Muon anomalous magnetic moment, two-Higgs-doublet model, and supersymmetry

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The recent measurement of the muon anomalous magnetic moment a_μ shows a 2.6σ deviation from the standard model value. We show that it puts strong constraints on the parameter space of the two-Higgs-doublet model (2HDM) II. The dominant contribution of the Higgs bosons comes at the two-loop level, and in order to explain the data it favors a pseudoscalar A with a light mass range and a large $\tan\beta$. At 95% C.L., the upper limit for m_A is 29 (55) (85) GeV for $\tan\beta = 30$ (45) (60), and $\tan\beta$ is bounded below at 17. This is in sharp contrast to the conclusion one draws from considering one-loop Higgs contributions alone. Finally, we also discuss the role of the Higgs contributions in the minimal supersymmetric standard model.

The recent result of the measurement of muon anomalous magnetic moment (a_μ) by the experiment E821 [1] at Brookhaven National Laboratory has reduced the error to a very small level. Comparing the data with the prediction from the standard model (SM) [2], one gets the deviation

$$\Delta a_\mu \equiv a_\mu^{\exp} - a_\mu^{SM} = 426\,(165) \times 10^{-11}, \qquad (1)$$

which may suggest the presence of contributions from physics beyond the SM. Taking the above numbers at face value, the range of Δa_μ at 95% C.L. ($\pm 1.96\sigma$) is given by

$$10.3 \times 10^{-10} < \Delta a_\mu < 74.9 \times 10^{-10}. \qquad (2)$$

Most of the extensions of the SM start with an extended Higgs sector, the simplest of which is the two-Higgs-doublet model (2HDM). In particular, the more interesting model II [3] shares the same Higgs structure as the minimal supersymmetric standard model (MSSM). Hence, going beyond the SM to look for extra contributions to a_μ, the 2HDM should be among the first to be examined seriously.

In this Rapid Communication, we investigate the contributions from the 2HDM II. Stringent constraints are thus obtained on the parameter space of the model. The results are summarized as follows. In the 2HDM II, the dominant contribution actually comes at the two-loop level, and in order to explain the data Δa_μ is preferred to come from the Higgs pseudoscalar A, the mass of which is required to be less than 29 (55) (85) GeV for $\tan\beta = 30$ (45) (60) by the 95% C.L. range of Eq. (2). Moreover, $\tan\beta$ has to be larger than 17. Recently, there has been some work on the same subject [4]; however, the paper only considered the one-loop contributions from the Higgs bosons, which become smaller than their corresponding two-loop contributions when the Higgs bosons are heavier than a few GeV and substantially smaller for heavier Higgs bosons. It is important to note that the

two-loop contributions considered here dramatically change the story of the Higgs sector contributions to a_μ and invalidate most of the results of the one-loop studies. Although we focus on the 2HDM II here, a similar conclusion holds for most models with possible large contributions from the Higgs sector. In models with flavor-changing Higgs couplings, in particular, there are potentially 1-loop contributions substantially larger than the flavor-conserving ones [5]. However, the Barr-Zee type 2-loop contributions would still have an important role to play and should be taken into consideration. This fact has often been overlooked in the literature.

In the MSSM, the dominant contribution comes from the chargino-sneutrino-loop diagrams and it has been shown [6] that to satisfy Δa_μ requires the gaugino mass and the smuon mass below about 600–800 GeV. Because of additional mass constraints on the scalars in MSSM, the total contribution from the Higgs bosons is not at a significant level and thus will not affect the conclusion of Ref. [6]. While the typical failure of the MSSM studies to address this kind of 2-loop contribution is a potential problem, our results here get rid of the worry, at least for the case of a more generic scalar mass spectrum.

Many other extensions of the SM have extra contributions to the a_μ. Some examples are additional gauge bosons [7], leptoquarks [8], and muon substructure [9]. However, not all of them can contribute in the right direction as indicated by the data. Thus, the $a_\mu^{\exp}$ measurement can differentiate among various models, and perhaps with other existing data can put very strong constraints on the model under consideration.

Given the mass bound on SM Higgs boson [10], the Higgs contribution to Δa_μ at one-loop level is negligible. However, it has been emphasized, in Ref. [11] for example, that for Higgs boson mass larger than about 3 GeV, the dominant Higgs contribution to a_μ actually comes from the two-loop Barr-Zee diagram (first discussed by Bjorken and Weinberg) [12] with a heavy fermion (f) running in the loop. A m_f^2/m_μ^2 factor could easily overcome the $\alpha/4\pi$ loop factor. The two-loop scalar contribution with a heavy fermion f, as shown in Fig. 1, is given by

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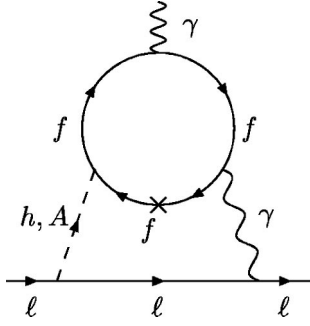
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FIG. 1. The dominant two-loop graph involving a scalar or a pseudoscalar boson that contributes to a_l.

$$\Delta a_l^h = -\frac{N_c^f \alpha^2}{4\pi^2 \sin^2 \theta_W} \frac{m_l^2 \lambda_l}{M_W^2} \mathcal{Q}_f^2 \lambda_f f\left(\frac{m_f^2}{m_h^2}\right), \qquad (3)$$

where

$$f(z) = \frac{1}{2} z \int_0^1 dx \frac{1-2x(1-x)}{x(1-x)-z} \ln\frac{x(1-x)}{z}, \qquad (4)$$

N_c^f represents the number of color degrees of freedom in f, and $\mathcal{Q}_f$ its electric charge. Here l denotes a generic lepton, m_h is the (scalar) Higgs boson mass, and λ_l and λ_f represent plausible modifications to the Higgs couplings of the fermions ($\lambda_l = \lambda_f = 1$ in the SM).

Reference [2] quoted an electroweak contribution, calculated up to two-loop level, of

$$a_\mu^{\mathrm{EW}} = 152(4) \times 10^{-11}. \qquad (5)$$

Included in this number is a Barr-Zee diagram contribution with a t loop. The numerical value of this result barely exceeds the order of 10^{-11} (see also Ref. [13]) and is negative for any reasonable value of m_h. Moreover, there are other purely bosonic two-loop contributions [14], in which the SM Higgs boson also plays a role. Nevertheless, all the contributions involving the SM Higgs boson are quite small [2,13,14]. When considering a model with an extended Higgs sector, a complete analysis would first require one to subtract the SM Higgs contribution and recalculate all the Higgs contributions. This is because the number of Higgs bosons, their effective couplings, and mass constraints would be different from the SM scenario. Nevertheless, in our study here, we only calculate the Higgs Barr-Zee diagram contributions and assume that this does give a very good approximation of a_μ from the diagrams involving Higgs bosons ($\Delta a_\mu^{\mathrm{Higgs}}$). Our rationale is as follows. We are interested in the region of parameter space where the Higgs contributions could explain the discrepancy of Eq. (1), or at least, in the case of models that have other important contributions, do play a substantial role in Δa_μ. Hence, we focus on the region where $\Delta a_\mu^{\mathrm{Higgs}}$ is at or close to the order of 10^{-9}. As we will see below, the possibility of having such largely enhanced Higgs contributions comes from the combined effect of coupling enhancements and weakened Higgs mass constraints. The coupling enhancement is only to be found

among the Yukawa couplings, thus illustrating the special importance of the Barr-Zee diagram considered.

In a model with an extended Higgs sector, we can write the fermion couplings of a neutral Higgs mass eigenstate ϕ^0 as

$$\mathcal{L}^{\bar{f}\phi^0 f} = -\lambda_f \frac{g m_f}{2M_W} \bar{f}\phi^0 f + i\gamma_5 A_f \frac{g m_f}{2M_W} \bar{f}\phi^0 f, \qquad (6)$$

where $\lambda_f(g m_f/2M_W)$ and $A_f(g m_f/2M_W)$ are the effective scalar and pseudoscalar couplings. The contribution of two-loop diagram (Fig. 1) to a_μ is then given by the sum of Eq. (3) (with $m_h = m_{\phi^0}$) and the pseudoscalar expression

$$\Delta a_l^A = \frac{N_c^f \alpha^2}{4\pi^2 \sin^2 \theta_W} \frac{m_l^2 A_l}{M_{W^2}} \mathcal{Q}_f^2 A_f g\left(\frac{m_f^2}{m_A^2}\right), \qquad (7)$$

(with $m_A = m_{\phi^0}$), where

$$g(z) = \frac{1}{2} z \int_0^1 dx \frac{1}{x(1-x)-z} \ln\frac{x(1-x)}{z}. \qquad (8)$$

Without CP violation in the Higgs sector, Eq. (3) or Eq. (7) gives directly the contribution from a scalar or a pseudoscalar, respectively. The corresponding contributions with the γ in Fig. 1 replaced by the Z^0 are suppressed by about two orders of magnitude [11], hence neglected here. We also skip the details about the similar contribution from a charged Higgs boson, which involves a $W^\pm$ boson and is thus suppressed also. Note that a light charged Higgs boson less than 80.5 GeV is ruled out by the CERN e^+e^- collider (LEP) experiments [10]. Moreover, analysis of its contribution to $b \to s\gamma$ leads to a much stronger lower bound—380 GeV, as claimed in Ref. [15], for instance.

The 2HDM has three physical neutral Higgs bosons: two scalars h and H, and the pseudoscalar A. For model II under consideration, the corresponding nonzero λ_f or A_f for $f = t, b$, and $l (= e, \mu, \tau)$ are given by

$h(\lambda_f)$:	$\dfrac{\cos\alpha}{\sin\beta}$	$-\dfrac{\sin\alpha}{\cos\beta}$	$-\dfrac{\sin\alpha}{\cos\beta}$
$H(\lambda_f)$:	$\dfrac{\sin\alpha}{\sin\beta}$	$\dfrac{\cos\alpha}{\cos\beta}$	$\dfrac{\cos\alpha}{\cos\beta}$
$A(A_f)$:	$\cot\beta$	$\tan\beta$	$\tan\beta$

respectively, in the standard notation [3]. What is particularly interesting phenomenologically is the enhancement of the couplings of $\bar{b}bH$, $\bar{b}bh$, $\bar{l}lH$, $\bar{l}lh$, $\bar{b}bA$, and $\bar{l}lA$ at large $\tan\beta$. In fact, the dominant contributions then come from the diagrams with a b or τ loop.

It was pointed out in Ref. [11] that the two-loop pseudoscalar contribution to a_μ is positive while the two-loop scalar contribution is negative in the large $\tan\beta$ region. Note that this is always true for the dominating contributions with a b or τ loop, independent of the scalar mixing angle α. The reverse happens in the corresponding one-loop contributions, but these one-loop contributions are suppressed relative to
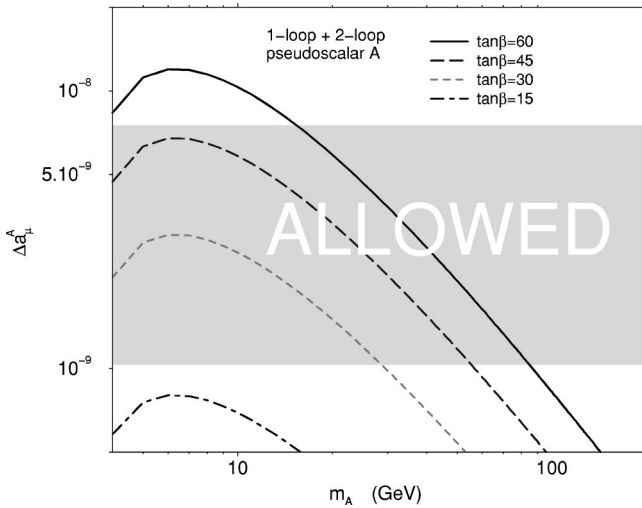
FIG. 2. Δa_μ^A (one loop and two loop summed) versus the pseudoscalar Higgs boson mass m_A for various values of $\tan\beta$.

the two-loop contributions. In fact, using the one-loop a_μ result in constraining the 2HDM II has been studied extensively [16]. Using the two-loop result, however, changes the story dramatically and invalidates most of the conclusions from the one-loop studies.

Applying the Δa_μ constraint to the 2HDM II, we need to suppress the scalar-Higgs (h and H) contributions, as it comes in the opposite direction as indicated by the data, relative to the pseudoscalar contribution. This is in direct contradiction to what is suggested in the one-loop studies. At large $\tan\beta$ both the dominating contributions from the scalars and the pseudoscalar scale roughly as $\tan\beta$. For the scalar part, we can adjust the mixing angle α to zero such that the contribution from the light Higgs boson h is negligible, and impose a large mass hierarchy between the scalar Higgs H and the pseudoscalar A. Then a relatively light mass for the pseudoscalar A will give a sufficiently large positive contribution to Δa_μ, or the required Δa_μ value could be used to obtain the admissible range for m_A. In Fig. 2, we show the contribution of Δa_μ^A from the pseudoscalar A versus m_A for various values of $\tan\beta$. We included the one-loop and two-loop pseudoscalar contributions. The shaded region is the 95% C.L. range of Eq. (2). The required range of m_A is then given by about 4–29 (15–85) GeV for $\tan\beta=30$ (60). Moreover, a $\tan\beta\gtrsim17$ is always required.

What happens when other mixing angles α are chosen? The light Higgs boson h will give a negative contribution to a_μ and thus offsets the pseudoscalar contribution. Therefore, the required range of m_A shifts to a lower value in order to accommodate the data. We show the contour plots of Δa_μ in the plane of (m_A,m_h) for a small and a large α at $\tan\beta=40$ and 60 in Fig. 3. The $\alpha=0$ limit, which corresponds to switching off the contribution from the scalar h, can be easily read off from the vertical asymptotes. On the other hand, $\alpha=\pm\pi/2$ corresponds to the maximal contribution from h. The contour plots show how the contribution from the scalar h affects the solution to a_μ. For example, at $\alpha=-\pi/2$ for $m_h\sim60$ GeV, the 95% C.L. required range of m_A lowers to 13–47 GeV for $\tan\beta=60$.

At this point, it is interesting to take into consideration other experimental constraints on Higgs masses. A collider search for the neutral Higgs bosons in the context of the 2HDM typically rules out a region of small m_h and m_A. In particular, an OPAL analysis [17] using the LEP II data up to $\sqrt{s}=189$ GeV excludes the regions $1<m_h<44$ GeV *and* $12<m_A<56$ GeV at 95% C.L., independent of α and $\tan\beta$. This is a conservative limit. Details of the exclusion region vary with α and $\tan\beta$, and go substantially beyond the rectangular box [17]. Essentially, the search for A relies on the process $e^+e^-\to A\,h$, therefore, if m_h is so large that this process becomes negligible for A production, there would be no limit on m_A. In addition, below $m_A\simeq5$ GeV, the direct search in $e^+e^-\to A\,h$ was not included because the detection efficiency vanishes and the total Z^0 width only provides very limited exclusion. The OPAL exclusion region is roughly sitting at the center on the m_A axis going up to about $m_h=60$ GeV in the plots of Fig. 3, and cuts out part of the admissible region of the Δa_μ solution. For m_h larger than the OPAL limit the admissible m_A range is roughly from 3 to 50 GeV for $\tan\beta=40$. At larger $\tan\beta$ (60 as the illustrated example), the range widens, especially at the upper end, but a middle range (about 4 to 15 here) is lost as Δa_μ gets too large. There is another admissible small window at very small m_h (a few GeV) with large m_A. This region is indeed dominated by the one-loop contribution from h, but even here, the two-loop contribution (from h) has an important role to play. This tiny m_h window and the similar solution with this kind of small m_A are in fact excluded by Upsilon decay [18] and some other processes [3].

In addition to the constraint from direct search, there are also other constraints on the masses of the Higgs bosons coming from the electroweak precision data. While a comprehensive treatment of the topic is really beyond the scope of the present study, we discuss below the basic features, using results from a recent paper [19]. The scope of the latter study is limited to the large $\tan\beta$ region and $\alpha=\beta\approx\pi/2$. There, the ratio m_h/m_A is constrained at 95% C.L. (based on the Bayesian approach) via the function

$$G\left(\frac{m_h^2}{m_A^2}\right)\geq-\left(\frac{39}{\tan\beta}\right)^2,$$

where

$$G(x)=1+\frac{1}{2}\left(\frac{1+x}{1-x}\right)\ln x.$$

Solving for m_h/m_A at $\tan\beta=60,45,30$, we obtain, respectively,

$$0.3\;\leq\;\frac{m_h}{m_A}\;\leq\;3.2$$

$$0.2\;\leq\;\frac{m_h}{m_A}\;\leq\;5.1$$
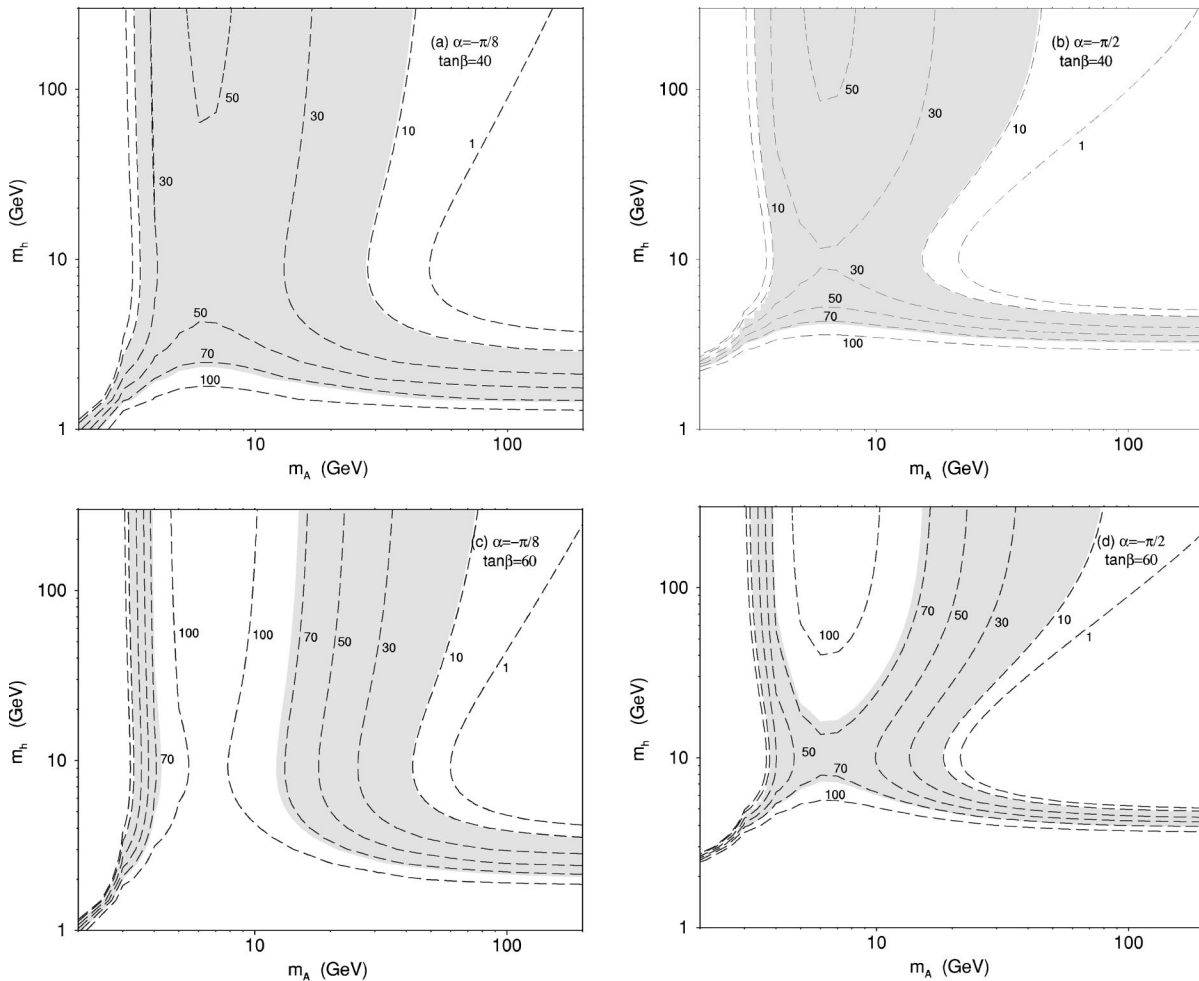
$$0.07\;\leq\;\frac{m_h}{m_A}\;\leq\;14.4.$$

FIG. 3. Contours of $\Delta a_\mu^{\text{Higgs}}$ in unit of 10^{-10} in the plane of (m_A, m_h) for (a) $\alpha = -\pi/8$ and $\tan\beta = 40$, (b) $\alpha = -\pi/2$ and $\tan\beta = 40$, (c) $\alpha = -\pi/8$ and $\tan\beta = 60$, and (d) $\alpha = -\pi/2$ and $\tan\beta = 60$.

We can see that the precision electroweak data prefer a region close to the diagonal of the m_h versus m_A plot. If one naively imposes the result of $0.3 \lesssim (m_h/m_A) \lesssim 3.2$ at $\tan\beta = 60$ onto Fig. 3(d), which is at a different α but with which a similar result is expected to be valid, together with the direct search limit $m_h \gtrsim 60$ GeV and the a_μ requirement, only a small "triangle" is left. This triangle is bounded by $m_h = 60$ GeV, $m_h/m_A = 3.2$ and the contour of $a_\mu = 10 \times 10^{-10}$ [labeled by "10" in Fig. 3(d)]. The surviving parameter space region, however, is in the more favorable "larger" mass area. In particular, it re-enforces our previous comment at the end of the last paragraph that the one-loop dominating tiny m_h window of solution to a_μ is ruled out. The situation for the other α values is expected to be similar. It will be very interesting to have the complete phenomenological analysis combining all the constraints on the 2HDM.

Finally, we discuss the role of the Higgs sector contributions to a_μ in the MSSM. The LEP bound on the Higgs boson masses is in the range 85–95 GeV [20]. From the above result, one may naively conclude that if a Higgs particle is just around the corner, it could have an important role to play in a_μ. However, there are some strong theoretical constraints on the relation of the Higgs boson masses in the

MSSM. At the large $\tan\beta$ value required, one has $m_h \lesssim m_A \simeq m_H$ [3]. Most of the Higgs contributions to a_μ cancel among themselves. Moreover, a small Higgs boson mass may require a μ parameter so small that the chargino/neutralino contributions to a_μ get far too large. In fact, we have checked and found no interesting solution within MSSM in which the Higgs contributions play a substantial role. In the admissible range of chargino/smuon masses found in Ref. [6], the total Higgs contribution is only about 1% of the SUSY contribution. The importance of this null result should not be underestimated.

We conclude that the new measurement on the muon anomalous magnetic moment constrains severely on the parameter space of the 2HDM II, and our results, including one-loop and two-loop contributions, change dramatically from the conclusion that one draws by using only one-loop results.

[1] MUON$-$2 Collaboration, H.N. Brown *et al.*, Phys. Rev. Lett. **86**, 2227 (2001).

[2] A. Czarnecki and W. Marciano, Phys. Rev. D **64**, 013014 (2001); see also S. Narison, Phys. Lett. B **513**, 53 (2001).

[3] See, for example, J.F. Gunion, H.E. Haber, G. Kane, and S. Dawson, *The Higgs Hunter's Guide* (Addison-Wesley, Reading, MA, 1990).

[4] A. Dedes and H. Haber, J. High Energy Phys. **05**, 006 (2001).

[5] R. Diaz, R. Martinez, and J. Rodriguez, Phys. Rev. D **64**, 033004 (2001); S.K. Kang and K.Y. Lee, hep-ph/0103064; E.O. Iltan, hep-ph/0103105.

[6] J. Feng and K. Matchev, Phys. Rev. Lett. **86**, 3480 (2001); L. Everett, G. Kane, S. Rigolin, and L. Wang, *ibid.* **86**, 3484 (2001); E. Baltz and P. Gondolo, *ibid.* **86**, 5004 (2001); U. Chattopadhyay and P. Nath, *ibid.* **86**, 5854 (2001); S. Komine, T. Moroi, and M. Yamaguchi, Phys. Lett. B **506**, 93 (2001); J. Hisano and K. Tobe, *ibid.* **510**, 197 (2001); J. Ellis, D. Nanopoulos, and K. Olive, *ibid.* **508**, 65 (2001); R. Arnowitt, B. Dutta, B. Hu, and Y. Santoso, *ibid.* **505**, 177 (2001).

[7] D. Choudhury, B. Mukhopadhyaya, and S. Rakshit, Phys. Lett. B **507**, 219 (2001); T. Huang, Z. Lin, L. Shan, and X. Zhang, Phys. Rev. D **64**, 071301 (2001); S. Gninenko and N. Krasnikov, Phys. Lett. B **513**, 119 (2001).

[8] D. Chakraverty, D. Choudhury, and A. Datta, Phys. Lett. B **508**, 103 (2001); U. Mahanta, *ibid.* **515**, 111 (2001); **511**, 235 (2001); K. Cheung, Phys. Rev. D **64**, 033001 (2001).

[9] K. Lane, hep-ph/0102131; X. Calmet, H. Fritzsch, and D. Holtmannspötter, Phys. Rev. D **64**, 037701 (2001).

[10] LEP Collaborations, T. Junk, hep-ex/0101015.

[11] D. Chang, W.-F. Chang, C.-H. Chou, and W.-Y. Keung, Phys. Rev. D **63**, 091301 (2001).

[12] J.D. Bjorken and S. Weinberg, Phys. Rev. Lett. **38**, 622 (1977); S.M. Barr and A. Zee, *ibid.* **65**, 21 (1990).

[13] A. Czarnecki, B. Krause, and W. Marciano, Phys. Rev. D **52**, R2619 (1995).

[14] A. Czarnecki, B. Krause, and W. Marciano, Phys. Rev. Lett. **76**, 3267 (1996); see also T.V. Kukhto, E.A. Kuraev, A. Schiller, and Z.K. Silagadze, Nucl. Phys. **B371**, 567 (1992).

[15] M. Ciuchini, G. Degrassi, P. Gambino, and G.F. Guidice, Nucl. Phys. **B527**, 21 (1998).

[16] For earlier works, see M. Krawczyk and J. Zochowski, Phys. Rev. D **55**, 6968 (1997), and references therein.

[17] OPAL Collaboration, G. Abbiendi *et al.*, Eur. Phys. J. C **18**, 425 (2001).

[18] CUSB Collaboration, P. Franzini *et al.*, Phys. Rev. D **35**, 2883 (1987).

[19] O. Lebedev, W. Loinaz, and T. Takeuchi, Phys. Rev. D **62**, 055014 (2000).

[20] See, for example, G.L. Kane and L.-T. Wang, Phys. Lett. B **488**, 383 (2000); M. Carena, J. Ellis, A. Pilaftsis, and C.E.M. Wagner, *ibid.* **495**, 155 (2000).