Interplay between weak and strong phases and direct *CP* violation from the charmless *B*-meson decays

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We present a general analysis on charmless B-meson decays $B\to\pi\pi$ and πK. It is noticed that the final state interactions and inelastic rescattering effects must be significant in order to understand the consistency of the current data. By using general isospin decompositions, the isospin amplitudes and the corresponding strong phases could be extracted from a global χ^2 fit of the experimental data. We emphasize that in general there are two, rather than one, relative strong phases in the decomposition. In the assumption of two equal strong phases as considered in the literature, the current data, especially the ones concerning $B\to\pi^0 K^{0(\pm)}$ decays, will imply a large isospin amplitude $|a^c_{3/2}|>40$, which is larger by a factor of 5 than the one from the naive factorization estimation. When two different strong phases are considered, all the isospin amplitudes can become, within the 1σ level, comparable with the theoretical values. We also show that the difference between the two strong phases cannot be too large and will be restricted by the most recent upper bound of $B\to\pi^0\pi^0$ decay. In any case, the strong phases are found to be large and the branching ratio of $B\to\pi^0\pi^0$ is likely to be enhanced by an order of magnitude in comparison with the one obtained from the naive factorization approach. Direct CP violations in all decay modes are also calculated and found to be close to the sensitivity of the present experiments.

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I. INTRODUCTION

Recently the CLEO Collaboration has reported measurements on the branching ratios of rare hadronic B decays $B\to\pi\pi,\pi K$ [1,2]. The data have attracted great interest from both theorists and experimentalists. The study of these channels will provide us with important insights into understanding the effects of electroweak penguin diagrams (EWP) in the B system [3] and final state interactions (FSIs) [4,5], as well as extracting the weak Cabibbo-Kobayashi-Maskawa (CKM) phase $\gamma=\arg(V_{ud}V^*_{ub}/V_{cd}V^*_{cb})$ [6–8,10]. It may also open a window for probing new physics [11].

From the current data, it is noticed that the branching ratio for $B\to\pi^+\pi^-$ is relatively small, $\mathrm{Br}(B\to\pi^+\pi^-)\sim 4$ (in units of 10^{-6}). The decays for $B\to\pi^+K^-,\pi^-\overline{K^0}$ have almost equal decay rates, i.e., $\mathrm{Br}(B\to\pi^+K^-)\simeq\mathrm{Br}(B\to\pi^-\overline{K^0})\sim 17$. While an unexpectedly large branching ratio for $B\to\pi^0\overline{K^0}$ decay was also observed, $\mathrm{Br}(B\to\pi^0\overline{K^0})\sim 14$. These measurements seem in conflict with calculations based on naive factorization hypotheses. For the first three decays, it was pointed out that the factorization approach may still be valid if one takes the weak phase γ to be greater than 90° [8]. With such a large γ, i.e., $\cos\gamma<0$, the interference between tree and penguin diagrams has opposite sign in $B\to\pi\pi$ and $B\to\pi K$ decays. Thus negative $\cos\gamma$ will suppress the decay rate for $B\to\pi\pi$ and enhance that for $B\to\pi^+K^-$. As a consequence, the almost equal decay rates for $B\to\pi^+K^-$ and $B\to\pi^-\overline{K^0}$ decay modes indicate the dominance of strong penguin diagrams. However, the large rate for $B\to\pi^0\overline{K^0}$ is not easily explained. Most recent analysis showed that a large FSI phase would be helpful to enhance the branching ratio for the $B\to\pi^0\overline{K^0}$ decay [4], but only considering the elastic rescatterings remains insufficient to obtain the large central value of the data.

To understand the measured data, besides some model-dependent calculations, a model-independent approach using a single relative strong phase has also been proposed for the study of $B\to\pi\pi,\pi K$ decays [4,9,8]. The ordinary factorization approach suffers from uncertainties due to hadronic matrix elements, such as the meson decay constants, B-meson form factors, and so-called effective color number N_c. The model-independent analyses may be more useful as more data become available. The approach based on the isospin SU(2) and approximate flavor SU(3) [12] symmetries of the strong interactions has been proposed to constrain [13,14] and extract the weak phase γ [7]. It has been noticed that the ratios between the CP-averaged decay rates, such as $R=\mathrm{Br}(B\to\pi^\pm K^0)/\mathrm{Br}(B\to\pi^0K^\pm)$, may provide us with important information on the weak phase γ. Most recently, it has been shown that the weak phase γ may be determined through three ratios among CP-averaged decay rates of $B\to\pi^+\pi^-$, $\pi^+\pi^0$, π^-K^+, and π^-K^0 decays [10], where two solutions were obtained at the 1σ level, one with positive $\cos\delta$ and negative $\cos\gamma$, i.e., relative small strong phase δ and large weak phase γ, and another with negative $\cos\delta$ and positive $\cos\gamma$. The latter with positive $\cos\gamma$ seems to be favored by solutions obtained from other constraints in the standard model but appears not to be as favorable as the one with negative $\cos\gamma$ studies of all the existing charmless decays are taken in account. However, there is still no complete analysis in the literature.

In this paper, we shall give a general analysis for all 7 decay modes of $B\to\pi\pi$ and πK. For that purpose, we will start from a general model-independent parametrization for all the decay amplitudes by considering both the isospin and simple diagrammatic decompositions. We will show that there are in general 15 independent variables. By assuming the SU(3) relations which appropriately account for SU(3) symmetry breaking effects, it allows us to reduce the 15 independent variables to 9. They consist of 6 isospin amplitudes and 2 relative strong phases as well as one weak phase γ. Note that once the relative strong phase is zero, $B\to\pi^-\bar{K}^0$ only receives contributions from penguin-type diagrams and the amplitudes of the isospin $I=1/2$ and $I=3/2$ amplitudes from tree-type diagrams cancel each other, which provides an additional constraint [8]. In fact, one of the isospin amplitudes becomes almost irrelevant due to the suppression factor of the CKM mixing element. With such a consideration, there are only 8 relevant unknown quantities with 5 isospin amplitudes and 3 phases. We show that the current 6 measured decay rates allow us to extract 6 unknown quantities as functions of two variables. The upper bound of the decay rate $B\to\pi^0\pi^0$ also provides a bound for the difference between the two strong phases. Once taking the numerical value of the weak phase γ to be the one obtained from other constraints in the standard model and fixing one of the strong phases, all the other parameters can be determined. With these determined parameters, we are then able to predict the branching ratio of the $B\to\pi^0\pi^0$ decay mode which is yet unmeasured due to the difficulty of its identification by the current detector. In addition we also present predictions for direct CP violations in all 7 decay channels of $B\to\pi\pi,\ \pi K$. In our numerical fitting, we have adopted the χ^2 analysis for the CLEO data in order to have a systematic treatment on the experimental errors.

In general, according to the Watson theorem, there are two independent relative strong phases associating with the isospin amplitudes. They are often assumed to be equal in the literature [4,8,9]. In this work, we shall make a more general analysis with two relative strong phases. It is shown that the equal phase assumption will result in large enhancement of isospin amplitude $a^c_{3/2}$ which will be 5 times larger than the one calculated from the factorization approach. The value of the strong phase is found to be $\delta\simeq\pm 95°$. These large values may imply large inelastic FSIs or indicate the possible new physics effects. However, if the two strong phases are different, the value of $a^c_{3/2}$ can be lower and is comparable with the usual factorization calculations.

It is remarkable to observe that within 1σ all 6 decay rates can be consistently fitted for a large range of the weak phase $0°<\gamma<180°$ for the above two cases. It is also of interest to note that one of isospin amplitudes and the strong phases have a weak dependence on the weak phase γ. Three isospin amplitudes show a moderate dependence on the weak phase γ. Only one isospin amplitude is sensitive to γ. In particular, the fitting values for the 4 usual isospin amplitudes considered in most of the literature could still be comparable with the ones obtained by using naive factorization approach. The resulting large strong phases may be regarded as a strong indication of large FSIs in $B\to\pi\pi,\pi K$ decays.

II. GENERAL FRAMEWORK

We begin with writing the decay amplitude of $B\to\pi\pi,\pi K$ in the following general form:

$$A^{\pi\pi(\pi K)}=\lambda_u^{d(s)}A_u^{\pi\pi(\pi K)}+\lambda_c^{d(s)}A_c^{\pi\pi(\pi K)}, \tag{1}$$

where $\lambda_u^{d(s)}=V_{ub}V^*_{ud(s)}$ and $\lambda_c^{d(s)}=V_{cb}V^*_{cd(s)}$ are the products of CKM matrix elements. The term proportional to $\lambda_t^{d(s)}=V_{tb}V^*_{td(s)}$ has been absorbed into the above two terms by using the unitarity relation, $V_{ub}V^*_{ud(s)}+V_{cb}V^*_{cd(s)}+V_{tb}V_{td(s)}=0$.

We also find it useful to adopt the isospin decomposition for the decay amplitudes

$$A^{u,c}_{\pi^-\pi^+}=\sqrt{\frac{2}{3}}a_0^{u,c}e^{i\delta_0}+\sqrt{\frac{1}{3}}a_2^{u,c}e^{i\delta_2}, \tag{2}$$

$$A^{u,c}_{\pi^0\pi^0}=\sqrt{\frac{1}{3}}a_0^{u,c}e^{i\delta_0}-\sqrt{\frac{2}{3}}a_2^{u,c}e^{i\delta_2}, \tag{3}$$

$$A^{u,c}_{\pi^-\pi^0}=-\sqrt{\frac{3}{2}}a_2^{u,c}e^{i\delta_2}, \tag{4}$$

$$A^{u,c}_{\pi^+K^-}=\sqrt{\frac{2}{3}}a_{1/2}^{u,c}e^{i\delta_{1/2}}+\sqrt{\frac{1}{3}}a_{3/2}^{u,c}e^{i\delta_{3/2}}, \tag{5}$$

$$A^{u,c}_{\pi^0\bar{K}^0}=\sqrt{\frac{1}{3}}a_{1/2}^{u,c}e^{i\delta_{1/2}}-\sqrt{\frac{2}{3}}a_{3/2}^{u,c}e^{i\delta_{3/2}}, \tag{6}$$

$$A^{u,c}_{\pi^0K^-}=-\sqrt{\frac{3}{2}}a_{3/2}^{u,c}e^{i\delta_{3/2}}-\frac{1}{\sqrt{2}}A^{u,c}_{\pi^-\bar{K}^0}, \tag{7}$$

with

$$A^{u,c}_{\pi^-\bar{K}^0}=\sqrt{\frac{2}{3}}b_{1/2}^{u,c}e^{\delta'_{1/2}}-\sqrt{\frac{1}{3}}a_{3/2}^{u,c}e^{i\delta_{3/2}}, \tag{8}$$

where $a_I^{u,c}$ and $b_I^{u,c}$ are the isospin amplitudes and δ_I and $\delta'_{1/2}$ are the strong phases due to final state interactions. In some of the literature the strong phase of the isospin amplitude $b^{u,c}_{1/2}$ is assumed to be equal to the one of $a^{u,c}$ for simplicity [4,8,9]. However, in the most general case, these strong phases are not necessarily the same, since they arise from the effective Hamiltonian with different isospin. The subscripts $I=0,\ 2,\ 1/2,\ 3/2$ denote the isospins of the amplitudes. The advantage of the isospin decomposition allows one to use SU(3) relations including leading order SU(3) breaking effects. In other words, the isospin amplitudes are assumed to satisfy the following relations:

$$a_0^{u,c}\simeq(f_\pi/f_K)a_{1/2}^{u,c},\quad a_2^{u,c}\simeq(f_\pi/f_K)a_{3/2}^{u,c},$$

$$\delta_0\simeq\delta_{1/2},\quad \delta_2\simeq\delta_{3/2}, \tag{9}$$

where f_π and f_K are the π,K meson decay constants with $f_\pi/f_K\simeq 0.8$. For convenience, we define two phase differences as follows:

$$\delta = \delta_{3/2} - \delta_{1/2},$$
$$\delta' = \delta'_{1/2} - \delta_{1/2}. \tag{10}$$

Practically, the decay amplitudes are evaluated by calculating various Feynman diagrams. In order to see how those isospin amplitudes receive contributions from diagrams, we also present a simple diagrammatic decomposition. The diagrams can in general be classified into six types denoted by T(tree diagram), C(color-suppressed tree diagram) P(QCD penguin diagram), P_{EW}(electroweak penguin diagram) and P_{EW}^{C}(color-suppressed electroweak penguin diagram) [15]:

$$A_{\pi^-\pi^+} = T + P + \frac{2}{3}P_{EW}^{C}, \tag{11}$$

$$A_{\pi^0\pi^0} = \frac{1}{\sqrt{2}}\left(-C + P - P_{EW} - \frac{1}{3}P_{EW}^{C}\right), \tag{12}$$

$$A_{\pi^-\pi^0} = \frac{1}{\sqrt{2}}(-T - C - P_{EW} - P_{EW}^{C}), \tag{13}$$

$$A_{\pi^+K^-} = T' + P' + \frac{2}{3}P'^{C}_{EW}, \tag{14}$$

$$A_{\pi^0\bar{K}^0} = \frac{1}{\sqrt{2}}\left(-C' + P' - P'_{EW} - \frac{1}{3}P'^{C}_{EW}\right), \tag{15}$$

where the primed and unprimed quantities are the amplitudes in $B \to \pi K$ and $B \to \pi\pi$ decays. They roughly differ by a factor $f_\pi / f_K \simeq 0.8$ when the SU(3) flavor symmetry breaking effects are considered.

Combining the two decompositions, it is straightforward to get the following relations:

$$a_{1/2}^{u,c} e^{i\delta_{1/2}} = \frac{1}{\sqrt{6}}(2T' - C' + 3P' - P'_{EW} + P'^{C}_{EW})^{u,c}, \tag{16}$$

$$a_{3/2}^{u,c} e^{i\delta_{3/2}} = \frac{1}{\sqrt{3}}(T' + C' + P'_{EW} + P'^{C}_{EW})^{u,c}. \tag{17}$$

From the above equations, one may easily see the relative magnitudes among those SU(2) invariant amplitudes. If the inelastic rescattering effects are small, T', C' will only contribute to the term proportional to λ_u^s. Therefore one may expect that $T'(C')^u \gg T'(C')^c$. This will lead to $a^u_{1/2(3/2)} \gg a^c_{1/2(3/2)}$. Since $a^c_{1/2}$ receives contributions from QCD penguins, while $a^c_{3/2}$ only gets contributions from EWP diagrams, one may conclude that $a^c_{1/2} \gg a^c_{3/2}$.

To obtain relations for the isospin amplitude $b^{u,c}_{1/2}$, one needs to be careful in adopting the diagrammatic decomposition implied by the naive factorization ansatz. This is because the resulting relative strong phase is zero in the factorization approach, i.e., $\delta = \delta_{3/2} - \delta_{1/2} = 0$ and $\delta' = 0$. As a consequence,

$$A_{\pi^0K^-} = \frac{1}{\sqrt{2}}\left(-T' - C' - P' - P'_{EW} - \frac{2}{3}P'^{C}_{EW}\right), \tag{18}$$

$$A_{\pi^-\bar{K}^0} = P' - \frac{1}{3}P'^{C}_{EW}. \tag{19}$$

The amplitudes with isospin $I = 1/2$ and $I = 3/2$ from tree-type graphs cancel each other in Eq. (19). Thus the total amplitude only receives contributions from penguin diagrams in this case, namely,

$$A^{u,c}_{\pi^-\bar{K}^0} = \sqrt{\frac{2}{3}}b^{u,c}_{1/2} - \sqrt{\frac{1}{3}}a^{u,c}_{3/2} = \left(P' - \frac{1}{3}P'^{C}_{EW}\right)^{u,c}. \tag{20}$$

Assuming t-quark dominance in the penguin loops, one finds from Eq. (20) that

$$A^u_{\pi^-\bar{K}^0} \simeq A^c_{\pi^-\bar{K}^0} \quad \text{or} \quad b^u_{1/2} \simeq b^c_{1/2} + \frac{1}{\sqrt{2}}a^u_{3/2} - \frac{1}{\sqrt{2}}a^c_{3/2}, \tag{21}$$

which may be assumed for simplicity to be approximately valid after considering final state interactions with nonzero strong phases. In the numerical calculations, we have checked that the amplitude $b^u_{1/2}$ is less important due to the strong suppression of the CKM factor (for instance, even taking $b^u_{1/2} \simeq b^c_{1/2}$, the results remain almost unchanged).

With the above analyses, let us provide an intuitive discussion of how to yield a large branching ratio for $B \to \pi^0 \bar{K}^0$ decay by appropriately choosing the isospin amplitudes. Note the fact that as $\lambda_u^s \ll \lambda_c^s$, one may roughly estimate the ratio between $\mathrm{Br}(B \to \pi^0\bar{K}^0)$ and $\mathrm{Br}(B \to \pi^+K^-)$ by neglecting the terms containing the CKM factor λ_u^s:

$$R = \frac{\mathrm{Br}(B \to \pi^0\bar{K}^0)}{\mathrm{Br}(B \to \pi^+K^-)} \simeq \left| \frac{\sqrt{\frac{1}{3}}a^c_{1/2} - \sqrt{\frac{2}{3}}a^c_{3/2}e^{i\delta}}{\sqrt{\frac{2}{3}}a^c_{1/2} + \sqrt{\frac{1}{3}}a^c_{3/2}e^{i\delta}} \right|^2, \tag{22}$$

with $\delta = \delta_{3/2} - \delta_{1/2}$. Neglecting $a^c_{3/2}$, the ratio may be simply given by $R \simeq \frac{1}{2}$, which is much smaller than the central value of the data, $R = 0.84$. It indicates that to enhance the decay rate of $B \to \pi^0\bar{K}^0$, the isospin amplitude $a^c_{3/2}$ should not be neglected. Its small value may provide a sizable contribution for a large value of $\delta > \pi/2$. This is because in this case there exists a constructive interference between $a^c_{1/2}$ and $a^c_{3/2}$ in $B \to \pi^0\bar{K}^0$ and a destructive interference in $B \to \pi^+K^-$. The situation is quite similar to the case for a large $\gamma > \pi/2$, which is considered to enhance $B \to \pi K$ and decrease $B \to \pi\pi$ decay rates. From Eq. (22), it is easily seen that the value of $a^c_{3/2}$ satisfies

$$a^c_{3/2} \geq \frac{\sqrt{2R} - 1}{\sqrt{2} + \sqrt{R}}a^c_{1/2} \simeq 0.12 a^c_{1/2}. \tag{23}$$

With the above considerations, there are only eight relevant quantities $a^{u,c}_{1/2}$, $a^{u,c}_{3/2}$, $b^c_{1/2}$, δ, δ', γ, which should be constrained by six measured decay rates and one upper bound. When taking the weak phase γ as a free parameter, the remaining six variables can be determined from six equations of Eqs. (2)–(7). As the errors in the current data remain considerably large, one may not take the central values of the data to be too serious. Thus by only using the central values of the data to determine the six variables may not be good enough. To take into account the experimental errors in a systematic way, we shall adopt a global χ^2 (least squares) analysis for the present data. This treatment allows us to obtain not only the central value but also the errors for the fitting amplitudes. Our fitting will be carried out by using the standard χ^2 analysis program package MINUIT [16].

III. RESULTS AND DISCUSSION

In order to compare with the values estimated from the factorization, it is necessary to explicitly see how large are the isospin amplitudes $a^{u,c}_{1/2,3/2}$ and $b_{1/2}$ from the factorization calculations; we present the relevant formulas for the $B \to \pi\pi, \pi K$ decay amplitudes with the assumption of factorization [18]:

$$A_{\pi^+K^-}=i\frac{G_F}{\sqrt{2}}f_K F_0^{B\pi}(m_K^2)(m_B^2-m_\pi^2)\{\lambda_u^s a_1-\lambda_t^s[a_4+a_{10}+(a_6+a_8)R_4]\},\quad (24)$$

$$A_{\pi^0\bar{K}^0}=-i\frac{G_F}{2}f_K F_0^{B\pi}(m_K^2)(m_B^2-m_\pi^2)\lambda_t^s\left(a_4+a_6R_5-\frac{1}{2}(a_{10}+a_8R_5)\right)-i\frac{G_F}{2}f_\pi F_0^{BK}(m_\pi^2)(m_B^2-m_K^2)\times\left(\lambda_u^s a_2-\lambda_t^s\frac{3}{2}(a_9-a_7)\right),\quad (25)$$

$$A_{\pi^0K^-}=-i\frac{G_F}{2}f_K F_0^{B\pi}(m_K^2)(m_B^2-m_\pi^2)\times\{\lambda_u^s a_1-\lambda_t^s[a_4+a_6R_4+(a_{10}+a_8R_4)]\}-i\frac{G_F}{2}f_\pi F_0^{BK}(m_\pi^2)(m_B^2-m_K^2)\times\left(\lambda_u^s a_2-\lambda_t^s\frac{3}{2}(a_9-a_7)\right),\quad (26)$$

$$A^c_{\pi^-\bar{K}^0}=-i\frac{G_F}{\sqrt{2}}f_K F_0^{B\pi}(m_K^2)(m_B^2-m_\pi^2)\lambda_t^s\times\left(a_4+a_6R_5-\frac{1}{2}(a_{10}+a_8R_5)\right),\quad (27)$$

where $f_{\pi,K}$ and $F^{B\pi,BK}$ are the decay constants and B-meson form factors, respectively, $R_4=2m_K^2/(m_b-m_u)(m_u+m_s)$, and $R_5=2m_K^2/(m_b-m_d)(m_d+m_s)$. In the flavor SU(2) limit, one has $R_4\simeq R_5\simeq 2m_K^2/(m_bm_s)$.

The expressions of the isospin amplitudes can be rewritten as follows:

$$a^u_{1/2}e^{i\delta_{1/2}}=\sqrt{\frac{2}{3}}A^u_{\pi^+K^-}+\sqrt{\frac{1}{3}}A^u_{\pi^0\bar{K}^0}=r\left[\sqrt{\frac{2}{3}}[a_1+a_4+a_{10}+(a_6+a_8)R_4]+\sqrt{\frac{1}{6}}\left[a_4+a_6R_5-\frac{1}{2}(a_{10}+a_8R_5)\right]-\sqrt{\frac{1}{6}}\left(a_2+\frac{3}{2}(a_9-a_7)X\right)\right],\quad (28)$$

$$a^c_{1/2}e^{i\delta_{1/2}}=\sqrt{\frac{2}{3}}A^c_{\pi^+K^-}+\sqrt{\frac{1}{3}}A^c_{\pi^0\bar{K}^0}=r\left[\sqrt{\frac{2}{3}}[a_4+a_{10}+(a_6+a_8)R_4]+\sqrt{\frac{1}{6}}\left(a_4+a_6R_5-\frac{1}{2}(a_{10}+a_8R_5)\right)-\sqrt{\frac{1}{2}\sqrt{\frac{3}{2}}}(a_9-a_7)X\right],\quad (29)$$

$$a^u_{3/2}e^{i\delta_{3/2}}=\sqrt{\frac{1}{3}}A^u_{\pi^+K^-}-\sqrt{\frac{2}{3}}A^u_{\pi^0\bar{K}^0}=r\sqrt{\frac{1}{3}}\left(a_1+a_2X+\frac{3}{2}(a_9-a_7)X+\frac{3}{2}(a_{10}+a_8)R_4\right),\quad (30)$$

$$a^c_{3/2}e^{i\delta_{3/2}}=\sqrt{\frac{1}{3}}A^c_{\pi^+K^-}-\sqrt{\frac{2}{3}}A^c_{\pi^0\bar{K}^0}=r\frac{\sqrt{3}}{2}[a_{10}+a_8R_4+(a_9-a_7)X],\quad (31)$$

and

$$\sqrt{\frac{2}{3}}b^c_{1/2}-\frac{1}{\sqrt{3}}a^c_{3/2}=A^c_{\pi^-\bar{K}^0}=r\left(a_4+a_6R_5-\frac{1}{2}(a_{10}+a_8R_5)\right),\quad (32)$$

where $r=(G_F/\sqrt{2})f_KF_0^{B\pi}(m_K^2)(m_B^2-m_\pi^2)$ and $X=(f_\pi/f_K)(F_0^{BK}/F_0^{B\pi})(m_B^2-m_K^2)/(m_B^2-m_\pi^2)$. In our numerical estimates, we will take $f_\pi=133$ MeV, $f_K=158$ MeV, $F_0^{B\pi}=0.36$, and $F_0^{BK}=0.41$. There remains a large uncertainty in

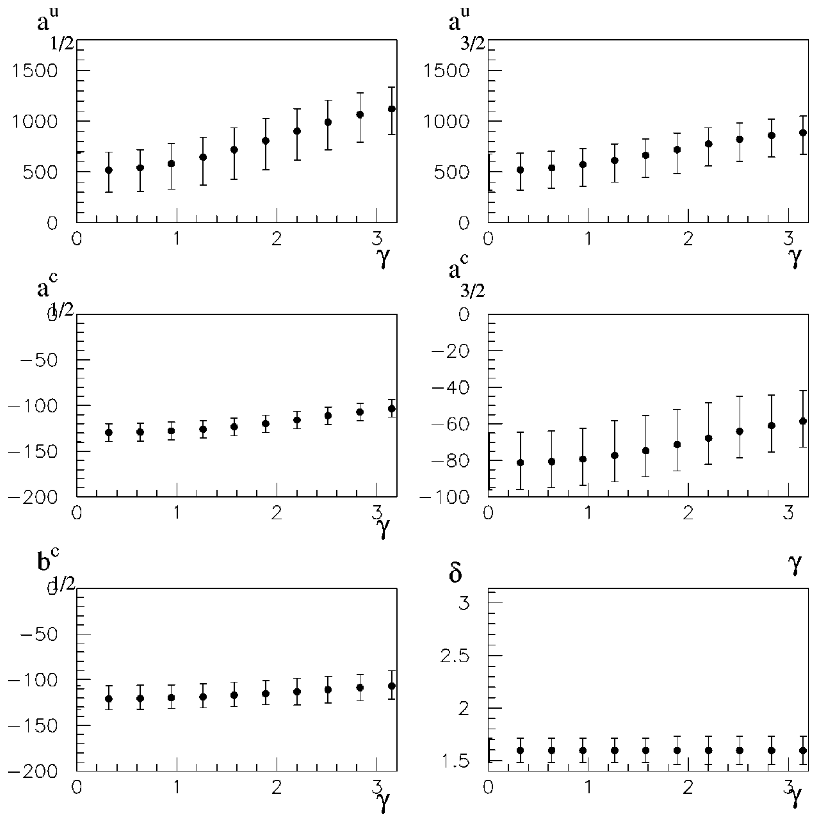

FIG. 1. The isospin amplitude fitted as a function of the weak phase γ from a χ^2 analysis of recent CLEO data. The vertical bars indicate the errors at the 1σ level. The strong phase δ' is set to zero.

strange quark mass m_s. For $m_s=(100–200)$ MeV, we find that the numerical values of those amplitudes are given by

$$a^u_{1/2}\simeq 818–846, \quad a^u_{3/2}\simeq 709,$$

$$a^c_{1/2}\simeq -(103–131), \quad a^c_{3/2}\simeq -7,$$

$$b^c_{1/2}\simeq -(72–100). \tag{33}$$

The results from the χ^2 fitting are shown in Figs. 1–3, where the six amplitudes as well as their errors at the 1σ level are obtained as functions of the weak phase γ with δ' fixed at $0,\pi/6,\pi/3$, respectively. The relative magnitudes of the amplitudes are consistent with the previous discussions. In our fit, the minimum value of χ^2 is found to be extremely low (typically $\chi^2_{min}\sim 0.5\times 10^{-12}$). This means that the χ^2 fits are highly consistent and the six amplitudes are actually extracted as the solutions of Eqs. (2)–(8). It can be seen from the figures that the γ dependence of $a^u_{1/2}$ is quite strong and the one of $a^c_{1/2}$ and $a^{u,c}_{3/2}$ is relatively weak. On the other hand, it may be used to extract the angle γ once one of those amplitudes can be determined or calculated in other independent ways. It is of interest to see that the γ dependence of the amplitude $b^c_{1/2}$ and the strong phase δ is weak, which shows that these two quantities are approximately fixed. The possibility of large δ was also suggested in Ref. [4] to explain the large branching ratio of the $B\rightarrow\pi^0\overline{K}^0$ decay. Recently, perturbative QCD calculations have also shown a large strong phase [17].

In Fig. 1 where the phase difference δ' is set to be zero as usual, the χ^2 fitting shows that the values of $a^u_{1/2}$ and $a^u_{3/2}$ may be comparable with the ones from the theoretical estimations only when the weak phase γ is large. Especially for $\gamma>2\pi/3$, the fitting values could coincide with the ones from factorization except for $a^c_{3/2}$. For $\gamma<2\pi/3$, the two amplitudes are smaller than the ones from naive factorization calculations. It appears that the factorization approach may become suitable for large weak phase γ. This phenomenon was observed by most of the analyses in the literature which neglects the isospin amplitude $a^c_{3/2}$. As a consequence, the resulting large value of γ seems to be in conflict with the one obtained from other constraints in the standard model. Before drawing the final conclusion, one can also notice that in the factorization approach, one yields a zero strong phase $\delta=0°$ which actually contradicts the general fitting value $\delta\simeq\pm 95°$. Therefore, the results of estimates based on the naive factorization approach should be unreliable, and the isospin amplitudes must receive additional large contributions. A large value for the relative strong phase $\delta\pm 95°$ implies that the final state interactions or inelastic rescattering effects must be significant.

We would like to stress that the most outstanding feature of the χ^2 analysis with $\delta'=0$ is that the isospin amplitude $a^c_{3/2}$ is likely to be relatively larger than the one estimated from the naive factorization calculations. The fitting central value of $a^c_{3/2}$ may be larger by a factor of 7–9. To explicitly see how the decay rates depend on the isospin amplitude $a^c_{3/2}$, we plot in Fig. 4 the six branching ratios of $B\rightarrow\pi\pi,\pi K$ decays as functions of $a^c_{3/2}$. It can be clearly seen that if $\delta'=0$, a small value of $a^c_{3/2}\sim -7$ is not able to reproduce all the CLEO data within the 1σ level, especially the

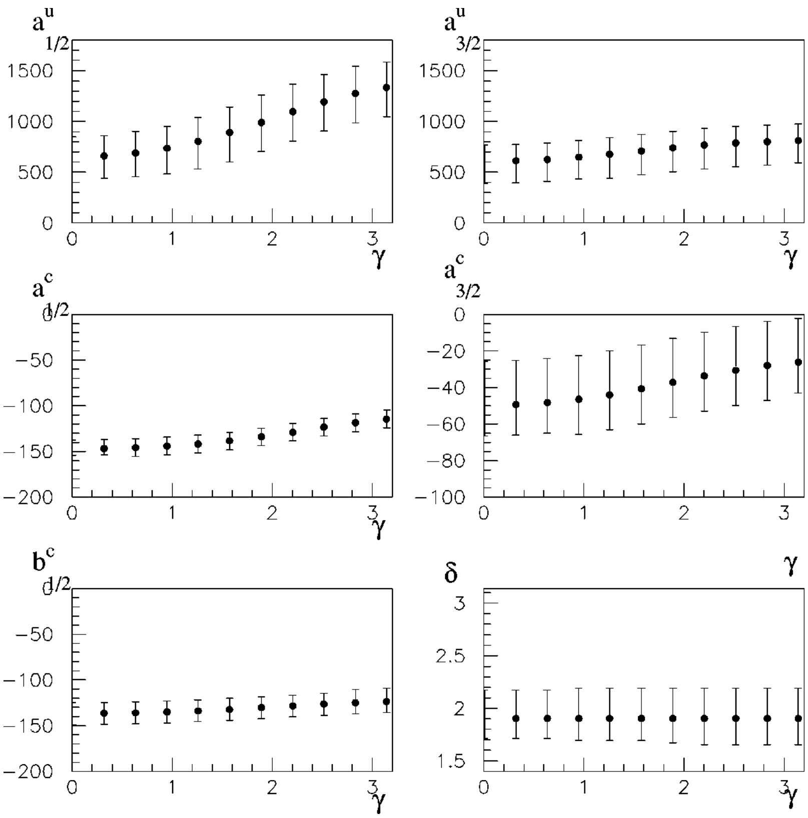

FIG. 2. The same as Fig. 1 but with $\delta' = \pi/6$.

data for the channels of $\pi^0\overline{K}^0$ and $\pi^0 K^-$. To consistently describe the whole data, we need a relative large value $a^c_{3/2} \sim -75$ for fitting the central value of the data which is about 10% of the largest one a^u_0.

Within the standard model, it seems difficult to enhance the isospin amplitude $a^c_{3/2}$ by an order of magnitude even when the inelastic FSI is involved; this is because the main inelastic channels, such as $B \to DD(DD_s, \eta_c K) \to \pi\pi(K)$,

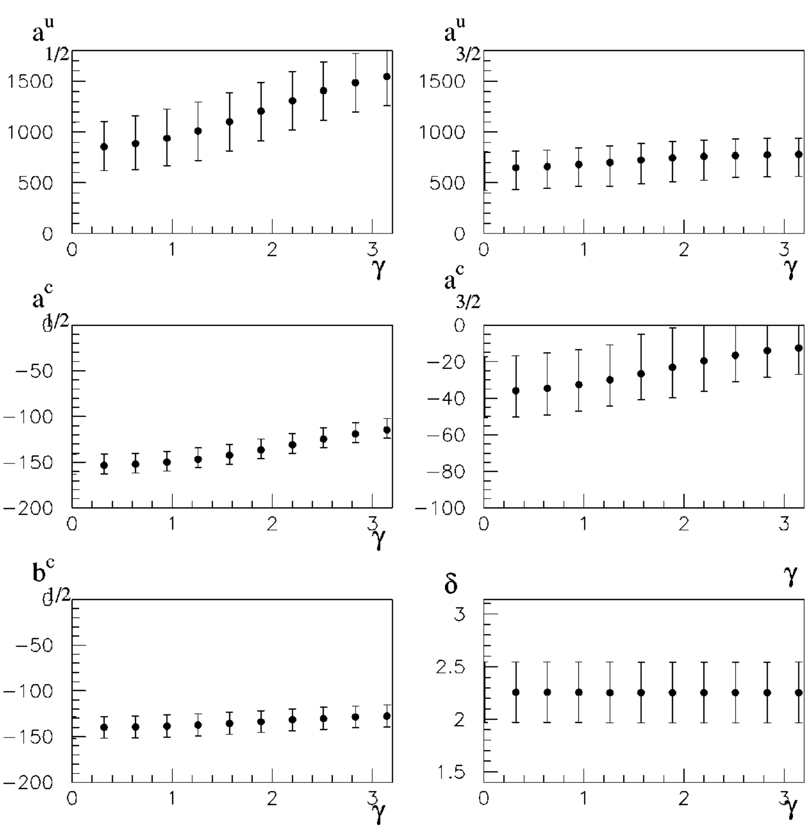

FIG. 3. The same as Fig. 1 but with $\delta' = \pi/3$.

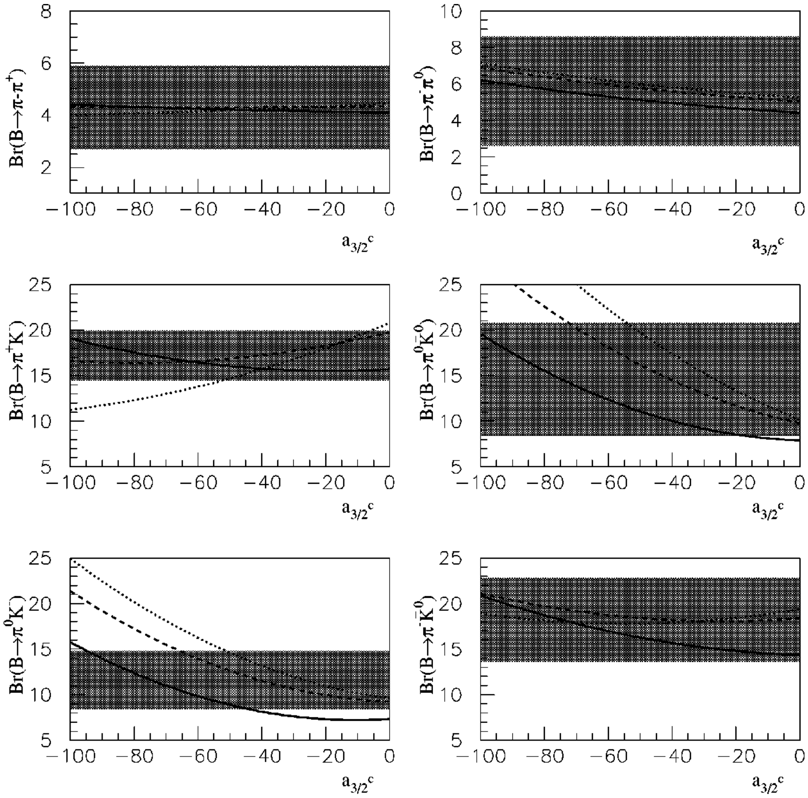

FIG. 4. The $a^c_{3/2}$ dependence of the six branching ratios (in units of 10^{-6}), for $\gamma=70°$. The other parameters are at their central values. The three curves in each plot correspond to $\delta'=0$ (solid lines), $\pi/6$ (dashed lines), $\pi/3$ (dot-dashed lines). The hatched bands indicate the errors (at 1σ) of the data.

only contribute to isospin-$\frac{1}{2}$ part of the decay amplitude. In the standard model (SM), it is known that the ratio $a^c_{3/2}/a^u_{3/2}$ can be determined without the hadronic uncertainties in the flavor SU(3) limit; this is because the ratio only depends on the short distance Wilson coefficients [19]. Thus a large value of $a^c_{3/2}/a^u_{3/2}$ may indicate the existence of new physics. While all models beyond the standard model must effectively provide large contributions to the electroweak penguins in order to enhance the isospin amplitude $a^c_{3/2}$, such models are supersymmetry (SUSY) with R parity violation, the Z' model, Z-mediated flavor changing neutral current (FCNC) models, etc.

Let us now consider the case that δ' is nonzero; the situation then becomes quite different. In Figs. 2 and 3, it is seen that $a^c_{3/2}$ decreases as the value of δ' increases. When δ' reaches $\pi/3$, $a^c_{3/2}$ will be consistent with the value yielded from the factorization approach. It is also noticed from the figures that a large δ' leads to large values of $a^u_{1/2}$ and δ. The enhancement of $a^u_{1/2}$ may be easily understood as the enhancement of $a^c_{3/2}$ due to final state interactions. Nevertheless, as will be discussed below, the values of δ' cannot be too large due to the constraint of the upper bound of the branching ratio of $B\to\pi^0\pi^0$.

When all the isospin amplitudes and strong phases are determined, one is able to predict the direct CP asymmetries for all the relevant decay channels. The direct CP asymmetry in $B\to\pi\pi,\pi K$ decays is defined in the standard way:

$$A_{CP}=\frac{\Gamma(\overline{B}\to\overline{f})-\Gamma(B\to f)}{\Gamma(\overline{B}\to\overline{f})+\Gamma(B\to f)}\equiv a^f_{\epsilon''}, \tag{34}$$

where f denotes the final state mesons. In Fig. 5 we plot several $|A_{CP}|$'s as functions of the weak phase γ with different value of δ'. When γ is near 90° and $\delta'=0$ one has $|A_{CP}(\pi^+K^-)|\sim 0.04$ which is in good agreement with the most recent CLEO data, $A_{CP}=-0.04\pm 0.16$ [2,20]. At this point, we have a reliable prediction for the direct CP violations in the following decay modes. The A_{CP}'s with $45°<\gamma<95°$ read

$$\begin{aligned}
&|A_{CP}(\pi^+K^-)|\simeq(2.5\text{–}4)\%,\quad |A_{CP}(\pi^0\overline{K^0})|\simeq(2.5\text{–}5)\%,\\
&|A_{CP}(\pi^0K^-)|\simeq(7.5\text{–}10)\%,\\
&|A_{CP}(\pi^-\overline{K^0})|\simeq(5\text{–}6.5)\%,\\
&|A_{CP}(\pi^+\pi^-)|\simeq(7.5\text{–}12.5)\%,\\
&|A_{CP}(\pi^0\pi^0)|\simeq(7.5\text{–}12)\%,
\end{aligned}\tag{35}$$

for $\delta'=0$, and

$$\begin{aligned}
&|A_{CP}(\pi^+K^-)|\simeq(5\text{–}8)\%,\quad |A_{CP}(\pi^0\overline{K^0})|\simeq(8\text{–}10)\%,\\
&|A_{CP}(\pi^0K^-)|\simeq(12\text{–}18)\%,\\
&|A_{CP}(\pi^-\overline{K^0})|\simeq(8\text{–}12)\%,\\
&|A_{CP}(\pi^+\pi^-)|\simeq(16\text{–}24)\%,\\
&|A_{CP}(\pi^0\pi^0)|\simeq(10\text{–}14)\%,
\end{aligned}\tag{36}$$

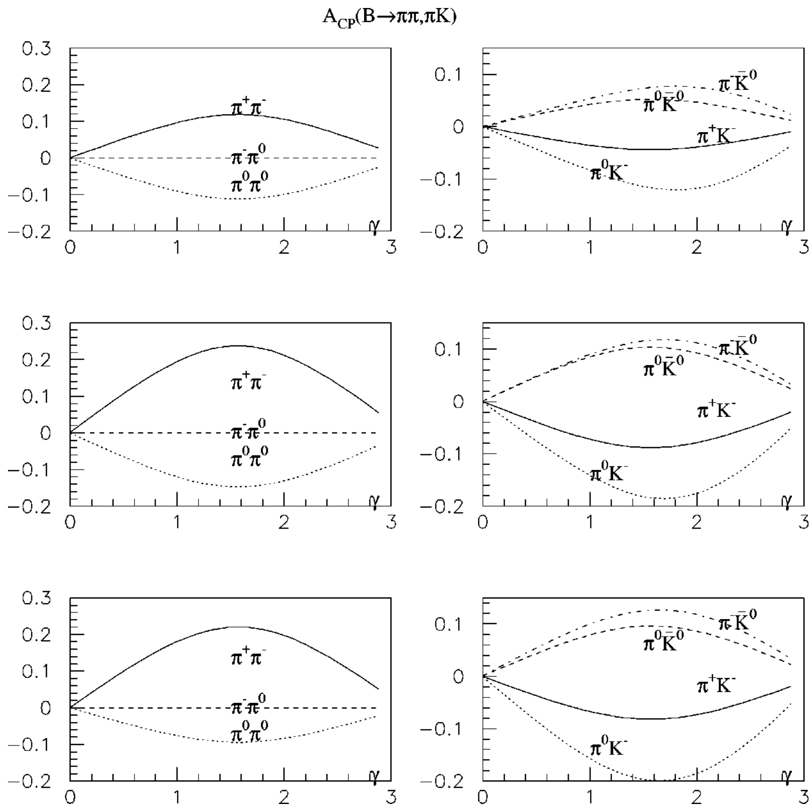

FIG. 5. The values of CP asymmetries A_{CP} vs weak phase γ with different $\delta'=0,\pi/6,\pi/3$ (from top to bottom). The curves corresponding to $|A_{CP}|$'s for π^0K^-, $\pi^+\pi^-$, $\pi^0\overline{K}^0$, π^+K^-, $\pi^-\overline{K}^0$, and $\pi^-\pi^0$ are indicated in the plot.

for $\delta'=30°$. In spite of the large δ, the smallness of the CP asymmetries in $B\to\pi^-\pi^0$ decay is due to the absence of the interference between tree and penguin diagrams.

There remains an unobserved decay mode in $B\to\pi\pi,\pi K$ decays, which is the decay mode $B\to\pi^0\pi^0$ [the CLEO Collaboration has already reported the indication of $\mathrm{Br}(B\to\pi^0\pi^-)\simeq 5.6$]. As all the relevant isospin amplitudes have been determined as functions of γ, it allows us to predict $B\to\pi^0\pi^0$ as a function of γ. It is interesting to note that our χ^2 analysis shows that the resulting branching ratio $\mathrm{Br}(B\to\pi^0\pi^0)$ is almost independent of the weak phase γ. Its value at $\delta'=0$ is close to the one of $B\to\pi^+\pi^-$ decay:

$$\mathrm{Br}(B\to\pi^0\pi^0)\sim 4.6\times 10^{-6}. \tag{37}$$

With δ' increasing, the branching ratio becomes larger and can reach $\sim 7(10)\times 10^{-6}$ when $\delta'=30°(60°)$. Such a large branching ratio is about an order of magnitude larger than the prediction based on factorization calculations. Most recently, the CLEO Collaboration reported an upper bound of $\mathrm{Br}(B\to\pi^0\pi^0)<5.7\times 10^{(-6)}$ [21]; this will impose a strong constraint on the value of δ' (see Fig. 6). It is seen that to be consistent with the data at the 1σ level, the upper bound of the branching ratio $\mathrm{Br}(B\to\pi^0\pi^0)$ limits δ' to be less than $\sim 50°$.

IV. CONCLUSIONS

In summary, we have made a general less model-dependent investigation on the charmless B-meson decays by using the χ^2 analysis based on the most recent CLEO data. We have used the most general isospin decomposition with two independent strong phases. All the isospin amplitudes in rare hadronic B decays $B\to\pi\pi,\pi K$ can be determined as functions of the weak phase γ and one strong phase δ'. The

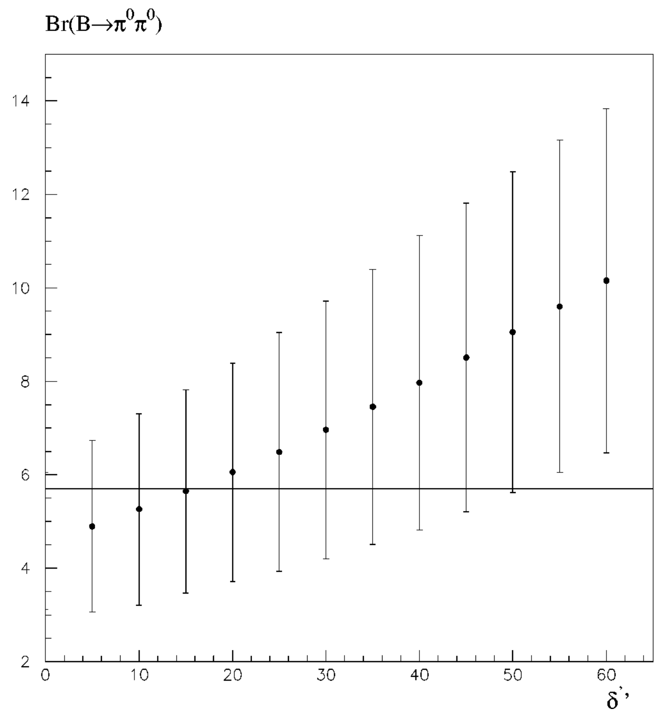

FIG. 6. The branching ratio of $B\to\pi^0\pi^0$ (in units of 10^{-6}) predicted as a function of δ' (in degrees). The solid line indicates the upper bound observed reported by the CLEO Collaboration.

effects of two equal and unequal strong phases are studied in detail. It is found that the isospin amplitude $b_{1/2}^{c}$ and the strong phase δ only slightly depend on the phase γ. An important observation under the equal strong phase assumption is the relative large isospin amplitude $a_{3/2}^{c}$ ($a_{3/2}^{c}\simeq -70$) where the central value is about 5 times greater than the one obtained from the factorization calculations. When the two strong phases are not equal, the allowed values of $a_{3/2}^{c}$ decrease as their difference, i.e., δ', increases. For the most general case with two rather than one large FSI strong phase, the magnitude of all the isospin amplitudes may be around the one estimated from the factorization approach. Nevertheless, one needs to find out the mechanism of producing large strong phases. This could directly be tested by measuring the branching ratio $\mathrm{Br}(B\to\pi^0\pi^0)$.

The direct CP asymmetries A_{CP}^{f} for all the relevant decay channels have also been given as functions of γ. The resulting numerical value for $A_{CP}^{\pi^+K^-}$ is consistent with the most recent data. When taking γ to be in a reasonable range $\gamma = 45°–95°$, we are led to the results given in Eqs. (35) and (36), which can be directly tested by experiments in the near future. A resulting large branching ratio $\mathrm{Br}(B\to\pi^0\pi^0)$ which is comparable with the one $\mathrm{Br}(B\to\pi^+\pi^-)$ will also provide an important and consistent test.

From the most general analysis presented in this paper, the data appear to strongly suggest that final state interactions and inelastic rescattering effects must be significant and play an important rule in the charmless $B\to\pi\pi,\pi K$ decays. Otherwise, our general analyses may be interpreted as hinting at the existence of new physics. For a more definite conclusion, one needs more precise data. The two B factories BaBaR and BELLE are expected to provide us with more information from charmless decays.

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[1] CLEO Collaboration, D. Cronin-Hennessy *et al.*, hep-ex/0001010.
[2] CLEO Collaboration, S. Chen *et al.*, Phys. Rev. Lett. **85**, 525 (2000).
[3] G. Kramer and W. F. Palmer, Phys. Rev. D **52**, 6411 (1995); N. G. Deshpande and X. G. He, Phys. Rev. Lett. **74**, 26 (1995).
[4] W. S. Hou and K. C. Yang, Phys. Rev. Lett. **84**, 4806 (2000).
[5] R. Fleischer, Eur. Phys. J. C **6**, 451 (1999).
[6] M. Gronau and J. L. Rosner, Phys. Rev. Lett. **73**, 21 (1994); **76**, 1200 (1996).
[7] M. Neubert and J. L. Rosner, Phys. Rev. Lett. **81**, 5076 (1998).
[8] N. G. Deshpande, X. G. He, W. S. Hou, and S. Pavaska, Phys. Rev. Lett. **82**, 2240 (1999); X. G. He, W. S. Hou, and K. C. Yang, *ibid.* **83**, 1100 (1999); W. S. Hou and K. C. Yang, Phys. Rev. D **61**, 073014 (2000).
[9] T. N. Pham, hep-ph/0009142; C. Isola and T. N. Pham, Phys. Rev. D **62**, 094002 (2000).
[10] Y. L. Wu and Y. F. Zhou, Phys. Rev. D **62**, 036007 (2000).
[11] X. G. He, C. L. Hsueh, and J. Q. Shi, Phys. Rev. Lett. **84**, 18 (2000); Y. L. Wu and Y. F. Zhou, Phys. Rev. D **61**, 096001 (2000).
[12] D. Zeppenfeld, Z. Phys. C **8**, 77 (1981); M. Savage and M. Wise, Phys. Rev. D **39**, 3346 (1989); **40**, 3127(E) (1989); M. Gronau, O. F. Hernandez, D. London, and J. L. Rosner, *ibid.* **52**, 6374 (1995); M. Gronau, O. F. Hernandez, D. London, and J. L. Rosner, *ibid.* **52**, 6356 (1995); X. G. He, Eur. Phys. J. C **9**, 443 (1999); M. Gronau and J. L. Rosner, Phys. Rev. D **61**, 073008 (2000).
[13] R. Fleischer and T. Mannel, Phys. Rev. D **57**, 2752 (1998).
[14] A. J. Buras and R. Fleischer, Eur. Phys. J. C **16**, 97 (2000).
[15] M. Gronau and J. L. Rosner, Phys. Rev. D **61**, 073008 (2000).
[16] F. James and M. Roos, Comput. Phys. Commun. **10**, 343 (1975); CERNLIB-writeup, http://wwwinfo.cern.ch/asdoc/shortwrupsdir/d506/top.html
[17] Y. Y. Keum, H.-N. Li, and A. I. Sanda, talk given at 28th International Workshop on Gross Properties of Nuclei and Nuclear Excitation: Hadrons in Dense Matter (HIRSCHEGG 2000), Hirschegg, Austria, 2000, hep-ph/0004173.
[18] A. Ali, G. Kramer, and C. D. Lü, Phys. Rev. D **58**, 094009 (1998).
[19] M. Neubert and J. Rosner, Phys. Lett. B **441**, 403 (1998).
[20] BaBar Collaboration, D. Hitlin *et al.*, plenary talk in ICHeP 2000, Osaka, Japan, 2000, Report No. SLAC-PUB-8540; Belle Collaboration, H. Aihara, plenary talk in ICHeP 2000, Osaka, Japan, 2000.
[21] CLEO Collaboration, plenary talk in ICHEP 2000, Osaka, Japan, 2000.