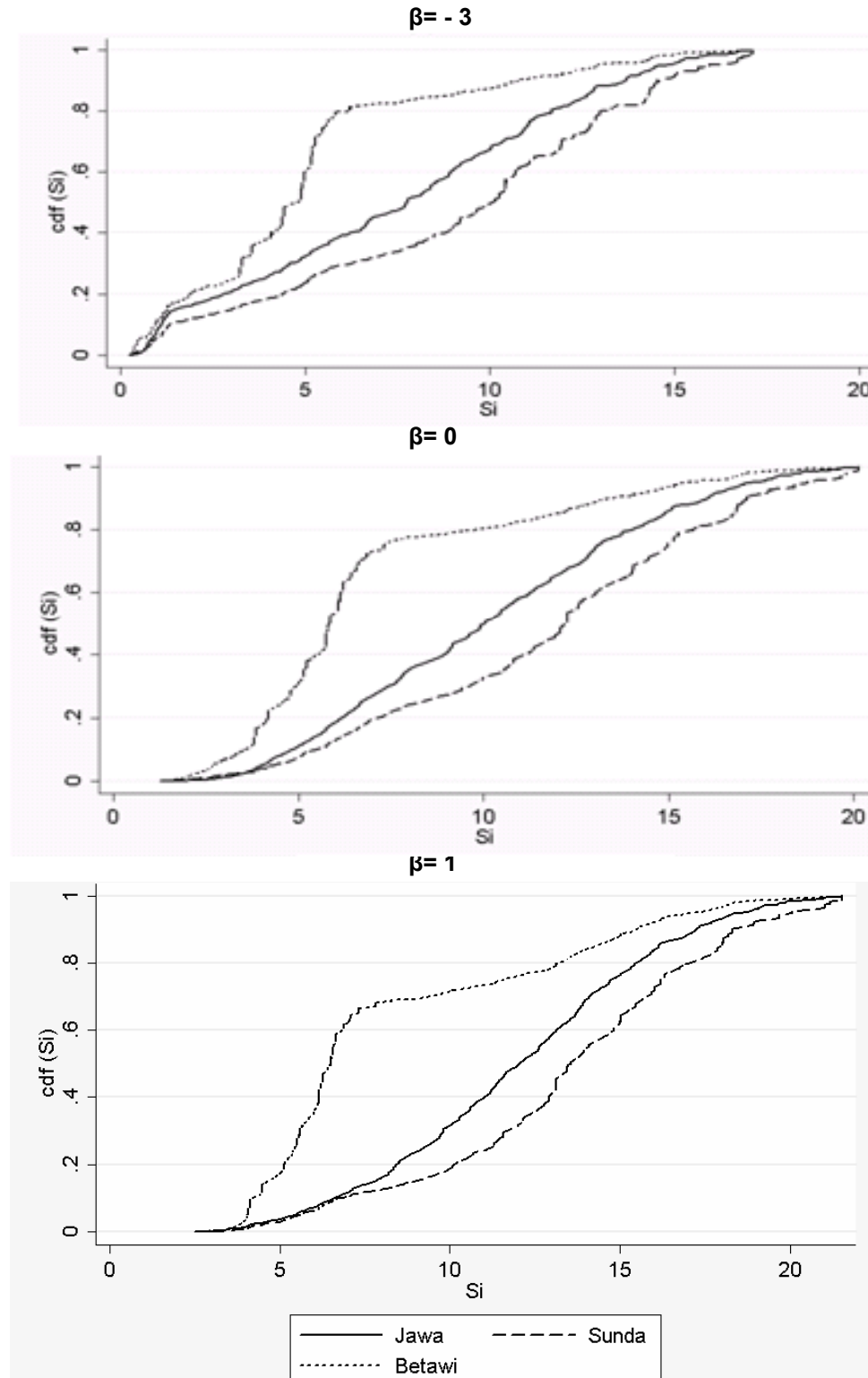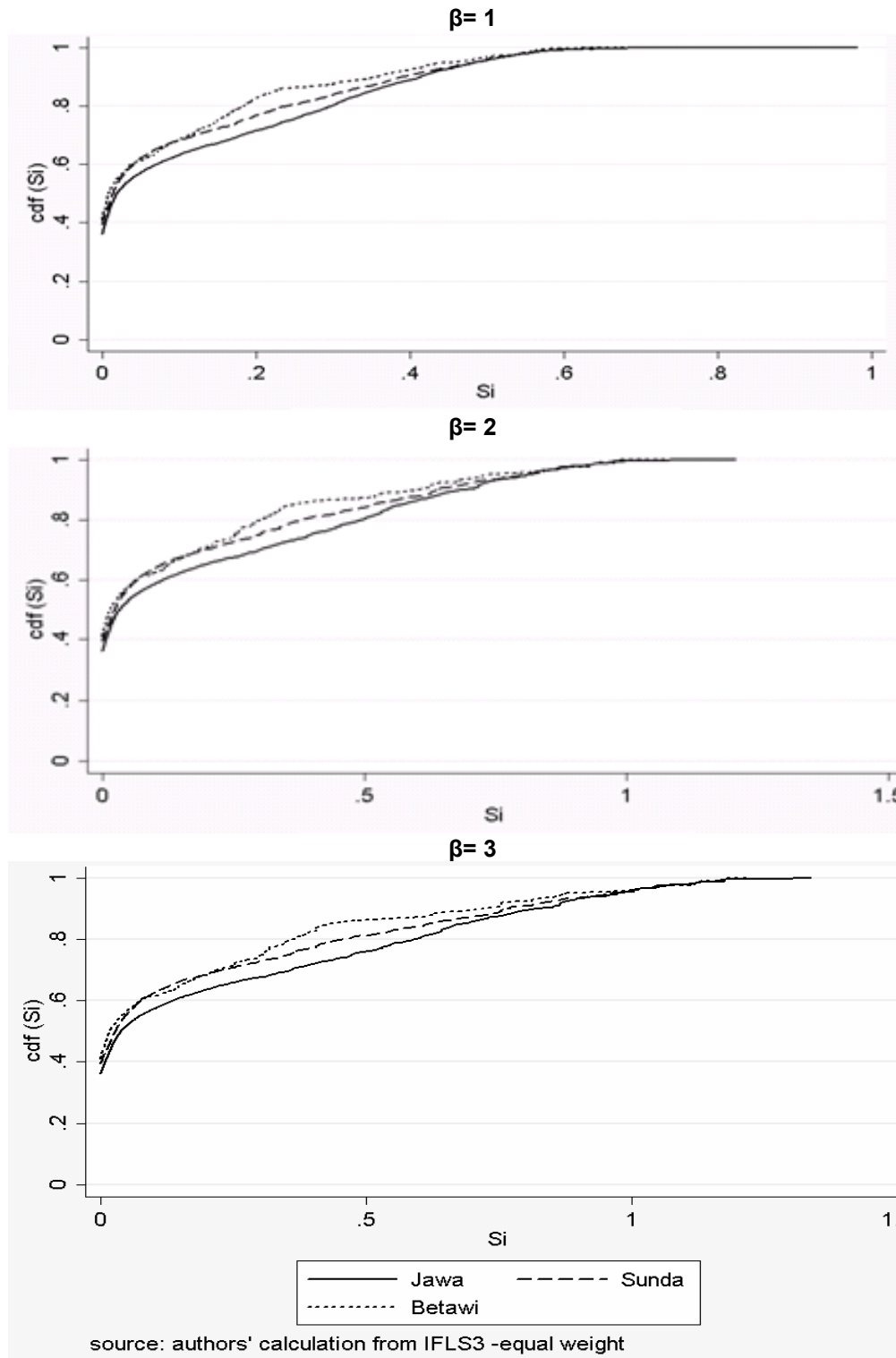Aggregate Poverty Line Approach (Strong Focus)

Figure 12: CDFs of aggregated well-being, by ethnic groups. Indonesia, 2000 Component Poverty Line Approach (Strong Focus)

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