Energy-Efficient Broadcasting with Cooperative Transmissions in Wireless Sensor Networks

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***Abstract*— Broadcasting is a method that allows the distributed nodes in a wireless sensor network to share its data efficiently among each other. Due to the limited energy supplies of a sensor node, energy efficiency has become a crucial issue in the design of broadcasting protocols. In this paper, we analyze the energy savings provided by a cooperative form of broadcast, called the Opportunistic Large Arrays (OLA), and compare it to the performance of conventional multi-hop networks where no cooperation is utilized for transmission. The cooperation in OLA allows the receivers to utilize for detection the accumulation of signal energy provided by the transmitters that are relaying the same symbol. In this work, we derive the optimal energy allocation policy that minimizes the total energy cost of the OLA network subject to the SNR (or BER) requirements at all receivers. Even though the cooperative broadcast protocol provides significant energy savings, we prove that the optimum energy assignment for cooperative networks is an NP-complete problem and, thus, requires high computational complexity in general. We then introduce several suboptimal yet scalable solutions and show the significant energy-savings that one can obtain even with the approximate solutions.**

***Index Terms*— Broadcasting, minimum energy control, communication systems, complexity theory, sensor networks.**

I. Introduction

WHILE the size of sensor devices are rapidly decreasing, the slow improvement of the energy density in batteries aggravates the problem of energy limitations in wireless sensor networks. These constraints increase the importance of energy-efficient designs in all aspects of the wireless sensor network, ranging from hardware devices [1] to information processing schemes [2] to networking protocols [3], all of which are mutually coupled. In this paper, we focus specifically on analyzing the energy efficiency of cooperative transmission techniques in a network broadcasting scenario.

Broadcasting was traditionally used in many network protocols as an efficient way to distribute control information throughout the network. For example, in ad-hoc networks, broadcasting was used for network discovery [4] to initiate the configuration of the network. In [5], flooding/broadcasting was also used as an efficient way of achieving reliable multicast in a highly dynamic or hostile environment. In particular, in wireless sensor networks, broadcasting can serve as an efficient solution for the sensors to share their local measurements among each other due to the robustness and the low complexity of the protocol.

With regard to the limited energy resources at each sensor, many research efforts have been focused on minimizing the energy expenditure for broadcasting either by reducing the number of redundant transmissions due to lack of coordination or by minimizing the total transmission energy required to maintain full connectivity in the network. Specifically, Wieselthier *et al* [6] proposed to exploit the broadcast nature of the wireless medium and allow the transmission of each sensor to simultaneously reach any receiver within its broadcast range. As a result, the total number of relays are reduced since more receivers are reached with each transmission. The advantage in energy efficiency provided by this scheme is referred to as the *wireless multicast advantage (WMA)*. Although the WMA allows us to derive energy assignments that reduce significantly the total energy expenditure in the network, the computational complexity of finding the minimum energy solution[1] for this scenario is NP-complete [7], [8]. Therefore, numerous approximate algorithms [6]–[8] have been proposed and shown to achieve reasonably good solutions and analytical bounds [7], [9]. A similar problem has been studied where sensors are restricted to a fixed power level when it is transmitting, which is called the *single-power power control* problem. The problem reduces to determining the minimum set of transmitting nodes such that the broadcast message is able to reach all destinations reliably. This problem can be formulated as a multipoint relaying problem [10] or the problem of reducing *broadcast redundancy* [11], which are computationally intractable.

In addition to the significant energy-savings provided by the WMA, we show, in the paper, that one can further reduce the total energy consumption by allowing cooperation among transmitters instead of having each receiver detect based only on the signal contribution coming from one transmitter. In fact, many work has been done in the field of user cooperation [12]–[16] where it has been shown to provide spatial diversity and achieve better BER performance for a fixed SNR value. Without cooperation, the simultaneous reception of signals from different transmitters are treated as interference and the signals that do not provide sufficient energy for symbol detection is discarded at the receiver. By utilizing the user cooperation strategy proposed in [12], we quantify the gain in energy-savings that cooperation can provide in a network broadcasting scenario, when compared to conventional multihop networks.

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[1]The minimum energy solution refers to the energy assignment for each transmitter that results in the minimum overall energy consumption.

The main contribution of this paper is to analyze the fundamental gain in terms of energy efficiency that is achievable with a novel form of cooperative broadcasting that allows each receiver to utilize the accumulation of signal energy from multiple transmitting nodes to enhance the detection at each node. The cooperation is provided through a system called the *Opportunistic Large Array* (OLA) [12] where network broadcasting is done through signal processing techniques at the physical layer (c.f. Section III). We will show that the OLA strategy achieves a lower minimum energy solution compared to the scheme proposed in [6] where no user cooperation is considered. A similar formulation of the minimum energy cooperative broadcasting problem was proposed independently in [17], [18] under a different system model, where the authors showed a similar gain in energy efficiency.

In order to quantify the total energy savings, we derive explicitly the optimum centralized energy assignment policy that determines both the transmission energy and scheduling of each sensor. We derived the gain analytically for a specific *unicast* example where all the nodes are aligned in one direction with uniform spacing and proved that the computational complexity of the optimum policy is NP-complete in general. Therefore, we prove two approximate algorithms, the *Cumulative Increment Algorithm (CIA)* and the *Cumulative Sum Increment Algorithm (CSIA)*, to demonstrate the great energy savings that can be obtained with cooperative broadcasting, even when using these suboptimal solutions. The algorithms proposed in this paper are centralized where the channels are known for all transmitter-receiver pairs. After the energy assignment is determined at the central processor, the assigned energy values and the scheduling of transmissions are then distributed to each individual sensor. Although a distributed algorithm is desirable in practice, our goal is to determine the fundamental limit in terms of energy savings that is achievable with the cooperative broadcast network.

In the following section, we extend upon the three node example given in [6] to illustrate the difference of our proposed strategy compared to the conventional multi-hop network. In Section III, we describe the Opportunistic Large Array (OLA) system that is used as the physical transmission strategy to achieve the cooperative gain proposed in Section II. Given the network topology and the scheduling of transmissions, we formulate the optimal energy assignment problem as a linear programming problem and obtain analytically the minimum energy solution for an explicit unicast example in Section IV. In Sections V and VI, we prove, through a graph-theoretic formulation, that the computational complexity of the proposed problem is NP-complete. Therefore, we propose two approximate algorithms in Section VII and show, through numerical comparisons, the significant gain that is achieved with the proposed strategies.

II. Cooperative Wireless Advantage

Let us consider a wireless sensor network where there is only one source trying to broadcast information to all the other nodes in the network in a multi-hop fashion. Given a common noise-level at each receiver, the reliability, i.e. error probability, of the symbol detection is determined by the energy (E) of the received signal. In conventional multi-hop wireless networks, broadcasting is achieved by sending the information from the source to all the other nodes in the network through a series of multi-hop transmissions. Each multi-hop transmission is formed by the concatenation of point-to-point links (where each link is constructed between a single transmitter and single receiver pair) and the reliability of the link depends only on the energy emitted by the corresponding transmitter. We will henceforth refer to this as the *point-to-point architecture*.

In the past, many wireless systems [19] inherit the structure of traditional wireline networks and apply strategies that assign separate costs to each point-to-point link in the network (*i.e.* link-based strategies). In terms of energy consumption, where the cost is considered as the transmission energy required to establish a transmitter-receiver link, the link-based assignment implies that the transmitter can reach only one receiver node with each transmission and separate transmissions are required to reach each individual receiver, even when they are located within the same broadcast range. In this case, the minimum energy broadcasting problem is equivalent to the minimum spanning tree (MST) problem where costs are assigned to every transmitter-receiver pair and the optimization is done on a link-basis. The solution can be found through the well-known Prim's or Kruskal's Algorithm [20]. However, this approach is clearly inefficient since it does not exploit the broadcast nature of the wireless medium.

In fact, Wieselthier *et al* [6] remarked that energy savings can be obtained by enabling the transmission of each sensor to reach multiple receivers simultaneously if the received signals contain sufficient signal energy for detection. With the ability to reach multiple receivers with a single transmission, the total number of transmissions required to broadcast the message throughout the network is significantly reduced[2]. This is referred to as the *Wireless Multicast Advantage (WMA)* [6], which is the only advantage that is exploited in most wireless networks to improve the energy efficiency in network broadcasting applications. However, if the receiver node is outside the transmission range of the transmitter and the energy received from that transmitter alone is not sufficient for reliable detection, the received energy is either neglected or considered as additional receiver noise. This property leads to an inefficient use of the energy.

In fact, the main intuition of our proposed strategy is that, in order to increase energy-savings, the residual energy of the signal emitted from distant transmitters should be accumulated along with the aggregation of signals from multiple near-by transmitters [c.f. Section III]. In this case, the reliability of the detection at each receiver node (*i.e.* the connectivity of the node) will be achieved by the cooperation of multiple relay nodes, instead of a single transmitter. This is referred to as the *"Cooperative Wireless Advantage" (CWA)* since it utilizes the two main features of cooperative communications: the forwarding of data by cooperating users and the combining of signals at the receiver. From a simple three node example as we show in the following, we prove that the CWA approach can significantly outperform the WMA approach in terms of energy efficiency.

[2] In this case, the cost of the point-to-point links originating from the same transmitter need only be considered once when calculating the total energy consumption.

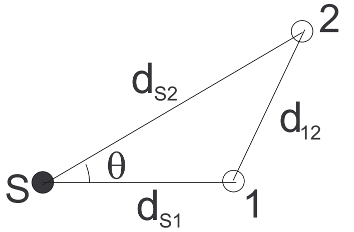


Fig. 1. Three node example. With network protocols that do not utilize any "advantage", the minimum total energy expenditure $E_{MST} = \min\{E_{S1} + E_{S2}, E_{S1} + E_{12}\}$ is the cost of the MST problem. Instead, when the WMA is applied, we can choose among two strategies: (a) S transmits with E_{S2} reaching both 1 and 2; (b) S transmits with E_{S1} and node 1 relays with E_{12}. Thus, the minimum total power is $E_{WMA} = \min\{E_{S2}, E_{S1} + E_{12}\}$ [6].

We illustrate the gain given by the CWA with the three node network as shown in Fig. 1. Let us consider a path loss model of $1/d^{\alpha}$, where d is the distance between the transmitter and the receiver, and α is the path loss exponent. Following the general convention, the connectivity of a node in the broadcast scenario is determined by the ability to receive reliably (with a desired error probability) the message transmitted by the source node. The requirement in error probability is translated into a minimum energy requirement at the receiver, where we say that a node is connected if and only if the received signal energy is sufficient to reach the energy threshold Φ_{th}. Assume, *w.l.o.g.*, that $\Phi_{th} = 1$. In this case, the minimum transmission energy needed for node i to reach node j is $E_{ij} = d_{ij}^{\alpha}$ where d_{ij} is the distance between nodes i and j.

Suppose that the transmission of each node is orthogonal to each other (*i.e.* the transmission of each sensor causes no interference on the other sensors' transmissions). In this case, the CWA will allow us to utilize for detection the addition of signal energy from multiple transmitters.[3] In the three node example shown in Fig. 1, there is one source S broadcasting to two destinations 1 and 2 where we assume, *w.l.o.g.*, that $d_{S1} < d_{S2}$. With the CWA, we can choose among two transmission strategies for broadcasting: (a) S transmits with the signal energy $E_{S2} = d_{S2}^{\alpha}$ reaching both nodes 1 and 2, since the signal transmitted to node 2 is received by node 1 simultaneously; and (b) S transmits with E_{S1} and node 1 relays the message by transmitting with the energy $(1 - \frac{E_{S1}}{d_{S2}^{\alpha}})E_{12}$, where $E_{S1} = d_{S1}^{\alpha}$ and $E_{12} = d_{12}^{\alpha}$. In (b), the signal emitted by the source will be received by both nodes 1 and 2, but the signal received at node 2 will not be able to reach the energy threshold required for reliable detection, *i.e.* $\frac{E_{S1}}{d_{S2}^{\alpha}} < \Phi_{th} = 1$ since $d_{S1} < d_{S2}$. As opposed to discarding the signal at node 2 (as done in the case with no transmitter cooperation), CWA utilizes the aggregation of this power to enhance the detection of the signal received from the relays, therefore, it is sufficient for the relay node, *i.e.* node 1, to provide node 2 with the remaining energy, *i.e.* $(1 - \frac{E_{S1}}{d_{S2}^{\alpha}})$, that is necessary to reach the receiver energy requirement Φ_{th}. Multiplying by the inverse of the path loss gain, *i.e.* $d_{12}^{\alpha} = E_{12}$ as defined above, we obtain the minimum transmission energy of node 1 as $(1 - \frac{E_{S1}}{d_{S2}^{\alpha}})E_{12}$. The minimum total energy consumption for CWA is $E_{CWA} = \min\{E_{S2}, E_{S1} + (1 - \frac{E_{S1}}{E_{S2}})E_{12}\}$. In fact, it is easy to verify that the optimal strategy would be to choose (a) when $d_{S1} > d_{S2}\cos\theta$ and choose (b) otherwise. The improvement of CWA, compared to the WMA, is the use of the residual energy in (b). Therefore, it follows that $E_{CWA} \leq E_{WMA} \leq E_{MST}$ (see Fig. 1). Through this simple example, we show how cooperation among transmitters can provide energy-savings for network broadcasting applications.

In the following section, we introduce a novel form of physical broadcasting protocol, *i.e.* the Opportunistic Large Arrays (OLA) [12] system, and show that the CWA can be attained with this transmission strategy.

III. OLA System Model

Let there be N nodes randomly deployed within a specified region. As in Section II, we consider a network with only one source broadcasting to the entire network where every other node serves as a part of the multiple-stage relay. In conventional point-to-point networks where there is no cooperation among users, the MAC and physical layers construct virtual transmission pipes that are contending for the channel even when they are transmitting the same information. Therefore, we argue that it is more efficient to integrate in the design the physical broadcasting property of wireless devices and utilize the concurrent signals from multiple relaying nodes to enhance the detection, instead of having these signals serve as interference to each other. In the following, we introduce the Opportunistic Large Array (OLA) as the physical transmission system that achieves the signal aggregation required to attain the CWA.

Consider an M-ary communication system where the source transmits a symbol pulse $p^{(m)}(t)$ out of an M-ary set of waveforms and that the waveforms have an average energy equal to 1, *i.e.* $\frac{1}{M}\sum_{m=0}^{M-1}\int |p^{(m)}|^2 dt = 1$. Each node in the network serves both as a destination of the broadcast message and, at the same time, as a relay that forwards the messages to other destinations. On a symbol-by-symbol basis[4], each node receives (either from the source or the relay node), reliably detects[5] and retransmits the same pulse as emitted by the source. The time required for a symbol to propagate throughout the network is defined as the symbol period T_s. In the remainder of this paper, we assume that node 1 is the source and that all the other nodes act as relays to the symbol transmitted by node 1. It is assumed that all nodes are synchronized to the transmission time slot of each symbol period T_s, either through the availability of the GPS signal or through distributed synchronization strategies [22]. When the m-th modulating waveform, *i.e.* $p^{(m)}(t)$, is transmitted from the source and all the other nodes relay correctly, the signal that arrives at the receiver front-end of node i is equal to

$$r_i(t) = s_i^{(m)}(t) + n_i(t) \quad (1)$$

[3]Orthogonality is not a constraint of the cooperative strategy that we propose, but it is an asymptotic assumption that is used to simplify our analysis. When the transmissions at each node are non-orthogonal, a constructive interference, as well as a destructive interference, will occur among the transmitted pulses. An interesting study on this subject has been provided in [21] where promising results of the non-orthogonal case is shown due to its beamforming gains that the orthogonal system cannot provide. However, we do not address this problem and refer the readers to [21].

[4]This is in contrast to standard multihop networks where the relaying operations are done on the packet-by-packet basis.

[5]For an analysis of error propagation, see [12]

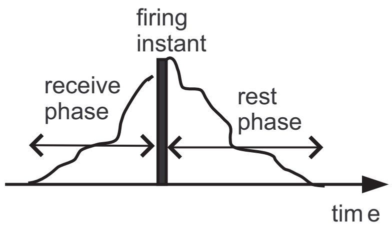


Fig. 2. The structure of the network signature $s_{i,m}(t)$ observed at the ith node and the corresponding actions of the ith node transceiver.

where

$$s_i^{(m)}(t) = \sum_{\substack{n=1 \\ n \neq i}}^{N} A_{i,n}\sqrt{\varepsilon_n} p^{(m)}(t - \tau_{i,n}), \qquad (2)$$

$\tau_{i,n}$ is the arrival time of the pulse transmitted by node n, ε_n is the energy transmitted by node n, $n_i(t)$ is the ith receiver AWGN with variance N_0, and $A_{i,n} = 1/d_{i,n}^{\alpha}$ is the channel gain between nodes i and n. In particular, the time τ_{ii} is the firing time of the transmission at node i. Due to causality, the detection at node i will utilize only the portion of the signal in (1) that arrived prior to its firing time τ_{ii}.

The aggregate waveform $s_i^{(m)}(t)$, corresponding to the mth symbol, is unique for each receiver i, therefore, it is referred to as the *network signature* of node i. Note that the signal received in (1) is analogous to a multi-path channel with impulse response

$$h_i(t) = \sum_{n \neq i} A_{i,n}\sqrt{\varepsilon_n}\delta(t - \tau_{i,n}),$$

and $s_i^{(m)}(t) = p^{(m)}(t) * h_i(t)$ is the aggregation of the active scattering performed by the relaying nodes. Given the signal up to time τ_{ii}, *i.e.* $\{s_i(t)\}_{t=-\infty}^{\tau_{ii}}$, the receiver node i detects the transmitted symbol by combining optimally the multi-path signals with a standard RAKE receiver [23] (the node can obtain an estimate of the waveform through training or blind estimation schemes [24]). In fact, the firing time τ_{ii} is determined solely by the energy contained in the signal received up to that point; therefore, except for the symbol-level synchronization, the estimation of the signature $\{s_i(t)\}_{t=-\infty}^{\tau_{ii}}$ is the only form of synchronization that is required in the OLA system. Compared to a packet network, no buffering of soft samples or higher-layer interventions (*e.g.* MAC or Network Layer) are required [12]. The readers are referred to [12] for a more detailed description of the OLA system.

In Fig. 2, we show an illustration of the signature waveform where the decaying shape of the signal is caused by the large path loss that distant signals may experience. During every symbol period, we can consider two main intervals tied to the evolution of the network signature: 1) the earlier *receive phase*, when the upstream waves of signals approach the node and, 2) the period after the *firing instant*, which we call the *rest phase*, is the period where the nodes are shut down to avoid the echoes from the downstream waves of signals. Only the signals received within the receive phase are used for signal detection, therefore, the received signal energy embedded in this phase must allow the receiver to detect the symbol reliably within a specified error probability.

Assume that the receiver has an ideal estimate of $h_i(t)$ and that the detection of the transmitted symbol is obtained by using a filter bank at the i-th receiver which incorporates the estimate of $h_i(t)$. Let us define the *energy phase* $\phi_i(\boldsymbol{\varepsilon}, p^{(m)}, t)$ as the accumulated signal energy received by node i up to time t when the pulse $p^{(m)}(t)$ is transmitted, i.e.

$$\begin{aligned} \phi_i(\boldsymbol{\varepsilon}, p^{(m)}, t) &\triangleq \int_0^t |s_i^{(m)}(u)|^2 du \qquad (3) \\ &= \sum_{k=1}^{N}\sum_{l=1}^{N} A_{i,k}A_{i,l}^{*}\sqrt{\varepsilon_k\varepsilon_l} \cdot R_{pp}^{(m)}(\tau_{i,l}, \tau_{i,k}; t) \end{aligned}$$

where $\boldsymbol{\varepsilon} \triangleq [\varepsilon_1, \cdots, \varepsilon_N]$ is the vector of energies assigned to each node and

$$R_{pp}^{(m)}(\tau_{i,l}, \tau_{i,k}; t) = \int_0^t p^{(m)}(u - \tau_{i,k})(p^{(m)}(u - \tau_{i,l}))^* du.$$

In order to guarantee a certain error probability performance, each relay node in the network will not decode or retransmit the symbol until sufficient energy is received. In this case, the firing instant for node i (defined as $t_{fi} = \tau_{ii}$) should be sufficiently large such that the integration in (3) exceeds the energy that is necessary to meet the probability of error requirement. Therefore, we define the connectivity (or the reliable detection) of a node as follows:

Definition 1 (Integrate-and-Fire Model / Connectivity): *Let t_{fi} be the* **firing instant** *of the i-th node and let T_s be the duration of the symbol period. The i-th node is connected and is allowed to rebroadcast if and only if $\exists t_{fi} \leq T_s$ such that*

$$\Phi_s \triangleq \frac{1}{M}\sum_{m=0}^{M-1} \phi_i(\boldsymbol{\varepsilon}, p^{(m)}, t_{fi}) \geq \Phi_{th} \qquad (4)$$

where Φ_s is the average symbol energy and Φ_{th} is the minimum received energy requirement.

The firing instants $\{t_{fi}\}_{i=1}^{N}$ are determined through training along with the estimation of the channel gain $h_i(t)$. Since $\phi_i(\boldsymbol{\varepsilon}, p^{(m)}, t)$ is the integral of a positive function, it is easy to show that the function increases monotonically with respect to t. If there exists at least one node that is not connected, *i.e.* $\frac{1}{M}\sum_{m=0}^{M-1} \phi_i(\boldsymbol{\varepsilon}, p^{(m)}, t) < \Phi_{th}$ for all t, then we say that the network is not fully connected. The notion of average symbol energy defined in (4) has often been used to determine the probability of error for M-ary modulations. Borrowing from [25], we provide several standard examples in the following: (1) the symbol error probability (SER) for the M-ary PAM signal can be expressed as SER $= \frac{2(M-1)}{M} Q\left(\sqrt{\frac{6\Phi_s}{(M^2-1)N_0}}\right)$, where $Q(x) = \int_x^\infty \frac{1}{\sqrt{2\pi}} e^{-x^2/2} dx$; (2) the SER for M-ary orthogonal signals is upper bounded with SER$\leq M \cdot Q(\sqrt{\Phi_s/N_0})$; (3) the SER for the M-ary PSK can be approximated as $2Q(\sqrt{\frac{2\Phi_s}{N_0}} \sin\frac{\pi}{M})$. We note that when the bandwidth of the transmission channel is limited, there is a positive probability that the pulses from different relays may overlap with each other which reduces the number of resolvable paths at the receiver and alters the geometric distances between the different modulation waveforms. In this case, the dependence between the received SNR and the SER curve is then different from node to node and also different for different sets of pulsing times. Therefore, in order to achieve a fixed SER, each node should have a different set of SNR thresholds which varies with the pulsing time of its upstream nodes. This shows that both the pulsing time and the pulse waveform are closely coupled with the minimum energy required to achieve full connectivity in the network. In order to simplify our analysis, we shall assume in the remainder of this paper that the bandwidth is sufficiently large such that the pulses will not overlap with each other. The problem is simplified since the minimum energy solution in this case depends only on the order of firing instead of the absolute pulsing time of each node.

Through the remainder of this paper, we shall assume, *w.l.o.g.*, that $\Phi_{th} \equiv 1$. The requirement on the received energy allows us to limit the effect of error propagation that may perturb the realization of the network signature. The discussion on error propagation is provided in [12] where the error is modelled as an interference to the received signal. A shift in the probability of error curve is observed where the desired BER performance is achieved with higher SNR. This results in a loss in energy efficiency which is also controlled by the energy threshold. A detailed discussion on error propagation is omitted in this paper, but is provided in [12]. However, we note that the error propagation exists in all multi-hop systems, especially when the network is operating under critical power.

In OLA, the received signal at each node consists of the aggregation of multiple pulse energies, as shown in (1); hence, we expect a significant decrease in the total energy required to guarantee full connectivity when compared to the conventional point-to-point multi-hop network. In addition, the symbol-by-symbol relaying structure and the signal combining at the receiver allows OLA to achieve network broadcasting without the MAC and Network layers and, thus, results in a lower end-to-end delay [12] compared to packet systems. Without the congestion in the MAC layer, additional energy-savings can be potentially attained in practice by avoiding the retransmission of packets that are necessary in conventional multi-hop networks. However, it is important to note that the OLA, in its present form, is an uncoded system which would require higher energy to achieve the same BER performance as a coded system. In principle, OLA can be applied with a symbol that belongs to a complex multidimensional lattice, or waveform codes, which are optimized to cope with the *opportunistic multipath* [12] that OLA introduces. We do not take into account these issues in our analysis since it is hard to quantify the energy savings provided by the reduction of retransmissions in the MAC layer compared to the loss in BER in an uncoded system. In fact, in this paper, we consider a centralized algorithm and, therefore, would have no loss of energy efficiency due to the MAC, since the optimal scheduling can be determined and assigned to the nodes. The comparison of these systems and the construction of a coded OLA system are beyond the scope of this work and are the subject of future investigation.

Note that the CWA can be provided by other protocols besides OLA. In fact, the energy efficiency of the CWA has been derived independently in the work by Maric and Yates [17], [18]. The main difference between the two strategies is that the system proposed in [17] utilizes packet-level cooperation while the OLA scheme employs symbol-by-symbol relaying [12]. More specifically, the authors in [17], [18] considered the use of coded packet transmissions where a source transmits a Gaussian codeword that is retransmitted by the relaying nodes once the received signal energy is sufficient for decoding reliably the codeword. Similar to OLA [12], the reliability of the decoding is achieved through the collection of unreliable copies from different nodes. In the packet system, a higher throughput can be attained by increasing the length of each codeword which then reduces the average broadcast delay experienced by each 'symbol' in the packet. However, a larger packet size also increases the latency of the broadcast and requires a significant increase in the buffer size and the complexity of the receiver signal processing.[6] One of the major advantages of OLA is that no buffering of soft samples (unreliable copies) or complex synchronization information is needed due to its instantaneous symbol-by-symbol relay. Furthermore, the system proposed in [17] assumes the availability of orthogonal transmission channels for all users. However, the bandwidth limitations will often require the system to use overlapping channels in practice when the network is dense.

In the following, we will also assume, for the simplicity of our analysis, that the pulses in the OLA system are non-overlapping in time, which is equivalent to the orthogonal assumption made by [17]. Although a loss in energy efficiency may be experienced when the orthogonality is not strictly attained, the implementation of the OLA system is not reliant on the assumption. By using a standard RAKE receiver, the only reduction in performance is in its diversity gain. In packet transmission, a similar effect can be achieved only with careful inter-node packet synchronization.

IV. Problem Formulation in Linear Programming

In this section, we analyze the minimum energy cooperative broadcasting problem for a particular network topology. We assume that each node transmits a BPSK signal pulse from the set $\{p(t), -p(t)\}$, where $\int |p(t)|^2 = 1$; and that the pulses received, as shown in (2), do not overlap with each other, *i.e.* the pulse duration T_p is much less than the propagation delay, and that each node has an ideal estimate of $h_i(t)$. We can effectively let T_p go to zero such that $p(t) = \delta(t)$, where $\delta(t)$ is the Dirac delta function. Following from (3), the energy phase of the received signal is then equal to the sum of the individual pulse energies, *i.e.*

$$\phi_i(\boldsymbol{\varepsilon}, t) \triangleq \sum_{k=1}^{N} \varepsilon_k |A_{i,k}|^2 u(t - \tau_{i,k}) = \sum_{k=1}^{N} \gamma_{i,k}(\varepsilon_k) u(t - \tau_{i,k}) \tag{5}$$

where $u(t)$ is the unit step function and $\gamma_{i,k}(\varepsilon_k)$ equals to $\varepsilon_k |A_{i,k}|^2$. The assumption that the pulses are non-overlapping is merely to simplify the analysis of the minimum energy consumption in OLA, the implementation of the system does not rely on this assumption.

Let the nodes be enumerated by their firing order, such that

[6] OLA requires a RAKE receiver that has as many fingers as the 'resolvable paths' which, in a narrow-band regime, can be much less than the number of relays. However, there is a loss of diversity in this case.

$t_{f1} < \cdots < t_{fN}$, where t_{fi} is the firing time of node i.[7] We assume that the arrival order of pulses at each receiving node is the same as the absolute firing order so that the connectivity of the nodes is now dependent only on *the order of firing*, instead of on the exact firing instants.

Assume that the optimal order of firing is known and the sensors are enumerated accordingly. The firing of node i will cause an energy increment of $\gamma_{ij}(\varepsilon_i)$, for all $j > i$, which is typically $\varepsilon_i/d_{ij}^{\alpha}$ when considering only the path loss model. Therefore, the SNR constraint at the receiver node k must be satisfied through the accumulation of the values $\gamma_{ik}(\varepsilon_i)$ from $i = 1$ to $k-1$. Hence, the optimization problem can be formulated as follows:

$$\begin{array}{ll} \text{minimize} & \sum_{i=1}^{N} \varepsilon_i; \\ \text{subject to} & \left\{ \begin{array}{ll} \sum_{i=1}^{k-1} \gamma_{i,k}(\varepsilon_i) \geq 1, & \text{for } k = 2, \cdots, N; \\ \varepsilon_i \geq 0, & \forall i \end{array} \right. \end{array} \tag{6}$$

The power control policy determined by the optimization problem is a centralized protocol where the channel is assumed to be known, *i.e.* the functions γ_{ij} is known. The use of the function γ_{ij} allows us to adapt the algorithms proposed in this paper to nonlinear systems that possess similar properties.

For the case where $\gamma_{ij}(\varepsilon_i)$ is a linear function of ε_i, *i.e.* $\gamma_{ij}(\varepsilon_i) = \beta_{ij}\varepsilon_i$, the constraints can be expressed in the linear form: $\mathbf{B}\boldsymbol{\varepsilon} \geq \mathbf{1}$ and $\boldsymbol{\varepsilon} \geq \mathbf{0}$, where $\boldsymbol{\varepsilon} = [\varepsilon_1, \cdots, \varepsilon_{N-1}]^T$, $\mathbf{1}$ is a $(N-1)$-dimensional vector with all 1 entries, and $\mathbf{B}$ is the $(N-1) \times (N-1)$ lower triangular matrix

$$\mathbf{B} = \begin{bmatrix} \beta_{1,2} & 0 & \cdots & 0 \\ \beta_{1,3} & \beta_{2,3} & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ \beta_{1,N} & \cdots & \cdots & \beta_{N-1,N} \end{bmatrix}.. \tag{7}$$

We note that the problem in (6) is a simple variant of the *linear programming problem* that can be solved simply with back-substitution starting from $k = 2$.

Although the minimum energy solution for any given firing order can easily be found through linear programming, the optimum solution still involves the search for the best feasible ordering of pulses that allows to achieve the minimum energy. In the following, we address a simple case where we are able to determine the minimum energy solution in closed form.

The Unicast Application: Consider the unicast case, as shown in Fig. 3, where nodes are placed on a straight line with equal spacing D. This case is interesting in that: 1) the arrival order of pulses at each receiver node is exactly the same as the firing order; 2) the firing order is trivially determined to be in the order of the distance between the source and the receiver nodes. This example also demonstrates the potential of using cooperative schemes when transmitting over a single route instead of broadcasting.

For a signal originating at the source node 1, the relaying nodes will fire in the order of the distance from the source node as enumerated in the figure. Thus, the constraint matrix $\mathbf{B}$ in (7) has the realization

[7]The firing order of the pulses does not necessarily coincide with the receiving order at each node due to the propagation delay of the signals through the medium. However, in our application, the pulses that arrive out of order (with respect to the firing order) origin from distant nodes that typically contribute with only a small amount of energy.

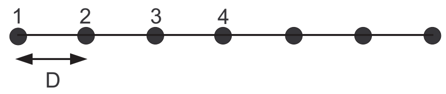


Fig. 3. The linear network with spacing D between neighboring nodes. Each node itself is considered as a layer of the signal propagation in OLA.

$$\mathbf{B} = \begin{bmatrix} \frac{1}{D^\alpha} & & & \\ \frac{1}{(2D)^\alpha} & \frac{1}{D^\alpha} & & \\ \vdots & \ddots & \ddots & \\ \frac{1}{[(N-1)D]^\alpha} & \cdots & \cdots & \frac{1}{D^\alpha} \end{bmatrix} \tag{8}$$

which is a Toeplitz matrix with the lower triangular structure. All the constraints corresponding to $\mathbf{B}$ must be satisfied with equality for this special case. Therefore, we can solve, by induction, the minimum energy for node k as follows

$$\varepsilon_k = D^\alpha - \sum_{i=0}^{k-1} \frac{\varepsilon_i}{(k+1-i)^\alpha}. \tag{9}$$

When the number of nodes in the network goes to infinity, the contribution of the signal energy from far away nodes fades out due to the path loss effect. The nodes that are infinitely downstream from the source will then require the same amount of energy contribution from all upstream nodes that are reasonably close-by, therefore, the transmitted signal energy of these downstream nodes will converge to the same value. This property is described in the following theorem with the proof shown in the Appendix I.

Theorem 1: Consider a linear network as shown in Fig. 3. For a fixed distance D and the path loss exponent $\alpha > 1$, the minimum energy at node n converges to $D^\alpha/\zeta(\alpha)$, *i.e.* $\varepsilon_\infty = \lim_{n\to\infty} \varepsilon_n = D^\alpha/\zeta(\alpha)$, where $\zeta(\alpha) = \sum_{i=1}^{\infty} 1/i^\alpha$ is the *Riemann zeta function*.

Specifically, for $D = 1$, the sequence of the minimum cooperative energy required at each node converges to the value $1/\zeta(\alpha)$, while in the point-to-point system $\varepsilon_n = 1$, $\forall\ n$. Therefore, for $\alpha = 2$, we obtain $\varepsilon_\infty = 0.6079$; but when $\alpha = 3$, we have instead $\varepsilon_\infty = 0.8319$. We note that the gain in using CWA is less significant when the path loss exponent is large. This is intuitively true since the large path loss effect will reduce the signal contribution of far away nodes. Therefore, the asymptotic energy ε_∞ should converge to 1 as $\alpha \to \infty$. The result in Theorem 1 shows quantitatively the significant gain in energy savings that is provided by CWA in an idealized network setting. The set of minimum energy solutions derived in this specific example can also be used to approximate the minimum energy solution for a random network where the nodes are distributed with a one-dimensional spatially Poisson distribution. Note that the total energy expenditure of the network is unbounded [26, Thm 2.3] when the distance between neighboring nodes are constant.

The centralized power control policy derived in this section relies on the knowledge of the channel state and the firing order of the nodes. However, in a general network scenario, the order of the firing is not known and it is, in fact, the main source of complexity in this algorithm. In the following section, we formulate our minimum energy problem into a graph theoretic framework and prove that deriving the optimal power control policy is in general an NP-complete problem.

V. Graph Theoretic Formulation of OLA

The minimum energy broadcasting problem using CWA can be formulated into a more general form using directed graphs with constraints on incoming flows at each node. Consider a directed graph $G = (V, E)$, where V is the set of vertices and E is the set of edges. Let $W \equiv \{\omega_1, \omega_2, \cdots, \omega_{|W|}\}$ be the set of energy levels that a node can transmit and assign to each vertex i an energy value $\varepsilon_i \in W$. Although, in general, the set of energy levels W can be equal to the set of positive real numbers, the practical constraints often restrict the set to be finite and the discussions in Section VI will be restricted to the case where W is finite. Let each edge, say the edge ij, be assigned an *energy flow* $\gamma_{ij} : W \to \mathbf{R}^+$ ($\mathbf{R}^+$ is the set of positive real numbers) which monotonically increases with respect to the energy value ε_i and that $\gamma_{ij}(0) = 0$. In a typical wireless scenario, a node can be modeled as a vertex in the set V and an edge ij exists for all vertices i and j due to the broadcast nature of the medium. If the channel gain considers only the path loss model $1/d^\alpha$, the energy flow $\gamma_{ij}(\varepsilon_i)$ on the edge ij is equal to $\varepsilon_i/d_{ij}^\alpha$.

As mentioned in Section III, each node is only able to receive signal energy coming from its upstream nodes because the signals coming from downstream nodes arrive during the *rest period* and cannot contribute to the signal detection, *i.e.* the digraph of interest cannot consist of any cycles. Let $s \in V$ be the source, then we say that a node $j \in V - \{s\}$ is *connected* as in Definition 1 if and only if the sum of the positive net flows directed toward node j exceeds the threshold 1, *i.e.* the energy received before the firing time of j is

$$\sum_{\{i \in V,\ \gamma_{ij}(\varepsilon_i) > 0\}} \gamma_{ij}(\varepsilon_i) \geq 1. \tag{10}$$

The problem is then generalized to finding the set of energy values $\{\varepsilon_i, \forall i\}$ such that $G = (V, E)$ contains a directed acyclic subgraph $G' = (V, E')$ rooted at the source s where the positive net flow at each node satisfies (10).

We note that the subgraph G' to our problem does not result in a tree structure as it is in the minimum energy broadcasting problem of conventional networks, because the energy flow from multiple transmitters can be combined at the receiver. The solution is rather in the form of a directed acyclic graph (DAG). For example, if there exists nodes i and j such that $t_{fi} < t_{fj}$ but $\tau_{ki} > \tau_{kj}$, then the edge from node i to the receiver node k would not be included in G' if the pulse received from node j contains sufficient energy for node k to reach the threshold. This formulation does not neglect the effect of propagation in the situations where the arrival of pulses is ordered differently than the firing order. In fact, the incoming edges of a node in the subgraph G' models the fact that the node can fire only after receiving pulses from the nodes on the other end of these edges.[8] With this general formulation, we can show that the computational complexity of this problem is intractable.

[8]As proven in [16], the NP-completeness of the problem is true even without considering the effect of propagation.

VI. Computational Complexity

The minimum energy for cooperative broadcasting involves the combinatorial problem of finding the optimum set of nodes $\mathcal{L}_i$ that contribute to the received signal at node i such that the set of energy values $\{\varepsilon_j \in W : j \in \mathcal{L}_i\}$ satisfy the energy requirement at node i. The possible combinations of the sets $\{\mathcal{L}_i, \forall\, i\}$ are exponential in the number of nodes. Hence, it is important that we gain insight into the computational complexity of the problem. For the case where W is a finite set of energy levels, we characterize the intractability of the minimum energy problem in terms of NP-completeness [27].

In complexity theory [27], the problems that are solvable in polynomial-time with deterministic algorithms belong to the class denoted by P. The problems that are solvable with nondeterministic algorithms belong to the NP class. It can be shown that $P \subseteq NP$. There is a class of problems among NP that are said to be NP-complete where every problem is polynomial-time reducible to any other problem in the same class. It is commonly believed that $P \neq NP$. If this indeed is true, then there does not exist a deterministic algorithm that solves the problem in polynomial-time. To prove that a problem is NP-complete, we must first prove that it belongs to the class NP, then find a polynomial-time reduction of a well-known NP-complete problem to our problem at hand. By proving our problem NP-complete, we know that it is unlikely to find a polynomial time algorithm that solves this problem, therefore, we direct our attention to heuristics that provide us reasonably good approximations (see Section VII).

The decision form of problems are the prototype of the problems studied in computation theory, therefore, we shall first map the problem of minimum energy broadcasting with CWA (MEB-CWA) to the following decision form:

Minimum Energy Broadcasting with CWA
Instance: For a directed graph $G = (V, E)$, the finite set of energy levels W, an energy assignment $\varepsilon_i : V(G) \to W$, the positive net flow $\gamma_{ij} : W \to \mathbf{R}^+$, $\forall\ i \neq j$ the source node s, and a positive constant $B \in \mathbf{R}^+$.
Question: Is there an energy assignment $[\varepsilon_1, \cdots, \varepsilon_{|V|}]$ such that the digraph G has an acyclic subgraph $G' = (V, E')$ where each node satisfies (10) and the constraint $\sum_{i \in V} \varepsilon_i \leq B$?

To prove the NP-completeness of the problem, it is sufficient to prove that a special case of the problem is NP-complete. Here, we choose a special case where we allow each node either to transmit with energy ε or do not transmit at all, *i.e.* the case where the set of transmission energies $W = \{0, \varepsilon\}$. This is referred to as the *Single Power MEB-CWA (SP-CWA)*. We will prove this problem to be NP-complete by the reduction of the 3-Conjunctive Normal Form Satisfiability problem (3-CNF SAT) [27] which is well known to be NP-complete:

3-Conjunctive Normal Form Satisfiability
Instance: For a set of boolean variables $\{x_1, x_2, \cdots, x_n\}$ and a collection of m conjunctive normal forms $\{C_1, C_2, \cdots, C_m\}$ where $C_j = a_{j1} \vee a_{j2} \vee a_{j3}$ such that the three literals $a_{j1}, a_{j2}, a_{j3} \in \{x_1, \bar{x}_1, x_2, \bar{x}_2, \cdots, x_n, \bar{x}_n\}$.
Question: Is there a truth assignment for the set of boolean

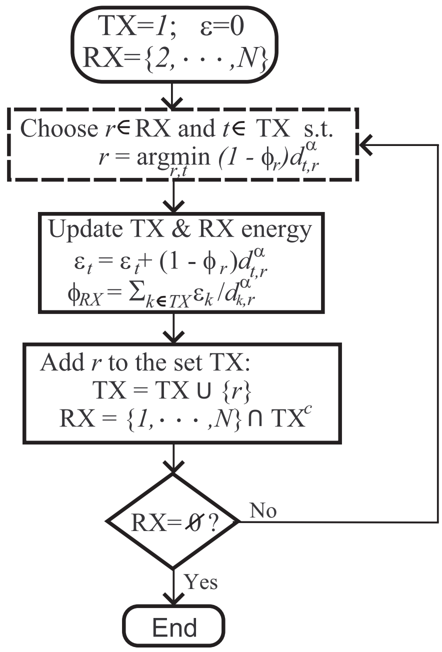


Fig. 4. The flowchart for the CIA. Let TX be the set of nodes that reliably received the symbol, RX the set that has not yet reached the energy threshold, ϕ_r the accumulated energy at node r, ε_t the energy allocated to transmitter t and $d_{t,r}$ is the distance between t and r.

variables such that all the clauses $\{C_1, C_2, \cdots, C_m\}$ are satisfied?

Theorem 2: The *MEB-CWA* problem is NP-complete.

Sketch proof: We prove this by reducing the 3-CNF SAT to the SP-CWA which is a special case of the MEB-CWA. This is similar to the NP-completeness proof of the minimum energy broadcast tree (MEBT) problem presented in [7]. The details are shown in Appendix II. ■

Remark *1:* We note that the proof of NP-completeness is restricted to the case where W is a finite set of energy values. However, the importance of this result holds for all practical purposes since the set of energy levels are always finite in practice. In [16], a proof of NP-completeness was provided for the optimal scheduling problem that is required to achieve the minimum energy solution.

VII. Approximate Algorithms with CWA

In this section, we propose heuristic methods to provide suboptimal but scalable solutions for the minimum energy broadcasting problem. Two algorithms are introduced: one is the *Cumulative Increment Algorithm (CIA)* and the other is the *Cumulative Sum Increment Algorithm (CSIA)*. Both CIA and CSIA are iterative algorithms that continuously update the energy assignment at each node in the network until the entire network is connected as per Definition 1. Let us denote by TX the set of nodes that are connected under the energy assignment during the current iteration of the algorithm; and let RX be the complement set of TX.

The CIA, as first proposed in [28] and illustrated in the flow chart in Fig. 4, increases iteratively the total transmission energy of the network such that, during each iteration, at least one node in the set RX can receive sufficient energy to reach the energy threshold and, thus, be added into the set TX. During each iteration, the node in RX that requires the minimum increment of the total transmission energy is selected and the energy assignments at each node in TX is updated accordingly. More specifically, as shown in Fig. 4, we define the cost for each receiver as $\min_{j\in \mathrm{TX}}(1-\phi_i)d_{ji}^{\alpha}$ where ϕ_i is the accumulated energy at the receiver node i and, thus, $(1-\phi_i)d_{ji}^{\alpha}$ is the increase of transmission energy required by node j in order to have the receiver node i reach the energy threshold. However, in OLA, any increment in the transmission energy will affect not only one single receiver node, but all the other receivers as well; therefore, considering only the single receiver that requires the least increment in transmission energy makes the CIA less efficient. Hence, in the CSIA design, we assign to each transmitter node, at each iteration, a weight value that is equal to the aggregate channel gain between the transmitter and the set of receivers, and update the selection process (in dashed box of Fig. 4) by choosing the transmitter with the maximum weight. This selection results in the largest total increase in the energy phase for all receivers. Specifically, by considering only the path loss effect in the channel gain, we define the weight of transmitter node i as

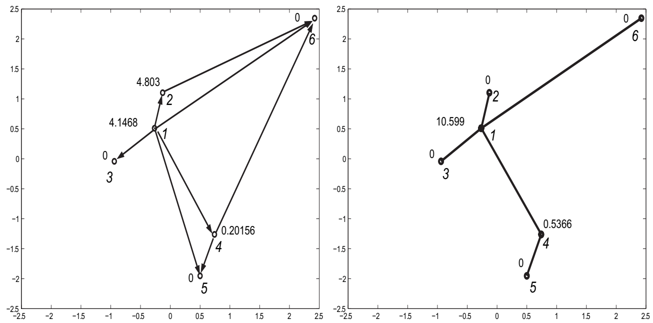
$$\mathrm{weight}_i \triangleq \sum_{j\in RX} d_{ij}^{\alpha}. \tag{11}$$

This method is equivalent to the *Greedy filling algorithm* proposed in [17], [18]. In Experiment 1, we compare the performance of these algorithms with the optimal solution and the well-known *Broadcast Incremental Power Algorithm (BIP)* [6] where no form of cooperation is utilized.

The algorithms proposed above allow each node to adjust its power arbitrarily without any limitations on the allowed power levels. For the purpose of practical implementation, we propose a class of Single Power (SP) power control algorithms where nodes either transmit with a fixed energy ε or do not transmit at all. Specifically, we propose two algorithms: (1) the Single Power Most Node Increment Algorithm (SPMNIA) and (2) the Single Power CSIA (SPCSIA) which follows similarly the CSIA. In SPMNIA, a node that has not yet been chosen to transmit is included into the set of transmitters at each iteration if its transmission allows the most amount of nodes to become connected. In SPCSIA, we choose the node that provides the largest amount of total energy increment among all receiver nodes. This is exactly the same as the CSIA algorithm except for the limitation on the adjustable power levels. The single-power power control problem is related to the problem of reducing broadcast redundancy [11] and the multipoint relay problem [10] in conventional wireless networks. In Experiment 2, we show that the SP algorithms provide reasonably good performance compared to the CSIA even while requiring only binary decisions.

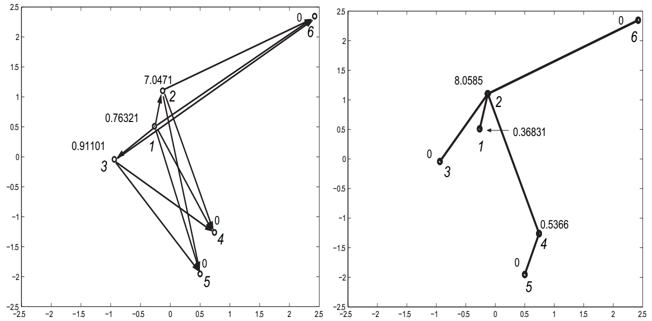
Experiment 1 (CWA Algorithms):

In Fig. 5(a), we show the directed acyclic broadcasting graph of the CIA solution where 6 nodes are randomly distributed with uniform distribution in a $5 \times 5\ m^2$ area. In this case, the CIA has the same solution as the CSIA, but the solutions may vary in general, especially when the number of



(a) CIA (total $= 9.1514$) (b) BIP (total $= 11.1356$)

Fig. 5. A 6 node example of both the CIA and the BIP algorithm.



(a) OLA (total $= 8.7213$) (b) MEBT (total $= 8.9634$)

Fig. 6. The optimum solution to a 6 node example of the OLA and the MEBT problem.

nodes are large. In Fig. 6(a), we obtained the directed acyclic graph of the optimum solution through exhaustive search which is outperforming CIA and CSIA by approximately 5% for this particular example. Comparing to the BIP, we show in Fig. 5 that the ratio of the minimum energy solution for BIP over that of CIA is equal to 1.2168. Also, the solution obtained by the BIP algorithm is approximately 24% higher than the optimum solution in Fig. 6(b) which is obtained through exhaustive search.

In Fig. 7, we considered a network of nodes randomly distributed in a $5 \times 5\ m^2$ area with uniform distribution and fixed a path loss exponent $\alpha = 2$. Since the CSIA maximizes the energy increment of all receiving nodes, it is expected that CSIA achieves an average total power less than the CIA. In Fig. 7, the total energy expenditure of the network decreases with the increase of the number of nodes when using the CWA algorithms while that of the BIP algorithm remains approximately the same. In fact, for the case of BIP, increasing the node density by a factor of two, the new solution would require approximately twice the number of nodes transmitting with half the amount of power to maintain coverage over the network area. This effect balances the total energy consumption of the network. Although doubling the node density has a similar effect on CWA algorithms, the total energy expenditure decreases because each transmission affects the entire area, therefore, the addition of each node will reduce the energy required for all other users.

In Figs. 8 and 9, we fix the node density to be 1 node$/m^2$ while nodes are still randomly distributed in a square area. In Fig. 8, the total energy expenditure increases almost linearly for the BIP, while the tail of the curve for CIA and CSIA are

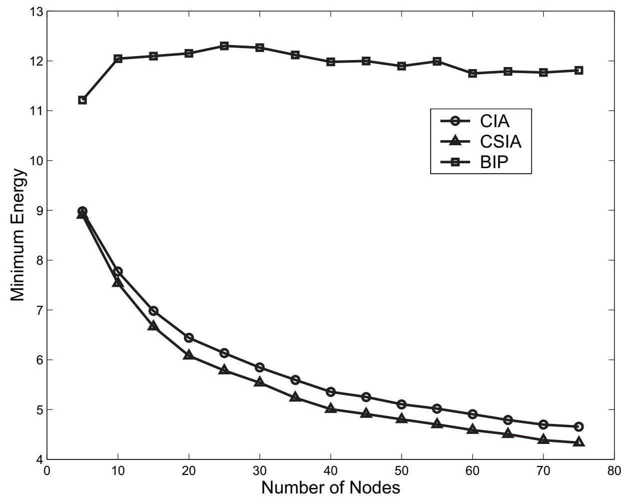


Fig. 7. The minimum energies obtained by the CIA, CSIA and the BIP are averaged over 200 realizations for every different number of nodes in the network.

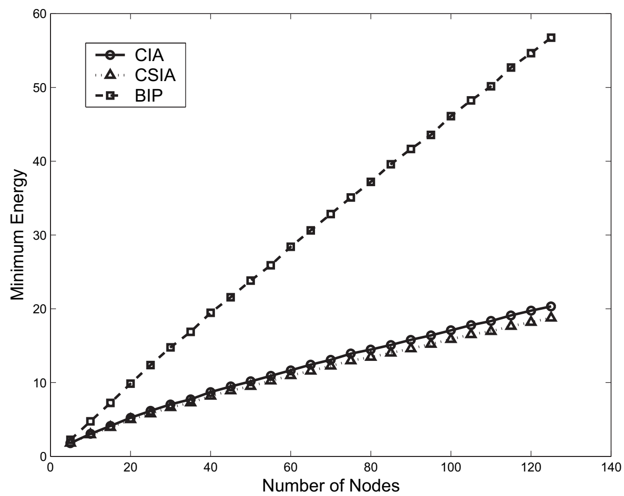


Fig. 8. The average total energy of the network when the node density is fixed to 1 node$/m^2$.

also approximately linear. Therefore, in Fig. 9, the average energy per node for the BIP is approximately constant, while that of CIA and CSIA decreases and tends to converge asymptotically, similar to that shown in the unicast example in Section IV (shown in Fig. 9 for $D = 0.5$ [9]). We can see that the CIA and CSIA converge to values similar to the unicast case while the unicast solution serves as an asymptotic upper bound for the optimum solution of the two dimensional network.

Experiment 2 (Single Power Algorithms):

The performance is evaluated for nodes randomly distributed in a fixed $5 \times 5 m^2$ area and that the nodes are allowed to transmit with power equal to 1. We compare the proposed SP schemes to the Multipoint Relay algorithm (MPR) proposed in [10], where no cooperation is utilized. In Fig. 10, we can see that the average per node energy spent by using SP power control outperforms the MPR by approximately $4\ dB$. In fact, the energy savings that we obtain by using signal accumulation is so significant that the SP schemes still outperform the BIP

[9]A two dimensional point poisson process of density ρ has average distance $1/2\sqrt{\rho}$ between a node and its closest neighbor.

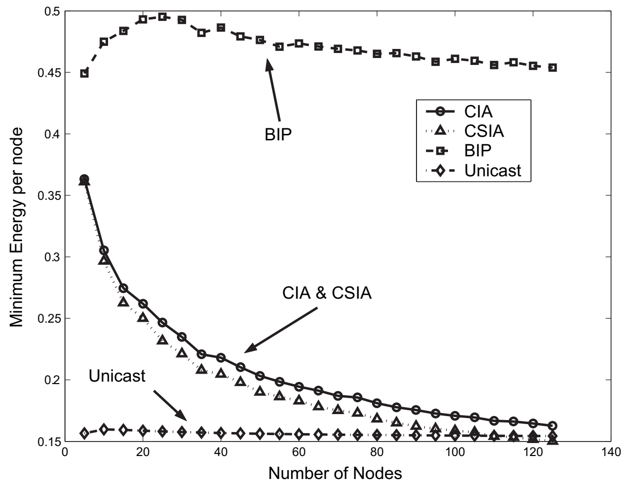


Fig. 9. The average energy expenditure per node when the node density is fixed to 1 node$/m^2$.

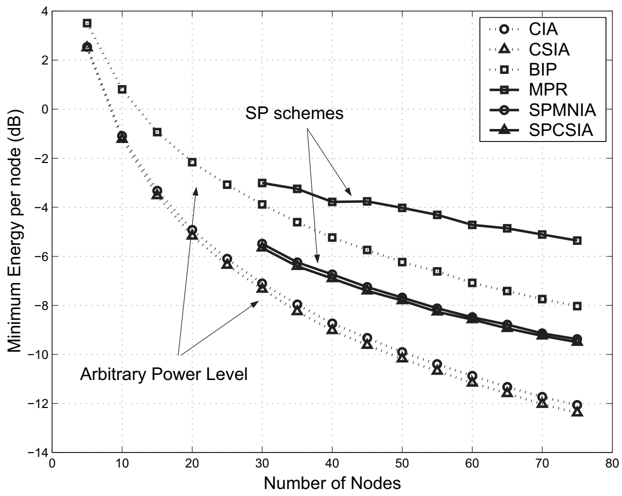


Fig. 10. The average energy expenditure per node for the CIA, CSIA, SPMNIA and SPCSIA. The average is taken over 200 network realizations for all cases.

algorithm by approximately $1.5\ dB$ when there are 50 nodes in the network. We note that the BIP algorithm does not have any limitations on its adjustable power levels. However, the connectivity of the network cannot be guaranteed when using a fixed transmission power, unless the node density or transmission power is sufficiently high. These experiments are evaluated for networks with over 30 nodes where the probability of obtaining *full connectivity* of the network is over 85%.

VIII. Conclusion

In this paper, we proposed and analyzed the gain in energy efficiency when utilizing the cooperative wireless advantage in a network broadcast scenario. We studied the energy-savings for a class of cooperation strategies that enable the receiver to accumulate the signal energy obtained from multiple transmitters to enhance the detection at the receiver. We focused our attention on centralized power control policies and on the physical layer model (the OLA system) provided in [12]. The minimum energy solution is derived in closed form for a simple unicast example while for the general case of a random network we show that the problem is NP-complete, much like the minimum energy problem in the point-to-point network. Therefore, we proposed the CIA and the CSIA as the centralized approximate algorithm to derive the power control policy. For practical purposes, we also derived equivalent algorithms for the single power scenario. Even with suboptimal solutions, we show that the minimum energy allocation achievable by utilizing the CWA gains significantly compared to the point-to-point network.

Appendix I
Proof of Theorem 1

Proof: In the following, we derive the asymptotic solution of the minimum energy when the constraints are satisfied with equality and show that this solution is the optimum solution.

Let's solve for $1 = \mathbf{B}\boldsymbol{\varepsilon}$, where $\mathbf{B}$ is an infinite matrix as shown in Section IV. We claim that the infinite Toeplitz matrix $\mathbf{B}$ has an asymptotically equivalent circulant matrix $\mathbf{B}_c$ [29] expressed as follows:

$$\mathbf{B}_c = \begin{bmatrix} \frac{1}{D^\alpha} & \frac{1}{[(N-1)D]^\alpha} & \cdots & \frac{1}{(2D)^\alpha} \\ \frac{1}{(2D)^\alpha} & \frac{1}{D^\alpha} & \ddots & \vdots \\ \vdots & \ddots & \ddots & \frac{1}{[(N-1)D]^\alpha} \\ \frac{1}{[(N-1)D]^\alpha} & \cdots & \cdots & \frac{1}{D^\alpha} \end{bmatrix}.$$

To prove that B_c is asymptotically equivalent to B, we must show that $\lim_{N\to\infty} |B - B_c| = 0$, where $|\cdot|$ is the Hilbert-Schmidt norm [29]. The weak norm of the difference is

$$\begin{aligned} |B - B_c|^2 &= \frac{1}{ND^{2\alpha}} \sum_{k=2}^{N-1} (k-1)(\frac{1}{k^\alpha})^2 \\ &< \frac{1}{ND^{2\alpha}} \sum_{k=1}^{N-1} \frac{1}{k^\alpha} < \frac{\zeta(\alpha)}{ND^{2\alpha}} \end{aligned}$$

Therefore, it is proven that $\lim_{N\to\infty} |B - B_c| = 0$.

The solution obtained by the equation $\mathbf{1} = \mathbf{B}_c\boldsymbol{\varepsilon}$ is asymptotically equivalent to the solution obtained from $\mathbf{1} = \mathbf{B}\boldsymbol{\varepsilon}$. From the elements of the circulant matrix, the sum of each row of infinite elements is equal to $\zeta(\alpha)$. Therefore, $\mathbf{1} = \mathbf{B}_c\mathbf{1}\frac{D^\alpha}{\zeta(\alpha)}$. Hence, $\boldsymbol{\varepsilon}' = \frac{D^\alpha}{\zeta(\alpha)}\mathbf{1}$ is a solution to $\mathbf{1} = \mathbf{B}_c\boldsymbol{\varepsilon}$. Since B_c is invertible, $\boldsymbol{\varepsilon}'$ must be the only solution.

Assume that $\boldsymbol{\varepsilon}^*$ is the optimal solution to the minimum energy problem and $\boldsymbol{\varepsilon}^* \neq \boldsymbol{\varepsilon}'$. Since $\boldsymbol{\varepsilon}^*$ is a solution, it follows that

$$1 \le \sum_{k=1}^{N-1} \{\mathbf{B}_c\}_{i,k}\varepsilon_k^*$$

for all i. From the fact that $\mathbf{B}_c\boldsymbol{\varepsilon}' = \mathbf{1}$, we have that

$$0 \le \sum_{k=1}^{N-1} \{\mathbf{B}_c\}_{i,k}(\varepsilon_k^* - \varepsilon_k')$$

for all i. Due to the circular structure of the matrix $\mathbf{B}_c$, we can derive the fact that

$$0 \le \sum_{i=1}^{N-1}\sum_{k=1}^{N-1} \{\mathbf{B}_c\}_{i,k}(\varepsilon_k^* - \varepsilon_k') = const. \cdot \sum_{k=1}^{N-1} (\varepsilon_k^* - \varepsilon_k')$$

where $const. = \sum_{i=1}^{N-1}\{\mathbf{B}_c\}_{i,k}$ for all k. However, this contradicts the assumption since having $\boldsymbol{\varepsilon}^*$ as the optimal solution implies that

$$\sum_{k=1}^{N-1}(\varepsilon_k^* - \varepsilon_k') = \sum_{k=1}^{N-1}\varepsilon_k^* - \sum_{k=1}^{N-1}\varepsilon_k' < 0.$$

Hence, $\boldsymbol{\varepsilon}^* = \boldsymbol{\varepsilon}'$. This proves that the minimum energy solution at node ε_n converges to $D^\alpha/\zeta(\alpha)$, i.e. $\varepsilon_\infty = \lim_{n\to\infty}\varepsilon_n = \frac{D^\alpha}{\zeta(\alpha)}$. ■

APPENDIX II
PROOF OF THEOREM 2

Proof: It is easy to see that MEB-CWA $\in$ NP, because a nondeterministic algorithm need only to guess a set of energy levels $\{\varepsilon_i : \varepsilon_i \in W, \ \forall i\}$ and verify in polynomial time whether the sum of the energies is less than the constant B, and whether all the nodes in the set $V - \{s\}$ satisfy the condition in (10). In the following, we show that the MEB-CWA problem is NP-complete by reducing the 3-CNF SAT to the SP-CWA problem which is a special case of the MEB-CWA with $W = \{0, \varepsilon\}$. We construct an instance of the SP-CWA problem such that the 3-CNF SAT instance is satisfiable if and only if the instance of the SP-CWA has a solution.

Suppose there are n boolean variables $x_1, x_2, \cdots, x_n$ and a collection of m CNF's $\{C_1, C_2, \cdots, C_m\}$. The corresponding SP-CWA instance for this 3-CNF SAT instance is constructed as follows:

1) Let there be a source node s with transmission energy equal to ε. For every boolean variable x_i, let there be 3 nodes x_i, $\bar{x}_i$ and X_i. For every CNF clause C_j, there is a corresponding node also denoted as C_j. Then, construct the digraph $G = (V, E)$ where the set of vertices V consists of all the above mentioned nodes, *i.e.* $V = \{s\} \cup \{x_i, \bar{x}_i, X_i, \ \forall i\} \cup \{C_j, \ \forall j\}$ and $E = E_1 \cup E_2$, where E_1 is the set of edges going from the source node to all the other nodes and E_2 is the set of edges going from the nodes $\{x_i, \bar{x}_i \ \forall i\}$ to the nodes $\{X_i, \ \forall i\}$ and $\{C_j, \ \forall j\}$, *i.e.* $E_1 = \{s\} \times \{V \setminus \{s\}\}$ and $E_2 = \{x_i, \bar{x}_i \ \forall i\} \times \{X_i, C_j \ \forall i, j\}$.
2) For the edges in E_1, we assign the flow from the source node to the nodes x_i and $\bar{x}_i$ to be equal to 1 when the source node transmits with energy ε, i.e. $\gamma_{sv}(\varepsilon) = 1$ for all $v \in \{x_i, \bar{x}_i, \forall i\}$, and let $\gamma_{sv}(\varepsilon) = \frac{1}{n+2}$, otherwise. For the edges in E_2, we assign the flow value 1 to edges running from nodes $v = x_i$ or $\bar{x}_i$ to the nodes X_j if $i = j$, and also to the nodes C_k if v is an element in the clause C_k. Therefore, let $\gamma_{vw}(\varepsilon) = 1$ for $v \in \{x_i, \bar{x}_i \ \forall i\}$ and $w \in \{X_i : v = x_i \text{ or } \bar{x}_i\} \cup \{C_j : v \in C_j\}$; and let $\gamma_{vw}(\varepsilon) = \frac{1}{n+2}$ for all the other edges in E_2. By definition of γ, we note that $\gamma_{vw}(0) = 0 \ \forall v, w$.
3) Let the constant $B = (n+1)\varepsilon$.

We claim that if there exists an energy assignment $[\varepsilon_1, \cdots, \varepsilon_{|V|}]$ such that a directed subgraph $G' = (V, E')$ exists with positive net flow at each node satisfying (10) and that $\sum_{i\in V}\varepsilon_i \le B$, then the 3-CNF SAT instance is satisfiable. A typical example of this directed acyclic subgraph is illustrated in Fig. 11 where the solid line represents the edges with flow 1, and the gray line represents those with flow $\frac{1}{n+2}$.

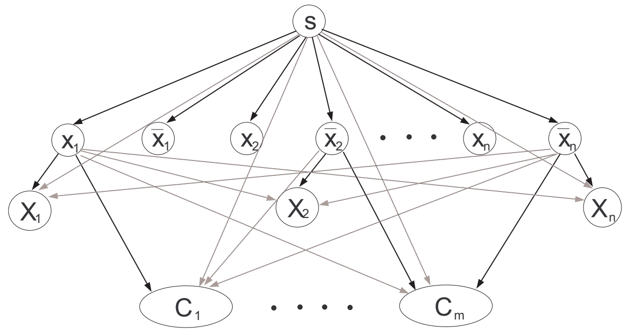


Fig. 11. An illustration of the constructed SP-CWA instance in the proof of Theorem 2. The flows on the edges represented by the solid line is equal to 1, while that of the dashed line is $\frac{1}{n+2}$.

The acyclic directed subgraph G' includes the edges from source s to all the nodes $x_1, \bar{x}_1, \cdots, x_n, \bar{x}_n$, because the edges coming from s are the only incoming edges of these nodes. Since the flow on these edges are equal to 1, the inequality (10) is satisfied. For a node X_i, there must exist an edge of either x_iX_i or $\bar{x}_iX_i$, but not both. We can prove this by contradiction. First of all, if none of these two edges exist, then at least $n+1$ of the elements in the set $\{x_j, \bar{x}_j \ \forall j \neq i\}$ must transmit. In fact, the flow from these $n+1$ nodes plus that of the source s allows the positive net flow of X_i to satisfy (10), because all the incoming edges in this case has flow of $\frac{1}{n+2}$. However, this would cause the total energy transmitted by the network to be $(n+2)\varepsilon$ which exceeds the constant $B = (n+1)\varepsilon$. Secondly, if both edges x_iX_i and $\bar{x}_iX_i$ exist, then the total energy would again exceed the value B if the nodes $X_j, \forall j$, is to satisfy the connectivity constraint in (10). Therefore, for all i, one and only one of the two nodes x_i or $\bar{x}_i$ is transmitting. Similarly, if $C_j = a_{j1} \vee a_{j2} \vee a_{j3}$ where $a_{j1}, a_{j2}, a_{j3} \in \{x_1, \bar{x}_1, x_2, \bar{x}_2, \cdots, x_n, \bar{x}_n\}$, then, by construction, there must exist, in G', at least one of the edges $a_{j1}C_j$, $a_{j2}C_j$ or $a_{j1}C_j$. We can also prove this by contradiction: if none of these edges exist then $\sum_{vw\in E(G')}\gamma_{vw}(\varepsilon) = \frac{n+1}{n+2} < 1$.

Let's assign values to the boolean variables such that x_i is true if node x_i transmits, i.e. x_i is not a leaf node in G', otherwise $\bar{x}_i$ transmits. For the subgraph G' that satisfies (10) and $\sum_{i\in V}\varepsilon_i \le B$, we claim that all the clauses $C_1, \cdots, C_m$ are true under the truth assignment of the boolean variables. If, for any value of j, the clause C_j is false, then the nodes corresponding to the literals of C_j should not be transmitting. But, there must exist an edge $a_{jk}C_j \in E(G')$ where a_{jk} is a literal in C_j. Therefore, by contradiction, C_j cannot be false. Hence, we have proven that for any energy assignment that answers yes to the decision problem of SP-CWA, the 3-CNF SAT problem is satisfiable.

Suppose that the 3-CNF SAT problem is satisfied with a truth assignment of the boolean variables $x_1, \cdots, x_n$, we show that there exists an energy assignment such that the SP-CWA problem is satisfied. First, let the source node s transmit energy ε. Then, let the node x_i transmit if the value of the boolean variable x_i is true, otherwise, let $\bar{x}_i$ transmit. Node X_i will readily satisfy the constraint in (10). Finally, for a clause $C_j = a_{j1} \vee a_{j2} \vee a_{j3}$, at least one of a_{jk} must be true, for $k \in$

$\{1,2,3\}$. Let a_{jk} be the literal that is true, then the node corresponding to node a_{jk} is transmitting with energy ε. From the construction, the flow $\gamma_{a_{jk}C_j}(\varepsilon) = 1$, which allows the node C_j to satisfy (10). Also, the total energy transmitted by the network is $(n+1)\varepsilon \leq B$. Therefore, for any truth assignment that satisfies the 3-CNF SAT problem, there is an energy assignment that allows the subgraph G' to satisfy (10) for all nodes and that $\sum_{i \in V} \varepsilon_i \leq B$.

It is easy to show that the construction of the SP-CWA instance from the 3-CNF SAT instance can be done in polynomial time in terms of n and m. Since the 3-CNF SAT problem is well-known to be NP-complete, it is proved that the SP-CWA problem is also NP-complete. Furthermore, since SP-CWA is a special case of the more general MEB-CWA problem, we have shown that MEB-CWA is NP-complete, too. ∎

References

[1] R. Min, M. Bhardwaj, S.-H. Cho, N. Ickes, E. Shih, A. Sinha, A. Wang, and A. Chandrakasan, "Energy-centric enabling technologies for wireless sensor networks," *IEEE Wireless Commun. Mag.*, vol. 9, pp. 28–39, Aug. 2002.

[2] A. Goldsmith and S. Wicker, "Design challenges for energy-constrained ad hoc wireless networks," *IEEE Wireless Commun. Mag.*, vol. 9, pp. 8–27, Aug. 2002.

[3] A. Ephremides, "Energy concerns in wireless networks," *IEEE Wireless Commun. Mag.*, vol. 9, pp. 48–59, Aug. 2002.

[4] J. Broch, D. A. Maltz, D. Johnson, Y.-C. Hu, and J. Jetcheva, "A performance comparison of multi-hop wireless ad hoc network routing protocols," in *Proc. ACM MobiCom 1998*.

[5] K. Obraczka, K. Viswanath, and G. Tsudik, "Flooding for reliable multicast in multi-hop ad hoc networks," *ACM Wireless Networks*, vol. 7, pp. 627–634, Nov. 2001.

[6] J. E. Wieselthier, G. D. Nguyen, and A. Ephremides, "On the construction of energy-efficient broadcast and multicast trees in wireless networks," in *Proc. IEEE INFOCOM 2000*, vol. 2, pp. 585–594.

[7] W. Liang, "Constructing minimum-energy broadcast trees in wireless ad hoc networks," in *Proc. 3rd ACM International Symposium on Mobile Ad Hoc Networking and Computing 2002*, pp. 112–122.

[8] M. Čagalj, J.-P. Hubaux, and C. Enz, "Minimum-energy broadcast in all-wireless networks: Np-completeness and distribution issues," in in *Proc. 8th Annual International Conference on Mobile Computing and Networking 2002*, pp. 172–182.

[9] P. J. Wan, G. Calinescu, X. Y. Li, and O. Frieder, "Minimum-energy broadcasting in static ad hoc wireless networks," *ACM Wireless Networks*, vol. 8, pp. 607–617, Nov. 2002.

[10] A. Qayyum, L. Viennot, and A. Laouiti, "Multipoint relaying for flooding broadcast messages in mobile wireless networks," in *Proc. 35th Annual Hawaii International Conference on System Sciences 2002*, pp. 3898–3907.

[11] Y.-C. Tseng, S.-Y. Ni, Y.-S. Chen, and J.-P. Sheu, "The broadcast storm problem in a mobile ad hoc network," *ACM Wireless Networks*, vol. 8, pp. 153–167, Mar. 2002.

[12] A. Scaglione and Y.-W. Hong, "Opportunistic large arrays: cooperative transmission in wireless multi-hop ad hoc networks for the reach back channel," *IEEE Trans. Signal Processing*, vol. 51, pp. 2082–2092, Aug. 2003.

[13] A. Sendonaris, E. Erkip, and B. Aazhang, "User cooperation diversity–part I: system description ," *IEEE Trans. Commun.*, vol. 51, no. 11, pp. 1927–1938, Nov. 2003.

[14] ——, "User cooperation diversity–part II: implementation aspects and performance analysis," *IEEE Trans. Commun.*, vol. 51, no. 11, pp. 1939–1948, Nov. 2003.

[15] J. Laneman, D. Tse, and G. Wornell, "Cooperative diversity in wireless networks: efficient protocols and outage behavior," *IEEE Trans. Inf. Theory*, vol. 50, pp. 3062–3080, Dec. 2004.

[16] I. Maric and R. D. Yates, "Cooperative multihop broadcast for wireless networks," *IEEE J. Sel. Areas Commun.*, vol. 22, pp. 1080–1088, Aug. 2004.

[17] ——, "Efficient multihop broadcast for wideband systems," in *Proc. DIMACS Workshop on Signal Processing for Wireless Transmission 2002*.

[18] ——, "Efficient multihop broadcast for wideband systems," in *Proc. Allerton Conference on Communications, Control and Computing 2002*.

[19] K. Makki, N. Pissinou, and O. Frieder, "Efficient solutions to multicast routing in communication networks," *ACM Mobile Networks and Applications*, vol. 1, pp. 221–232, Oct. 1996.

[20] T. H. Cormen, C. E. Leiserson, R. L. Rivest, and C. Stein, *Introduction to Algorithms, Second Edition.* MIT Press, 2001.

[21] B. Sirkeci-Mergen, A. Scaglione, and G. Mergen, "Asymptotic analysis of multi-stage cooperative broadcast in wireless networks," submitted to *IEEE Trans. Inf. Theory*, Mar. 2005.

[22] Y.-W. Hong and A. Scaglione, "A scalable synchronization protocol for large scale sensor networks and its applications," *IEEE J. Sel. Areas Commun.*, vol. 23, no. 5, pp. 1085–1099, May 2005.

[23] G. L. Stüber, *Principles of Mobile Communications, Second Edition*. Kluwer Academic Publishers, 2001.

[24] S. M. Kay, *Fundamentals of Statistical Signal Processing: Estimation Theory*. Prentice Hall, 1993.

[25] J. G. Proakis, *Digital Communications, Fourth Edition.* New York: McGraw Hill, 2001.

[26] A. Vecchio, "A bound for the inverse of a lower triangular toeplitz matrix," *SIAM J. Matrix Analysis and Applications*, vol. 24, no. 4, pp. 1167–1174, 2003.

[27] M. R. Garey and D. S. Johnson, *Computers and Intractability: A Guide to the Theory of Np-Completeness*. W. H. Freeman & Co., 1979.

[28] Y.-W. Hong and A. Scaglione, "Energy-efficient broadcasting with cooperative transmission in wireless sensory ad hoc networks," in *Proc. Allerton Conference 2003*.

[29] R. Gray, "Toeplitz and circulant matrices: a review," Information System Laboratory, Department of Electrical Engineering, Stanford University, 2001.

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