# Lepton polarization asymmetry in $\boldsymbol{B} \rightarrow \boldsymbol{K}^{(*)} \boldsymbol{l}^{+} \boldsymbol{l}^{-}$ 

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#### Abstract

We investigate the longitudinal lepton polarization asymmetry in the exclusive processes $B \rightarrow K l^{+} l^{-}$and $B \rightarrow K^{*} l^{+} l^{-}$. We include both short- and long-distance contributions to the asymmetry in our discussions. We find that average values of the polarization asymmetries of the muon and $\tau$ for $B \rightarrow K^{(*)} \mu^{+} \mu^{-}$and $B \rightarrow K^{(*)} \tau^{+} \tau^{-}$are $-0.8(-0.7)$ and $-0.2(-0.5)$, respectively. [S0556-2821(96)05321-0]


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## I. INTRODUCTION

As is well known, the study of $B$-meson physics is important in the determination of the elements of the Cabibbo-Kobayashi-Maskawa (CKM) [1] matrix of charged-current weak couplings. In addition, it could shed light on physics beyond the standard model. Recently, much interest [3] has been centered on rare $B$-meson decays induced by the flavorchanging neutral current $b \rightarrow s$ transition due to the CLEO measurement of the radiative $b \rightarrow s \gamma$ decay [2]. In the standard model, these rare processes do not occur at tree level and appear only at the quantum (loop) level. The shortdistance (SD) contributions to the decays involving the $b \rightarrow s$ transition are dominated by loops with the top quark and they are basically free of the uncertainties in the CKM parameters. The rare $B$-meson decays are, thus, a good probe of heavy top quark physics as well as physics beyond the standard model.

It has been pointed out by Ali et al. [4] that, in the standard model, the measurement of the forward-backward asymmetry of the dileptons in the inclusive decays $b \rightarrow s l^{+} l^{-}$provides information on the short-distance contributions dominated by the top quark loops. Recently, it has been emphasized by Hewett [5] that the longitudinal lepton polarizations, which are other parity-violating observables, are also important asymmetries. In particular, the $\tau$ polarization in $b \rightarrow s \tau^{+} \tau^{-}$mode could be accessible to the $B$ factories currently under construction. It is interesting to note that the dilepton forward-backward asymmetry of the exclusive decays $B \rightarrow M l^{+} l^{-}$is identically zero when $M$ are pseudoscalar mesons such as $\pi$ and $K$, but nonzero when $M$ are vector mesons such as $\rho$ and $K^{*}$. However, the longitudinal lepton polarizations for the exclusive modes are nonzero for both cases of pseudoscalar and vector mesons. In this paper we will examine these lepton polarization asymmetries for the exclusive decays of $B \rightarrow K l^{+} l^{-}$and $B \rightarrow K^{*} l^{+} l^{-}$, in which we only concentrate on the muon and $\tau$ dileptonic modes since the electron polarization is hard to be measured experimentally. In our discussions, we will use the results of the relativistic quark model by the light-front formalism [6,7] on the form factors in the hadronic matrix elements between the $B$ meson and the kaons.

The paper is organized as follows. In Sec. II, we give the general effective Hamiltonian in the standard model for the rare dilepton decays of interest. We study the exclusive decays of $B \rightarrow K l^{+} l^{-}$and $B \rightarrow K^{*} l^{+} l^{-}$in Sec. III and Sec. IV,
respectively. Our conclusions are summarized in Sec. V.

## II. EFFECTIVE HAMILTONIAN

The effective Hamiltonian relevant to $b \rightarrow s l^{+} l^{-}$based on an operator product expansion is given by [8]

$$
\begin{equation*}
\mathcal{H}_{\mathrm{eff}}=\frac{4 G_{F} \lambda_{t}}{\sqrt{2}} \sum_{i=1}^{10} C_{i}(\mu) O_{i}(\mu) \tag{1}
\end{equation*}
$$

where $G_{F}$ denotes the Fermi constant, $\lambda_{t}=V_{t b} V_{t s}^{*}$ are the products of the CKM matrix elements. The $O_{i}(\mu)$ are the operators

$$
\begin{gather*}
O_{1}=\left(\bar{s}_{\alpha} \gamma^{\mu} L b_{\alpha}\right)\left(\bar{c}_{\beta} \gamma_{\mu} L c_{\beta}\right), \\
O_{2}=\left(\bar{s}_{\alpha} \gamma^{\mu} L b_{\beta}\right)\left(\bar{c}_{\beta} \gamma_{\mu} L c_{\alpha}\right), \\
O_{3}=\left(\bar{s}_{\alpha} \gamma^{\mu} L b_{\alpha}\right)\left[\left(\bar{u}_{\beta} \gamma_{\mu} L u_{\beta}\right)+\cdots+\left(\bar{b}_{\beta} \gamma_{\mu} L b_{\beta}\right)\right], \\
O_{4}=\left(\bar{s}_{\alpha} \gamma^{\mu} L b_{\beta}\right)\left[\left(\bar{u}_{\beta} \gamma_{\mu} L u_{\alpha}\right)+\cdots+\left(\bar{b}_{\beta} \gamma_{\mu} L b_{\alpha}\right)\right], \\
O_{5}=\left(\bar{s}_{\alpha} \gamma^{\mu} L b_{\alpha}\right)\left[\left(\bar{u}_{\beta} \gamma_{\mu} R u_{\beta}\right)+\cdots+\left(\bar{b}_{\beta} \gamma_{\mu} R b_{\beta}\right)\right], \\
O_{6}=\left(\bar{s}_{\alpha} \gamma^{\mu} L b_{\beta}\right)\left[\left(\bar{u}_{\beta} \gamma_{\mu} R u_{\alpha}\right)+\cdots+\left(\bar{b}_{\beta} \gamma_{\mu} R b_{\alpha}\right)\right], \\
O_{7}=\frac{e}{16 \pi^{2}} m_{b}\left(\bar{s}_{\alpha} \sigma^{\mu \nu} R b_{\alpha}\right) F_{\mu \nu}, \\
O_{8}=\frac{e^{2}}{16 \pi^{2}}\left(\bar{s}_{\alpha} \gamma^{\mu} L b_{\alpha}\right) \bar{l} \gamma_{\mu} l, \\
O_{9}=\frac{e^{2}}{16 \pi^{2}}\left(\bar{s}_{\alpha} \gamma^{\mu} L b_{\alpha}\right) \bar{l} \gamma_{\mu} \gamma_{5} l, \\
O_{10}=\frac{g}{16 \pi^{2}} m_{b}\left(\bar{s}_{\alpha} \sigma^{\mu \nu} T_{\alpha \beta}^{a} R b_{\beta}\right) G_{\mu \nu}^{a}, \tag{2}
\end{gather*}
$$

where $R(L)=\left(1 \pm \gamma_{5}\right) / 2$. Here $F_{\mu \nu}$ and $G_{\mu \nu}^{a}$ are the electromagnetic and strong interaction field strength tensors, and $e$ and $g$ are the corresponding coupling constants, respectively. In the standard model with the dimension six operator basis, $O_{1}$ and $O_{2}$ are current-current operators, $O_{3}, \ldots, O_{6}$ are usually QCD penguin operators, $O_{7}$ and $O_{10}$ are magnetic penguin operators, and $O_{8}$ and $O_{9}$ are semileptonic electroweak
operators. The $C_{i}$ in Eq. (1) are the corresponding Wilson coefficients. They are calculated first at a renormalization scale $M_{W}$ and then scaled down to a mass of order $m_{b}$ using the renormalization group equations. It is known that the coefficients $C_{3}, \ldots, C_{6}$ are small and, thus, the contributions of the corresponding operators can be neglected. Moreover, the operator $O_{10}$ does not contribute to the decay of $b \rightarrow s l^{+} l^{-}$. Hence, the relevant operators of our study are $O_{1}, O_{2}, O_{7}, O_{8}$, and $O_{9}$, respectively. At the scale $\mu=M_{W}$, one has [9]

$$
\begin{gather*}
C_{1}\left(M_{W}\right)=0, \\
C_{2}\left(M_{W}\right)=-1, \\
C_{7}\left(M_{W}\right)=\frac{1}{2} A(x), \\
C_{8}\left(M_{W}\right)=\frac{1}{\sin ^{2} \theta_{W}} B(x)+\frac{-1+4 \sin ^{2} \theta_{W}}{\sin ^{2} \theta_{W}} C(x) \\
+D(x)-\frac{4}{9}, \\
C_{9}\left(M_{W}\right)=\frac{-1}{\sin ^{2} \theta_{W}} B(x)+\frac{1}{\sin ^{2} \theta_{W}} C(x), \tag{3}
\end{gather*}
$$

where $x=m_{t}^{2} / M_{W}^{2}$ and

$$
\begin{align*}
A(x)= & x\left[\frac{2 x^{2} / 3+5 x / 12-7 / 12}{(x-1)^{3}}-\frac{3 x^{2} / 2-x}{(x-1)^{4}} \ln x\right] \\
& B(x)=\frac{1}{4}\left[\frac{-x}{x-1}+\frac{x}{(x-1)^{2}} \ln x\right] \\
& C(x)=\frac{x}{4}\left[\frac{x / 2-3}{x-1}+\frac{3 x / 2+1}{(x-1)^{2}} \ln x\right] \\
D(x)= & \frac{-19 x^{3} / 36+25 x^{2} / 36}{(x-1)^{3}} \\
& +\frac{-x^{4} / 6+5 x^{3} / 3-3 x^{2}+16 x / 9-4 / 9}{(x-1)^{4}} \ln x \tag{4}
\end{align*}
$$

Because $O_{1}$ and $O_{2}$ produce dilepton via virtual (vector) photon, they can be incorporated into $O_{8}$ in the later calculation. From Eq. (1) and Eq. (2), we obtain the amplitude for the inclusive process $B \rightarrow X_{s} l^{+} l^{-}$[5]:

$$
\begin{align*}
M= & \frac{G_{F} \alpha}{\sqrt{2} \pi} \lambda_{t}\left(C_{8}^{\mathrm{eff}} \overline{s_{L}} \gamma_{\mu} b_{L} \bar{l} \gamma^{\mu} l+C_{9} \bar{s}_{L} \gamma_{\mu} b_{L} \bar{l} \gamma^{\mu} \gamma_{5} l\right. \\
& \left.-2 C_{7} m_{b} \bar{s}_{L} i \sigma_{\mu \nu} \frac{q^{\nu}}{q^{2}} b_{R} \bar{l} \gamma^{\mu} l\right) . \tag{5}
\end{align*}
$$

The coefficients $C_{8}^{\text {eff }}, C_{9}$, and $C_{7}$ are given by [10]

$$
\begin{gather*}
C_{7}\left(m_{b}\right)=\eta^{-16 / 23}\left[C_{7}\left(M_{W}\right)-\frac{58}{135}\left(\eta^{10 / 23}-1\right) C_{2}\left(M_{W}\right)\right. \\
\left.-\frac{29}{189}\left(\eta^{28 / 23}-1\right) C_{2}\left(M_{W}\right)\right] \\
C_{8}^{\mathrm{eff}}\left(m_{b}\right)= \\
C_{8}\left(m_{b}\right)+\left[3 C_{1}\left(m_{b}\right)+C_{2}\left(m_{b}\right)\right] \\
\times\left(h\left(\hat{m}_{c}, \hat{s}\right)-\frac{3}{\alpha^{2}} \kappa\right. \\
\left.\times \sum_{V_{i}=J / \psi, \psi^{\prime}} \frac{\pi \Gamma\left(V_{i} \rightarrow l l\right) M_{V_{i}}}{q^{2}-M_{V_{i}}+i M_{V_{i}} \Gamma_{V_{i}}}\right)  \tag{6}\\
C_{9}\left(m_{b}\right)=C_{9}\left(M_{W}\right)
\end{gather*}
$$

where

$$
\begin{gather*}
C_{1}\left(m_{b}\right)=\frac{1}{2}\left(\eta^{-6 / 23}-\eta^{12 / 23}\right) C_{2}\left(M_{W}\right) \\
C_{2}\left(m_{b}\right)=\frac{1}{2}\left(\eta^{-6 / 23}+\eta^{12 / 23}\right) C_{2}\left(M_{W}\right) \\
C_{8}\left(m_{b}\right)=C_{8}\left(M_{W}\right)+\frac{4 \pi}{\alpha_{s}\left(M_{W}\right)}\left[-\frac{4}{33}\left[1-\eta^{-11 / 23}\right)\right. \\
\left.+\frac{8}{87}\left(1-\eta^{-29 / 23}\right)\right] C_{2}\left(M_{W}\right) \tag{7}
\end{gather*}
$$

Here $\quad \hat{m}_{c}=m_{c} / m_{b}, \quad \eta=\alpha_{s}\left(m_{b}\right) / \alpha_{s}\left(M_{W}\right), \quad \alpha=e^{2} / 4 \pi, \quad \hat{s}$ $=q^{2} / m_{b}^{2}$ with $q^{2}$ being the invariant mass of the dilepton, and $h\left(\hat{m}_{c}, \hat{s}\right)$ arising from the one-loop contributions of $O_{1}$ and $O_{2}$ is given by

$$
\begin{align*}
h(z, \hat{s})= & -\frac{4}{9} \ln z^{2}+\frac{8}{27}+\frac{4}{9} y-\frac{2}{9}(2+y) \sqrt{|1-y|} \\
& \times\left[\Theta(1-y)\left(\ln \frac{1+\sqrt{1-y}}{1-\sqrt{1-y}}+i \pi\right)\right. \\
& \left.+\Theta(y-1) 2 \arctan \left(\frac{1}{\sqrt{y-1}}\right)\right], \tag{8}
\end{align*}
$$

where $y \equiv 4 z^{2} / \hat{s}$. The last term of $C_{8}^{\text {eff }}\left(m_{b}\right)$ in Eq. (6) is the long-distance (LD) contribution mainly due to the $J / \psi$ and $\psi^{\prime}$ resonances [11], and the factor $\kappa$ must be chosen such that $\kappa\left[3 C_{1}\left(m_{b}\right)+C_{2}\left(m_{b}\right)\right] \simeq-1$ in order to reproduce correctly the branching ratio [4]

$$
\begin{equation*}
\mathcal{B}(B \rightarrow J / \psi X \rightarrow X l \bar{l})=\mathcal{B}(B \rightarrow J / \psi X) \mathcal{B}(J / \psi \rightarrow l \bar{l}) \tag{9}
\end{equation*}
$$

## III. LONGITUDINAL LEPTON POLARIZATION IN $\boldsymbol{B} \rightarrow \boldsymbol{K} \boldsymbol{l}^{+} \boldsymbol{l}^{-}$

The hadronic matrix elements of the operators $O_{1}, O_{2}$, $O_{7}, O_{8}$, and $O_{9}$ between the $B$ meson and the pseudoscalar $K$ meson are given in terms of form factors as $[6,7,10,12]$

$$
\begin{align*}
& \quad\left\langle p_{K}\right| \bar{s} \gamma_{\mu}\left(1 \mp \gamma_{5}\right) b\left|p_{B}\right\rangle=F_{+}\left(q^{2}\right) P_{\mu}+F_{-}\left(q^{2}\right) q_{\mu}  \tag{10}\\
& \left\langle p_{K}\right| \overrightarrow{s i} \sigma_{\mu \nu} q^{\nu}\left(1 \pm \gamma_{5}\right) b\left|p_{B}\right\rangle
\end{align*}
$$

$$
\begin{equation*}
=\frac{1}{m_{B}+m_{K}}\left[P_{\mu} q^{2}-\left(m_{B}^{2}-m_{K}^{2}\right) q_{\mu}\right] F_{T}\left(q^{2}\right), \tag{11}
\end{equation*}
$$

where $P \equiv p_{K}+p_{B}$ and $q \equiv p_{K}-p_{B}$. In this paper, we use the form factors given by the relativistic constituent quark model in Refs. [6] and [7]. The model is based on the light front formalism, in which the form factors $F_{+}, F_{-}$, and $F_{T}$ are taken to be approximately as

$$
\begin{equation*}
F\left(q^{2}\right)=\frac{F(0)}{1-q^{2} / \Lambda_{1}^{2}+q^{4} / \Lambda_{2}^{4}} \tag{12}
\end{equation*}
$$

Here the parameters $\Lambda_{1}$ and $\Lambda_{2}$ are determined by the first and second derivative of $F\left(q^{2}\right)$ at $q^{2}=0$. The values of $F(0)$ and $\Lambda_{1}, \Lambda_{2}$ for the various form factors used can be found in Table I of Ref. [7]. When lepton mass (for $l=e$ or $\mu$ ) is negligible, $q_{\mu}$ terms in Eqs. (10) and (11) give no contributions to the decay rate and which is the case studied in Refs. [6] and [7]. However, the contributions have to be considered for the $\tau$ dileptonic channel of $B \rightarrow K \tau^{+} \tau^{-}$. In this case, one cannot take $F_{-}$as zero. The form factor $F_{-}$has been examined in Ref. [13], and it is found to be

$$
\begin{equation*}
F_{-} \simeq-F_{+}\left(m_{B}^{2}+m_{K}^{2}\right) /\left(m_{B}^{2}-m_{K}^{2}\right) \tag{13}
\end{equation*}
$$

$F_{-}$can be extracted from other methods, such as that from heavy quark symmetry (HQS) [14]. In this method, one has

$$
\begin{equation*}
F_{-}=F_{+}+\frac{2 m_{B} F_{T}}{m_{B}+m_{K}} \tag{14}
\end{equation*}
$$

The differential decay rate is given by

$$
\begin{align*}
& \frac{d \Gamma\left(B \rightarrow K l^{+} l^{-}\right)}{d \hat{s}} \\
& =\frac{G_{F}^{2}\left|\lambda_{t}\right|^{2} m_{B}^{5} \alpha^{2}}{3 \cdot 2^{9} \pi^{5}} \mathcal{D} \phi^{1 / 2}\left[\left[\left|C_{8}^{\mathrm{eff}} F_{+}-\frac{2 C_{7} F_{T}}{1+\sqrt{r}}\right|^{2}\right.\right. \\
& \left.\quad+\left|C_{9} F_{+}\right|^{2}\right] \phi\left(1+\frac{2 t}{\hat{s}}\right)+12\left|C_{9}\right|^{2} t\left[(1+r-\hat{s} / 2) F_{+}^{2}\right. \\
& \left.\left.\quad+(1-r) F_{+} F_{-}+\frac{1}{2} \hat{s} F_{-}^{2}\right]\right] \tag{15}
\end{align*}
$$

where

$$
\begin{gather*}
t=m_{l}^{2} / m_{B}^{2}, \quad r=m_{K}^{2} / m_{B}^{2}, \quad \hat{s}=q^{2} / m_{B}^{2} \\
\phi=(1-r)^{2}-2 \hat{s}(1+r)+\hat{s}^{2}, \quad \mathcal{D}=\sqrt{1-4 t / \hat{s}} \tag{16}
\end{gather*}
$$

We note that the rate in Eq. (15) is a general form with $m_{l} \neq 0$ and it agrees with that given in Ref. [15]. The differential branching ratios, $d B / d \hat{s} \equiv d \Gamma /\left(\Gamma_{\text {tot }} d \hat{s}\right)$, as a function of $\hat{s}$ for $B \rightarrow K \mu^{+} \mu^{-}$and $B \rightarrow K \tau^{+} \tau^{-}$decays are displayed in Figs. 1(a) and 1(b), respectively. Here we have used that $m_{t}=180 \mathrm{GeV}$ and $\Gamma_{\text {tot }} \simeq m_{b}^{5} G_{F}^{2}\left|V_{c b}\right|^{2} /\left(64 \pi^{3}\right)$ with $m_{B}$ $\simeq m_{b}=5 \mathrm{GeV}$. The dashed and solid lines in Figs. 1(a) and 1(b) represent the results with and without the LD contributions from the resonance states, respectively. It is noted that the solid line for the decay of $B \rightarrow K \tau^{+} \tau^{-}$in Fig. 1(b) is similar to the corresponding figure in Ref. [15].


FIG. 1. Differential branching ratios for (a) $B \rightarrow K \mu^{+} \mu^{-}$and (b) $B \rightarrow K \tau^{+} \tau^{-}$as a function of $\hat{s}=q^{2} / m_{B}^{2}$ with $m_{t}=180 \mathrm{GeV}$. The dashed and solid lines correspond to the results with and without the LD contributions from the resonance states, respectively.

For the dilepton decays of the $B$ meson, the longitudinal polarization asymmetry (LPA) of the lepton, $P_{L}$, is defined as

$$
\begin{equation*}
P_{L}(\hat{s})=\frac{d \Gamma_{h=-1} / d \hat{s}-d \Gamma_{h=1} / d \hat{s}}{d \Gamma_{h=-1} / d \hat{s}+d \Gamma_{h=1} / d \hat{s}} \tag{17}
\end{equation*}
$$

where $h=+1(-1)$ means right (left) handed $l^{-}$in the final state and $d \Gamma / d \hat{s}$ means the differential decay rate of the $B$ meson. In the standard model, the polarization asymmetries in Eq. (17) for $B \rightarrow K l^{+} l^{-}$and $B \rightarrow K^{*} l^{+} l^{-}$come from the interference of the vector or magnetic moment and axialvector operators. For the decay of $B \rightarrow K l^{+} l^{-}$, it is found that
$P_{L}(\hat{s})=2 \mathcal{D} \phi F_{+} C_{9}\left(F_{+} \operatorname{Re} C_{8}^{\mathrm{eff}}-2 \frac{F_{T}}{1+\sqrt{r}} C_{7}\right) / R$,
where $R$ is given by


FIG. 2. Longitudinal polarization asymmetries of (a) the muon in $B \rightarrow K \mu^{+} \mu^{-}$and (b) the $\tau$ in $B \rightarrow K \tau^{+} \tau^{-}$as a function of $\hat{s}=q^{2} / m_{B}^{2}$ with $m_{t}=180 \mathrm{GeV}$. Legend is the same as in Fig. 1.

$$
\begin{align*}
R= & \left(\left|C_{8}^{\mathrm{eff}} F_{+}-\frac{2 C_{7} F_{T}}{1+\sqrt{r}}\right|^{2}+\left|C_{9} F_{+}\right|^{2}\right) \phi\left(1+\frac{2 t}{\hat{s}}\right) \\
& +12\left|C_{9}\right|^{2} t\left[(1+r-\hat{s} / 2) F_{+}^{2}+(1-r) F_{+} F_{-}+\frac{1}{2} \hat{s} F_{-}^{2}\right] . \tag{19}
\end{align*}
$$

In Figs. 2(a) and 2(b), taking $m_{t}=180 \mathrm{GeV}$, we show the LPAs for $B \rightarrow K l^{+} l^{-}$as a function of $\hat{s}$ with $l=\mu$ and $\tau$, respectively. The LPAs for both cases vanish at the thresholds and oscillate in the resonance regions of $q^{2} \simeq M_{\psi\left(\psi^{\prime}\right)}^{2}$ and they also reach zero at the end points. However, we note that if $m_{l}=0$, the LPA in Eq. (18) has a finite value at the maximal end point of $\hat{s}_{\max }=\left(1-m_{K} / m_{B}\right)^{2}$. From Figs. 2(a) and 2(b), we find that, away from the resonance regions and end points, $P_{L}(\hat{s})$ of the muon dilepton channel is around -0.9 , while that of the $\tau$ is between -0.2 and -0.3 for $0.6 \leqslant \hat{s} \leqslant 0.8$. The average values of the LPAs are -0.8 and -0.2 , respectively. Finally, we remark that Figs. 1 and 2 were drawn by using the form factor $F_{-}$in Eq. (13). The curves in Figs. 1 and 2 are nearly unchanged if one uses the form factor of $F_{-}$formulated in Eq. (14).

## IV. LONGITUDINAL LEPTON POLARIZATION <br> IN $\boldsymbol{B} \rightarrow \boldsymbol{K}^{*} \boldsymbol{l}^{+} \boldsymbol{l}^{-}$

The hadronic matrix elements of the operators in Eq. (5) between the external states $B$ and $K^{*}$ for the exclusive decay of $B \rightarrow K^{*} l^{+} l^{-}$are [7]

$$
\begin{align*}
\left\langle p_{K^{*}}\right| \bar{s} \gamma_{\mu}\left(1 \mp \gamma_{5}\right) b\left|p_{B}\right\rangle= & \frac{1}{m_{B}+m_{K^{*}}}\left[i V\left(q^{2}\right) \epsilon_{\mu \nu \alpha \beta} \epsilon^{* \nu} P^{\alpha} q^{\beta}\right. \\
& \pm A_{0}\left(q^{2}\right)\left(m_{B}^{2}-m_{K^{*}}^{2}\right) \epsilon_{\mu}^{*} \pm A_{+}\left(q^{2}\right) \\
& \left.\times\left(\epsilon^{*} P\right) P_{\mu} \pm A_{-}\left(q^{2}\right)\left(\epsilon^{*} P\right) q_{\mu}\right] \tag{20}
\end{align*}
$$

$$
\left\langle p_{K^{*}}\right| \overline{\sin } \sigma_{\mu \nu} q^{\nu}\left(1 \pm \gamma_{5}\right) b\left|p_{B}\right\rangle=i g\left(q^{2}\right) \epsilon_{\mu \nu \alpha \beta} \epsilon^{* \nu} P^{\alpha} q^{\beta}
$$

$$
\pm a_{0}\left(q^{2}\right)\left(m_{B}^{2}-m_{K}^{2}\right)
$$

$$
\times\left[\epsilon_{\mu}^{*}-\frac{1}{q^{2}}\left(\epsilon^{*} q\right) q_{\mu}\right]
$$

$$
\pm a_{+}\left(q^{2}\right)\left(\epsilon^{*} P\right)
$$

$$
\begin{equation*}
\times\left[P_{\mu}-\frac{1}{q^{2}}(P q) q_{\mu}\right] \tag{21}
\end{equation*}
$$

where $P=p_{B}+p_{K^{*}}$ and $q=p_{B}-p_{K^{*}}$. The terms corresponding to $A_{-}$in Eqs. (20) and (21) are important only for the mode of $B \rightarrow K^{*} \tau^{+} \tau^{-}$. Similar to Eq. (13), one has that [13]

$$
\begin{equation*}
A_{-} \simeq-A_{+}\left(m_{B}^{2}+m_{K^{*}}^{2}\right) /\left(m_{B}^{2}-m_{K^{*}}^{2}\right) . \tag{22}
\end{equation*}
$$

From Eqs. (5), (20), and (21) we obtain the differential decay rate of $B \rightarrow K^{*} l^{+} l^{-}$as

$$
\begin{align*}
\frac{d \Gamma\left(B \rightarrow K^{*} l^{+} l^{-}\right)}{d \hat{s}}= & \frac{G_{F}^{2} m_{B}^{5}\left|\lambda_{t}\right|^{2}}{8 \pi^{3}} \frac{\alpha^{2}}{16 \pi^{2}} \mathcal{D} \phi^{1 / 2} \\
& \times\left[\left(1+\frac{2 t}{\hat{s}}\right)\left(\frac{\hat{s}}{m_{B}^{2}} \alpha+\frac{\beta}{3} \phi\right)+t \delta\right] \tag{23}
\end{align*}
$$

where the definitions of $\alpha$ and $\beta$ are the same as in Ref. [10] with choosing zero values of $C_{7^{\prime}}, C_{8^{\prime}}$, and $C_{9^{\prime}}$ and

$$
\begin{align*}
\delta= & \frac{\left|C_{9}\right|^{2}}{2(1+\sqrt{r})^{2}}\left\{-2 \phi|V|^{2}-3(1-r)^{2}\left|A_{0}\right|^{2}\right. \\
& +\frac{\phi}{4 r}[2(1+r)-\hat{s}]\left|A_{+}\right|^{2}+\frac{\phi \hat{s}}{4 r}\left|A_{-}\right|^{2} \\
& \left.+\frac{\phi}{2 r}(1-r) \operatorname{Re}\left(A_{0} A_{+}^{*}+A_{0} A_{-}^{*}+A_{+} A_{-}^{*}\right)\right\} . \tag{24}
\end{align*}
$$

The forms of $r$ and $\phi$ are the same as that in Eq. (16) with the replacement $m_{K} \rightarrow m_{K^{*}}$. The differential branching ratios as a function of $\hat{s}$ for $B \rightarrow K^{*} \mu^{+} \mu^{-}$and $B \rightarrow K^{*} \tau^{+} \tau^{-}$de-



FIG. 3. Same as Fig. 1 but for (a) $B \rightarrow K^{*} \mu^{+} \mu^{-}$and (b) $B \rightarrow K^{*} \tau^{+} \tau^{-}$.
cays are shown in Figs. 3(a) and 3(b), respectively, with $m_{t}=180 \mathrm{GeV}$. In Eq. (23), the lepton mass is kept to be nonzero and it is consistent with the result given in [10] for the $B \rightarrow K^{*} l^{+} l^{-}$decay by taking the limit of $m_{l}=0$.

From Eq. (17), the LPA in $B \rightarrow K^{*} l^{+} l^{-}$is found to be

$$
\begin{equation*}
P_{L}(\hat{s})=\frac{\mathcal{D}}{3} \frac{C_{9}}{1+\sqrt{r}}\left(\frac{\operatorname{Re} C_{8}^{\mathrm{eff}}}{1+\sqrt{r}} S_{\mathrm{av}}-\frac{2 C_{7}}{\hat{s}} S_{\mathrm{at}}\right) / R \tag{25}
\end{equation*}
$$

where

$$
\begin{gather*}
R=\left(1+\frac{2 t}{\hat{s}}\right)\left(\frac{\hat{s}}{m_{B}^{2}} \alpha+\frac{\beta}{3} \phi\right)+t \delta, \\
S_{\mathrm{av}}=V^{2} F_{V}+A_{0}^{2} F_{A}^{0}+A_{+}^{2} F_{A}^{+}+A_{0} A_{+} F_{A}^{0+}, \\
S_{\mathrm{at}}=g V F_{V}+a_{0} A_{0} F_{A}^{0}+\frac{a_{0} A_{+}+a_{+} A_{0}}{2} F_{A}^{0+}+a_{+} A_{+} F_{A}^{+}, \tag{26}
\end{gather*}
$$



FIG. 4. Same as Fig. 2 but for (a) $B \rightarrow K^{*} \mu^{+} \mu^{-}$and (b) $B \rightarrow K^{*} \tau^{+} \tau^{-}$.

$$
\begin{gather*}
F_{V}=\hat{s} \phi, \\
F_{A}^{0}=\frac{1}{8 r}(1-r)^{2}\left[(1-r)^{2}-2 \hat{s}+10 r \hat{s}+\hat{s}^{2}\right], \\
F_{A}^{+}=\frac{1}{8 r} \phi^{2}, \\
F_{A}^{0+}=\frac{1}{4 r} \phi(1-r)(1-r-\hat{s}) . \tag{27}
\end{gather*}
$$

In Figs. 4(a) and 4(b), we plot the LPAs as a function of $\hat{s}$ for $B \rightarrow K^{*} \mu^{+} \mu^{-}$and $K^{*} \tau^{+} \tau^{-}$, respectively, with $m_{t}=180$ GeV . From Fig. 4, we see that $P_{L}\left(B \rightarrow K^{*} l^{+} l^{-}\right)$for both $l=\mu$ and $\tau$ vanish at the minimal end point of $\hat{s}=4 m_{l}^{2} / m_{B}^{2}$ due to the kinematic factor $\mathcal{D}$ in Eq. (25). However, in contrast with the cases in $B \rightarrow K l^{+} l^{-}$shown in Figs. 2(a) and 2(b), $P_{L}\left(B \rightarrow K^{*} l^{+} l^{-}\right)$with $l=\mu$ and $\tau$ are not zero at the maximum of $\hat{s}_{\max }=\left(1-m_{K^{*}} / m_{B}\right)^{2}$. This is due to the fact that the numerator in Eq. (18) has extra zero from $\phi$ at $\hat{s}_{\text {max }}$ whereas it does not contain $\phi$ in Eq. (25). Similar to the cases of $B \rightarrow K l^{+} l^{-}, P_{L}\left(B \rightarrow K^{*} \mu^{+} \mu^{-}\right)$has a large negative value over most of the allowed range of $\hat{s}$, with an average value $\left\langle P_{L}\left(B \rightarrow K^{*} \mu^{+} \mu^{-}\right)\right\rangle=-0.7$ while for the $\tau^{+} \tau^{-}$channel, the average $\tau$ polarization is
$\left\langle P_{L}\left(B \rightarrow K^{*} \tau^{+} \tau^{-}\right)\right\rangle=-0.5$. Finally, we note that our results in Figs. 3 and 4 are insensitive to the values of $A_{-}$predicted in the different form factor models.

## V. CONCLUSIONS

We have investigated the longitudinal polarization asymmetries of the muon and $\tau$ in the exclusive processes of $B \rightarrow K l^{+} l^{-}$and $B \rightarrow K^{*} l^{+} l^{-}$. The average values of the polarization asymmetries of the muon and $\tau$ are found to be -0.8 and -0.2 for $B \rightarrow K \mu^{+} \mu^{-}$and $B \rightarrow K \tau^{+} \tau^{-}$, and -0.7 and -0.5 for $B \rightarrow K^{*} \mu^{+} \mu^{-}$and $B \rightarrow K^{*} \tau^{+} \tau^{-}$, respectively. These polarization asymmetries provide valuable information on the flavor changing loop effects in the standard model. From Figs. 1 and 3, we find that the total integrated branching ratios of $B \rightarrow K^{(*)} l^{+} l^{-}$are $0.5(1.4) \times 10^{-6}$ and 1.3
(2.2) $\times 10^{-7}$ with $l=\mu$ and $\tau$, respectively. Experimentally, to measure an asymmetry $A$ of a decay with the branching ratio $B$ at the $n \sigma$ level, the required number of events is $N=n^{2} /\left(B A^{2}\right)$. For example, to observe the $\tau$ polarizations at the both exclusive channels of $B \rightarrow K^{(*)} \tau^{+} \tau^{-}$, one needs at least $1.8 \times 10^{7} n^{2} B \bar{B}$ decays. Therefore, at the $B$ factories under construction, some of the asymmetries could be accessible.

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