A Sensitivity-Enhanced Optical Heterodyne Polarimeter for the Measurement of Optical Rotation Angle of Chiral Media

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ABSTRACT

A novel heterodyne polarimeter is designed to measure the concentration of chiral media. Optical common-path for the interference of the TE and TM waves, after a polarizer, is set up in our system to reduce noises from the environment. A phase-variable waveplate is placed behind the sample to enhance the phase signal change of rotation of polarization introduced by the sample itself. This enhancement can be excess two orders in amplitude when the retardation of the phase-variable plate is set close to 180 degree. With this polarimeter, the measurement of optical rotation angle with high sensitivity of 6.5×10^{-4} degree experimentally can be achieved when phase retardation of the phase-variable waveplate is 178.5 degree. We expect that, by further improvement, it can be applied in noninvasive blood glucose concentration monitoring for diabetics in the future.

Keywords: Optical rotation, polarimeter, heterodyne, chiral media

1. INTRODUCTION

In the early nineteen-century, Arago and Biot observed optical activity separately. As known, it’s a property of chiral media to make the polarization plane of linear light rotated due to the phase velocity difference between the right and left circularly polarized light that are decomposition of linear light. The easiest setup to measure optical rotation angles is employing two crossed polarizers. There are disadvantages usually existing with this method, involving the fluctuation of the light source, the influence of background noise, and the SNR related highly with the extinction ratio of analysers. Thus the accuracy, sensitivity, and stability of this setup are not good enough to be applied to detect the variation of small optical rotation angles below 10^{-2} degree. Until now many optical methods for measuring small optical rotation angles have been investigated because of potential for achieving the noninvasive detection of blood glucose concentration of diabetics. Optical amplitude or intensity measurement with Faraday rotators is a common method for usage. Compared to two crossed polarizers, due to application of modulation technique the resolution is improved. In 1997 Chou et al. presented an amplitude sensitive measurement method with optical common-path and heterodyne interferometer that the signal of small optical rotation angle is multiplied by two light amplitudes. The sensitivity of concentration 10 mg/dl in 1 cm optical path length can be obtained with this setup. Afterward Chou et al. employed this idea in non-invasive glucose monitoring in vivo. An alternative kind of the signal could be detected is optical phase. This kind of phase-measurement method is better than that of measuring amplitude or intensity in stability and sensitivity, because the fluctuation of intensity is avoided. However a true phase measurement is not sensitive enough to detect the small optical rotation angle. Thus the application of this method with phase measurement developed is limited, and can’t be used in lower blood glucose measurement. Later a new type of polarimeter is developed to measure the chiral parameter and average refractive index simultaneously. Although this kind of polarimeter shows a good accuracy, the possibility of noninvasive blood glucose monitoring is lower.

In this study, we employ a phase sensitivity-enhanced polarimeter for measuring small rotation angles. The main concept is using a great slope to enhance the sensitivity of optical rotation angle measurement. We adopt the optical common-path and heterodyne interferometric setup to reduce the noise, and use a phase-variable waveplate to be our sensitivity-enhanced sensor for optical rotation angle. The refractive index variance of different concentration of chiral solution will not affect the phase signal because of the optical common-path interference configuration. The change of phase signals is related only to the optical rotation angle, and the d- or l-rotatory of the chiral media also can be determined in this polarimeter.

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Fig. 1. The scheme of the optical heterodyne polarimeter. M: mirrors; HWP: half wave plate; BS: beamsplitter; AOM: acousto-optics modulator; PBS: polarizing beamsplitter; S: sample; \( \delta \)-WP: phase-variable wave plate; P: polarizer; PhD: photodetector.

Fig. 2. The simulated relationship between the phase difference \( \phi \) and orientation angle \( \theta \) with different phase retardation \( \delta \) of the phase-variable wave plate (\( \delta = 0, 45, 90, 178, \) and 180 deg). It can be observed that when \( \delta \) is close to 180 deg, some regions have a great slope.

2. PRINCIPLE

In order to progress this experiment, we adopt a configuration of a Mach-Zehnder structure to set up our optical common-path heterodyne polarimeter, and use a phase-variable waveplate (\( \delta \)-WP) to be a sensor for optical rotation angle (Fig. 1). A beamsplitter (BS1) separates the laser light to propagate in each channel of the Mach-Zehnder structure. The different driving frequencies of two acousto-optics modulators (AOMs) are \( \omega_1 \) and \( \omega_2 \), respectively. Due to the specification of AOMs, the polarization of incident and output light are horizontal and vertical, respectively. Thus to construct a heterodyne light source similar to the configuration in ref. [10], two half wave plates (HWP) are needed. HWP1 rotates the polarization of incident light to be the mode of a TM wave, and HWP2 is used to return the polarization plane of the beam after AOM1 in the upper channel to be the mode of a TM wave. A polarizing beamsplitter (PBS) is used to combine two beams from two channels, and a heterodyne light source is established now. Thus the heterodyne light source includes two waves with crossed polarization, TE and TM. In order to clarify the theoretical analysis, the calculus of Jones matrices is used herein. For convenience, the +z-axis is chosen to be along the light propagation direction and the y-axis is vertical direction. The electric vector of the heterodyne light source is:

\[
\vec{E}_{\text{h}} = A(\omega_1, \omega_2) \vec{E}_i
\]  

(1)

where \( A(\omega_1, \omega_2) \) is the Jones matrix of AOMs, and \( \vec{E}_i \) is the incident laser light.

To measure the concentration of glucose solution, a heterodyne light source is separated into a reference beam and a measurement beam, respectively. Between two light paths, the reference beam in turn meets a polarizer (P1) and a photodetector (PhD1). The signal from PhD1 is sent to the reference input of a phase lock-in amplifier. The other beam named the measurement beam passes through the sample (S), \( \delta \)-WP, and polarizer (P2) in turn. The transmission axis of two polarizers P1 and P2 are adjusted at 45 deg with respect to the x-axis. Finally, the second photodetector (PhD2) detects the measurement signal treated as an input signal of a phase lock-in amplifier. The phase difference between reference and measurement light will be shown in the display of the lock-in amplifier. The Jones vector of the measurement beam is given as:

\[
\vec{E}_{\text{m}} = P(45^\circ)C(\delta, \theta)S(\beta) \vec{E}_0
\]

(2)

where \( P(45^\circ) \), \( C(\delta, \theta) \), \( S(\beta) \), and \( \vec{E}_0 \) are Jones matrices, respectively, represented the polarizer whose azimuth is at 45 deg, \( \delta \)-WP which phase retardation is \( \delta \) deg and azimuth is at \( \theta \) deg, sample resulting optical rotation angle with \( \beta \) deg, and heterodyne light source. All values of matrices are shown as following:
\[ E_i = \begin{bmatrix} 1 \\ e^{i\omega_i t} \end{bmatrix}, \quad A(\omega_1, \omega_2) = \begin{bmatrix} e^{i\omega_1 t} & 0 \\ 0 & e^{i\omega_2 t} \end{bmatrix}, \quad S(\beta) = \begin{bmatrix} \cos \beta & \sin \beta \\ -\sin \beta & \cos \beta \end{bmatrix}. \]

\[ C(\delta, \theta) = \begin{bmatrix} \cos \frac{\delta}{2} + i \sin \frac{\delta}{2} \cos 2\theta \\ i \sin \frac{\delta}{2} \sin 2\theta \\ i \sin \frac{\delta}{2} \sin 2\theta \\ \cos \frac{\delta}{2} - i \sin \frac{\delta}{2} \cos 2\theta \end{bmatrix} \cdot P(45^\circ) = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} \]

We neglect the initial phase of laser output and phase difference due to transmission and reflectance of the BS here, because it can be eliminated by calibration. And the phase difference resulting from the light path length between TE and TM waves also can be excluded due to the optical common-path configuration.

Substituting those matrixes above into Eq. (2), we get:

\[
\begin{align*}
\vec{E}_m &= [\cos (\frac{\delta}{2} \cos \beta \sin \beta) + i \sin (\frac{\delta}{2} \cos \beta \cos 2\theta)] e^{i(\omega_1 + \omega_2) t} \\
&\quad + [\cos (\frac{\delta}{2} \cos \beta + \sin \beta) - i \sin (\frac{\delta}{2} \cos \beta \cos 2\theta) - \sin (\beta \cos 2\theta)] e^{i(\omega_1 - \omega_2) t},
\end{align*}
\]

The intensity of the measurement beam is given as:

\[ I_m = \vec{E}_m \times \vec{E}_m^* = k_{s1}^2 + k_{s2}^2 + 2 k_{s1} k_{s2} \cos [(\omega_1 - \omega_2) t + \phi] \]

where

\[ k_{s1} = (1 - \cos^2 \frac{\delta}{2} \sin 2\beta + \sin^2 \frac{\delta}{2} \sin(4\theta + 2\beta))^\frac{1}{2}, \]

\[ k_{s2} = (1 + \cos^2 \frac{\delta}{2} \sin 2\beta - \sin^2 \frac{\delta}{2} \sin(4\theta + 2\beta))^\frac{1}{2}, \]

and

\[ \phi = \tan^{-1}\left[ \frac{\sin \delta \cos 2\theta}{\cos^2 \delta \cos 2\beta - \sin^2 \frac{\delta}{2} \cos(4\theta + 2\beta)} \right]. \]

Similarly the electric vector of the reference beam could be expressed as:

\[ \vec{E}_r = P(45^\circ) \vec{E}_0, \]

and the intensity is given:

\[ I_r = \vec{E}_r \times \vec{E}_r^* = 1 + \cos [(\omega_1 - \omega_2) t] \]

The phase lock-in amplifier could compare two signals, (5) and (10), and extract the phase difference \( \phi \) shown on the LCD display. Due to the range of the phase lock-in amplifier is defined between \(-180\) and \(180\) deg, we should change range of the calculated phase signal that defined between \(0\) and \(360\) deg to the range between \(-180\) and \(180\) deg. We could reach our purpose with simple mathematical formulas. We can simulate the relationship between \( \delta \) and \( \theta \) in the condition of \( \beta = 0 \) (Fig. 2). It can be observed that some regions are linear and have an increasing slope when \( \delta \) is close to \(180\) deg. To observe more clearly, we simulate the relationship in the condition of \( \delta = 178\) deg. The regions near \( \theta = 22.5, 67.5, 112.5, 157.5, 202.5, 247.5, 292.5, \) and \(337.5\) deg could be thought as linear areas, and the absolute value of the slope (named multiplying power \(|P|\)) is so great. Thus we could expect when the sample with optical activity is placed, large signal change of the phase difference will be observed. As given the concentration of glucose solution \( C (g \cdot dl^{-1}) \) is defined by the following relationship:

\[ C = \frac{\beta}{L \times [\alpha]_{rpm}} \]

where \([\alpha]_{rpm} \) (deg \cdot dl \cdot cm^{-1} \cdot g^{-1}) is the specific rotation depending on temperature \( T \), wavelength \( \lambda \), and pH value, \( \beta \) (deg) is the optical rotation angle, and \( L \) (cm) is the light path length through the sample. To test our system, we can measure the signal of phase difference \( \phi_0 \) with the doubly distilled water (ddH\(_2\)O, it means \( \beta = 0 \)) treated as the sample.
at first to check the phase retardation of \( \delta \)-WP, and then proceed to measure samples with different concentration at the sharp region. According to formula (11), we know that \( C \) is linearly proportional to \( \beta \). When measuring different concentration of glucose solution resulting in different \( \beta \), we would obtain different phase signal \( \phi_c \). By simulating the relationship between optical rotation angle and the phase difference \( \Delta \varphi = \phi_c - \phi_0 \), we can get a linear relationship. The plot shown in Fig. 3 is the simulated result under the conditions of \( \theta = 22.5 \) deg and \( \beta \) from 0 to 0.01 deg by each step of 0.001 deg. We could estimate roughly the slope about 80. We have known that some regions are linear and have an increasing slope when \( \delta \) is close to 180 deg from Fig. 2. Here we give a definition as following to describe this slope named multiplying power:

\[
P = \frac{\Delta \varphi}{\Delta \theta}
\]  

(12)

The plot of is the simulated result of the relationship between \( P \) and \( \delta \) is shown in Fig. 4. The \( P \) is increased greatly with \( \delta \) approach to 180 degree. Hence, we can utilize the idea of a great slope to procure a phase sensitivity-enhanced measurement theoretically.

3. EXPERIMENT SETUP AND RESULTS

In order to confirm the feasibility of this setup (Fig. 1), we chose the glucose solution to be our optically active medium. A stabilized He-Ne laser with wavelength 632.8 nm (STP 901, Melles Griot) and two AOMs (FS040-2E-NI1, Gooch & Housego) were used to produce the heterodyne light source. The dual frequency source (DFE-404A4, IntraAction) supplied the two driving frequencies, 40 kHz and 40.06 kHz, respectively, to two AOMs. Thus the beat frequency was 60 kHz. We employed the manner of serial dilution to measure different concentrations of glucose solutions from 0 to 500 mg/dl. A phase lock-in amplifier (SR850, Stanford Research System) with resolution of 0.01 deg was used to extract the phase signal that is due to optical rotation angle. The doubly distilled water was used to be the zero concentration for calibration of the zero-point. At first, we tuned the phase retardation of the \( \delta \)-WP at 178.5 deg and the orientation angle of \( \delta \)-WP at the region near 22.5 deg by 0.04 deg for each step. The measured relationship between \( \varphi \) and \( \theta \) was shown in Fig. 5. Due to the effect of the initial phase of laser and phase difference resulting from BS, experimental data can’t be compared with simulation directly. In order to simplify the comparison of experimental data and the simulation result, we set values by subtracting the first point value to obtain the phase different signal. In other words, we set the first point to be the zero-point. After this correction, the experimental data and the simulation result were matched. From Fig. 5, we know that the slope is about -204. With simple calculation to correct the rotationless of the polarizer P2, we obtained the absolute value of the multiplying power is 108 with \( \delta = 178.5 \) deg. We first prepared 1000mg/dl of glucose.
solution as our sample and diluted it with the same volume of ddH$_2$O sequentially to obtain a series of samples with half concentration in each step (i.e. 500, 250, ... mg/dl). Due to the larger variation of signal in the linear region we chose larger time constant (e.g. 1s) of the phase lock-in amplifier to reduce variation. Each concentration was measured in 10s with sampling rate 1 Hz. The plot shown in Fig. 6 is the experimental result with different concentration. As known, 100 mg/dl of glucose solution gets 0.004 degree of rotation in a 1-cm path length when temperature is at 20°C. Compared this value with our result, we obtained a factor of two in the optical rotation angle. We contributed this to the high temperature effect (i.e. 26°C). The standard deviation obtained from data at each concentration is not over 0.07 degree so the sensitivity $\Delta \theta$ of our system is about $6.5 \times 10^{-4}$ degree with $P=108$ by the definition as following:

$$\Delta \theta = \frac{\theta_{nl}}{P}$$  \hspace{1cm} (13)

However, the phase-drift in our system was also observed. This is a concerned limitation of the system performance. The reason of this problem may be caused by the thermal instability of the system. Thus, the concentrations below 31.25 mg/dl are difficult to be resolved.

4. CONCLUSIONS

We presented a sensitivity-enhanced heterodyne polarimeter for the measurement of small optical rotation angle of chiral media. A $\delta$-WP was employed in our setup to enhance the change of rotation of polarization introduced by the sample itself. The configurations of the common-path and heterodyne interferometer were employed in our system. The optical common-path interferometer with TE and TM waves can reduce the disturbance from surrounding environment, and the heterodyne interferometer can avoid the 1/f noise. The high sensitivity of about $6.5 \times 10^{-4}$ deg is achieved in our system when the phase retardation $\delta=178.5$ deg. The detection limit is 31.25 mg/dl. Our achievement is the best one among other polarimeters with phase measurement. Thus, our sensitivity-enhanced polarimeter has potential to be a tool for noninvasive glucose monitoring. However, the phase-drift in our system limits the system performance. It’s a critical issue for the in vivo measurement.

ACKNOWLEDGMENTS

This work was supported by the National Science Council under grand No. NSC-92-2215-E-007-026.

REFERENCES


