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### **A Classical-Marxian Model Of Education, Growth And Distribution**

By

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# A CLASSICAL-MARXIAN MODEL OF EDUCATION, GROWTH AND DISTRIBUTION\*

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**Abstract:** This paper develops a classical-Marxian macroeconomic model to examine the growth and distributional consequences of education. First, the role of education in skill formation is considered and it is shown that an expansion in education will promote growth and have beneficial distributional effects within the working class, but it will redistribute income from workers to capitalists. Second, the model is extended analyze the broader political economic consequences of education on class relations and class conflict. The model suggests the importance of a progressive type of education rather than one which weakens the power workers, for it allows for equitable growth outcomes which improve the position of workers as a whole and reduces inequality within workers. Finally, the model shows that education leads to multiple equilibria and it stresses the importance of providing suitable incentives to workers for taking advantage of greater education access, without which the economy can be caught in a low-skill trap.

**JEL classification system:** E2, E11, O41, J31.

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## **1. Introduction**

The role of education – and what is called human capital – has received a great deal of attention in the literature on economic growth in recent decades. The role of education in skill formation, and the resultant division of the labor force into high- and low-skilled workers, has also been widely examined in the literatures on income distribution and international trade. This analysis has been conducted empirically and theoretically, and also entered popular discussions. Neoclassical models of growth (including the neoclassical endogenous growth models) see education as promoting growth by making the productivity of labor increase more rapidly, and improving income distribution by increasing wages, although different rates of skill formation – through education – between different groups are sometimes argued to exacerbate income inequality. Surprisingly, however, education has received little or no attention in classical-Marxian theories of growth and distribution, despite the obvious relevance it has for the dynamics of the capitalist economy and the fair amount of attention being given to it in broader political economy discussions. While much of this discussion is in relation to the ideological role of education, some of it also relates to growth. For instance, some commentators have associated the relative neglect of the education sector with the profit squeeze and the decline in productivity growth after the period of the so-called Golden Age of Capitalism which ended in the late 1960s.<sup>1</sup>

The purpose of this paper is to develop a simple classical-Marxian model of growth and income distribution in which the primary role of education is to convert low-skilled workers into high-skilled workers, and with which broader political-economy considerations may also be addressed. The classical-Marxian approach to growth has thus

far focused mainly on the role of capital accumulation due to saving by capitalists (see Foley and Michl, 1999, for an exposition), and due to technological change brought about by the response of capitalist firms to labor market conditions (see, for example, Duménil and Lévy, 2003; Flaschel, 2009). It is not obvious what role education would play in the growth process. Increases in education, by increasing labor productivity, will not necessarily increase output and its growth as in neoclassical models with fully employed labor, because of the existence of unemployed workers in the classical-Marxian framework. Regarding income distribution, increases in productivity due to education need not improve wages in the presence of unemployment.<sup>2</sup> Indeed, the classical-Marxian framework has not been concerned with the effect of education and skill formation on distribution, focusing instead on the question of how income is distributed between workers who receive wages and capitalists who receive profits and the consequences of this for the dynamics of capital accumulation,<sup>3</sup> with some attention being given to landlords and rent (following Ricardo's analysis of the tripartite division of output and income into rent, wages and profits)<sup>4</sup> and to financial capitalists (following Marx's discussion of the division of the surplus or profits between financial capitalists who receive interest income and industrial capitalists who receive net profit).<sup>5</sup> How will the entry of education and human capital affect the dynamics of distribution? Will it blur the distinction between workers and capitalists by allowing workers to become capitalists (with human capital)? Or will the classical-Marxian distinction between workers and capitalists continue to have a central role to play? Will limited access to education widen the inequality between people who obtain higher levels of education and those who do not?

The rest of this paper proceeds as follows. Section 2 will discuss the major decisions that have to be made before we can develop a classical-Marxian model of education, growth and distribution. Section 3 describes the structure of the model. Section 4 examines the dynamics of the model by analyzing its behavior in the short and long runs. Section 5 concludes.

## **2. Education in a classical-Marxian framework**

This section discusses how the model of this paper incorporates education in a classical-Marxian model of growth and distribution, by considering in turn three key questions that have to be answered before we can develop our classical-Marxian model with education. First, how do we characterize a classical-Marxian economy? Second, what exactly is the role of education in the economy? Third, what determines changes in the level of education? We will examine each question in order to describe and justify how choices regarding these questions are made in the model of this paper. Given the wide attention these issues have received in the neoclassical literature, we will find it useful to compare the choices made in this paper to the choices typically made in neoclassical models. This section also briefly discusses the overall implications of education for growth and distribution in the neoclassical approach to growth and comments on how the role may be different in a classical-Marxian approach which incorporates the features implied by our answers to the three questions just posed.

We characterize the classical-Marxian economy as having two key features. First, it is one in which: there are two *basic* classes, capitalists and workers; the distribution of income is influenced by the state of class struggle between them and there are unlimited supplies of labor; capitalists save a fraction of their income while workers consume their

entire income; all saving in the economy is automatically invested since there is no problem of aggregate demand; and saving and investment implies capital accumulation which expands output and employment. The simplest classical-Marxian model has only two classes – capitalists who own the capital stock and workers who earn only wage income which is a fixed share of total production, and there are fixed coefficients of production. The second, and essential, characteristic of the classical-Marxian approach is that growth depends on saving and capital accumulation and since saving is done only (or at least primarily) by capitalists, a more unequal distribution of income which increases the share of profits in income, leads to higher growth. Of course, this simple characterization has to be amended to take into account the role of education and the existence of high- and low-skilled workers.

This characterization of the economy may be contrasted to the neoclassical one of both the Solovian and the new or endogenous growth variety. In this approach, the economy always has fully employed labor, due to perfect wage-price flexibility and the substitution between capital and labor. In the Solow (1956) model, in steady state growth, with the assumption of diminishing returns to capital, the rate of growth depends only on the rate of growth of effective labor supply (which is the sum of the rate of growth of labor supply and the rate of labor-augmenting technological change) and not on the saving behavior of the economy. In the new growth theory approach, diminishing returns to capital are offset by externalities which improve technology due to a number of factors, including increases in education and the accumulation of human capital (as in Lucas, 1988). Consequently, in these models increases in saving increase the rate of growth of the economy permanently, as in the classical-Marxian approach, although in

the latter this is possible because of the existence of unemployed workers and in the former because of increases in productivity with fully employed labor.<sup>6</sup>

Regarding what exactly education does in the economy, we initially adopt the simple view that it transforms low-skilled workers into high-skilled workers. This in turn raises the question: what is the difference between low- and high-skilled workers in terms of their functions in the economy? Our view of low-skilled workers is that they are simply an input into the production of the final good, while high-skilled workers have a more complex role in the economy. Our view of high-skilled workers is that they also serve as an input in the production of the final good, but as a distinct factor of production from low-skilled workers, the elasticity of substitution between the two kinds of labor being relatively low. But in addition, and more importantly, high-skilled workers have a number of other functions: they increase the efficiency of both low-skilled and high-skilled workers through the process of innovation, and they also help in the process of education, as family members, mentors or educators. We take the view that low-skilled workers are employed in routine production activities, while high-skilled workers are innovators. Unger (2007, p. 96-97), who distinguishes such roles in terms of an idea about the mind, expresses it as follows:

We know how to repeat some of our activities, and we do not know how to repeat others. As soon as we learn how to repeat an activity we can express our insight in a formula and embody the formula in a machine ... The not yet repeatable part of our activities – the part for which we lack formulas and therefore also machines – is the realm of innovation, the front line of production. In this realm, production and discovery become much the same thing.

We therefore assume that education converts workers who could only do repetitive activities into discoverers and innovators, although they continue to be engaged in some

routine activities, activities which are qualitatively different from those of low-skilled workers.

This formulation of the role of education is to be found in the writings of some of the classical economists, including Smith (1776, p. 282) and McCulloch (1825, p. 122). McCulloch, in fact, emphasized, much more than his contemporaries, the role of education and the diffusion of knowledge in increasing growth through technological change (see O'Brien, 1975, p. 217).<sup>7</sup> It is now also a fairly standard one in neoclassical growth theory, which stresses that education and the accumulation of human capital increases productivity growth in the economy. This productivity-enhancing role of education can be contrasted to other approaches which focus of the role of education as job-screening or mere labeling systems. However, there are differences between the standard neoclassical approach and ours.

The view usually adopted in neoclassical growth models with education (see, for instance, Uzawa, 1965, Lucas, 1988) is that there is no essential difference between workers who are educated and those who are not. Raw (or unskilled) labor accumulates human capital through education and becomes skilled labor, the result of which is to augment their productive power. One worker becomes more than a worker in efficiency units, and therefore receives a higher wage. In this approach, workers are qualitatively the same, and can be shifted between the educational system and actual goods production. This is in contrast to our approach, in which education converts workers into high-skilled workers who are qualitatively different.

Our approach is actually closer to another neoclassical approach which is more popular in the trade-theoretic literature rather than in the growth literature, which takes



low-skilled and high-skilled labor to be qualitatively different and distinct inputs. The more traditional capital- (or land-)labor distinction of the Heckscher-Ohlin-Samuelson approach has been replaced by the low-skilled/high-skilled labor distinction. Models in this vein have been used to examine the implication of trade liberalization for the relative wages of skilled and unskilled workers. In Wood's (1994) application of this approach to North-South trade, the North, being skilled-labor abundant, exports the skilled-labor-intensive good, and the South, being unskilled-labor abundant, exports the unskilled-labor-intensive good. If there are restrictions to trade, trade liberalization will increase the disparity between the wages of skilled and unskilled workers in the North (and reduce it for the South). Trade between the rich North and the poor South can also lead to uneven development by raising skilled labor wages or the return to human capital accumulation in the North, and reducing it in the South, and to the extent that human capital accumulation drives technological change and growth, the result is unequal growth (Stokey, 1991). This approach has been used in empirical work to examine the actual increase in the wage of high-skilled labor to that of low-skilled labor in developed countries such as the US and the focus on the distribution of income between capital and labor income has yielded to that of the distribution between high-skilled and low-skilled labor. Despite stressing the qualitative difference between workers with different skill levels, our approach differs from these because it assumes that high-skilled and low-skilled workers are not only qualitatively different *inputs* used in producing the final good, but because they have qualitatively different *roles* in the economy. In some models the roles are different because high-skilled workers produce differentiated intermediate goods, whereas low-skilled workers combine the intermediate goods to produce the final

good (see Dutt, 2005). The model of this paper combines the two approaches – allowing both low- and high-skilled workers to be inputs into the production of a single good – but also allowing only high-skilled workers to have a role in inducing productivity growth and in inducing the process of education as family members, educators and mentors.

Our formulation of the role of education so far is a fairly narrow one, being much more specific than the multidimensional role given to it in the general classical, Marxian and radical approaches to education. This literature incorporates broad issues such as the role of education in weakening the position of workers by dividing them into groups based on their level of education, in creating and strengthening the perception of upward socio-economic mobility and thereby increasing tolerance for income inequality, indoctrination and socialization, and easing the process of the extraction of labor (and hence labor productivity and profits) from labor power (see Bowles and Gintis, 1975, 1976). In fact, Marx and Engels noted the possibility of the division of the labor movement into factions, one consisting of “the mass of workers living in real proletarian conditions” which was revolutionary, and the other comprising “the petty bourgeois members and the labor aristocracy” which was reformist. Subsequent Marxist writers have echoed these ideas and taken the argument further. In his writings during World War I Lenin (1914-15, p. 161) followed Marx and Engels to write that “certain strata of the working class (the bureaucracy in the labour movement and the labour aristocracy ...) as well as their petit-bourgeois fellow travellers ... served as the main social support of these tendencies to opportunism and reformism”. Although these ideas did not always refer specifically to the role of education in creating and maintaining differences within the labor force, they recognized the role that such divisions could play in weakening the

relative power of workers, a theme that has been more fully examined by subsequent Marxist scholars (see Giddens, 1973, for instance). Not all classical economists took this view, however. Education, McCulloch (1825, p. 134) believed, would show workers “how closely their interests are identified with those of their employers, and with the preservation of tranquility and good order”. Smith (Smith, 1776, p. 782) argued that economic growth with the division of labor in which workers performed simple repetitive tasks would make workers “as stupid and ignorant as it is possible for a human creature to become. The torpor of his mind renders him not only incapable of relishing or bearing a part in any rational conversation, but of conceiving any generous, noble, or tender sentiment, and consequently of forming any just judgment concerning many even of the ordinary duties of private life. Of the great and extensive interests of his country he is altogether incapable of judging ....” He argued that the spread of education would reverse these tendencies. According to this view in modern times education may have a role in creating more informed political participants and a discerning electorate. Even Marx (1867, p. 453) saw education as “the *only* method of producing *fully developed human beings*”, although he was thinking not education in the form actually existing in his time but of a purely proletarian education.

Turning to our third question, regarding the determinants of the change in the stock of education, in our approach the rate of change in the number of people educated, or the rate of change of education for short, depends positively on the stock of high-skilled workers (to capture the influence of more educators, families with high-skilled workers and mentors), positively on the wage of high-skill workers relative to that of low-skill workers, and positively on the access to education which captures factors such

as the degree of openness of the education system and the availability of educational loans.

Our approach differs from the neoclassical approach which focuses on the choice individuals make regarding whether to become educated or not, or the amount of education they will obtain. This approach makes the amount of human capital accumulated depend on individual preferences (reflected, for instance, in their rate of time preference) and the returns to schooling (that is, how much a worker can increase his or her wages by obtaining more education). Since in our approach the wage differential affects the rate of education, it is not inconsistent with the choice approach. However, it stresses other factors, such as the degree of access to education, and the wage differential may reflect increases in the opportunity to obtain education because of subsidies provided by businesses who react to the relative cost of educated workers. Our approach is therefore less specific than the neoclassical one, but we consider this lack of specificity to be a virtue because it opens up space for other determinants of the spread of education, which are crowded out in the neoclassical approach.

Our purpose in the remaining sections of the paper is to develop a classical-Marxian model of growth and distribution with three classes – capitalists, high-skilled workers and low-skilled workers – in which education transforms low-skilled workers into high-skilled workers who, in addition to being an input into production, contribute to the rate of growth of the efficiency of both high-skilled and low-skilled workers and in which education can have a broader political economy role by influencing the state of class struggle. Our main goal will be to examine how education, and increasing access to it, affects growth and distribution in the economy.

The implications of introducing education into an orthodox neoclassical model in which the economy grows with its labor fully employed, with education augmenting the effective amount of labor, and with individuals choosing freely how much education they receive, on growth and distribution are relatively straightforward. A change in a parameter which increases the spread of education will increase growth by increasing the effective supply of labor which is fully employed. In fact, education and human capital accumulation was shown early on to increase the rate of growth of the economy in the model developed by Uzawa (1965). Later, new growth theory has relied on education to offset the tendency of the marginal product of capital to decline by augmenting the effective supply of labor endogenously, to make long-run growth endogenous (as in Lucas, 1988). Lucas's model, in fact implies, that government intervention to increase the accumulation of human capital will increase the rate of growth of the economy in comparison to the perfectly competitive decentralized equilibrium, because of the externalities that result from human capital. Neoclassical models usually do not have much to say about the distribution of income, because they often assume that all individuals or families are identical. But because they view education as increasing the productivity of labor, which in turn increases the wage, the effect of the education of workers is a positive one. From a broader perspective, in which capitalists and workers are distinguished within the neoclassical framework, human capital can be interpreted as making workers into capitalists, and seems to blur the distinction between capitalists and workers and make irrelevant distributional conflicts and inequality. Workers who choose to get educated can increase their human capital and improve their lot, an avenue that may have been closed for workers through the accumulation of capital.

The shift in attention from physical and financial capital to human capital in discussions of inequality, as is reflected even in popular accounts of American cultural trends (see Brooks, 2000), can be taken to imply that income disparities will fall. It can be argued that it is easier to get education and become a human capitalist than to get enough capital to become a traditional capitalist; that while theoretically, there is no limit to the amount of capital one can accumulate, the possibility of human capital expansion has limits; that education cannot be bequeathed from generation to generation, at least as easily as financial wealth. These claims, however, are controversial. Intergenerational externalities in education and bequests of financial wealth can allow rich families to have advantages in accumulating human capital. Entry to education may not be a matter of choice, but be restricted by various means, such as legacy admissions and credit market imperfections. Human capital can be used to obtain virtually unlimited amounts of financial capital.

Some neoclassical models have been developed to explain the implications of education for inequality, and some have incorporated some of these criticisms. Bénabou (1996), for instance, shows how minor differences in education technology, wealth and preferences can result in widening disparities between income groups that cumulate over time. The driving force in models of persistent inequality are either differences in preferences (such as the rate of time preference), technology of education (which makes education more effective in increasing skills in some people than in others) and wealth differences which allow different levels of credit-financed investment in human capital. Choice plays an important role, in addition to market imperfections. The relation between inequality, education and growth can also be affected by broader political economy

issues. Perotti (1993) develops a model in which human capital formation has positive externalities, so that the pattern of income distribution affects growth through its effect on political equilibrium that is conducive to policies promoting human capital investment.<sup>8</sup> While models such as these show that the neoclassical optimizing and full employment approach to growth dynamics does not necessarily imply that education will remove inequalities and promote growth, they do not represent the mainstream neoclassical view on the positive effects of education on growth and distribution.

In a classical-Marxian growth model the favorable effects of education on growth and distribution are not guaranteed. Education may raise the productivity of workers, but if all workers are not fully employed, the growth in productivity may increase unemployment rather than growth. Moreover, access to education may not be a matter of choice for individuals, but may be restricted to upper-income groups, thereby increasing inequality between the rich and the poor people, exacerbating class stratification. Moreover, education may serve the ideological function of socializing people into accepting large inequalities among people and in creating the impression of high degrees of income mobility. While these issues have been widely discussed, unlike the importance they have received in the neoclassical growth-theoretic literature, there has been very little effort in incorporating education and its effects into heterodox growth models which examine how education affects growth and income distribution.<sup>9</sup> The neglect of the distributional consequences of education in growth models is particularly problematic, given the theoretical and policy relevance of income distribution.

### 3. Structure of the model

In developing a classical model of education, growth and distribution we draw on the classical-Marxian perspective, which examines growth determined by saving and capital accumulation with unemployment of labor. The general approach we follow is to extend standard models in the classical-Marxian tradition discussed in the previous section to incorporate two kinds of labor – high-skilled and low-skilled, the quantities employed of which are given by  $H$  and  $L$ , and which receive real wages  $w_H$  and  $w_L$ . We define the ratio of skilled to unskilled wage as

$$\sigma = w_H/w_L, \tag{1}$$

which represents the skill premium, so that typically one would expect  $\sigma > 1$ . With capital, we therefore have three inputs used in the economy, which correspond to three classes in society, capitalists who own physical capital and organize production, high-skilled workers with one unit of high-skilled labor and low-skilled workers with one unit of low-skilled labor. We examine a closed economy in which the government has no fiscal functions, and firms produce one good which can be used both for consumption and for (capital) investment. The rest of this section presents the main assumptions made in the model and derives some preliminary implications.

The technology is as follows. We suppose that there is only one sector and productivity increases – which derive from learning-by-doing processes and innovation activity by high-skilled workers, and depend on the amount of educated workers – are non-rival and spread across firms instantaneously. Production uses fixed coefficients input-output relations with capital and a mixture of high-skilled and low-skilled labor as inputs into production. The productivity of high-skilled and low-skilled labor is given at a



point in time by  $A_H$  and  $A_L$ , respectively, and the maximum output that can be produced by a unit of capital is  $k$ . High and low-skilled workers employed in the production of the consumption good are partially substitutable. To be specific, the production function of the standard firm is:

$$Y = \min [kK, f(A_L L, A_H H)], \quad (2)$$

where  $Y$  is the output of the good,  $K$  is the amount of capital, and  $f$  is homogenous of degree one, which is consistent with the fixed coefficients structure. This function is in line with standard heterodox assumptions in rejecting the substitutability between labor and capital, but in principle it allows for some substitutability between the two types of labor. In the rest of this paper, for the sake of analytical convenience, and without significant loss of generality, we shall make the following assumption.<sup>10</sup>

**Assumption 1 (A1).**  $Y = \min \{kK, [(A_L L)^\rho + (A_H H)^\rho]^{1/\rho}\}$ , with  $\rho < 0$ .

As a first step to a more complete analysis, we shall assume that high-skilled workers are more productive at all  $t$ , and that their productivity advantage remains constant over time.

This is formally stated in the next assumption.

**Assumption 2 (A2).** There is a scalar  $\mu \geq 1$ , such that  $A_H = \mu A_L$ , all  $t$ .

This assumption encompasses the special case with  $\mu = 1$ , at all  $t$ , and it allows one to analyse the comparative statics of increases in productivity differentials on growth, distribution, and the relative composition of the labor force. Further, to assume that  $\mu$  is constant seems reasonable (if not necessary) in a steady state, such that if any loss of generality occurs, this only has to do with the analysis of the transition path.

Given A1 and A2, the optimal demands for high-skilled and low-skilled labor by profit-maximizing, perfectly competitive firms (we consider one representative firm, with all firms being identical) are as follows.<sup>11</sup>

$$H^D = \frac{kK}{\left[ \left( \frac{w_H}{w_L} \right)^{-\frac{\rho}{\rho-1}} \mu^{\frac{\rho}{\rho-1}} + 1 \right]^{\frac{1}{\rho}} A_L \mu} = \frac{b(\sigma)K}{A_L}, \quad (3)$$

where  $\sigma = (w_H/w_L)$ , and  $b(\sigma) = k\mu^{-1} \left[ \sigma^{\frac{\rho}{1-\rho}} M + 1 \right]^{\frac{1}{\rho}}$ , where  $M = \mu^{\frac{\rho}{\rho-1}}$ . Similarly,

$$L^D = \frac{kK}{\left[ 1 + \left( \frac{w_H}{w_L} \right)^{\frac{\rho}{\rho-1}} \mu^{-\frac{\rho}{\rho-1}} \right]^{\frac{1}{\rho}} A_L} = \frac{c(\sigma)K}{A_L} \quad (4)$$

where  $c(\sigma) = k \left[ \sigma^{-\frac{\rho}{1-\rho}} M^{-1} + 1 \right]^{\frac{1}{\rho}}$ . Given (A1), it follows that  $b' < 0$  and  $c' > 0$ . Further, as

$\sigma$  tends to zero,  $b(\sigma)$  tends to infinity and as  $\sigma$  tends to infinity,  $b(\sigma)$  tends to  $k/\mu$ , and the function  $b$  is inelastic for all  $\sigma$ . The function  $b(\sigma)$  is shown in Figure 1, where  $\underline{b} = k/\mu$ .

The markets for the two kinds of workers are as follows. Low-skilled workers are in unlimited supply, and along standard neo-Marxian lines we assume that the real wage of these workers is determined exogenously by the relative bargaining power of low-skilled workers and firms, or what has been called the “state of class struggle”.<sup>12</sup> We parameterize this state in terms of the real wage of low-skilled workers in terms of their efficiency factor, so that given the state of class struggle, an increase in  $A_L$  will result in a proportionate increase in  $w_L$ . The market for high-skilled workers is flexprice, and the

skill premium adjusts in response to the excess demand for high-skilled workers, given the supply of these workers at a point in time, denoted as  $H^s$ , and given  $w_L$ . The low-skilled worker wage serves as a reference point, and given the skill premium, high low-skilled wages increase the high-skilled wage proportionately. We therefore make the following assumption.

**Assumption 3 (A3).** There exists a given scalar,  $\lambda$ , such that  $w_L = \lambda A_L$ . Further, given  $H^s$ , at any  $t$ ,  $\sigma$  solves  $H^s = b(\sigma)K/A_L$ .

Given the assumptions on the labor market, in what follows we shall use the symbols  $H$  and  $L$ , to denote the quantities of high- and low-skilled workers traded. The level of  $\lambda$  is determined by the relative bargaining power of low-skilled workers, and can be thought of as representing the state of class struggle. This parameter will be a determinant of what is left for capitalists to pay high-skilled workers and for their profits. To be precise, the share of low-skilled workers in total income is given by  $\lambda c(\sigma)k$ . Given  $H$ ,  $K$ , and  $A_L$ ,  $\sigma$  is given in the short run, so that  $\lambda$  determines the low-wage share. The remainder is left for distribution to capitalists and high-skilled workers.

As concerns employment, a variable of interest will be the skill composition of employed workers (a proxy of the skill composition of the labor force), which we will capture with the variable  $H/L$ . Given A3, it follows that at any point in time  $H/L = b(\sigma)/c(\sigma)$ , so that by the properties of  $b(\sigma)$  and  $c(\sigma)$ , the skill composition of employed labor is strictly decreasing in  $\sigma$ .<sup>13</sup>

We formalize the relationship between the use of high-skilled labor and labor productivity growth by assuming that the rate of growth of labor productivity of high-skilled workers depends positively on the amount of high skilled labor in efficiency units

as a ratio of the stock of capital. With a simple linear functional form, and denoting rates of growth by overhats, we assume that:

**Assumption 4 (A4).** There exist positive scalars  $\tau_0$  and  $\tau_1$  such that

$$\hat{A}_H = \tau = \tau_0 + \tau_1 (A_H H/K). \quad (5)$$

Here we measure high-skilled labor input as a ratio of capital stock as a scaling factor representing the size of the productive economy. We assume that all firms are identical, so that, for instance,  $A_H$  can be thought of as representing average productivity of high-skilled workers. Thus, although there may be externalities involved here, they are not required.

Because  $A_H$  and  $A_L$  are in general different, in principle equation (5) would not be sufficient to describe the behavior of labor productivity over time. By A2, however, it follows that  $\hat{A}_L = \hat{A}_H$ , all  $t$ , and we can write

$$\hat{A}_L = \tau_0 + \tau_1 \mu \frac{(A_L H)}{K} \quad (5a).$$

In other words, we conceptualize innovations as non-rival products of learning-by-doing with an immediate spillover to low-skilled workers, or as high-skilled workers developing new methods of production which increase low-skilled worker productivity.<sup>14</sup>

Low-skilled labor is converted into high-skilled labor through the process of education. The dynamics of the stock of high-skilled labor  $H$  is given by the following assumption.

**Assumption 5 (A5).** The supply of high-skilled labor  $H$  changes over time according to

$$dH/dt = \theta g(\sigma) H. \quad (6)$$

There exists a value  $\sigma_{min} \geq 1$  such that  $g(\sigma) = 0$  for all  $\sigma \leq \sigma_{min}$ . For all  $\sigma > \sigma_{min}$ , the function  $g$  is strictly increasing, convex, and differentiable.

According to A5, the change in the stock of high-skilled workers depends on three things. First, it depends on the demand for education which, in turn, depends positively on the skill premium, which increases the ‘return’ to education. Second, it depends on the size of the stock of high-skilled workers, both by increasing the availability of mentors and educators, and by increasing the support for, and access to, education (for instance, a higher stock implies a higher number added from high-skilled worker families). Third, it depends on a parameter,  $\theta$ , which captures the openness of the education system, either through government policy or through the degree of exclusivity of the education system and also, indirectly, the functioning of credit markets, in their role of financing education (with a lower  $\theta$  incorporating more severe credit constraints). Easier access to low-cost public education and greater access to student loans and grants, and a more open private education system which is less elitist on the basis of class and income would increase  $\theta$ . A value of  $\theta$  equal to zero would correspond to an extremely backward society, in which knowledge and skills are not created, and therefore are not transmitted, so that the stock of human capital is stationary.

To be sure, there are many different factors which determine the influence that the education system (and more generally the transmission of knowledge in a society) has on the creation of skills. We regard the parameter  $\theta$  as a parsimonious way to model such influences, and thus potentially the role of public policy in the creation of skills. It can be seen as a black box, a convenient way of modeling the multifaceted influence of education on the dynamics of human capital.

We will keep our analysis as general as possible and, apart from some mild regularity conditions, we will not specify an explicit functional form for  $g$ . The only theoretical restriction concerns the definition of  $\sigma_{min}$ : A5 incorporates the intuition that no one will seek education if the wage premium falls below a certain level. This seems rather reasonable at a theoretical and empirical level. It can be related to Smith's (1776, p. 118-19) explanation of the "difference between the wages of skilled labour and those of common labour" based on the principle that "it must be expected, over and above the usual wages of common labour, will replace to him the whole expence of his education, with at least the ordinary profits of an equally valuable capital." While Smith supposed that the supply of (high-)skilled workers would expand to make the actual wage equal to their "natural" wage, given limitations on education opportunities and other factors, we take them to provide a floor. It allows us to analyze some interesting issues concerning the relation between labor-augmenting innovation, education and growth, and in particular the existence of low-skill equilibria in which the economy may be trapped.

We make the following assumption about consumption and saving behavior in the economy.

**Assumption 6 (A6).** Workers – both high-skilled and low-skilled – do not save, but consume their entire income; capitalists save a fixed fraction,  $s$ , of their profits.

The income of profit recipients, or capitalists, is given by

$$rK = Y - w_L L - w_H H, \quad (7)$$

where  $r$  is the rate of profit. Total consumption expenditure in the economy is therefore given by

$$C = (1-s)rK + w_L L + w_H H \quad (8)$$

This implies that saving is given by the standard equation,

$$S = srK. \quad (9)$$

Finally, regarding investment, we have the following.

**Assumption 7 (A7).** Saving and investment are identically equal.

Capitalists save in order to invest, so that saving and investment are always equal. This version of Say's law is a standard assumption of the classical-Marxian approach. Equation (9) and A7 imply

$$I = srK. \quad (10).$$

The assumption implies that there is no effective demand problem, so that, given the existence of unemployed low-skilled workers, we have

$$Y = kK. \quad (11)$$

This macroeconomic condition justifies the microeconomic profit-maximizing decision made by each firm to produce at full capacity, as noted earlier.

#### 4. Education, growth and distribution with given class struggle parameter

We examine the dynamics of the model by considering two runs, for now assuming that  $\lambda$ , the distributional or class-struggle parameter affecting the share of income going to low-skilled workers, is exogenously given. In the short run we assume that the levels of  $K$ ,  $H$  and  $A_L$  are fixed, and the equations of the model solve for  $Y$ ,  $L$ ,  $\sigma$ ,  $r$  and  $I$  from equations (3), (4), (7), (10), and (11). The rate of profit is given by

$$r = k - \frac{w_L c(\sigma)}{A_L} - \sigma \frac{w_L}{A_L} \frac{A_L H}{K} \quad (12)$$

For given levels of  $A_L$ ,  $K$ , and  $H$ , we may solve for the equilibrium value of  $\sigma$  as shown in Figure 1. This is given by<sup>15</sup>

$$\sigma = b^{-1}(A_L H/K). \quad (13)$$

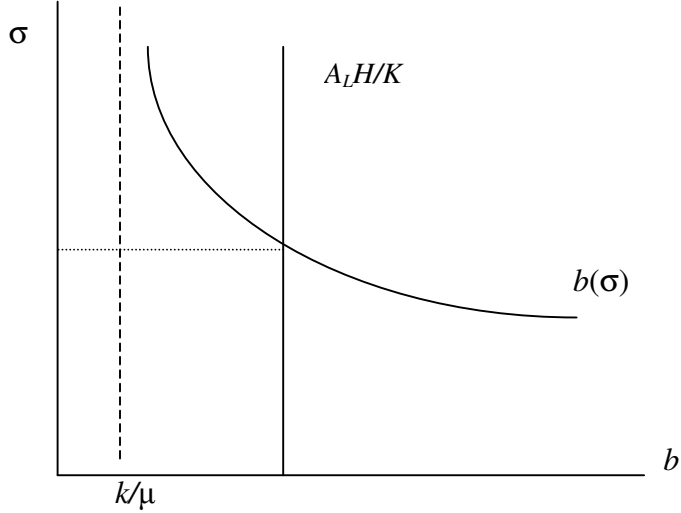


Figure 1. Determination of the skill premium in the short run.

In the long run we allow  $K$ ,  $H$  and  $A_L$  to change. Assuming away the depreciation of capital without loss of generality, the change in capital stock is given by

$$dK/dt = I \quad (14)$$

and changes in  $H$  and  $A_L$  are governed by equations (6) and (5a).

It is convenient to examine the time path of the economy in the long run by defining the state variable  $h = A_L H/K$ . Because the rate of growth of  $h$  is given by

$$\hat{h} = \hat{A}_L + \hat{H} - \hat{K}, \quad (15)$$

we can substitute from equations (5a), (6), and (11) through (14), to obtain, using the definition of  $h$ ,

$$\hat{h} = \tau_0 + \tau_1 \mu h + \theta g(\sigma(h)) - s[k - \lambda c(\sigma(h)) - \sigma(h)\lambda h], \quad (16)$$

where the function  $\sigma(\cdot) = b^{-1}(\cdot)$ .



The economy is defined as being in long-run equilibrium when  $\hat{h} = 0$ , that is,  $h$  is stationary. The determination of both short- and long-run equilibrium can be examined in terms of Figure 2. In the four-quadrant diagram, the south-west quadrant represents equation (3), essentially setting  $b=h$  from Figure 1, showing the relation

$$\sigma = b^{-1}(h). \quad (3')$$

The north-west quadrant shows the technological change curve, shown by equation (5a). The south-east quadrant shows the capital accumulation curve and the high-skilled labor accumulation curves, where the former is obtained from equations (10)-(12) and (14), and is given by

$$\hat{K} = s[k - \lambda c(\sigma) - \sigma \lambda b(\sigma)] = sk \left\{ 1 - \lambda \left[ \sigma^{-\frac{\rho}{1-\rho}} M^{-1} + 1 \right]^{\frac{1}{\rho}} - \sigma \lambda \mu^{-1} \left[ \sigma^{\frac{\rho}{1-\rho}} M + 1 \right]^{\frac{1}{\rho}} \right\} \quad (17)$$

and the latter is given, from equation (6), by

$$\hat{H} = \theta g(\sigma) \quad (6')$$

It is worth noting that  $\hat{K}$  is a decreasing and strictly convex function of  $\sigma$ . Further, if  $\sigma = 0$ ,  $\hat{K} = s(1 - \lambda)k$  and as  $\sigma$  becomes infinitely large, the growth rate of capital becomes minus infinity. Therefore by continuity there is a value of  $\sigma$ , call it  $\sigma^*$ , such that  $\hat{K} = 0$ .

$$\text{If } sk \left[ 1 - \lambda \frac{M^{\frac{1}{\rho}} + M^{-1}}{[M + 1]^{\frac{1}{\rho}}} \right] > 0, \text{ then } \sigma^* > 1.$$

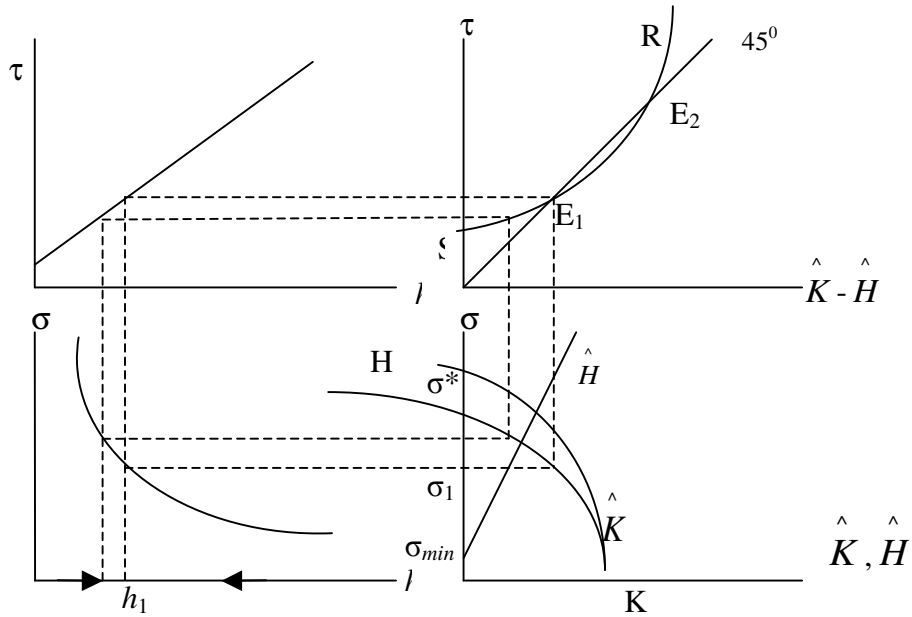


Figure 2: the long-run dynamics and equilibrium.

Subtracting the value of  $\hat{H}$  from that of  $\hat{K}$  for each value of  $\sigma$  gives the KH curve shown in the same quadrant.

In the short run,  $h$  is given, and the south-west quadrant solves for the short-run equilibrium value of  $\sigma$ . The north-west quadrant then determines  $\tau$ . The south-east quadrant determines the values of  $\hat{K}$  and  $\hat{H}$ . The north-east quadrant shows the resultant levels of  $\tau = \hat{A}_L$  and  $\hat{K} - \hat{H}$ , which is a point on the short-run equilibrium curve SR. Other initial values of  $h$  trace out other possible short-run equilibria, and hence the entire SR curve. Points on it above the  $45^\circ$  line imply that  $\tau = \hat{A}_L > \hat{K} - \hat{H}$ , or that  $h$  rises over time. In the long run  $h$  increases, yielding another short-run equilibrium on the SR curve till long-run equilibrium is attained at the intersection  $E_1$  of the SR curve and the  $45^\circ$  line

(provided the SR line is flatter than the 45<sup>0</sup> line at E<sub>1</sub>, which is required for stability).

Long-run equilibrium is attained at  $h_1$ .

Formally, the long-run equilibria of this economy are described by the next Propositions. Proposition 1 proves the existence of multiple equilibria, one of which with a positive skill premium.<sup>16</sup>

**Proposition 1 (The long-run equilibria).** Under (A1)-(A7), if

$\tau_0 + \tau_1 k [M + 1]^{-\frac{1}{\rho}} < sk \left[ 1 - \lambda \mu^{-1} [M + 1]^{-\frac{1}{\rho} + 1} \right]$ , then there are two long-run equilibria, one

with  $\sigma_1 > 1$  and one with  $\sigma_2 < 1$ . The former equilibrium is dynamically unstable,

whereas the latter is dynamically stable.

**Proof.** The long-run equilibrium requires

$$\tau_0 + \tau_1 k \left[ \sigma^{\frac{\rho}{1-\rho}} M + 1 \right]^{-\frac{1}{\rho}} + \theta g(\sigma) = sk \left\{ 1 - \lambda \left[ \sigma^{\frac{\rho}{1-\rho}} M^{-1} + 1 \right]^{-\frac{1}{\rho}} - \sigma \lambda \mu^{-1} \left[ \sigma^{\frac{\rho}{1-\rho}} M + 1 \right]^{-\frac{1}{\rho}} \right\}.$$

If  $\sigma = 0$ , then the LHS goes to infinity and the RHS is equal to  $(1 - \lambda)sk$  so that LHS >

RHS. If  $\sigma$  tends to infinity, then by Assumption 5 the LHS tends to a positive value

strictly greater than  $\tau_0 + \tau_1 k$ , while the RHS tends to minus infinity so that again LHS >

RHS. If the condition in the antecedent of the proposition is true, this implies that at  $\sigma =$

1, LHS < RHS, and thus by continuity the two curves must intersect at least twice at  $\sigma_1 >$

1 and at  $\sigma_2 < 1$ . They intersect exactly twice by the convexity of  $g$ , which makes the LHS

a strictly convex function of  $\sigma$ , whereas the RHS is a monotonically decreasing and

strictly convex function of  $\sigma$ . Stability follows in the usual manner. Q.E.D.

It is worth noting that Proposition 1 could have been proved focusing on equation (16) and noting that, under the premises of the Proposition, the function  $\hat{h}$  is strictly convex and it is equal to zero at  $h_1 = b(\sigma_1)$  and  $h_2 = b(\sigma_2)$ , where  $h_2 > h_1$ . Furthermore,  $\hat{h} > 0$  for all  $h < h_1$  and  $h > h_2$ , whereas  $\hat{h} < 0$  for all  $h_1 < h < h_2$ ,

Two points are worth making about Proposition 1. Firstly, the necessary condition for the existence of a long-run equilibrium is more likely to hold the lower  $\tau_0, \tau_1, \lambda$ , and the higher  $s, k$ . In other words, if technical progress is too strong, or if profits and capital accumulation, are too slow, then the dynamics of innovation may dominate and lead the economy to an explosive path. Secondly, and perhaps surprisingly, the equation of motion of the stock of high-skilled workers, and a fortiori the education system, has no effect on the existence, multiplicity or stability properties of the long-run equilibria. The education system plays a crucial role, instead, in the determination of the main features of the long-run equilibrium path of the economy.

The next result analyzes the effect of education policies on growth, employment, and distribution.

**Proposition 2 (Education, growth and distribution).** Assume (A1)-(A7). Assume

$$\tau_0 + \tau_1 k [M + 1]^{-\frac{1}{\rho}} < sk \left[ 1 - \lambda \mu^{-1} [M + 1]^{-\frac{1}{\rho} + 1} \right]$$

and assume that  $\sigma_1 > \sigma_{min}$ . At the long-run equilibrium with  $\sigma_1 > 1$ , an increase in  $\theta$  implies that in equilibrium the level of human capital increases, intra-worker inequality decreases, the skill composition of employed labor increases, physical and human capital grow at a higher rate, and the rate of profit increases.

**Proof.** At the long-run equilibrium

$$\tau_0 + \tau_1 k \left[ \sigma_1^{\frac{\rho}{1-\rho}} M + 1 \right]^{\frac{1}{\rho}} + \theta g(\sigma_1) = sk \left\{ 1 - \lambda \left[ \sigma_1^{\frac{\rho}{1-\rho}} M^{-1} + 1 \right]^{\frac{1}{\rho}} - \sigma_1 \lambda \mu^{-1} \left[ \sigma_1^{\frac{\rho}{1-\rho}} M + 1 \right]^{\frac{1}{\rho}} \right\}.$$

Because  $\sigma_1 > \sigma_{min}$ , an increase in  $\theta$  implies an upward shift of the LHS. Then – given the condition in the antecedent of the proposition – the LHS and the RHS intersect at  $\sigma'_1 < \sigma_1$ . The rest of the proposition immediately follows. Q.E.D.

It is worth noting that under the conditions of the proposition, the size of the decrease in the high-value equilibrium skill premium  $\sigma_1$  (and of the changes in the other variables) will depend on the dimension of the increase in  $\theta$ . If, instead,  $\sigma_{min} \geq \sigma_1 > 1$  then no change in  $\theta$  will affect the high-value equilibrium, nor – for analogous reasons – the low-value equilibrium at  $\sigma_2$ . This argument can be summarized in the next proposition.

**Proposition 3 (The low-skill trap).** Assume (A1)-(A7). Assume

$$\tau_0 + \tau_1 k [M + 1]^{\frac{1}{\rho}} < sk \left[ 1 - \lambda \mu^{-1} [M + 1]^{\frac{1}{\rho} + 1} \right] \text{ and assume that } \sigma_{min} \geq \sigma_1 > 1, \text{ At the long-}$$

run equilibrium with  $\sigma_1 > 1$ , an increase in  $\theta$  yields no change in either the short or the long-run equilibrium.

Intuitively, if there are no people who wish to get educated because the skill premium is so low, increasing access (by increasing  $\theta$ ) to education does not help.

The dynamics of the economy described in the previous propositions can be examined in Figure 3, which shows the growth rates of labor productivity,  $A_L$ , the number of skilled workers,  $H$ , and capital stock,  $K$ , as functions of  $h$ . The first two terms on the right hand side of equation (16) show the rate of growth of labor productivity: this rate increases with  $h$  as the ratio of skilled workers in efficiency units relative to capital stock

increases, implying a positively sloped  $\tau = \hat{A}_L$  curve. The third term, which represents the rate of growth of the number of skilled workers, shows that as  $h$  increases, this rate falls as the skill premium falls, implying a negatively-sloped  $\hat{H}$  curve. The last term represents the rate of growth of the capital stock. An increase in  $h$  implies that the skill premium falls, so that the rate of profit rises because total labor costs decrease. We add up the rates of growth of  $A_L$  and  $H$  to obtain the  $\hat{A}_L + \hat{H}$  curve, which first decreases and then eventually increases with  $h$ . The long-run equilibria are determined at  $h_1$  and  $h_2$ , corresponding, respectively, to  $\sigma_1$  and  $\sigma_2$  where  $\hat{A}_L + \hat{H} = \hat{K}$ , so that  $\hat{h} = 0$ .

The long-run equilibrium  $h_1$  is stable: if we start from  $h < h_1$ , for instance,  $\hat{A}_L + \hat{H} > \hat{K}$ , so that  $h$  will increase till it reaches  $h_1$ . The long-run equilibrium  $h_2$ , instead, is unstable: if  $h > h_2$  initially,  $h$  will increase indefinitely, with increases in the rate of capital accumulation.

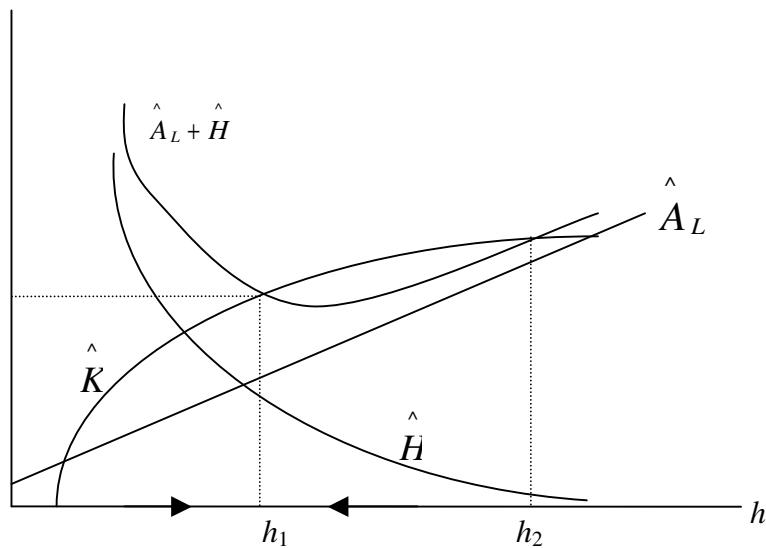


Figure 3: the long-run dynamics and equilibrium.

Focusing on the stable long-run equilibrium  $h_1$ , we can consider the effects of shifts in the main parameters of the model. A parameter of central interest for us is  $\theta$ , which represents the “openness” of the education system. A (sufficiently small) increase in  $\theta$  shifts the  $\hat{H}$  curve in Figure 2 to the right, increasing the short-run equilibrium value of  $\hat{H}$ . It therefore shifts the KH line to the left, and the SR curve also up and to the left. In the long run,  $h$  begins to increase, and the new long-run equilibrium will occur at a higher level of  $h$ . The same result is shown in Figure 3, where the increase in  $\theta$  shifts the  $\hat{H}$  and  $\hat{A}_L + \hat{H}$  curves upwards, making them steeper, at all values  $h$  such that  $k/\mu < h < h_{min}$ , where  $h_{min} = b(\sigma_{min})$ . The long-run equilibrium levels of capital accumulation, labor productivity growth and education accumulation will all increase, as can be verified from the figure. The rate of growth of the economy would therefore increase with  $\theta$ . The share of income going to low-skilled workers,  $\lambda c(\sigma)k$ , increases because the number of low-skilled workers employed is a positive function of  $\sigma$ , even though the state of class struggle is given by assumption (and so is the productivity of capital). The share of income going to high-skilled workers is given by  $\sigma(h)\lambda h/k$ . When  $\theta$  increases, we have already found that the long-run equilibrium level of  $h$  will be higher. But the rise in  $h$  will be proportionately less than the fall in  $\sigma$ , given the inelasticity of the  $b()$  function, so that  $\sigma(h)h$  will fall. Indeed, the fall in the share of income going to high-skilled workers is only partially compensated by the increase in the share going to low-skill workers and both the rate of profit and the share of total income going to the capitalists,  $r/k$ , will rise. Thus income is redistributed from high-skilled workers to low-skilled workers and from

all workers to capitalists. This, of course, is why the rate of capital accumulation in the economy is speeded up, as we saw earlier. However, the rate of growth of high-skilled workers, that is, the rate at which low-skilled workers (or the unemployed) become high-skilled workers, and obtain a higher wage, increases, since  $h$  increases and at any  $h$ ,  $\hat{H}$  is higher. The rate of growth of the real wage of low-skilled workers also rises with a higher rate of technological change. The rate of growth of low-skilled employment is given by  $\hat{L} = \hat{c}(\sigma) + \hat{K} - \hat{A}_L$ . Since in long-run equilibrium  $\hat{c}(\sigma) = 0$  as  $\sigma$  becomes stationary, and  $\hat{H} = \hat{K} - \hat{A}_L$ , it follows that  $\hat{H} = \hat{L}$ . The rate of growth of low-skilled employment therefore also increases. Thus, the condition of low-skilled workers, in terms of their real wage and employment growth, is unequivocally improved.

It is worth noting that, by using similar arguments, it is possible to draw analogous conclusions concerning changes in the other parameters of the model. If the condition in the antecedent of the propositions holds, then a sufficiently small improvement in the conditions of technical change will have the same effect as an improvement in the educational system. Focusing only on the high value equilibrium, we can summarize this finding in the next proposition.

**Proposition 4 (Technical progress).** Assume (A1)-(A7). Assume

$$\tau_0 + \tau_1 k [M + 1]^{-\frac{1}{\rho}} < sk \left[ 1 - \lambda \mu^{-1} [M + 1]^{-\frac{1}{\rho} + 1} \right].$$

At a long-run equilibrium with  $\sigma_1 > \sigma_{min}$ ,

there is a sufficiently small increase in  $\tau_0$ ,  $\tau_1$  such that in equilibrium the level of human capital increases, intra-workers inequality decreases, the skill composition of employed labor increases, physical capital grows at a higher rate, and the rate of profit increases.



Unlike changes in the education system, however, changes in the processes generating technological innovations will shift the *whole*  $\hat{A}_L + \hat{H}$  curve and this it will also affect the long-run equilibrium with  $\sigma_2 < \sigma_{min}$ .

Instead, if the condition in the antecedent of the propositions holds, a sufficiently small deterioration in the bargaining position of workers, and a sufficiently small increase in the savings rate will increase the growth rate of the main state variables while increasing the equilibrium value of the skill premium. Focusing only on the high value equilibrium, we can summarize this finding in the next proposition.

**Proposition 5 (Savings and productivity).** Assume (A1)-(A7). Assume

$$\tau_0 + \tau_1 k [M + 1]^{-\frac{1}{\rho}} < sk \left[ 1 - \lambda \mu^{-1} [M + 1]^{-\frac{1}{\rho} + 1} \right].$$

At a long-run equilibrium with  $\sigma_1 > \sigma_{min}$ ,

there is a sufficiently small decrease in  $\lambda$ , and a sufficiently small increase in  $s$  such that in equilibrium the level of human capital measured by  $h$  decreases, intra-workers inequality increases, the skill composition of employed labor decreases, physical capital and the high-skilled labor force grow at a higher rate, the rate of profit increases and the rate of productivity growth (of both kinds) of labor falls.

Graphically, the changes described in Proposition 5 imply a shift in the  $\hat{K}$  curve and thus it will also affect the long-run equilibrium with  $\sigma_2 < \sigma_{min}$ . A decrease in  $\lambda$ , the low wage-productivity ratio, will increase the rate of profit by lowering the wages of both types of workers. Similarly, an increase  $k$ , the productivity of capital, will increase the rate of profit despite increases in the employment of high- and low-skilled labor, for a given  $h$ , since firms maximize profits. In both cases, the  $\hat{K}$  curve will shift upwards, and

also shift the  $\hat{H}$  upwards for a given  $h$  because the increase in the demand for high-skilled labor increases  $\sigma$ . The  $\hat{A}_L$  curve is unaffected.

The effect of an increase in  $\mu$ , the productivity differential between high- and low-skilled labor can be discussed analogously. Such a change reduces  $M$ , increasing the demand for high-skilled labor and reducing it for low-skilled labor at a given  $h$ . The rate of profit increases since firms maximize profits, shifting the  $\hat{K}$  curve upwards. The increase in the demand for high-skilled labor increases  $\sigma$  at a given  $h$ , increasing  $\hat{H}$  and shifting its curve up. The increase in  $\mu$  increases  $\hat{A}_L$  for a given  $h$  also shifting its curve upwards. In both cases the effect on the equilibrium level of  $h$  is unclear, although the rate of capital accumulation must increase. The effect on the rate of productivity growth is unclear, although it is more likely to increase when  $\mu$  increases than when  $k$  increases, since in the former case the  $\hat{A}_L$  shifts up.

If technological change responds strongly to  $h$ , or if exogenous technical progress is too strong, the  $\hat{A}_L + \hat{H}$  curve may either intersect the  $\hat{K}$  curve at a value  $h_1$  such that  $\sigma_{min} \geq \sigma(h_1) > 1$ , or lie entirely above the  $\hat{K}$  curve. In the former case, the  $\hat{H}$  curve lies entirely above (and to the left of) the  $\hat{K}$  curve: this is the situation described in Proposition 3, and an increase in  $\theta$  which shifts the  $\hat{H}$  up as described above has no effect on the equilibria. In the latter case the condition in the antecedent of the propositions is violated and the economy is on an explosive path whereby starting with any value of  $h$ , the economy will experience increases in  $h$  and increasing rates of capital accumulation and technological change. If  $\sigma$  falls too low, increases in  $H$  will no longer

occur, but  $h$  will keep increasing as technological change occurs faster than capital accumulation. This kind of knowledge-driven increase in knowledge, however, is unlikely to occur in practice, and the  $\tau$  function is likely to flatten out, so that a stable equilibrium will be attained. Similar problems arise in economies with very low saving rates and capital productivity, or very high  $\lambda$ , which shift the  $\hat{K}$  curve downwards.

### 5. Education, growth and distribution with variable conditions of class struggle

The analysis has so far assumed that the low wage-productivity ratio,  $\lambda$ , is unchanged, given by the ‘state of class struggle’. This assumption may be questionable, given that it requires the real wage of unskilled workers to grow at the same rate as labor productivity. To examine the possible implications of changes in  $\lambda$  we need to make assumptions about its dynamics. We proceed by assuming that low-skilled workers have a fixed target ratio between wages and productivity which they try to achieve by pushing up their real wage, but they are not fully able to increase their real wage at the same rate as productivity growth. To formalize this, assume that

**Assumption 8 (A8).** The real wage of low-skilled workers changes according to

$$\hat{w}_L = \delta_1(\lambda^* - \lambda) + \delta_2 \tau, \quad (18)$$

where  $\lambda^* = \lambda^*(\theta)$ ,  $\delta_1 > 0$ , and  $1 > \delta_2 > 0$ .

The value  $\lambda^*$  is the target ratio of low-skilled workers, which is taken to depend on the educational access parameter, and it may be interpreted as reflecting normative considerations (e.g. criteria of ‘just pay’). While we assume that educational access will influence  $\lambda^*$ , we do not specify the exact way in which it affects workers’ attitudes in bargaining and conflict. Below, we will consider different scenarios. A8 states that the

rate of growth of the real wage depends positively on the extent to which actual ratio of  $\lambda$  falls short of the target, and on the growth of the productivity of labor, since workers demand and receive at least a part of the fruits of higher labor productivity,<sup>17</sup> Since, from the definition of  $\lambda$  we have

$$\hat{\lambda} = \hat{w}_L - \tau \quad (19)$$

substituting equations (5a) and (18) into (19) we get

$$\hat{\lambda} = \delta_1(\lambda^* - \lambda) - (1 - \delta_2)(\tau_0 + \tau_1 h) \quad (20)$$

This equation implies that an increase in  $h$ , by increasing the rate of productivity growth, will reduce the rate of growth of the wage-productivity ratio because workers are unable to increase their real wage to capture the full gains from productivity growth. An increase in the wage-productivity ratio will reduce its rate of increase because workers are closer to their target.

Equations (16) and (20) give us a dynamic system involving the two state variables  $h$  and  $\lambda$ . From equation (20) we see that the  $\hat{\lambda}=0$  isocline is a negatively-sloped straight line: starting from the locus, an increase in  $\lambda$  reduces  $\hat{\lambda}$ , making it negative, so that a reduction in  $h$  is required to increase it and make it return to zero. More precisely, the equation of the  $\hat{\lambda}=0$  isocline is

$$\lambda = \lambda^* - \frac{1 - \delta_2}{\delta_1} (\tau_0 + \tau_1 h) \quad (21)$$

In order to analyze the  $\hat{h}=0$  isocline, note, first of all, that by Propositions 1-2 above we know that for an equilibrium with positive real wages and a positive skill

premium to exist it must be the case that  $\tau_0 + \tau_1 k [M + 1]^{-\frac{1}{\rho}} < sk$ . In what follows we are going to assume that the latter condition is satisfied. Secondly, since the term  $sk \left[ 1 - \lambda \mu^{-1} [M + 1]^{-\frac{1}{\rho} + 1} \right]$  is monotonically decreasing in  $\lambda$ , let  $\bar{\lambda}$  be the value of  $\lambda$  that solves  $\tau_0 + \tau_1 k [M + 1]^{-\frac{1}{\rho}} = sk \left[ 1 - \bar{\lambda} \mu^{-1} [M + 1]^{-\frac{1}{\rho} + 1} \right]$ . Then, by Proposition 1, we know that for all  $\lambda \in [0, \bar{\lambda})$ , there exist two values  $(h_1, h_2)$ , with  $h_1 < h_2$ , such that  $\hat{h} = 0$ , whereas if  $\lambda = \bar{\lambda}$ , there exists one value  $\bar{h} = b(1) = k \mu^{-1} [M + 1]^{-\frac{1}{\rho}}$  such that  $\hat{h} = 0$ . We also know that, for all  $\lambda \in [0, \bar{\lambda})$ , at  $h_1$ ,  $\frac{d\hat{h}}{dh} < 0$  whereas at  $h_2$ ,  $\frac{d\hat{h}}{dh} > 0$ , and if  $\lambda = \bar{\lambda}$ , then at  $h_1$  it must be  $\frac{d\hat{h}}{dh} = 0$ . Therefore it follows from equation (16) that the  $\hat{h} = 0$  isocline is increasing for all  $h$  and then decreasing, and it reaches a maximum at  $h_1$ . It can also be proved that the  $\hat{h} = 0$  isocline is concave in  $\lambda$ .

From equation (16) we see that an increase in  $\lambda$  reduces the rate of profit and the rate of accumulation by increasing the payments to both kinds of workers, and hence increases  $\hat{h}$ . At the stable equilibrium  $h_1$ , the effect of  $h$  on technological change is low, and an increase in  $h$  reduces  $\hat{h}$ , so that the  $\hat{h} = 0$  isocline is positively sloped. At the unstable equilibrium  $h_2$ , the effect of  $h$  on technological change is high, and an increase in  $h$  increases  $\hat{h}$ , so that the  $\hat{h} = 0$  isocline is negatively sloped.

In order to prove our main result concerning the long-run equilibria of the general model, we need some notation. Let  $\lambda^{\max}$  be the highest value of  $\lambda$  in the  $\hat{\lambda} = 0$  isocline. By

equation (21),  $\lambda^{\max} = \lambda^* - \frac{1 - \delta_2}{\delta_1} \tau_0$ . Let  $h^{\max}$  be similarly defined:  $h^{\max} = \frac{\delta_1}{\tau_1(1 - \delta_2)} \lambda^* -$

$\frac{\tau_0}{\tau_1}$ . In the  $(h, \lambda)$  plane,  $\lambda^{\max}$  and  $h^{\max}$  correspond, respectively, to the vertical and

horizontal intercepts of the  $\hat{\lambda} = 0$  isocline. Finally, let  $h_1(0)$  and  $h_2(0)$  denote the two values of  $h$ , with  $h_1(0) < h_2(0)$ , such that  $\hat{h} = 0$  when  $\lambda = 0$ : by Proposition 1 we know that they exist, and they are such that  $h_1(0) = b(\sigma_1)$  with  $\sigma_1 > 1$  and  $h_2(0) = b(\sigma_2)$ , with  $\sigma_2 < 1$ . Furthermore, by A5, it follows that  $h_2(0) = (sk - \tau_0)/\tau_1\mu$ .

The next Proposition provides sufficient conditions for the existence of economically meaningful long-run equilibria.

**Proposition 6 (Long-run equilibria).** Assume (A1)-(A8). Assume

$\tau_0 + \tau_1 k [M + 1]^{-\frac{1}{\rho}} < sk$ . If  $\lambda^{\max} < \bar{\lambda}$  and  $h_1(0) \leq h^{\max} < h_2(0)$ , then there exists a long-run equilibrium with  $\sigma_1 > 1$ . If  $\lambda^{\max} < \bar{\lambda}$  and  $h^{\max} \geq h_2(0)$ , then there exist two long-run equilibria, one stable with  $\sigma_1 > 1$  and one unstable with  $\sigma_2 < 1$ . Furthermore, at the stable equilibrium the wage-productivity ratio is higher, and the stock of human capital is lower than at the unstable equilibrium.

**Proof.** 1. First, note that if  $\tau_0 + \tau_1 k [M + 1]^{-\frac{1}{\rho}} < sk$ , then there always exist combinations of the parameters such that the conditions  $\lambda^{\max} < \bar{\lambda}$  and  $h^{\max} > h_2(0)$  can both hold. (To see this, it is sufficient to choose  $\lambda^* < \bar{\lambda}$  and  $\delta_2$  sufficiently close to one.)

2. The existence of the two equilibria follows noting that because  $\lambda^{\max} < \bar{\lambda}$  and  $h^{\max} > h_2(0)$ , and given the monotonicity of the two isoclines, the two curves intersect twice: once in the increasing part of the  $\hat{h}=0$  isocline and once in the decreasing part of it.

3. The stability properties of the two equilibria follow from the properties of the two isoclines. First, for all  $\lambda \in [0, \bar{\lambda})$ , we know that there exist two values  $(h_1, h_2)$ , with  $h_1 < h_2$ , such that  $\hat{h}=0$ . We also know from the analysis in the previous section that, for any given  $\lambda$ ,  $\hat{h} < 0$ , for all  $h_1 < h < h_2$ , whereas  $\hat{h} > 0$ , for all  $h \notin [h_1, h_2]$ . Next, for all  $h \in [0, h^{\max}]$ , let  $\lambda' = \lambda^* - \frac{1-\delta_2}{\delta_1}(\tau_0 + \tau_1 h)$ . For all  $\lambda > \lambda'$ ,  $\hat{\lambda} < 0$ , whereas  $\lambda < \lambda'$ ,  $\hat{\lambda} > 0$ . This proves the desired claim.

4. The claims concerning the equilibrium values of the main variables follows from step 2 of the proof and Proposition 1 above. Q.E.D.

*Remark:* given  $\lambda^{\max} < \bar{\lambda}$ , and the monotonicity of the  $\hat{\lambda}=0$  isocline, it follows that in equilibrium the wage-productivity ratio is going to be strictly smaller than  $\bar{\lambda}$ , and we need not consider the singular case of an equilibrium occurring at  $\bar{\lambda}$ .

It is worth noting that the condition  $\lambda^{\max} < \bar{\lambda}$  is more likely to hold the lower  $\lambda^*$ ,  $\delta_1$ , and  $\delta_2$ . If conditions in the labor market are particularly conflictive, instead, it may happen that the  $\hat{\lambda}=0$  isocline lies entirely above the  $\hat{h}=0$  isocline and no equilibrium exists. In this case, the economy eventually settles on a path with ever-increasing  $h$  which takes the economy, eventually, to  $\lambda = 0$ . This is unlikely to happen, however, since the class struggle variable cannot be expected to go to zero; the parameters in equation (18)

can be expected to change, shifting the  $\hat{\lambda}=0$  isocline down, thereby producing a stable interior equilibrium.

Figure 4 shows the long-run dynamics of this model. The long-run equilibria occur at the intersections of the  $\hat{h}=0$  and  $\hat{\lambda}=0$  curves. The top-left equilibrium [E<sub>1</sub>] is (asymptotically) stable, and the economy will converge cyclically to the equilibrium, as can be seen from the arrows. The bottom-right equilibrium [E<sub>2</sub>] instead is unstable.<sup>18</sup>

We may now analyze the effect of education on the equilibria of the model. Consider first the  $\hat{h}=0$  isocline. From Proposition 2, it follows that an increase in  $\theta$  will shift the upward sloping part of the  $\hat{h}=0$  curve to the right, as shown by the dotted line, because it increases  $\hat{h}$  at given values of  $h$  and  $\lambda$ , for all values of  $h$  such that  $\sigma = \sigma(h) < \sigma_{\min}$ . Instead for all values of  $h$  such that  $\sigma = \sigma(h) \geq \sigma_{\min}$  the  $\hat{h}=0$  isocline – including all the downward sloping part of it – will not move.

The effect of an increase in  $\theta$  on the  $\hat{\lambda}=0$  isocline will depend on the effect of education on the workers' perception of a fair, or otherwise appropriate,  $\lambda^*$ . If education is genuinely progressive in that it facilitates the self-development of individuals, making them more conscious of their rights, and of their nature as social beings, then the function  $\lambda^* = \lambda^*(\theta)$  may be increasing in  $\theta$ . In this case, the  $\hat{\lambda}=0$  isocline will shift to the right. The combined effect on the two curves implies that the equilibrium value of  $h$  at the high-equilibrium [E<sub>1</sub>] unambiguously increases, and thus the skill premium falls, whereas the wage-productivity ratio may increase or decrease depending on the relative strength of the two effects.



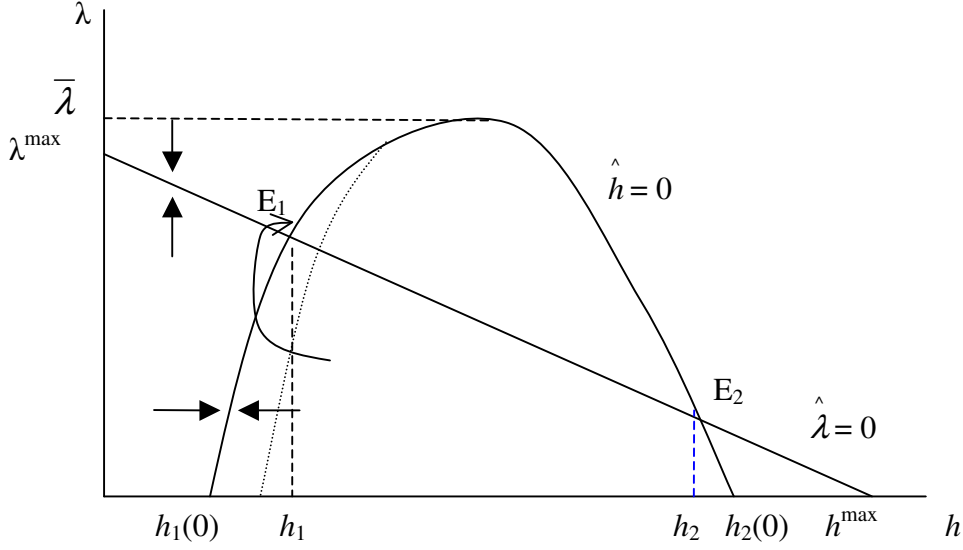


Figure 4. Long-run dynamics and equilibria of the model with variable  $\lambda$

The previous arguments can be summarized in the next proposition.

**Proposition 7 (Progressive role of education).** Assume (A1)-(A8). Assume  $\tau_0 + \tau_1 k [M + 1]^{-\frac{1}{\rho}} < sk$ . If  $\lambda^{\max} < \bar{\lambda}$ , and  $\lambda^*(\theta)$  is an increasing function of  $\theta$ , then at a long-run equilibrium with  $\sigma_1 > \sigma_{\min}$ , there is a sufficiently small increase in  $\theta$  such that a new equilibrium is reached with  $1 < \sigma'_1 < \sigma_1$  and a higher  $h$ . The equilibrium wage-productivity ratio may decrease or increase.

If instead education is important in maintaining the hegemony of the ruling classes by inculcating an ideology of resignation and moderation, by increasing the tolerance for inequality and creating the perception of greater upward mobility than what actually exists, or by undermining the unity of the working class through the creation of what has been called labor aristocracy, then the function  $\lambda^* = \lambda^*(\theta)$  may be decreasing in  $\theta$ . In this case, the  $\hat{\lambda} = 0$  isocline will shift to the left. The combined effect on the two

curves implies that the equilibrium value of  $\lambda$  at the high-equilibrium  $[E_1]$  unambiguously decreases, whereas the change in the equilibrium value of  $h$  (and thus of the skill premium) is indeterminate depending on the relative strength of the two effects.

**Proposition 8 (Education as ideology).** Assume (A1)-(A8). Assume  $\tau_0 + \tau_1 k [M + 1]^{-\frac{1}{\rho}} < sk$ . If  $\lambda^{\max} < \bar{\lambda}$ , and  $\lambda^*(\theta)$  is a decreasing function of  $\theta$ , then at a long-run equilibrium with  $\sigma_1 > \sigma_{\min}$ , there is a sufficiently small increase in  $\theta$  such that a new equilibrium is reached with  $\sigma'_1 > 1$  and a lower  $\lambda$ , whereas the equilibrium level of  $h$  may decrease or increase.

The long-run equilibrium effects on the rates of capital accumulation, productivity growth and the distribution of income between the three classes depends on the direction of change in  $h$  and  $\lambda$ . If there is an increase in  $h$  and a small change in  $\lambda$ , which is more likely to occur if  $\lambda^*$  is increasing in  $\theta$ , the result will be a fall in the wage premium, a rise in the rate of capital accumulation and a rise in the rate of technological change, and little change in the state of class struggle. However, if  $\lambda^*$  is decreasing in  $\theta$ , it is more likely that  $h$  will not change much while  $\lambda$  will decrease. The latter will increase the rate of capital accumulation (unless the possible fall in  $h$  reduces it sufficiently), but low-skilled workers will get weakened in the class struggle, and there will be little change in the wage premium.

Two final remarks are worth making about the effect of education in the economy with endogenous class struggle. First, in Propositions 7 and 8 we have focused only on the stable equilibrium  $[E_1]$  with a skill premium above one because it arguably represents the economically relevant case. The effect of education on the unstable equilibrium with the skill premium below one is analyzed in analogous fashion. Second, if at the high

equilibrium  $\sigma_{\min} \geq \sigma_1 > 1$ , then an improvement in the openness of the educational system will have unambiguous positive or negative effects on  $h$  and on the wage productivity ratio depending on whether education plays a progressive, or regressive role.

**Proposition 9 (Education in the low-skill trap).** Assume (A1)-(A8). Assume

$\tau_0 + \tau_1 k [M + 1]^{\frac{1}{\rho}} < sk$ . Suppose that  $\lambda^{\max} < \bar{\lambda}$  and at a long-run equilibrium  $\sigma_{\min} \geq \sigma_1 >$

1. If  $\lambda^*(\theta)$  is an increasing function of  $\theta$ , then there is a sufficiently small increase in  $\theta$  such that a new equilibrium is reached with  $1 < \sigma'_1 < \sigma_1$ , a higher  $h$ , and a higher wage-productivity ratio. If  $\lambda^*(\theta)$  is a decreasing function of  $\theta$ , then there is a sufficiently small increase in  $\theta$  such that a new equilibrium is reached with  $1 < \sigma_1 < \sigma'_1 \leq \sigma_{\min}$  a lower  $h$ , and a lower wage-productivity ratio.

## 5. Conclusion

This paper has developed a classical model which has allowed us to examine the growth and distributional consequences of greater openness in the education system. In the model the resultant expansion of education allows more low-skilled workers to become high-skilled workers if they want to obtain education and which, in terms of broader political economy considerations, can affect the state of class struggle. In so doing, this paper has attempted to fill a lacuna in the literature on the classical-Marxian approach, which has neglected the formal analysis of the effects of education and skill formation on distribution and growth, an issue which many observers find to be a central feature of contemporary capitalist knowledge-based economies.

The model shows that an expansion in education will promote growth and have beneficial distributional effects within the working class, but not along standard orthodox

lines. For instance, while in neoclassical full employment models education has a positive effect on output and growth directly by increasing effective labor supply, in the classical model of this paper, the growth effect is the consequence of distributional changes. The model also stresses the importance of providing suitable incentives to workers for taking advantage of greater education access, without which the economy can be caught in a low-skill trap. Finally, when extended to endogenize the state of class struggle, the model suggests the importance of a progressive type of education, rather than one which weakens the power workers, in order to obtain an equitable growth outcome which improves the position of workers as a whole and reduces inequality among them.

The model developed here is a simple one which should be modified in various ways to check the robustness of the results. Several simple extensions of the models may be particularly interesting: allowing high-skilled workers to save and hold capital, and thereby have mixed class interests; allowing the wage premium to change slowly with the possibility that some high-skilled workers find low-skilled jobs (being chosen above low-skilled jobs); distributional effects of labor market conditions; introducing different levels of educations (such as primary, secondary, and higher education); and allowing aggregate demand issues to enter into the distribution of output and growth, as in post-Keynesian heterodox models. We leave these issues for further research.

## Appendix : Proofs of the main claims.

### 1. The derivation of labor demands.

Given Assumption 1, profit maximization yields

$$Y = kK$$

and

$Y = [(A_L L)^\rho + (A_H H)^\rho]^{1/\rho}$ , where L, H are chosen so as to solve the following problem

$$\text{Min } w_L L + w_H H$$

subject to  $Y^\rho = [(A_L L)^\rho + (A_H H)^\rho]$ .

The Lagrangean is:  $\Lambda = w_L L + w_H H + \lambda[Y^\rho - [(A_L L)^\rho + (A_H H)^\rho]]$ .

The first order conditions are:

$$w_L = \lambda \rho A_L^\rho L^{\rho-1} \quad (\text{A1})$$

$$w_H = \lambda \rho A_H^\rho H^{\rho-1} \quad (\text{A2})$$

$$Y^\rho = [(A_L L)^\rho + (A_H H)^\rho] \quad (\text{A3})$$

From (A1) and (A2), it follows that

$$(H/L) = (w_H/w_L)^{1/(\rho-1)} (A_H/A_L)^{-\rho/(\rho-1)}. \quad (\text{A4})$$

Then, by Assumption 2, we have  $A_H = \mu A_L$ , and equation (A4) becomes

$$(H/L) = (w_H/w_L)^{1/(\rho-1)} (\mu)^{-\rho/(\rho-1)}. \quad (\text{A4}')$$

Then, condition (A3) can be written as

$$Y^\rho = L^\rho [A_L^\rho + (A_H H/L)^\rho] = L^\rho \left[ A_L^\rho + A_H^\rho \left( \frac{w_H}{w_L} \right)^{\frac{\rho}{\rho-1}} \mu^{-\frac{\rho^2}{\rho-1}} \right],$$

or, given the assumption  $A_H = \mu A_L$ , and using  $Y = kK$

$$L = \frac{kK}{\left[1 + \left(\frac{w_H}{w_L}\right)^{\frac{\rho}{\rho-1}} \mu^{-\frac{\rho}{\rho-1}}\right]^{\frac{1}{\rho}} A_L}. \quad (\text{A5})$$

Similarly, using equation (A4'),

$$H = \frac{kK}{\left[1 + \left(\frac{w_H}{w_L}\right)^{\frac{\rho}{\rho-1}} \mu^{-\frac{\rho}{\rho-1}}\right]^{\frac{1}{\rho}} A_L} \left(\frac{w_H}{w_L}\right)^{\frac{1}{\rho-1}} \mu^{-\frac{\rho}{\rho-1}}$$

or

$$H = \frac{kK}{\left[\left(\frac{w_H}{w_L}\right)^{-\frac{\rho}{\rho-1}} \mu^{\frac{\rho}{\rho-1}} + 1\right]^{\frac{1}{\rho}} A_L \mu} \quad (\text{A6})$$

By substituting  $\sigma = (w_H/w_L)$ , and noting that  $\rho < 0$ , one obtains equations (3) and (4).

It is worth noting that optimal total labor costs are

$$w_L L^* + w_H H^* = \frac{kK}{A_L} \left[1 + \sigma^{-\frac{\rho}{1-\rho}} M^{-1}\right]^{\frac{1}{\rho}} \left[w_L + w_H \sigma^{\frac{1}{1-\rho}} M^{-1}\right],$$

where  $M = \mu^{\frac{\rho}{\rho-1}}$ . The latter expression implies

$$w_L L^* + w_H H^* = kK \frac{w_L}{A_L} \left[1 + \sigma^{-\frac{\rho}{1-\rho}} M^{-1}\right]^{\frac{1}{\rho}+1},$$

which in turn implies that total labor costs are increasing in  $\sigma$ .

## 2. The main properties of labor demands.

Let  $M = \mu^{\frac{\rho}{\rho-1}}$ . The function  $b$  can be written as follows  $b(\sigma) = k\mu^{-1} \left[ \sigma^{\frac{\rho}{1-\rho}} M + 1 \right]^{\frac{1}{\rho}}$ .

Therefore, omitting the constants  $k\mu^{-1}$ , if  $\rho < 0$ , then

$$b'(\sigma) = -\frac{1}{\rho} \frac{\rho}{1-\rho} \sigma^{\frac{\rho}{1-\rho}-1} M \left[ \sigma^{\frac{\rho}{1-\rho}} M + 1 \right]^{\frac{1}{\rho}-1} = -\frac{1}{1-\rho} \sigma^{\frac{2\rho-1}{1-\rho}} M \left[ \sigma^{\frac{\rho}{1-\rho}} M + 1 \right]^{\frac{1+\rho}{\rho}} < 0$$

Thus, as  $\sigma$  tends to zero,  $b'(\sigma)$  tends to minus infinity and as  $\sigma$  tends to infinity  $b'(\sigma)$  tends to zero. Furthermore

$$b''(\sigma) =$$

$$-\frac{1}{1-\rho} \frac{2\rho-1}{1-\rho} \sigma^{\frac{2\rho-1}{1-\rho}-1} M \left[ \sigma^{\frac{\rho}{1-\rho}} M + 1 \right]^{\frac{1+\rho}{\rho}} + \frac{1}{1-\rho} \sigma^{\frac{2\rho-1}{1-\rho}} M \frac{\rho}{1-\rho} \sigma^{\frac{\rho}{1-\rho}-1} M \frac{1+\rho}{\rho} \left[ \sigma^{\frac{\rho}{1-\rho}} M + 1 \right]^{\frac{1+\rho}{\rho}-1}$$

or

$$b''(\sigma) = \frac{1}{(1-\rho)^2} \sigma^{\frac{2\rho-1}{1-\rho}-1} M \left[ \sigma^{\frac{\rho}{1-\rho}} M + 1 \right]^{\frac{1+\rho}{\rho}-1} \left\{ (2-\rho) \sigma^{\frac{\rho}{1-\rho}} M - 2\rho + 1 \right\}.$$

Thus, the latter expression is positive for all  $\sigma$  (and  $M$ ), yielding a strictly convex function.

Similarly, consider  $c(\sigma) = k \left[ \sigma^{-\frac{\rho}{1-\rho}} M^{-1} + 1 \right]^{\frac{1}{\rho}}$ . Omitting the constant  $k$ , we obtain

$$c'(\sigma) = \frac{1}{\rho} \frac{\rho}{1-\rho} \sigma^{-\frac{\rho}{1-\rho}-1} M^{-1} \left[ \sigma^{-\frac{\rho}{1-\rho}} M^{-1} + 1 \right]^{\frac{1}{\rho}-1} = \frac{1}{1-\rho} \sigma^{-\frac{1}{1-\rho}} M^{-1} \left[ \sigma^{-\frac{\rho}{1-\rho}} M^{-1} + 1 \right]^{\frac{1+\rho}{\rho}} > 0$$

and

$$c''(\sigma) =$$

$$-\frac{1}{(1-\rho)^2} \sigma^{-\frac{1}{1-\rho}-1} M^{-1} \left[ \sigma^{-\frac{\rho}{1-\rho}} M^{-1} + 1 \right]^{\frac{1+\rho}{\rho}} + \frac{1}{1-\rho} \sigma^{-\frac{1}{1-\rho}} M^{-1} \frac{\rho}{1-\rho} \sigma^{-\frac{\rho}{1-\rho}-1} M^{-1} \frac{1+\rho}{\rho} \left[ \sigma^{-\frac{\rho}{1-\rho}} M^{-1} + 1 \right]^{\frac{1+\rho}{\rho}-1}$$

or

$$c''(\sigma) = \frac{1}{(1-\rho)^2} \sigma^{-\frac{1}{1-\rho}-1} M^{-1} \left[ \sigma^{-\frac{\rho}{1-\rho}} M^{-1} + 1 \right]^{\frac{1+\rho}{\rho}-1} \left\{ \rho \sigma^{-\frac{\rho}{1-\rho}} M^{-1} - 1 \right\}$$

Therefore, if  $\rho < 0$ , then  $c''(\sigma) < 0$  for all  $\sigma$ , yielding a strictly concave function, and as  $\sigma$  tends to zero  $c(\sigma)$  tends to  $k$  and as  $\sigma$  tends to infinity  $c(\sigma)$  tends to infinity. Furthermore, as  $\sigma$  tends to zero,  $c'(\sigma)$  tends to infinity and as  $\sigma$  tends to infinity  $c'(\sigma)$  tends to zero.

Finally, consider the inverse function  $\sigma(h) = \mu \left[ \left( \frac{k}{\mu h} \right)^\rho - 1 \right]^{\frac{1-\rho}{\rho}}$ . Note that in order to

guarantee the nonnegativity of the skill premium, it must be  $h > k/\mu$ . Then note that

$$\sigma'(h) = -\mu(1-\rho) \left( \frac{k}{\mu h} \right)^\rho h^{-1} \left[ \left( \frac{k}{\mu h} \right)^\rho - 1 \right]^{\frac{1-\rho}{\rho}-1}$$

and

$$\sigma''(h) = \mu(1-\rho) \left( \frac{k}{\mu h} \right)^\rho h^{-2} \left[ \left( \frac{k}{\mu h} \right)^\rho - 1 \right]^{\frac{1-\rho}{\rho}-2} \left\{ (1+\rho) \left[ \left( \frac{k}{\mu h} \right)^\rho - 1 \right] + (1-2\rho) \left( \frac{k}{\mu h} \right)^\rho \right\}$$

and therefore  $\sigma(h)$  strictly decreasing and strictly convex for all  $h > k/\mu$ .

### 3. The capital accumulation equation.

First of all, let us consider  $\hat{K}$  as a function of  $\sigma$ . Since  $\rho < 0$ , then if  $\sigma = 0$ , then  $\sigma b(\sigma) = 0$  and  $c(\sigma) = k$ , and thus  $\hat{K} = s(1-\lambda)k$ . As  $\sigma$  becomes infinitely large,  $b$  tends to  $k$  and  $c$  tends to infinity, so that the growth rate of capital becomes infinitely negative.



Furthermore, noting that  $\frac{d[\sigma b(\sigma) + c(\sigma)]}{d\sigma} = b(\sigma)$ , it follows that

$$\frac{d\hat{K}}{d\sigma} = -\frac{s\lambda k}{\mu} \left[ \sigma^{\frac{\rho}{1-\rho}} M + 1 \right]^{\frac{1}{\rho}} < 0,$$

$$\frac{d^2\hat{K}}{d\sigma^2} = \frac{s\lambda k}{\mu(1-\rho)} \sigma^{\frac{\rho}{1-\rho}-1} M \left[ \sigma^{\frac{\rho}{1-\rho}} M + 1 \right]^{\frac{1}{\rho}-1} > 0.$$

Note that if the relevant lower bound is  $\sigma = 1$ , then  $b(1) = k\mu^{-1}[M+1]^{-\frac{1}{\rho}}$  and  $c(1) =$

$k[M^{-1}+1]^{-\frac{1}{\rho}} = kM^{\frac{1}{\rho}}[M+1]^{-\frac{1}{\rho}}$  so that the growth rate of capital is

$$\hat{K} = sk \left[ 1 - \lambda M^{\frac{1}{\rho}} [M+1]^{-\frac{1}{\rho}} - \lambda \mu^{-1} [M+1]^{-\frac{1}{\rho}} \right] = sk \left[ 1 - \lambda \frac{M^{\frac{1}{\rho}} + \mu^{-1}}{[M+1]^{\frac{1}{\rho}}} \right] = sk \left[ 1 - \lambda \mu^{-1} [M+1]^{-\frac{1}{\rho}+1} \right].$$

Finally, if  $\rho < 0$ , by continuity there is a value of  $\sigma$ , call it  $\sigma^*$ , such that  $\hat{K} = 0$ . If

$sk \left[ 1 - \lambda \mu^{-1} [M+1]^{-\frac{1}{\rho}+1} \right] > 0$ , then  $\sigma^* > 1$ . More precisely,  $\sigma^*$  is the solution of

$$1 = \lambda \left[ \sigma^{-\frac{\rho}{1-\rho}} M^{-1} + 1 \right]^{\frac{1}{\rho}} + \sigma \lambda \mu^{-1} \left[ \sigma^{\frac{\rho}{1-\rho}} M + 1 \right]^{\frac{1}{\rho}} \Leftrightarrow \left[ \sigma^{\frac{\rho}{1-\rho}} M + 1 \right]^{\frac{1}{\rho}} = \lambda \mu^{-1} \sigma \left[ \sigma^{\frac{\rho}{1-\rho}} M + 1 \right]$$

or  $\left[ \sigma^{\frac{\rho}{1-\rho}} M + 1 \right]^{\frac{1-\rho}{\rho}} \mu \sigma^{-1} = \lambda$ , which is equivalent to  $\left[ \sigma^{-\frac{\rho}{1-\rho}} M^{-1} + 1 \right]^{\frac{1-\rho}{\rho}} = \lambda$ . Note that  $\lambda$

could be bigger or smaller than one and therefore the latter equality can hold if  $\rho < 0$ .

Next, let us consider  $\hat{K}$  as a function of  $h$ , with  $\hat{K} = s[k - \lambda c(\sigma(h)) - \sigma(h)\lambda h]$ . From the

previous analysis it follows that  $\frac{d\hat{K}}{dh} = -s\lambda h\sigma'(h)$  or

$$\frac{d\hat{K}}{dh} = s\lambda\mu(1-\rho)\left(\frac{k}{\mu h}\right)^\rho \left[\left(\frac{k}{\mu h}\right)^\rho - 1\right]^{\frac{1-\rho}{\rho}-1} > 0, \text{ for all } \rho \text{ and all } \left(\frac{k}{\mu h}\right)^\rho > 1.$$

Therefore

$$\frac{d^2\hat{K}}{dh^2} = s\lambda\mu(1-\rho)\left\{-\rho\left(\frac{k}{\mu h}\right)^\rho h^{-1}\left[\left(\frac{k}{\mu h}\right)^\rho - 1\right]^{\frac{1-\rho}{\rho}-1} - \left(\frac{k}{\mu h}\right)^\rho (1-2\rho)\left(\frac{k}{\mu h}\right)^\rho h^{-1}\left[\left(\frac{k}{\mu h}\right)^\rho - 1\right]^{\frac{1-\rho}{\rho}-2}\right\}$$

or

$$\frac{d^2\hat{K}}{dh^2} = -s\lambda\mu(1-\rho)\left(\frac{k}{\mu h}\right)^\rho h^{-1}\left[\left(\frac{k}{\mu h}\right)^\rho - 1\right]^{\frac{1-\rho}{\rho}-2}\left\{\rho\left[\left(\frac{k}{\mu h}\right)^\rho - 1\right] + \left(\frac{k}{\mu h}\right)^\rho (1-2\rho)\right\},$$

or

$$\frac{d^2\hat{K}}{dh^2} = -s\lambda\mu(1-\rho)\left(\frac{k}{\mu h}\right)^\rho h^{-1}\left[\left(\frac{k}{\mu h}\right)^\rho - 1\right]^{\frac{1-\rho}{\rho}-2}\left\{\left(\frac{k}{\mu h}\right)^\rho (1-\rho) - \rho\right\},$$

and the latter expression is negative for all  $\rho < 0$ .

#### 4. The equation of motion of h.

Let us consider the growth rate of h as a function of h:

$$\hat{h} = \tau_0 + \tau_1\mu h + \theta g(\sigma(h)) - s[k - \lambda c(\sigma(h)) - \sigma(h)\lambda h].$$

Then it follows that

$$\frac{d\hat{h}}{dh} = \tau_1\mu + \theta g'(\sigma)\sigma'(h) + s\lambda h\sigma'(h)$$

If  $\rho < 0$  then as  $h \rightarrow k/\mu$ , then  $\hat{h} \rightarrow \infty$  and  $\frac{d\hat{h}}{dh} \rightarrow -\infty$ , whereas as  $h \rightarrow \infty$ , then  $\hat{h} \rightarrow \infty$

and  $\frac{d\hat{h}}{dh} \rightarrow \tau_1\mu$ . Furthermore, if  $g$  is convex, then it follows from the properties of the capital accumulation equation that  $\hat{h}$  is strictly convex for all  $\rho < 0$ .

### 5. Dynamic stability of the model with endogenous $\lambda$ .

The elements of the Jacobian matrix of the dynamic system given by equations (16) and (20) are given by

$$\frac{\partial \hat{h}}{\partial h} = \tau_1\mu + \theta g' \sigma' + s[\lambda \sigma'(c+h) + \lambda \sigma]$$

$$\frac{\partial \hat{h}}{\partial \lambda} = s[c(\sigma(h)) + \sigma(h)h] > 0$$

$$\frac{\partial \hat{\lambda}}{\partial h} = -(1 - \delta_2)\tau_1 < 0$$

$$\frac{\partial \hat{\lambda}}{\partial \lambda} = -\delta_1\lambda < 0$$

Only the first element cannot be definitely signed. However, since it was shown in the

text that the  $\hat{h} = 0$  is positively sloped at the long-run equilibrium at  $E_1$ , its sign must be

negative, given the positive sign of  $\frac{\partial \hat{h}}{\partial \lambda}$ . It immediately follows that in the neighborhood

of  $E_1$  the trace of the Jacobian is negative and its determinant is positive, satisfying the

sufficient conditions for stability. It can also be shown that in the neighborhood of  $E_2$

this stability condition is not satisfied.

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## NOTES

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<sup>1</sup> Glyn et. al. (1990) argue that the slowdown in productivity growth after the late 1960s can be traced at least in part to the growing tendency, found even during the period of the Golden Age, to exclude the mass of unskilled workers from the search for new technologies, which was conducted in specialist divisions like research and development departments, while technological improvements require greater involvement of production workers. This exclusion can be explained partly by the adoption of organizational methods of command and control, and the slowdown of the expansion of education and skill formation among workers. Lipietz and Leborgne (1996) link these economic problems with the abandonment of investment in education.

<sup>2</sup> Marx, for instance, argued that education would not have a positive effect of wages when the educated labor became unemployed due to the introduction of machinery because the skilled unemployed would bid down wages competing for the remaining jobs (Marx and Engels, 1975-2005, Vol. VI, p. 427).

<sup>3</sup> See, for example, the classic analyses by Goodwin (1967) and Foley (1986). More recent contributions include Flaschel (2009). The focus on the capitalist/workers divide is motivated by the centrality of exploitation theory in the Marxian approach. For a thorough, recent analysis of the relation between exploitation and distributive issues, see Veneziani (2007) and Yoshihara (2010).

<sup>4</sup> See Pasinetti (1960) for a classic treatment. Recent models including land and natural resources in a classical-Marxian vein include Foley and Michl (1999), Foley (2003) and Petith (2008).

<sup>5</sup> See Dutt (1989).

<sup>6</sup> It may be noted that both the classical-Marxian and neoclassical approaches to growth assume that all saving is automatically invested, so that there are no problems due to the lack of sufficient aggregate demand for goods.

<sup>7</sup> See O'Donnell (1985) for a review of classical ideas on education, and Dutt and Veneziani (2010) for a more systematic discussion of classical ideas on the role of education in growth and distribution. The latter work also develops a simpler and specialized version of the model developed more rigorously in the rest of this paper.

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<sup>8</sup> Roemer (2006) also develops a model with optimizing individuals, human capital accumulation and voting for educational funding with two parties to show how, depending on the degree of altruism and educational technology, a variety of outcomes with different implications for the evolution of inequality are possible.

<sup>9</sup> A recent text on growth theory written mainly from a classical-Marxian perspective, by Foley and Michl (1999) has no discussion of the role of education in economic growth. We are, in fact, unaware of any heterodox dynamic model of growth and distribution, let alone a classical-Marxian one, which analyzes the role of education, apart from a post-Keynesian model in Dutt (2010).

<sup>10</sup> All our main results can be generalized to the case with  $\rho > 0$ , provided elasticity of substitution is sufficiently low.

<sup>11</sup> For a complete formal derivation, see the Appendix.

<sup>12</sup> This is the standard interpretation of the classical-Marxian approach to growth adopted in most the literature mention earlier, including Goodwin (1967), Marglin (1984), Dutt (1990), Foley and Michl (1999) and Duménil and Lévy (2003). Some of these contributions refer to the approach as the neo-Marxian one.

<sup>13</sup> Of course, in the short run, given  $H$  and  $L$ , the skill composition of the labor force determines  $\sigma$ .

<sup>14</sup> This specification may be compared with similar specifications in the neoclassical literature on education, innovation and economic growth. Uzawa's (1965) pioneering model of education and economic growth assumes that the rate of labor productivity growth is a positive function of the ratio of employment in the education sector to the total labor force, Lucas's (1988) formulation assumes that the change in human capital (which increases labor productivity in production) depends positively on the stock of human capital and on the time workers spend on education as a ratio of total labor time. Our formulation is different from these formulations in that we not have a separate education sector and do not consider the proportion of time a worker spends on education (in comparison to time spent in productive employment), but is similar in that it makes productivity growth depend positively on the size of the educated labor force in relation to the size of the economy (measured by capital stock rather than the labor force, since we allow for unemployment), and an index of labor productivity. Our analysis is also similar to that of Aghion and Howitt (1998) who assume that the (expected) rate of arrival of new innovations depends positively on the level of employment in the research and development sector (the size of the economy being fixed because



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the labor force is held constant and because there is no capital in the basic model). The model, however, does not take education into account.

<sup>15</sup> By A1, the function  $\sigma = b^{-1}(h)$  can be written as  $\sigma(h) = \mu \left[ \left( \frac{k}{\mu h} \right)^\rho - 1 \right]^{\frac{1-\rho}{\rho}}$ , where  $\sigma(h)$  is strictly

decreasing and strictly convex for all  $h \in (k/\mu, \infty)$ .

<sup>16</sup> It can be proved that if  $\rho > 0$ , then there exists a unique, asymptotically stable long-run equilibrium with a positive skill premium.

<sup>17</sup> These dynamics could be made to depend on inflationary dynamics of price and money wage changes, but we abstract from these complications for simplicity. See, for instance, Dutt (1990) for a discussion.

<sup>18</sup> It may be noted that the stable equilibrium is the economically meaningful one: the unstable equilibrium has  $\sigma_2 < 1$ , which will not occur in the real economy.