Employment Turnover and Unemployment Insurance

John Hassler*   José Vicente Rodríguez Mora†

JEL classification number: J6, J65.

This version: Januari 1997

Abstract

Two features distinguish European and US labor markets. First, most European countries have substantially more generous unemployment insurance. Second, the duration of unemployment and employment spells are substantially higher in Europe - employment turnover is lower. We show that self-insurance, i.e., saving and borrowing, is a good substitute for unemployment insurance when turnover is high as in US. If the insurance system is less than perfectly actuarially fair, the employed median voter he will then prefer to self-insure instead of having unemployment insurance if turnover is high. We also show high unemployment insurance make unemployed more willing to wait for a job with low separation rates. This could make both high turnover/low insurance (US) and low turnover/high insurance (Europe) stable equilibria. Low turnover also leads to a strong divergence between the long and short run interest of the employed. In absence of devices such that the median voter can bind future voters to some level of insurance, the voting cycle must thus be long in order to support a high level of insurance.

* Institute for International Economic Studies, Stockholm University, S-106 91 Stockholm, Sweden. Email: John.Hassler@iies.su.se. Telephone: +46-8-162070. John Hassler thanks the people at the Minneapolis Federal Reserve Bank for their hospitality.

† Department of Economics, Universitat Pompeu Fabra, Ramon Trias Fargas 25-27, 08005 Barcelona, Spain. Email: sevimora@gobi.upf.es. Telephone: +34-3-5421755. JVRM thanks the people at the Institute for International Economic Studies for their hospitality.

	Inflow*	**Outflow***
Belgium	0.14	2.7
Denmark	0.29	6.3
Finland	1.49	36.5
France	0.32	3.7
Germany	0.25	6.1
Italy	0.14	1.8
Netherlands	0.28	6.8
Spain	0.35	1.7
Sweden	0.66	28.5
U.K.	0.51	6.1
USA	2.45	41.4

* As % per month of source population.

Source: OECD Employment Outlook, July 1995.

Table 1: Flows In and Out of Unemployment 1985

1 Introduction

A comparison between the labor markets in US and Europe reveals two distinct differences. First, unemployment insurance is much more generous in most European countries than in US. The generosity is reflected both in terms of replacement ratios and the length of time an unemployed is entitled to the benefits.[1] Second, the flows in and out of unemployment are much higher in the US than in Europe. In Table 1 we show figures of the flow from employment and from unemployment during 1985. The flows are expressed as the percentage of employed/unemployed that lost/found a job in an average month in 1985. In Germany, for example, only .25% of the employed lost their jobs in an average 1985 month. The figure for US was about 10 times higher. Similarly, 6.1% of the unemployed found a job in Germany[2] each month while the percentage in US was almost seven times higher.[3] The low flow out of employment means that the average risk of loosing a job is low in Europe while high in US. But then, why do we observe very comprehensive unemployment insurance systems in Europe while not in US?

In this paper we try to answer three questions. First, does a low employment turnover[4]

[1]The OECD Job Study [7] computes an index of the generosity of the unemployment benefits for 20 OECD countries. According to this index, Denmark and Netherlands have the most generous systems while US and Japan have the least generous. Due to multidimensional differerences in the structure of the systems, any ranking can, of course, be ambigous. However, it seems rather clear that most European countries have more generous unemployment insurance than US has.

[2]Note, however, the high flows out of unemployment in Finland and Sweden. This may reflect the (unsustainably) low unemployment rates these countries had in 1985. For Sweden, a lot of the flow is likely to be to different unemployment programs in which the individuals are not counted as openly unemployed. By 1993, the outflow in Finland had fallen to 13.9 and in Sweden to 11.6.

[3]There is, of course, substantial variation in the flow rates over age groups and industries. Disaggregating over age groups and industries, the higher rotation in US seems to prevail, however. See OECD Employment Outlook [8].

[4]We use the word turnover to denote the flow rates to and from unemployment. This should be

mean that employed would prefer a high unemployment insurance that is payed by taxing employed? This is the downward pointing arrow in Figure 1. Second, if we allow employment turnover to be a choice variable, how is that choice affected by the level of unemployment benefits (the upward pointing arrow)? Third, if people can vote sequentially on the level of unemployment benefits, is the length of the voting cycle important for voting outcomes?

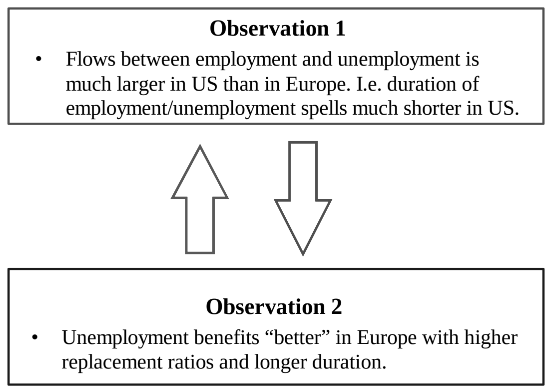

Figure 1: Issues

To answer the first question we first show that it is important to recognize that individuals in real life can self-insure, i.e., use saving and borrowing as a substitute for unemployment insurance. To illustrate the importance of this we start our analysis with a simple search model without saving. We show that such a model implies that the willingness of an employed person to pay for unemployment insurance increases in flow rates to and from unemployment. That is, large flows between unemployment and employment, as in the US, would tend to increase the level of insurance that employed people would prefer.

We will see that this result crucially depends on the assumption that individuals consume their wage when employed and cannot borrow when unemployed. The unemployment insurance thus has to function both as a means to smooth income and as true insurance. Allowing for a credit market where the individuals can save or borrow turns this result upside down. Low turnover means that income shocks associated with unemployment are more persistent than when turnover is high. Highly persistent shocks imply that uncertainty over long periods of time is high, which creates an insurance motive. On

distinguished from job rotation which typically is used for the flows on the labor market inclusive of flows of employed from one job to another. The level of job rotation defined in this way is not very different between US and Europe. (See, for example, Bertola and Rogersson [2] for a model of job-to-job rotation).

the other hand with low persistence the problem is more to smooth income between short but frequent unemployment and employment periods; something that can be done effectively by saving and borrowing. Consequently saving is a good substitute to insurance when spells short, but much worse when spells are long.[5]

We assume that a median voter (who is employed) can choose an unemployment insurance that is fixed forever (an assumption later to be dropped). We show that the median voter will choose a low (high) replacement ratio if he lives in a high (low) turnover economy. We also show that all coming generations of median voters will find that the choice of the first is optimal also to them. We have thus established a motive for the employed to introduce generous unemployment benefits *when employment turnover is low*.

Now turn to the second question. Suppose that unemployed individuals can choose between two strategies. Either, to look for a safe job that is expected to last long but it is difficult to find, or to look for jobs that are unsafe but easier to find. We think of this as a choice between being specialized in doing a specific task or being more of a generalist, although many complementary interpretations are possible. The relative attractiveness of being specialized or non-specialized, is affected by the generosity of the unemployment benefits. Ceteris paribus, higher unemployment benefits make less costly to take the longer search period associated with being specialized. This creates an implication from the level of unemployment benefits to the degree of turnover.

The causal relations between unemployment benefits and employment turnover can now be described by the two arrows in Figure 1. They create a circle which allows for the existence of multiple equilibria. Unemployed choose to be specialized (non-specialized) because unemployment insurance is high (low), the employed choose high (low) unemployment insurance because they expect to have long (short) spells of unemployment and employment.

In order to address the last of our three questions we drop the assumption that the median voter can fix insurance level for ever. Instead we assume that a vote is taken with regular intervals. The median voter can then only choose the level of insurance until next vote takes place, *not forever*. With low turnover, insurance is important mostly in the long run, because in the short run the employed is relatively sure of keeping his job. But if the voting cycle has short periods between votes, the median voter can only influence the insurance level during the near future, when he does not care much about insurance. Since the median voter has to bear the cost of the insurance he will thus vote for low or no insurance. We show that the voting cycle may have to be very long for the sequential voting outcome to be close to the permanent insurance case.

In a low turnover economy everybody would benefit from having a high insurance level. However, the tension between the short run good and the long run best becomes more

[5] Gruber [3] analyzes this consumption smoothing effect of unemployment insurance.

severe, so the long run best becomes more difficult to achieve. Broadway and Wildasin [1] discuss pension systems and note that, if voters do not anticipate any effects of their own votes, young non-altruistic voters would not vote for high pensions paid by high taxes on labor if the period until next vote is short relative to the time remaining to his retirement. Here a similar mechanism is at work.

The paper is organized in the following way. In section 2 we construct the basic model we will use in the paper. In section 2.1 the individuals are not allowed to save and borrow, an assumption that is relaxed in section 2.2. In section 2.3 we allow the unemployed to choose different turnover levels and establish the possibility of multiple equilibria. In section 3 we analyze the effects of introducing sequential voting and section 4 concludes the paper.

2 Preferences for Unemployment Insurance

2.1 A flow Model Without Savings

Consider the following discrete time search model. The individuals receive a net income of w_e when working. When not working, the individual receives unemployment benefits denoted w_u. If an individual is employed there is an exogenous probability q that he will loose his job between the current and next period. Similarly, any unemployed may get a job the next period with probability h. The only state variable for the individual is the employment status l that can take the values e and u, denoting employed and unemployed.

The value function $V(l)$, i.e., the sum of expected future discounted utility must satisfy

$$\begin{aligned} V(e) &= U(w_e) + \tfrac{1}{1+r}((1-q)V(e) + qV(u)) \\ V(u) &= U(w_e) + \tfrac{1}{1+r}((1-h)V(u) + hV(e)) \end{aligned} \tag{1}$$

where $U(\cdot)$ is the per period utility function and r is the subjective discount rate. It is then straight forward to verify that the expected utility in the two states is given by

$$\begin{aligned} V(e) &= \tfrac{(1+r)(r+h)U(w_e)+qU(w_u)}{r(r+h+q)} \\ V(u) &= \tfrac{(1+r)(r+q)U(w_u)+hU(w_e)}{r(r+h+q)} \end{aligned} \tag{2}$$

After substituting $U(\cdot)$ for the CARA utility function $-e^{-\gamma c}$, we can use the value function to study individual preferences over different unemployment insurance schemes. We will, in particular, look at unemployment insurance that is financed by a pay-roll tax τ, which is required to be non-negative. The amount paid in taxes, minus proportional administration costs $a \geq 0$ is distributed to the unemployed.[6] We will focus on steady

[6] We make the realistic assumption that the insurance system do not distribute exactly as much as

states so that the share of unemployment is constant. In this model the only state variable for the individuals is the employment status. There will thus only be conflicting interests with respect to the level of τ between employed and unemployed. We will assume that the employed are decisive and we will thus concentrate on their preferences over τ[7].

Let d be the dependency ratio, i.e., the ratio of unemployed to employed. In a steady state the flow of individuals to and from unemployment pool per unit of time must be equal. This implies that $d = q/h$. The net wage and the unemployment benefits satisfy

$$\begin{aligned} w_e &\equiv w(1-\tau) \\ w_u &\equiv \tfrac{w\tau(1-a)}{d} \end{aligned} \tag{3}$$

where w is the gross wage. Now we can maximize $V(e)$ with respect to τ. The first order condition for a maximum is

$$\tau = \frac{d}{1-a-d}\left[1 + \frac{1}{\gamma w} ln \left(\frac{q(1-a)}{dr+q}\right)\right] \tag{4}$$

Here we should note that with no administration costs the optimal tax rate is $\tau = \frac{d}{1-d}\left[1 + \frac{1}{\gamma w} ln \left(\frac{q}{dr+q}\right)\right]$. The first term, $\frac{d}{1-d}$, corresponds to full insurance. This is deviated from by the second term in brackets, which is negative provided that the discount rate is strictly positive.

Result 1 *The optimal insurance level (strictly) increases in turnover when individuals have no access to capital markets for borrowing and saving and discounting is (strictly) positive.*

Proof: The derivative of (4) with respect to the separation rate, holding q/h constant at d can be written

$$\frac{rd^2}{\gamma q w(1+d-a)(q+rd)} > 0. \tag{5}$$

With strictly positive discounting full insurance is suboptimal because when moving from perfect insurance the increased risk has only second order negative effects. On the

it receives. Our prefered interpretation of a is administration costs but we could also interpret it as representing a transfer element between different categories of labor. We can, for example, assume that the median voter faces a lower risk of loosing his job than the average probability. In the following we will see that a is important for the analysis.

[7] For now, we are analyzing their welfare for different τ given that it is held constant for ever. The value of τ that maximizes the welfare of the currently employed if held constant may, however, not be attainable in a political equilibrium with short voting cycles.We will return to this issue below.

period	1 month
r	3 % per year
γ	2
w	1
h_{low}	1/18 per month
h_{high}	1/6 per month
d	10%
q_{low}	$h_{low}/10$
q_{high}	$h_{high}/10$
a	0 and 3 %

Table 2: Parameters

other hand, the reduced tax gives more money to spend today while the loss comes in the future when it is valued lower. This has positive first order effects on the value function of the employed. With lower separation rates, this latter effect is stronger since the unemployment period is expected to come further away in the future.

2.2 Allowing Savings

Now assume that the individuals can save and borrow but not privately insure the unemployment risk. On their financial assets, denoted A_t, they receive an interest rate which, for simplicity, coincides with their subjective discount rate r. In addition to the employment status l, also the amount of financial assets now enter as an arguments of the value function. The finite horizon value functions are given by

$$V_t(A_t, l_t) = \max_{c_s} E_t \sum_{s=t}^{T} (1+r)^{-s+t} e^{-\gamma c_t}$$

$$s.t. \begin{cases} A_{t+1} = (1+r)(A_t + w_t - c_t), \\ A_t \text{ given}, \\ A_{T+1} \geq 0. \end{cases} \tag{6}$$

with

$$w_t = \begin{cases} w(1-\tau) \text{ if } l_t = e \text{ (employed)} \\ \frac{w\tau(1-a)}{d} \text{ if } l_t = u \text{ (unemployed)} \end{cases}$$

$$l_{t+1} = \begin{cases} e \text{ with probability } (1-q) \text{ if } l_t = e \\ u \text{ with probability } q \text{ if } l_t = e \\ u \text{ with probability } (1-u) \text{ if } l_t = u \\ e \text{ with probability } h \text{ if } l_t = u \end{cases} \tag{7}$$

In the appendix we show that as the horizon T goes to infinity the value functions converge to

$$\begin{aligned} V(A_t, e) &= -\tfrac{1+r}{r} e^{-\gamma\left(\frac{r}{1+r}A_t + c_e\right)} \\ V(A_t, u) &= -\tfrac{1+r}{r} e^{-\gamma\left(\frac{r}{1+r}A_t + c_u\right)} \end{aligned} \tag{8}$$

where c_e and c_u are constants that are determined from the first order condition of the Bellman equation. We also show that consumption equals the annuity value of financial assets plus a constant that depends on the current employment status, i.e.,

$$c_t = \begin{cases} \frac{r}{1+r}A_t + c_e, \text{if employed} \\ \frac{r}{1+r}A_t + c_u, \text{if unemployed} \end{cases} \tag{9}$$

In appendix A.2 we show that there always exists an unique solution to the maximization problem. Additionally we show in that the constants in 9 satisfy

$$w(1-\tau) \ > c_e > c_u > \frac{w\tau(1-a)}{d}$$

which implies that the individual saves when employed and dissaves when unemployed.

An important feature of the value functions is that wealth only enters the value functions through the multiplicative term $e^{-\gamma\frac{r}{1+r}A_t}$. We can then define

$$V(l) \equiv V(A_t\,, l)e^{\gamma\frac{r}{1+r}A_t} \tag{10}$$

Clearly, we can then use $V(l)$ to find individual preferences over different values of τ . Since $V(l)$ is independent of A_t preferences over different values of τ are also independent of A_t.[8]

To illustrate the analysis let us consider a numerical example. We have used the parameter values in Table 2. In the upper panel of Figure 2. We have plotted the expected utility of an employed for different values of the replacement ratio, i.e., the ratio of net wages to unemployment benefits, when the administrative loss a is zero.[9] In the high turnover case, the hiring rate h is set to give an expected duration of unemployment of 6 months.[10] In the low turnover case the duration is 18 months. The separation

[8] This, of course, results from the use of the constant absolute risk aversion function in conjunction with a risk level that is constant in absolute size.

[9] As seen from the expression for the value functions, $V(e)$ and $V(u)$ are monotone transformations of c_e and c_u so we plot the latter as functions of the replacement ratio.

[10] We use continuous time for convenience when we calculate the expected value and variance of the duration.

rates are proportional to the hiring rates so that the dependency ratio, defined as the ratio of unemployed to employed, is 10% in both cases. We see that the value of the replacement ratios that maximizes utility of the employed approximately coincide. The maximum is such that the unemployment compensation is 69% and 66% of the net wage while employed.

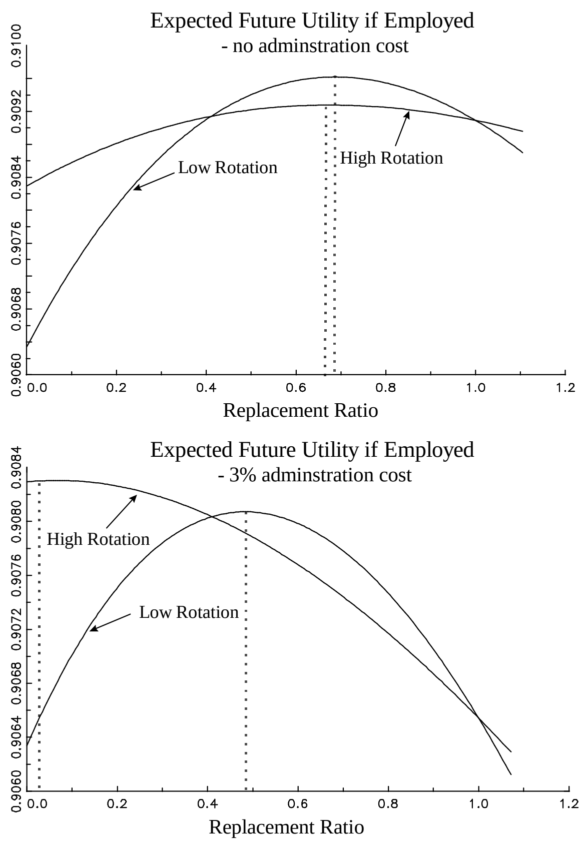

Figure 2: Value functions of employed

There is, however, an important difference between the two curves - the curve in the high turnover case is much flatter. This implies that the value of the insurance, i.e., utility loss of not having an unemployment insurance, is smaller in the high turnover case. Equivalently, a small administrative loss in the system will change the preferred insurance level more when turnover is high. This is illustrated in the second panel in the figure. There we have calculated the expected utility for the two cases, now setting a equal to 3%. In the high turnover case, we get a dramatic reduction in the optimal unemployment compensation, which now falls to as low as 3%. With just slightly higher administration costs, the optimal level falls to zero. With low turnover the effect is much smaller. Optimal unemployment compensation falls to 48%.

Why is it then, that insurance is more important in the low turnover case? An intuitive explanation is that low turnover means that an income shock associated with a job loss is more persistent than with high turnover. It is well known that saving and borrowing is a good substitute to insurance when shocks have low persistence. Note also that the variance of the length of the unemployment spells equals $1/h^2$ and thus increases with the expected length of the unemployment period. Due to the law of large number an individual that expects to see many spells of unemployment/employment during some given horizon faces less uncertainty than his fellow with few but longer spells of unemployment. With high turnover the individual's problem is largely to translate his variable income into a smooth consumption stream. This can be done by the insurance system but equally well by saving and using the credit market. A small inefficiency in the insurance system, like the administrative costs, makes the capital market preferable. With low turnover on the labor market the opposite is true, now insurance is important since one unusually long unemployment spell, with large effects on lifetime utility, is much more likely. For example, the probability that a currently unemployed has to wait more than three years before finding a job is 13.5% in the low turnover case but only 0.2% in the high turnover. The probabilities of more than five years of unemployment are 3.6% and 0.005% respectively.

Now let us consider how the optimal tax and insurance rates vary with employment turnover. The following two results show that this cannot be a monotonic relation.

Result 2 *If turnover is zero the value of unemployment insurance that maximizes the utility of the employed workers is zero.*

Proof: In appendix A.4

The intuition of this is straightforward; if an individual is working and employment turnover is zero, taxes and unemployment insurance is a pure transfer to the unemployed. The employed thus prefer the lowest possible tax rate, which is assumed to be zero.

Result 3 *As turnover with a given level of unemployment increases to infinity the value of the unemployment insurance that maximizes the utility of unemployed workers converges to zero when savings are allowed.*

Proof: In appendix A.3.

The last results holds also when administration costs are zero, although then the employed (and unemployed) are indifferent to the level of insurance.

The implication of these results is that intermediate degrees of employment turnover is required for the employed to support unemployment insurance. The intuition is here as follows. As rotation goes to infinity, the value functions in the two states converge, i.e., the current employment state has zero impact on the value of expected future utility. Then, when consumers have access to a capital market for consumption smoothing, consumption will be independent of employment status. Insurance is then of no value. This is certainly not the case when no capital market exists, since in this case consumption in the two states by assumption differs when unemployment insurance is imperfect.

We now know that the optimal (for the employed) tax level is zero both when the turnover is zero and when it is very high. Both common sense and our previous simulations indicate that there are parameter levels for which the optimal tax rate is positive. The following result, and some numerical simulations will shed light on the relation between the optimal tax rate and intermediate rates of turnover.

Result 4 *Let δ denote the tax level corresponding to full insurance, and D the relative utility in the two states, i.e., let $\delta \equiv \frac{1}{1+\frac{1-a}{d}}$ and $D \equiv \exp\{\gamma(c_e - c_u)\}$. Additionally define $\tilde{D}$ implicitly from*

$$\delta\ (1+r) = \delta\,\frac{(1-h)\ \tilde{D}}{(1-h)\ \tilde{D}+h} + (1-\delta)\ \frac{q\,\tilde{D}}{1-q+q\,\tilde{D}}$$

Then, in an interior maximum the optimal tax rate satisfies

$$\tau = \delta - \delta\frac{\ln\left(\tilde{D}^{1+r}\right) + \ln\left(1-q+q\ \tilde{D}\right) - \ln\left((1-h)\ \tilde{D}+h\right)}{\gamma\ r\ w} < \delta \tag{11}$$

and if $\delta\,(1+r) > 1$ the non-negativity constraint on τ will bind so the optimal tax and insurance rate are zero.

Proof: *In appendix A.4*

Unfortunately (11) defines a very nonlinear function and we can only establish analytically that for some parameters ranges it increases and for others it decreases. To analyze the characteristics of the relation between turnover, administration costs and optimal tax rates we thus have to resort to numerical examples. In the upper panel of Figure 3 we plot the optimal replacement ratio for employed workers against turnover, as measured by the hiring rate, for different values of the administration costs. The other parameters are given in Table 2 and the separation rate q is set to $h/10$ so that the dependency ratio and thus unemployment is kept constant regardless of the rate of turnover. The lower panel shows the same relation but now with turnover measured by the expected duration of the unemployment period.

In Figure 3 we see that for all levels of administrative costs, the optimal tax level is very steep and increasing for low rates of turnover. The highest optimal tax rates are achieved for low rates of turnover, and that these maxima are achieved at a lower rates of turnover the higher the administrative costs are. For the four examined values of the administration cost (0, .03, .10, and 0.30), the maximum replacement ratio occurs at turnover rates corresponding to an unemployment duration of 18, 80, 143 and 250 months.

From that point on the optimal tax level is monotonically decreasing with turnover.[11] We should also note that the decreasing portion of the schedules in the upper panel of figure 3 is flatter the lower is the administration cost. This reflects, as was previously stated, that as the turnover rate increases, saving, i.e., self-insurance, becomes a better substitute for unemployment insurance. So, the higher the rate of turnover, the larger is the substitution towards self-insurance, i.e., the fall in the optimal replacement ratio, for a given increase in administration costs.

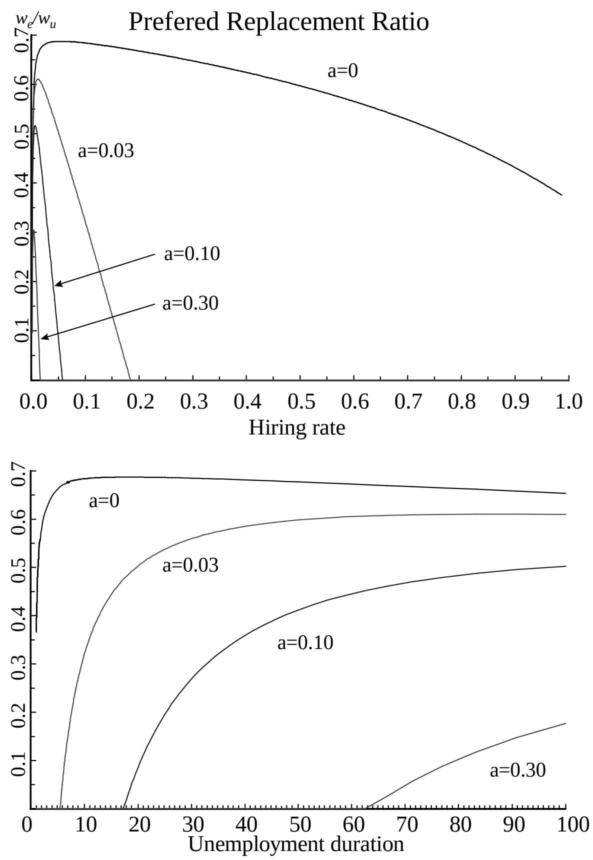

Figure 3: Prefered replacement ratio of employed for different administration costs and turnover rates.

To understand our results it is useful to think of two aspects of unemployment insurance: insurance and redistribution.[12] The insurance aspect creates a positive motive for unemployment insurance. The redistribution aspect of the insurance, on the other hand, creates a negative motive for (the egoistic) employed. At zero turnover unemployment insurance is a pure redistributive scheme. The redistribution aspect then becomes

[11] We have not been able to prove that the relation between turnover and optimal taxes is single-peaked, nor have we found any combinations of parameters such that this is not the case.

[12] This is clearly exposed in Wright [6]

monotonically less important as turnover increases. The insurance motive for unemployment insurance, on the other hand, is zero when turnover is zero. For low turnover rates it becomes high since income shocks in this case are very persistent. The insurance motive then falls as turnover increases. As noted previously, this is so because at higher turnover rates self-insurance via saving and borrowing becomes better as a substitute for collective insurance.

2.3 Multiple Equilibria

In the previous section we took the hiring and separation rates as exogenous. Here we will make them choice variables of the entrants to the labor market. The aim is to construct a stylized model where the current level of turnover is determined by choices made by optimizing agents.[13] For simplicity we assume that the labor market entrants can choose between a low and a high turnover strategy. We think of this as a choice of which type of human capital to acquire, specialized or general. Specialized human capital is demanded in a traditional sector where the matching process is more complicated so both hiring and separation rates are low and wages are higher. General human capital is demanded in a high turnover sector. If a person choose the low turnover strategy the probability of finding a job is low (h_{low} per period) but the jobs are relatively safe with a low separation rate (q_l per period). With the high turnover strategy the probability per period of finding a job is higher (h_{high}) but the separation rates (q_{high}) are also higher. If one gets a job, the wages are w_l and w_h with $w_{low} \geq w_h$.

Each period new workers enter the labor force. Before entering the labor market the individuals have to make a permanent choice between the two turnover strategies. Clearly, the level of unemployment benefits is one of the crucial parameters determining which of these two strategies is optimal. The higher is the unemployment insurance, the less costly is it to wait for the secure job in the traditional sector.

The level of unemployment benefits, in turn, is determined by the employed. As discussed above, individual preferences over τ will only differ depending on whether they are unemployed or not. We assume that the employed are politically decisive and that they choose a $\tau \geqslant 0$ that is fixed forever thereafter to maximize their expected utility. On the agenda is thus not any proposals to set unemployment insurance to different levels conditioning on different events or dates. Given this the chosen tax rate is the one that maximizes $V(e;\tau)$ over τ. The assumption that taxes and insurance levels are set forever and that the turnover choice of an individual cannot be reversed may by critical for the results and we will discuss this in the concluding section.

The issue is now whether we in this setup can generate two equilibria, each supporting the current level of turnover. We will look for history dependent equilibria with the

[13] Hall [4] notes that the duration of unemployment periods depends positively on the duration of the previous job which gives some support for the assumption of a permanent choice of rotation.

following property.

- Given the insurance implied by the vote made by the current median voter, the current labor market entrants will make the same turnover choice as the current median voter did.

With a slight abuse of notation we will call such an equilibrium stable if later generations of median voter would not want to change the level of insurance if they where given a chance to do so.

Consider first the employed in a low turnover equilibrium where the employed have jobs with low lay-off probabilities. In the previous section we found that they would set the tax rate is 4.85% giving a replacement ratio of just below 50% given the parameters in Table 2 with a set to 3%. In the upper panel of Figure 4 this replacement ratio is marked by a vertical line.

Now turn to the unemployed. They can choose between working in any of the two sectors. The expected utility of the two strategies is depicted with solid curves in the upper panel of Figure 4. We assume that the unemployment insurance agency cannot discriminate between individuals based on their rotation choice. The unemployment benefits is thus $\tau w(1-a)/d$ where w is the (average[14]) wage among employed. We use the same parameter values as before except that we set the wage for low rotation workers (w_{low}) to 1.02.

We see that the schedule for the utility of choosing the low turnover sector is steeper than the other schedule. So for high enough values of τ and correspondingly high unemployment benefits, choosing the traditional low employment turnover strategy is dominant. As we see the replacement ratio chosen by the employed is above the point at which the curves of the two strategies cross, which occurs at a replacement ratio of 35%. At the replacement ratio chosen by the employed, low turnover yields higher utility. Low turnover is thus an equilibrium. Furthermore, we should note that a decrease in the wage in the traditional low turnover sector shifts the whole schedule for that choice downwards. So, low turnover is an equilibrium if wages in the low turnover sector are higher than $\underline{w}$ which would produce an expected utility of the low turnover strategy depicted by the dotted curve. Later generations of employed face an identical problem that the current generation so they would not like to make a once and for all change in the benefit level. The equilibrium is thus stable in that sense.

Now turn to the other potential equilibrium, where the currently employed are in the high turnover sector. As shown in the previous section, they choose a tax rate to 0.7% corresponding to unemployment benefits of just below 7% of the wage rate when the

[14] In the equilibria we consider all individuals do the same and do consequently have the same wage when employed.

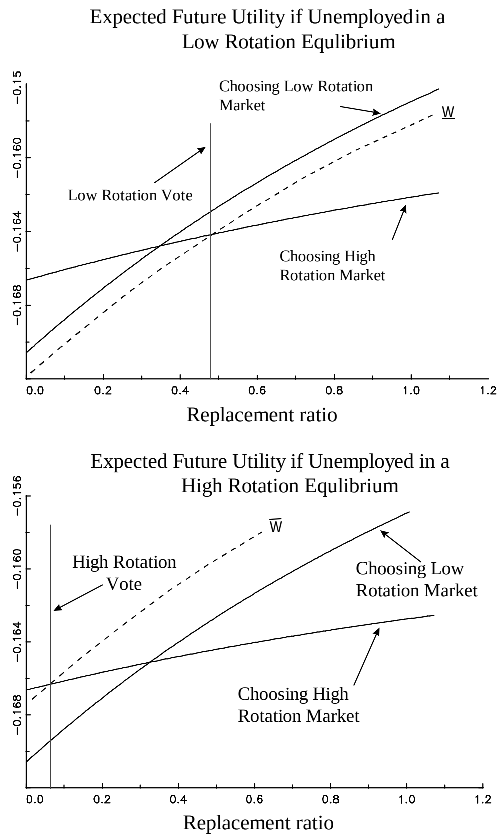

Figure 4: Valuefunctions of unemployed

administrative cost is 3%.[15] This level of benefits is marked by a vertical line in the lower panel of Figure 4. The two solid curves in the figure denotes the expected utility for the two turnover choices for different replacement ratios when the wage premium is 2% in the low turnover sector. We see the replacement rate is below the level at which the two solid curves cross, which occurs when unemployment benefits are 35%. Given the choice of unemployment benefits that the employed make, the entrants to the labor market thus choose the high turnover strategy. The high turnover equilibrium is thus an equilibrium and it is stable in the sense that coming generations of employed do not wish to make a once and for all change in insurance. The highest wage premium in the low turnover sector such that the high turnover equilibrium is supported is $\overline{w}$.

The result of this sections can be summarized.

Result 5 *If the wage in the low turnover sector is higher than* $\underline{w}$ *low turnover is a stable equilibrium. If the wage in the low turnover sector is lower than* $\overline{w}$ *high turnover is also a*

[15] The careful reader might recall that we have increased the wage to 1.02 which, given constant relative risk aversion, changes the optimal tax rate. The change is, however, neglible.

stable equilibrium. There exists a set of parameter values such that $\overline{w} > \underline{w}$ in which case both low and high turnover are history dependent equilibria.

3 Sequential Voting

3.1 Voting over one period

Let us now consider the case when the median voter chooses the tax rate $\tau \geqslant 0$ and the corresponding unemployment benefits each period. We assume that the median voter decides what tax rate and unemployment benefits are going to apply next period, i.e., before he knows whether he has become unemployed or not. We also assume that voting takes place each period and that future median voters can not be constrained by any binding arrangements but are free to vote however they prefer. In this section we treat the level of turnover as exogenous.

In addition to assets and employment status, the value functions now clearly depends on both the tax rate that is determined in the current periods and applies in the next and on the tax rates that will apply thereafter. Let $V(A_t, e, \tau, \tau^e)$ denote the value function for the employed median voter who at time t expects taxes to be set to τ^e from $t+2$ and himself sets the tax rate to τ for $t+1$.[16] In the appendix we show that we can write

$$\begin{aligned} V(A_t, e, \tau, \tau^e) &= -e^{-\gamma \frac{r}{1+r} A_t} \frac{1+r}{r} e^{-\gamma c_{e,1}} \\ V(A_t, u, \tau, \tau^e) &= -e^{-\gamma \frac{r}{1+r} A_t} \frac{1+r}{r} e^{-\gamma c_{u,1}} \end{aligned} \tag{12}$$

where $c_{e,1}$ and $c_{u,1}$ are functions of the parameters of the problem and τ and τ^e. Consumption is given by

$$c_t = \begin{cases} \frac{r}{1+r} A_t + c_{e,1} & \text{if employed} \\ \frac{r}{1+r} A_t + c_{u,1} & \text{if unemployed} \end{cases} \tag{13}$$

From (12) we see that expected utility is proportional to $e^{-\gamma \frac{r}{1+r} A_t}$ so preferences over τ are independent of wealth. As before, this makes it easy to identify the median voter as any of the employed. Furthermore, this implies that there is no strategic motive involved in voting. Changing the tax rate for $t+1$ only affects the asset distribution in the future. Since assets are irrelevant for preferences over tax rates, the current median voter cannot affect future votes if we restrict the attention to Markov strategies. He thus only have to consider what is his preferred tax rate until next vote. This implies that the median voter solves

[16] The tax rate in the current period is also set to τ^e, although it is trivial to change that assumption. With some tedious work we could compute the value function for all possible sequences of tax rates. This is, however, not necessary for our purpose.

$$\max_\tau V(A_t, e, \tau, \tau^e). \tag{14}$$

A time consistent dynamic voting equilibrium must then have the property that if the current employed median voter believes that the decision on the tax rate is going to be τ^e from the next period and onwards he votes for τ^e today. We thus require that the solution to (14) equals τ^e .

The first issue is now whether the tax rate that is optimal for the currently employed if was fixed forever, defined as τ^*, can be sustained in a sequential voting equilibrium. The following result states that this is not the case.

Result 6 *The temptation to deviate from a strictly positive long run optimal insurance level by reducing it for the next period is strictly positive. A positive long run optimal insurance can thus never be sustained when the median voter sets the tax rate for one period at a time.*

Proof: In appendix A.6.

This result implies that there is a tension between the long and short run interest of the employed. It turns out that this tension may be quite strong. For the ranges of the parameter values we have considered, the median voter prefers the corner solution $\tau = 0$ for all expectation about future τ . When this is true, we have a much stronger version of the previous result, namely the only possible time consistent dynamic voting equilibrium is thus zero unemployment insurance.

It may also be of interest to quantify the temptation to deviate from the long run optimal insurance and set it to zero during the next period. We are in particular interested in how this temptation varies with the degree of turnover. To do this we compute the optimal long run tax rate τ^* for different values of the turnover, holding unemployment constant as before. We then calculate the calculate $V(A_t, e, 0, \tau^*)$, i.e., we calculate $c_{e,1}$ when next period's tax and insurance are zero but are set to the long run optimum thereafter.

Knowing $c_{e,1}$it is straightforward to calculate the equivalent variation of a one period deviation to zero insurance. From 12 and 13 we see that a cash transfer of $(c_{e,1} - c_e)\frac{1+r}{r}$ yields the same consumption and utility increase as $(c_{e,1} - c_e)$. The former value can thus be interpreted as the equivalent variation, EV. We then calculate the cost of the next periods insurance, i.e., its expected discounted price $(1-q)\frac{\tau^*}{1+r}$, denoted P.

In Figure 5 we plot $(P - EV)/P$ against the expected lengths of the unemployment period $(1/h)$. The separation rate, q is as before adjusted to keep unemployment constant. If the depicted ratio is unity, the equivalent variation of removing the insurance for one period is zero, i.e., the insurance is worth its price. If the ratio is zero, on the other hand the employed view the insurance during the coming period as a pure transfer to the unemployed, i.e., they do not value the insurance component at all. For intermediate values they assign some value to the insurance component. We say that the temptation to deviate is larger the lower is $(P - EV)/P$.

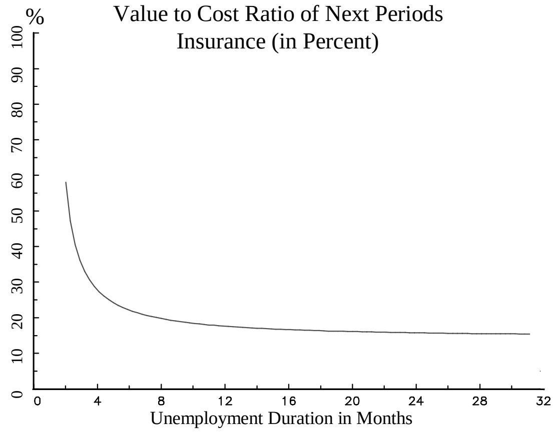

Figure 5: One period temptation to deviate

In the figure we see that the temptation to deviate increases as turnover decreases. For the highest degrees of turnover, the value of the insurance is around half its price. For longer durations most of the price of unemployment insurance is a pure transfer to the unemployed, generating a substantial temptation to deviate.

Result 7 *The temptation to deviate from the long run optimal insurance level and set it to zero for the next period increases as the turnover decreases.*[17]

3.2 Voting over several periods

Now let us consider the intermediate case when taxes can be fixed for some finite number $s > 1$ of periods. As above, we assume that the tax rate is set one period before it starts to apply, so the voter does not know his employment status he has when the tax rate he votes for starts to apply. The tax rate that is determined at t thus applies to $t+1 \ldots t+s$. In the appendix we derive the value functions for the case when the tax rate is set to τ for s periods and thereafter set to τ^e , where τ and τ^e are allowed to take any value $\geqslant 0$. In addition to assets and employment status, the value functions clearly depends on s, τ and τ^* . We show in the appendix that the value functions have the following form

[17] We have not been able to prove this analytically so the generality of this result remains to be established.

$$\begin{aligned} V(A_t, e, \tau, \tau^e, s) &= -e^{-\gamma \frac{r}{1+r} A_t} \frac{1+r}{r} e^{-\gamma c_{e,s}} \\ V(A_t, u, \tau, \tau^e, s) &= -e^{-\gamma \frac{r}{1+r} A_t} \frac{1+r}{r} e^{-\gamma c_{u,s}} \end{aligned} \tag{15}$$

where $c_{e,s}$ and $c_{u,s}$ depend on parameters, s, τ and τ^e. Consumption at the beginning of the s periods is given by

$$c_t = \begin{cases} \frac{r}{1+r} A_t + c_{e,s} & \text{if employed} \\ \frac{r}{1+r} A_t + c_{u,s} & \text{if unemployed} \end{cases} \tag{16}$$

To analyze preferences over tax rates τ for given levels of s and τ^e we can thus disregard wealth and voting is non-strategic. In the previous section we found that with a one period voting cycle the median voter always preferred zero taxes and unemployment insurance. Increasing the voting cycle increases the insurance motive by increasing the risk of being unemployed for a longer and longer period during which the unemployment benefit in question is to apply. With a longer voting cycle we would thus expect the median voter to become more favorable to high taxes and unemployment insurance during the next voting cycle. How the length of the voting cycle affects insurance preferences may also depend on the turnover rate, since the difference between the long and short run interests of the employed depends on turnover.

To study consider the following experiment. Set τ^e to the value preferred by an employed agent in the low and high turnover economies if it was to be fixed forever. As we saw in previous sections, without administrative losses in the system this tax rate corresponded to a replacement ratios of 69% and 66%. Let us now consider whether an employed would prefer taxes to be set to zero during the coming s periods (months) rather than being kept at τ^e all periods. Clearly, this will depend in the horizon, and we expect that for a long enough horizon, τ^e may be preferred to zero. In the previous section we found that if s is 1, zero taxes is preferred while if s was infinity τ^e (as well as all other lower taxes) was strictly preferred to zero.

In Figure 6 we plot the temptation to deviate from the long run optimal insurance, represented by the consumption increase it would generate $(c_{e,s} - c_e)$ at the time of the deviation against s, expressed in years for the two cases. We should note two things here:

- First, in both turnover cases the median voters prefer zero insurance during the coming voting period also if the voting cycle is quite long. We actually need voting cycles in excess of 20 years for the median voter to prefer the long run optimal insurance over zero.

- Second, the value the employed would attach to a deviation to zero insurance for one voting period for shorter voting cycles is substantially higher when turnover is

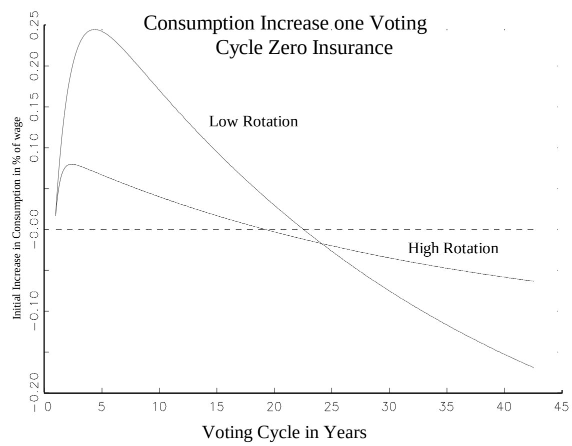

Figure 6: Temptation to deviate

low. When the voting cycle is around 5 years, the value of deviating from the long run high insurance optimum, as measured by the increase in consumption, is several times higher in the low turnover case. The employees' temptation to deviate from a high insurance is thus particularly strong when turnover is low.

We may now look for a time consistent dynamic voting equilibrium. Here this means that if the median voter, who is employed, expects a tax rate of τ^* from the next voting period and onwards, he votes for the same tax rate now and thus sets $\tau = \tau^e$ for the coming s periods. We thus reuire that a time consistent dynamic voting equilibrium for the tax rate τ^e satisfies

$$\tau^e = \arg\max \tau V(A_t, e, \tau, \tau^e, s) \tag{17}$$

From the previous results we expect that the voting cycle has to be rather long to generate non-trivial tax rates and unemployment benefits. We thus use a 15 year voting cycle. In the two panels of Figure 7, we plot the value functions represented by $c_{e,s}$ for the two turnover cases, which we, as above, take to be corresponding to an expected unemployment duration of 18 and 6 months. The straight lines represent the expected utility of τ^e corresponding to a replacement ratio of 18.3 and 27.8% forever. The curved lines are the expected utility for different values of τ represented by their corresponding replacement ratios when the replacement ratio after the current voting cycle of 15 year are 18.3 and 27.8% respectively. If 18.3 and 27.8% satisfies (17) it must be that the maximum of the curved lines is achieved at these replacement ratios and that the curved lines' maxima are tangents to the straight lines. We see that this is true in graphs. Two things should be noted. First, 15 years voting cycles is not enough to support particularly high replacement ratios. Second, the higher temptation to deviate in the low turnover case that was depicted in figures 5 and 6 translates into a lower supportable replacement ratio in the low turnover case.

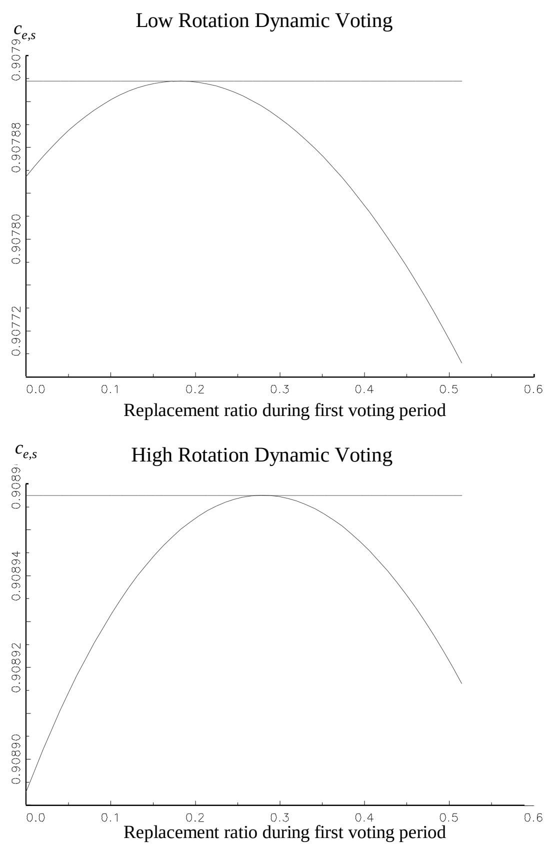

Figure 7: Dynamic voting equilibrium with 15 year voting cycle.

For short voting cycles no positive unemployment insurance can be sustained in a dynamic voting equilibrium with Markov strategies.[18] With a four year voting cycle, the median voter always prefers zero insurance during the coming voting period. With higher insurance in the future, the expected utility of an employed increases, and particularly so in the low turnover case, but he still prefers zero insurance during the coming four years.

As in the previous section, we can evaluate the temptation to deviate to zero insurance for the coming voting cycle from the long run optimal τ^* by studying the consumption increase such a deviation would generate. We thus use 16 and express the temptation to deviate as the consumption increase it would generate as a function of the level turnover. In Figure 8 we report this for different lengths of the voting cycle. We see that the

[18] If we relax the Markov assumption this may, of course, not be true.

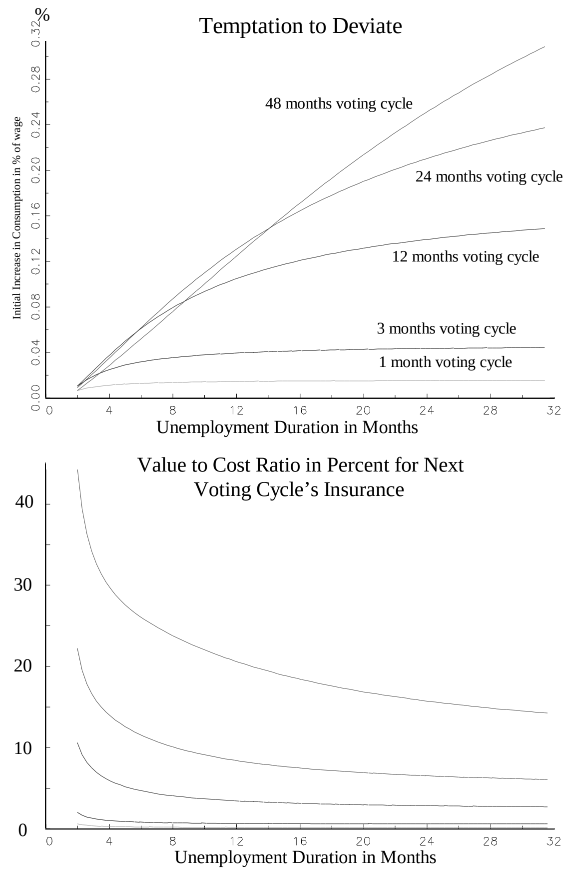

Figure 8: Temptation to deviate and unemployment duration

temptation to deviate increases when turnover decreases. We also see that the temptation to deviate tends to increase with the length of the voting cycle when turnover is low enough. Our interpretation of this is the following. Fix the voting cycle. As turnover then decreases, the value of the insurance during the next voting cycle decreases and the temptation to deviate thus increases. The temptation to deviate can, however, never be larger than the cash value of the insurance premiums payed during the voting cycle. This puts an upper limit on the temptation to deviate. This limit increases proportionally as the voting cycle increases. As turnover decreases this upper limit is approached. When turnover is short relative the voting cycle the insurance component is more important which may produce a non-monotonicity of the temptation to deviate with respect to the voting cycle as can be seen in Figure 8.

4 Concluding Discussion

Before making some concluding remarks let us summarize what we believe are the main results of our analysis.

First, the degree of flow in and out of unemployment is important for preferences over different levels of unemployment insurance. Low turnover means that the income shock associated with a job loss is highly persistent. In this case self-insurance via a capital market is a bad substitute for unemployment insurance. It is then in the long run interest of the employed (as well as, of course, of the unemployed) to have an unemployment insurance with high replacement ratios, also when the system is inefficient or actuarially unfair for other reasons.

Second, given that the degree of turnover directly or indirectly is a choice variable, it will be affected by the unemployment insurance. High unemployment insurance would tend to make the unemployed search longer. Under some circumstances this could create multiple equilibria, one with high insurance and low turnover and one with low insurance and high turnover.

Third, with sequential voting and no possibility to fix the insurance system forever, the tension between the long and short run interest of the employed is stronger with low turnover rates. The expected utility increases more for the employed if unemployment insurance is introduced the lower is turnover. However, the lower the turnover the stronger is the immediate temptation to reduce the insurance and the taxes that finance it during the near future. Low turnover also increases the tension between employed and unemployed by increase the difference in relative expected utility.

We have shown that the first result above critically depended on the assumption that people can save and borrow. In our model we made the rather unrealistic assumption that everybody has access to a perfect capital market. We thus need to consider the potential consequences of borrowing limits and other capital market imperfections. Such assumptions would complicate the model substantially, in particular, preferences over the insurance level would depend on current assets, thus making it more difficult to find the median voter. Probably more important is that if preferences depend on assets the current median voter can change the behavior of future median voter by affecting the wealth distribution via the insurance system. Finding a dynamic voting equilibrium is then difficult at best (see Krusell et al. [5]).

We do, however, believe that such modifications of the model would not change the first result qualitatively. When borrowing limits exist the persistence in the shocks becomes even more important. With low persistence, borrowing constraints are less likely to be binding and would not affect substantially precautionary savings. On the other hand, with high persistence, a borrowing constraint may have a substantial effect on precautionary savings and expected utility if no insurance exists. Our conjecture is thus that borrowing constraints would tend to enforce the mechanism discussed in this paper by making self-insurance an even worse substitute for unemployment insurance when employment turnover is low. Other capital market imperfections should have similar effects since alternatives to the capital market, like borrowing from friends and family or driving down household durables, work as a reasonable substitute for the capital market unless shocks are too persistent.

For convenience we have relied on the exponential (constant absolute risk aversion) utility function, which, together with the assumptions of constant wages and infinite horizons produces value functions of relatively simple form. That we use this function is one reason for us to stress that the quantitative results have to be taken at face value. We do not think that our model is particularly good at pinpointing quantitative results. For this, simulation, in, for example, a stochastic growth model is more suitable. Certainly, quantities are important for the issues we address. We have, however, in this paper tried to illustrate mechanisms that potentially may be important quantitatively, rather than providing any direct quantitative results.

Our second conclusion was that high unemployment insurance could contribute to a lower degree of turnover. In the model we let the choice of turnover be made by the labor market entrants. With higher unemployment insurance the unemployed can "afford" to wait longer before accepting a job offer, thus reducing the hiring rate. This could have a positive effect on the productivity of the match so that wages as well as the duration of the match increases. The assumption that the turnover choice is made by the entrants before they find a job is made mostly for convenience. We do not believe that the mechanism we point to is the only or even the most important mechanism. Certainly the low firing rates in Europe may to a large extent be due to job security legislation lobbied for by incumbent employees. Introducing this into the model complicates the analysis since the voter then would have a two dimensional choice to make. We do, however, think that if the employees could affect job turnover their choice of turnover would be affected by the insurance level in the same direction as in our model.

We have assumed that the choice of turnover is made once. We believe that the main results would be unchanged if this choice could be reversed as long as there is a sufficiently high cost associated with this. It may, of course, be the case that this cost has to be high to support the low turnover equilibria. With a low cost of changing, an optimal strategy of the employed in a low turnover economy could be to set a low insurance level and if they become unemployed they switch to a high turnover strategy.

We have not analyzed the issue of multiple equilibria in the case of sequential voting. In principle this should be possible, but the main complication is that the turnover choice of the unemployed creates a link between the votes of different generations of employed. By changing the insurance rate today, the turnover choice will be altered which may affect future votes. This indirect effect of the vote has to be taken into account when computing the optimal choice of the tax rate in the sequential voting case.

The third finding was that the short run and long run interest of the employed tend to diverge as employment turnover is reduced. This implies that it may be difficult to sustain an insurance system with high replacement ratios when turnover is low even though this is in the long run interest of the employed (and, of course, the unemployed). The starting point of this paper was the observation that unemployment insurance is generous and turnover is low in Europe. Our third finding implies that our model is not fully capable of explaining this. The first two findings provide with a motive but the third shows that the motive may not be enough when binding long run arrangements cannot be committed. We do think, however, that our third finding points in an interesting direction. It shows that in Europe the employed have more to gain than their American fellows by building institutions that facilitate long run arrangements. Similarly, the stronger tension between the short run interest of employed and unemployed may also make it more important for the employed to try to build institutions where the unemployed have important weight. We think that universal unemployment insurance coverage, unions and political labor parties all are means that can help achieve such long run social contracts. Such institutions may thus be more likely to develop in low turnover economies.

A Appendix

A.1 Consumption and Value Functions with Infinite Voting Cycle

To derive the infinite horizon solution we start with the finite horizon problem and let the horizon go to infinity. At $T-1$ the value function for an employed individual is given by

$$\begin{aligned} V(A_{T-1}, e) &= \max_{c_{T-1}} \{ -e^{-\gamma c_{T-1}} \\ &\quad - \frac{1}{1+r}[(1-q)e^{-\gamma((1+r)(A_{T-1}+w_e-c_{T-1})+w_e)} \\ &\quad + qe^{-\gamma((1+r)(A_{T-1}+w_e-c_{T-1})+w_u)}]\} \end{aligned} \tag{18}$$

with first order condition

$$\begin{aligned} e^{-\gamma c_{e,T-1}} &= (1-q)e^{-\gamma((1+r)(A_{T-1}+w_e-c_{e,T-1})+w_e)} \\ &\quad + qe^{-\gamma((1+r)(A_{T-1}+w_e-c_{e,T-1})+w_u)} \end{aligned} \tag{19}$$

Now define, $R_s \equiv (1+r)^s \left(\sum t = 0^s (1+r)^t\right)^{-1}$. This is the annuity factor; $R_s A_t$ is the maximum constant consumption level that can be supported with financial assets A_t and s periods left until the last period. The solution to the first order condition (19) can be written as $R_1 A_{T-1} + c_{e,T-1}$. Substituting into the first order condition and simplifying yields

$$
\begin{aligned}
e^{-\gamma R_1 A_{T-1}+c_{e,T-1}} &= (1-q)e^{-\gamma(R_1 A_{T-1}+(1+r)(w_e-c_{e,T-1})+w_e)} \\
&\quad +qe^{-\gamma(R_1 A_{T-1}+(1+r)(w_e-c_{e,T-1})+w_u)} \\
e^{-\gamma c_{e,T-1}} &= (1-q)e^{-\gamma((1+r)(w_e-c_{e,T-1})+w_e)} \\
&\quad +qe^{-\gamma((1+r)(w_e-c_{e,T-1})+w_u)}
\end{aligned} \tag{20}
$$

Using this in the value function yields

$$
\begin{aligned}
V(A_{T-1},e) &= -e^{-\gamma(R_1 A_{T-1}+c_{e,T-1})} - \frac{1}{1+r}e^{-\gamma(R_1 A_{T-1}+c_{e,T-1})} \\
&= -R_1^{-1}e^{-\gamma(R_1 A_{T-1}+c_{e,T-1})}
\end{aligned} \tag{21}
$$

and repeating this for problem of an unemployed agent at $T-1$ yields $V(A_{T-1},u) = -R_1^{-1}e^{-\gamma(R_1 A_{T-1}+c_{u,T-1})}$.

Iterating backwards and using $c_{T-s} = R_s A_{T-s} + c_{e,T-s}$ for employed and $c_{T-s} = R_s A_{T-s} + c_{u,T-s}$ for unemployed, it is easy to verify that

$$
\begin{aligned}
V(A_{T-s},e) &= -R_s^{-1}e^{-\gamma\left(R_s A_{T-s}+c_{e,T-s}\right)} \\
V(A_{T-s},u) &= -R_s^{-1}e^{-\gamma\left(R_s A_{T-s}+c_{u,T-s}\right)}
\end{aligned} \tag{22}
$$

with $c_{e,T-s}$ and $c_{u,T-s}$ satisfying

$$
\begin{aligned}
e^{-\gamma c_{e,T-s}} &= (1-q)e^{-\gamma\left(R_{s-1}(1+r)(w_e-c_{e,T-s})+c_{e,T-s+1}\right)} \\
&\quad +qe^{-\gamma\left(R_{s-1}(1+r)(w_e-c_{e,T-s})+c_{u,T-s+1}\right)} \\
e^{-\gamma c_{u,T-s}} &= (1-h)e^{-\gamma\left(R_{s-1}(1+r)(w_u-c_{u,T-s})+c_{u,T-s+1}\right)} \\
&\quad +he^{-\gamma\left(R_{s-1}(1+r)(w_u-c_{u,T-s})+c_{e,T-s+1}\right)}.
\end{aligned} \tag{23}
$$

The limiting value functions when $s \to \infty$ is

$$
\begin{aligned}
V(A_t,e) &= -\frac{1+r}{r}e^{-\gamma\left(\frac{r}{1+r}A_t+c_e\right)} \equiv e^{-\gamma\frac{r}{1+r}A_t}V(e) \\
V(A_t,u) &= -\frac{1+r}{r}e^{-\gamma\left(\frac{r}{1+r}A_t+c_u\right)} \equiv e^{-\gamma\frac{r}{1+r}A_t}V(u)
\end{aligned} \tag{24}
$$

with c_e and c_u satisfying

$$
\begin{aligned}
e^{-\gamma c_e} &= (1-q)e^{-\gamma(r(w_e-c_e)+c_e)} + qe^{-\gamma(r(w_e-c_e)+c_u)} \\
e^{-\gamma c_u} &= (1-h)e^{-\gamma(r(w_u-c_u)+c_u)} + he^{-\gamma(r(w_u-c_u)+c_e)}
\end{aligned}
\tag{25}
$$

which, as shown in the next section, always has a unique solution for c_e and c_u. Consumption is given by

$$
c_t = \begin{cases} \frac{r}{1+r} A_t + c_e \text{ if } l_t = e \text{ (employed)} \\ \frac{r}{1+r} A_t + c_u \text{ if } l_t = u \text{ (unemployed)} \end{cases}
\tag{26}
$$

From (26) we see that all individuals consume the annuity value of A_t plus a constant which only depends on employment status. All individuals with equal employment status will thus save (or borrow) equal shares of their wage, regardless of wealth.

A.2 Existence and Uniqueness of the Value Function

Let us now show that there always exists a unique solution to (25). Define :

$$
\begin{aligned}
x &= \exp\{-\gamma\, c_e\} \\
y &= \exp\{-\gamma\, c_u\} \\
D = \tfrac{y}{x} &= \exp\{\gamma\ (c_e - c_u)\}
\end{aligned}
\tag{27}
$$

and

$$
\begin{aligned}
W &= \exp\{\gamma\, r w_e\} \\
B &= \exp\{\gamma r w_u\}
\end{aligned}
$$

D is the ratio of the utility of an unemployed to the utility of an employed if they have the same assets. We can then rewrite (25) as

$$
\begin{aligned}
W\ x^r &= (1-q) + q\, D \\
B\, y^r &= (1-h) + h\, D^{-1}
\end{aligned}
\tag{28}
$$

Giving

$$
\frac{W}{B} = D^r\ \frac{(1-q) + q\, D}{(1-h) + h\, D^{-1}}
\tag{29}
$$

Note that (28) and (29) only have one unknown, D. If we find a solution for this equation we have also found x and y (given (28)), and consequently c_e and c_u, as well as $V(L)$ and $V(U)$. Now let the function $A(D)$ be defined by the RHS of (29). It is easy to see that $\frac{\partial A(D)}{\partial D} > 0$ and

$$\begin{aligned} A(1) &= 1 \\ \lim_{D \to 0} A(D) &= 0 \\ \lim_{D \to \infty} A(D) &= \infty \end{aligned} \tag{30}$$

Given that the left hand side of (29) is constant, and the right hand side monotonically increasing and with a range that goes form 0 to ∞, the solution to (29) has to exist and be unique. Additionally the solution to (29) requires that

$$\begin{aligned} &\tfrac{W}{B} > 1 \Leftrightarrow D^* > 1 \\ &\tfrac{W}{B} = 1 \Leftrightarrow D^* = 1 \\ &\tfrac{W}{B} < 1 \Leftrightarrow D^* < 1 \end{aligned} \tag{31}$$

If the employed are politically decisive, the utility will be higher for employed than for unemployed. This implies that the net wages are higher than unemployment benefits so $\frac{W}{B} > 1$. Given this, it is easy to observe that $w_e > c_e > c_u > w_u$. This follows from, $D > 1$ and (28)

$$\begin{aligned} w_e - c_e &= \frac{\log\left((1-q) + q\ D\right)}{\gamma\ r} > 0 \\ w_u - c_u &= \frac{\log\left(1 - h + h\ D^{-1}\right)}{\gamma\ r} < 0. \end{aligned} \tag{32}$$

A.3 Optimal taxes with infinite turnover

Consider the continuous time version of the model where h represents the instantaneous hiring rate, which is allowed to take any positive value. Take a currently unemployed person. Denote the time until he finds a job by τ which is a stochastic variable with a density function $f(\tau) = he^{-h\tau}$. Let us consider an unemployed individual who during his current unemployment period follows the (potentially) suboptimal plan of consuming $w_u + rA_t$. When he finds his next job he reverts to the optimal behavior. Denote the conditional value function of this individual $W(A_t, u, \tau)$ where τ denotes his expected utility if he finds a job exactly τ units from now. We then have

$$W(A_t,u,t) = -\int_0 \tau e^{-rs} e^{-\gamma(rA_t+w_u)} ds + e^{-r\tau} V(A_t,e) \qquad (33)$$
$$= e^{-\gamma(rA_t+w_u)} \frac{1-e^{-r\tau}}{r} + e^{-r\tau} V(A_t,e)$$

Clearly the unconditional value function, denoted $W(A_t,u)$ satisfies

$$V(A_t,u) \geq W(A_t,u) \equiv \int_0^\infty W(A_t,u,\tau) f(\tau) d\tau$$

So:

$$\begin{aligned} W(A_t,u) &= -\int_0^\infty \left(e^{-\gamma(rA_t+w_u)} \frac{1-e^{-r\tau}}{r} + e^{-r\tau} V(A_t,e) \right) he^{-h\tau} d\tau \\ &= \frac{-e^{-\gamma(rA_t+w_u)}}{r} \int_0^\infty \left(1-e^{-r\tau}\right) he^{-h\tau} d\tau + V(A_t,e) \int_0^\infty e^{-r\tau} he^{-h\tau} d\tau \qquad (34) \\ &= \frac{-e^{-\gamma(rA_t+w_u)}}{r} \left(1 - \frac{h}{r+h}\right) + V(A_t,e) \frac{h}{r+h} \end{aligned}$$

Now take the limit of both sides as $h \to \infty$. We then see that

$$\lim_{h\to\infty} W(A_t,u) = \lim_{h\to\infty} V(A_t,e) \qquad (35)$$

Now we have that $V(A_t,e) \geq V(A_t,u) \geq W(A_t,u)$ since W denotes a sub-optimal plan and wage is assumed to be higher when employed than when unemployed. So

$$\lim_{h\to\infty} V(A_t,u) = V(A_t,e) \qquad (36)$$

This means that uncertainty disappears and consumption in the two states converges to the same level. The conclusion is thus that the role for insurance disappears as turnover increases to infinity. If saving and borrowing are not allowed, the value functions in the two states may also converge as turnover goes to infinity. Consumption in the two states cannot converge in this case, however, so the role for insurance as a means of smoothing consumption remains.

A.4 Tax level that maximizes the Utility of the employed Workers.

To maximize the utility of the employed workers over τ is equivalent to maximize c_e. From (28), we have

$$c_e = (1-\tau)\ w - \frac{\ln(1-q+q\ D)}{\gamma\ r}. \tag{37}$$

Now note that $w_e - w_u = w(1-\tau) - w\tau\frac{1-a}{d} = w(1-\frac{\tau}{\delta})$ where δ is the tax rate that corresponds to full insurance. Using this in (29) and taking logs we obtain

$$\tau = \delta - \delta\frac{\ln\left(D^{1+r}\right) + \ln(1-q+q\ D) - \ln((1-h)\ D+h)}{\gamma\ r\ w}. \tag{38}$$

This establishes a monotonic relation between τ and D. Certainly, the employed will never want to have τ higher than δ (full insurance), and non negativity of τ implies that D can not be higher than some value $\bar{D} > 1$, which is the unique value that satisfies

$$\ln\left(\bar{D}^{1+r}\right) + \ln\left(1-q+q\ \bar{D}\right) - \ln\left((1-h)\ \bar{D}+h\right) = \gamma\ r\ w.$$

Substituting τ from (38) in (37) we can express the utility of the employed agents as a function of, D, the relative utility in the two states.

$$c_e = w\ (1-\delta) + \frac{\left[\delta\ \ln\left(D^{1+r}\right) - \left[\delta\ \ln((1-h)\ D+h) + (1-\delta)\ \ln(1-q+q\ D)\right]\right]}{\gamma\ r} \tag{39}$$

Consequently the problem is reduced to maximize equation (39) for values of D belonging to the interval $\left[1,\bar{D}\right]$. The first and second derivatives of (39) are

$$\frac{\partial c_e}{\partial D} = \frac{1}{D}\left[\delta\ (1+r) - \left\{\delta\ \frac{(1-h)\ D}{(1-h)\ D+h} + (1-\delta)\ \frac{q\,D}{1-q+q\,D}\right\}\right] \tag{40}$$

and

$$\frac{\partial^2 c_e}{\partial D^2} = -\frac{\frac{\partial c_e}{\partial D}}{D} - \frac{1}{D}\left[\delta\ \frac{(1-h)\ h}{\left[(1-h)\ D+h\right]^2} + (1-\delta)\ \frac{(1-q)\ q}{\left[1-q+q\,D\right]^2}\right] \tag{41}$$

Now let

$$J\,(D) = \delta\ \frac{(1-h)\ D}{(1-h)\ D+h} + (1-\delta)\ \frac{q\,D}{1-q+q\,D} \tag{42}$$

It is clear that $J\,(D)$ is monotonically increasing with $J\,(0) = 0$ and $\lim_{D\to\infty} J\,(D) = 1$. Consequently:

- If $\delta\,(1+r) > 1$ there is no interior maximum for c_e, since its first derivative is always positive. The non negativity of taxes implies that the tax level that employed agents would prefer would be zero (and $D = \bar{D}$).

- If $\delta(1+r) <$ and the turnover is zero (that is the probabilities q and h are both zero), then $J(D) = \delta < \delta(1+r)$ for all values of D. So again the tax level that employed agents would prefer would be zero (and $D = \bar{D}$).

- If $\delta(1+r) < 1$ and the turnover is positive (that is the probabilities q and h are both strictly positive) there is always an *unique* value of D that makes the first derivative of c_e equal to zero, and in this point the second derivative is negative. Let's call this value $\tilde{D}$ ($J\left(\tilde{D}\right) = \delta(1+r)$). Note that $\tilde{D} > 1$, because $J(1) = \delta(1-h) + (1-\delta)q < \delta(1+r)$.

  - If $\bar{D} < \tilde{D}$ (this is: $\delta(1+r) > J\left(\bar{D}\right)$), then c_e is increasing in the whole interval $\left[1, \bar{D}\right]$, consequently the employed agents would maximize their utility with $D = \bar{D}$ and $\tau = 0$.
  - If $\bar{D} > \tilde{D}$ (this is: $\delta(1+r) > J\left(\bar{D}\right)$), then c_e achieves its global maximum at the feasible point $\tilde{D}$, and the tax level that maximizes the utility function of the employed agents is:

$$\tau = \delta - \delta \frac{\ln\left(\tilde{D}^{1+r}\right) + \ln\left(1 - q + q\,\tilde{D}\right) - \ln\left((1-h)\,\tilde{D} + h\right)}{\gamma\, r\, w} < \delta \tag{43}$$

By solving $J(D) = \delta(1+r)$, we can obtain a closed form value for $\tilde{D}$:

$$\tilde{D} = \frac{1}{2}\left(\sqrt{b^2 - 4c} - b\right) \tag{44}$$

Where (denoting $Q = \frac{1-q}{q}$ and $H = \frac{h}{1-h}$)

$$b = \frac{\delta - \delta(1+r)}{1 - \delta(1+r)} Q + \frac{1 - \delta - \delta(1+r)}{1 - \delta(1+r)} H \tag{45}$$

and

$$c = -\frac{\delta(1+r)}{1 - \delta(1+r)}\, H\, Q. \tag{46}$$

A.5 Consumption and Value Functions With Finite Voting Cycles

Assume there is a voting cycle so that the tax rate can be fixed for s period. The decision about the tax rate is set at least one period in advance of when it becomes in effect.

The tax rate that is determined at t thus applies to $t+1 \ldots t+s$. We derive the value functions when the tax rate is set to τ for s periods and thereafter set to τ^* , where τ and τ^* are allowed to take non-negative value. The tax rate in the current period is also set to τ^*, although it is trivial to change that assumption. The aim of this section is to show how preferences over τ vary with s. To do this we need to find the value functions in the two employment states as functions of s, τ, τ^* and A_t. We derive the value functions $V(A_t, l, \tau, \tau^*, s)$ recursively for $s = \{1, 2, \ldots\}$.

Now redefine

$$\begin{aligned} w_e &\equiv w(1-\tau^*) \\ w_u &\equiv w\tau^*(1-a)/d \end{aligned} \tag{47}$$

With this notation, we can view $w(\tau^* - \tau)$ as an extra cash transfer to the employed the next period. For the unemployed, the corresponding extra transfer is $w\frac{(\tau-\tau^*)(1-a)}{d}$. When $s = 1$ the value functions are then given by

$$\begin{aligned} V(A_t, e, \tau, \tau^*, 1) = \; & \max_{c_t} \{ -e^{-\gamma c_t} \\ & -\tfrac{1}{1+r}[(1-q)V(A_{t+1} + w(\tau^* - \tau), e) \\ & +qV(A_{t+1} + w\tfrac{(\tau-\tau^*)(1-a)}{d}, u)]\} \\ & s.t.\;\; A_{t+1} = (1+r)(A_t + w_e - c_t) \end{aligned} \tag{48}$$

$$\begin{aligned} V(A_t, u, \tau, \tau^*, 1) = \; & \max_{c_t} \{ -e^{-\gamma c_t} \\ & -\tfrac{1}{1+r}[(1-h)V(A_{t+1} + w\tfrac{(\tau-\tau^*)(1-a)}{d}, u) \\ & +hV(A_{t+1} + w(\tau^* - \tau), u)]\} \\ & s.t.\;\; A_{t+1} = (1+r)(A_t + w_u - c_t) \end{aligned} \tag{49}$$

where $V(\cdot, \cdot)$ are the infinite voting cycle value functions previously derived. Now let us guess that the following solution to the consumption problem at time t, given τ, τ^* and s has the following form

$$c_t = \begin{cases} \frac{r}{1+r} A_t + \widetilde{c}_{e,s} & \text{if employed} \\ \frac{r}{1+r} A_t + \widetilde{c}_{u,s} & \text{if unemployed} \end{cases} \tag{50}$$

Simplifying, using the explicit form of $V(\cdot, \cdot)$ and using the budget constraint we get

$$
\begin{aligned}
V(A_t,e,\tau,\tau^*,1) &= e^{-\gamma\frac{r}{1+r}A_t}\max_{\widetilde{c}_{e,1}}\{-e^{-\gamma\widetilde{c}_{e,1}} \\
&\quad -\tfrac{1}{r}[(1-q)e^{-\gamma(\frac{r}{1+r}w(\tau^*-\tau)+r(w_e-\widetilde{c}_{e,1})+c_e)} \\
&\quad +qe^{-\gamma(\frac{r}{1+r}\frac{w(\tau-\tau^*)(1-a)}{d}+r(w_e-\widetilde{c}_{e,1})+c_u)}]\} \\
V(A_t,u,\tau,\tau^*,1) &= e^{-\gamma\frac{r}{1+r}A_t}\max_{\widetilde{c}_{e,1}}\{-e^{-\gamma\widetilde{c}_{u,1}} \\
&\quad -\tfrac{1}{r}[(1-h)e^{-\gamma(\frac{r}{1+r}\frac{w(\tau-\tau^*)(1-a)}{d}+r(w_u-\widetilde{c}_{u,1})+c_u)} \\
&\quad +he^{-\gamma(\frac{r}{1+r}w(\tau^*-\tau)+r(w_u-\widetilde{c}_{u,1})+c_e)}]\}
\end{aligned} \tag{51}
$$

Let the choice variables without tildes denote their optimized values. The first order conditions are then

$$
\begin{aligned}
e^{-\gamma c_{e,1}} &= (1-q)e^{-\gamma(\frac{r}{1+r}w(\tau^*-\tau)+r(w_e-c_{e,1})+c_e)} \\
&\quad +qe^{-\gamma(\frac{r}{1+r}\frac{w(\tau-\tau^*)(1-a)}{d}+r(w_e-c_{e,1})+c_u)} \\
e^{-\gamma c_{u,1}} &= (1-h)e^{-\gamma(\frac{r}{1+r}\frac{w(\tau-\tau^*)(1-a)}{d}+r(w_u-c_{u,1})+c_u)} \\
&\quad +he^{-\gamma(\frac{r}{1+r}w(\tau^*-\tau)+r(w_u-c_{u,1})+c_e)}.
\end{aligned}
$$

which are satisfied for all A_t for the proper choice of $c_{e,1}$ and $c_{u,1}$ thus confirming (50). Using the first order conditions, the value functions are

$$
\begin{aligned}
V(A_t,e,\tau,\tau^*,1) &= -e^{-\gamma\frac{r}{1+r}A_t}\frac{1+r}{r}e^{-\gamma c_{e,1}} \\
V(A_t,u,\tau,\tau^*,1) &= -e^{-\gamma\frac{r}{1+r}A_t}\frac{1+r}{r}e^{-\gamma c_{u,1}}
\end{aligned} \tag{52}
$$

Now continuing recursively we find that for $s>1$ periods the value functions are

$$
\begin{aligned}
V(A_t,e,\tau,\tau^*,s) &= e^{-\gamma\frac{r}{1+r}A_t}\max_{\widetilde{c}_{e,s+1}}\{-e^{-\gamma\widetilde{c}_{e,s}} \\
&\quad -\tfrac{1}{r}[(1-q)e^{-\gamma(\frac{r}{1+r}(A_t+w(\tau^*-\tau))+r(w_e-\widetilde{c}_{e,s+1})+c_{e,s-1})} \\
&\quad +qe^{-\gamma(\frac{r}{1+r}\frac{w(\tau-\tau^*)(1-a)}{d}+r(w_e-\widetilde{c}_{e,s+1})+c_{u,s-1})}]\} \\
V(A_t,u,\tau,\tau^*,s) &= e^{-\gamma\frac{r}{1+r}A_t}\max_{\widetilde{c}_{u,s+1}}\{-e^{-\gamma\widetilde{c}_{u,s}} \\
&\quad -\tfrac{1}{r}[(1-h)e^{-\gamma(\frac{r}{1+r}\frac{w(\tau-\tau^*)(1-a)}{d}+r(w_u-\widetilde{c}_{u,s})+c_{u,s-1})} \\
&\quad +he^{-\gamma(\frac{r}{1+r}w(\tau^*-\tau)+r(w_u-\widetilde{c}_{u,s})+c_{e,s-1})}]\}
\end{aligned} \tag{53}
$$

with value functions

$$
\begin{aligned}
V(A_t,e,\tau,\tau^*,s) &= -e^{-\gamma\frac{r}{1+r}A_t}\frac{1+r}{r}e^{-\gamma c_{e,s}} \\
V(A_t,u,\tau,\tau^*,s) &= -e^{-\gamma\frac{r}{1+r}A_t}\frac{1+r}{r}e^{-\gamma c_{u,s}}
\end{aligned} \tag{54}
$$

We see that the value functions in (54) are linear in the term $e^{-\gamma \frac{r}{1+r} A_t}$. Preferences over tax rates for the coming s periods are thus independent of wealth, just as in the case of constant tax rates.

A.6 Temptation to Deviate

Let $V(A_t, ., \tau_1, \tau_2)$ denote the value function if next periods tax is set to τ_1 and thereafter to τ_2 forever. Also let $V(A_t, ., \tau) \equiv V(A_t, e, \tau, \tau)$. Now define τ^* as the tax rate that maximizes $V(A_t, e, \tau)$. We then have that

$$\frac{\partial V(A_t, e, \tau^*, \tau^*)}{\partial \tau_1} = \frac{1}{1+r}[(1-q)(-w)U'(c(A_{t+1}, e) + qw\frac{1-a}{d}U'(c(A_{t+1}, u)]. \tag{55}$$

Now we want to show that (55) is strictly negative so that the temptation to deviate is strictly positive. First assume that the optimal tax rate is positive so that we have an interior optimum. Since τ^* maximizes $V(A_t, e, \tau)$ it satisfies

$$\begin{aligned} 0 &= \frac{\partial V(A_t, e, \tau^*)}{\partial \tau} \\ &= \frac{1}{1+r}[(1-q)\left((-w)U'(c(A_{t+1}, e) + \frac{\partial V(A_{t+1}, e, \tau^*)}{\partial \tau}\right) \\ &\quad +q\left(w\frac{1-a}{d}U'(c(A_{t+1}, u) + \frac{\partial V(A_{t+1}, u, \tau^*)}{\partial \tau}\right)]. \end{aligned} \tag{56}$$

where we use that also $\frac{\partial V(A_{t+1}, e, \tau^*)}{\partial \tau} = 0$. From this follows that

$$\frac{\partial V(A_t, e, \tau^*, \tau^*)}{\partial \tau_1} = 0 - q\frac{\partial V(A_{t+1}, u, \tau^*)}{\partial \tau} \tag{57}$$

where the partial derivative is with respect to the first τ, i.e., the tax rate next period. So what remains is to show that $\frac{\partial V(A_{t+1}, u, \tau^*)}{\partial \tau}$ is strictly positive. To do this we first note that from (56) follows that

$$U'(c(A_{t+1}, e) = \frac{q}{(1-q)w}\left(w\frac{1-a}{d}U'(c(A_{t+1}, u) + \frac{\partial V(A_{t+1}, u, \tau^*)}{\partial \tau}\right). \tag{58}$$

For an unemployed we have

$$\frac{\partial V(A_t,u,\tau^*)}{\partial\tau} = \frac{1}{1+r}[h\left((-w)U'(c(A_{t+1},e)+\frac{\partial V(A_{t+1},e,\tau)}{\partial\tau}\right) + (1-h)\left(w\frac{1-a}{d}U'(c(A_{t+1},u)+\frac{\partial V(A_{t+1},u,\tau)}{\partial\tau}\right)]. \quad (59)$$

Using (58) we get

$$\begin{aligned}
&\frac{\partial V(A_t,u,\tau^*)}{\partial\tau} \\
=\ & \frac{1}{1+r}[h\left(\frac{q}{q-1}\left(w\frac{1-a}{d}U'(c(A_{t+1},u)+\frac{\partial V(A_{t+1},u,\tau^*)}{\partial\tau}\right)\right) \\
& +(1-h)\left(w\frac{1-a}{d}U'(c(A_{t+1},u)+\frac{\partial V(A_{t+1},u,\tau^*)}{\partial\tau}\right)] \\
=\ & \frac{1-q-h}{(1+r)(1-q)}\left[w\frac{1-a}{d}U'(c(A_{t+1},u)+\frac{\partial V(A_{t+1},u,\tau^*)}{\partial\tau}\right].
\end{aligned}$$

From which we find that

$$\frac{\partial V(A_t,u,\tau^*)}{\partial\tau} = w\frac{1-a}{d}\sum_{s=1}^{\infty}k^s U'(c(A_{t+s},u) > 0$$

where $k \equiv \frac{1-q-h}{(1+r)(1-q)}$ and $U'(c(A_{t+s},u)$ is conditional on the median voter being unemployed from $t+1$ to at least $t+s$.

References

[1] Broadway, R. and D. Wildasin, (1989), "Voting Models of Social Security Determination", in Gustavsson, B.A. and N.A. Klevmarken (eds), *The Political Economy of Social Security,* North-Holland.

[2] Bertola, G., and R. Rogerson, (1996), "Institutions and Labor Reallocation", mimeo.

[3] Gruber, Jonathan, (1994), "The Consumption Smoothing Benefits of Unemployment Insurance", NBER Working Paper No. 4750.

[4] Hall. R. E., (1995), "Lost Jobs", Brookings Papers on Economic Activity, 1:95.

[5] Krusell, Per, Vincenzo Quadrini and José Víctor Ríos Rull, (1996), "Politico-Economic Equilibrium and Economic Growth", Journal of Economic Dynamics and Control, forthcoming.

[6] Wright, Randall, (1986) “The redistributive Roles of Unemployment Insurance and the Dynamics of Voting”. Journal of Public Economics 31.

[7] OECD, (1994), ”The OECD Jobs Study, Facts Analysis Strategies”, OECD.

[8] OECD, (1995), ”Employment Outlook, July 1995”, OECD.