Characteristics of the Stress Intensity Factor of a Circumferential Crack in a Cylinder under Radial Temperature Distribution*

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Characteristics of the stress intensity factor of a circumferential crack in a cylinder under radial temperature distribution, which can be regarded as linear, were investigated systematically. The simplified method previously developed by the authors enabled our systematical approach. The investigation was conducted to comprehend the previously reported fact on the stress intensity factor. That is, the stress intensity factor under a given linear temperature distribution tends to decrease monotonously as the crack becomes longer than a specific value. It was shown that this tendency was a fundamental characteristic of the stress intensity factor for the problem and it was concluded that the cause of this was the moment redistribution due to the increase in crack length. In addition, it was pointed out that the stress intensity factor of the crack for a specific cylinder length was larger than that for an infinite length.

Key Words: Fracture Mechanics, Stress Intensity Factor, Thermal Stress, Cylindrical Shell and Crack Arrest

1. Introduction

There are interesting experimental data which indicate a surface circumferential crack inside a hollow cylinder, shows tendency of crack arrest when the inside of the cylinder is subjected to cyclic cooling from uniform temperature[1]. If the crack propagation rate in this situation fits the Paris law, this tendency may be explained by investigating the characteristics of the stress intensity factor (SIF). As the test pieces used in these experiments were too short to satisfy the long cylinder assumption which has been often used for obtaining the SIF for the structure, the effect of cylinder length on the SIF has to be considered. In addition, the SIF for the problem is affected not only by cylinder configuration but also by edge restraint, cooling rate, etc. So the effects of these factors on the SIF should be systematically evaluated to grasp the characteristics of the SIF and to understand the crack arrest tendency through them.

Keeping these situations in mind, we derived a closed form equation of the SIF for a circumferential crack in a cylinder (with its edges rotation-restrained) subjected to radial temperature distribution. Solution was given for a case temperature distribution can be regarded as linear, as a first step[2],[3]. The closed form equation enables us to evaluate the effects that cylinder length and crack location have on the SIF easily and systematically. As the results of parametric study by using the equation, it was reported that i) the SIF shows its maximum when the crack is located at the midpoint of the cylinder length[3] and ii) the SIF for the problem decreases monotonously as the crack becomes longer than a specific value[2].

Though the tendency ii) of the SIF described above is for a specific radial temperature distribution, this information is expected to contribute in making a reasonable maintenance rule for structural components, once their characteristics are fully understood. For example, a guideline for the crack to show a tendency towards crack arrest may be obtained which will enable us to omit fatigue analysis in maintenance. From this point of view, the characteristics of the SIF

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are studied analytically in detail by utilizing the assumption of linear radial temperature distribution. It is assumed that the crack is located at the midpoint of the cylinder length, taking the fact i) into account.

In the following sections, the ideas and basic ways of thinking applied in studying the characteristics of the SIF are described first. Then the effects the structural parameters have on SIF are studied.

2. Closed form SIF of a Circumferential Crack in a Cylinder under Linear Radial Temperature Distribution

In this section, the closed form SIF of circumferential crack in a cylinder under radial temperature distribution[2][3], which can be regarded as linear, is reviewed as the preparation of studying its characteristics.

The problem under consideration is described in Fig. 1. The cylinder with a circumferential crack located at the midpoint of its length is subjected to a radial temperature distribution $T(\eta)$, and the edges are rotation-restrained. The edges can move freely in the axial direction, as encountered in practical problems. The material of the cylinder is assumed to be homogeneous with isotropic and temperature independent physical properties. Bernoulli-Euler assumption that sections which are plane and perpendicular to the axis before loading remains so after loading, is applied. Here, as the temperature difference from the cross section average temperature $T(\eta)-T_{avg}$ gives the desired SIF[3], we will focus on the temperature difference distribution $T(\eta)-T_{avg}$, and rewrite this as $T(\eta)$. Thus,

$$\int_{-W/2}^{+W/2} T d\eta = 0 \qquad (1)$$

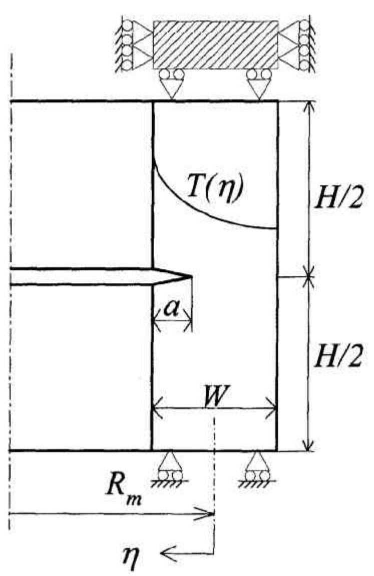

Fig. 1 A cylinder with a circumferential crack under radial temperature distribution

In case $T(\eta)$ can be regarded as linear, desired SIF for the problem K_{cyl} can be evaluated by the following closed form equation[2].

$$K_{cyl}=\frac{\psi_f\cdot\lambda_{cyl0}}{\lambda_{cyl0}+\phi_f\cdot(1-\psi_f)\cdot\lambda_{bf}^{*}}\cdot\left\{\frac{(-M_t)}{Z}\sqrt{\pi a}\cdot F_M(\xi)\right\} \qquad (2)$$

Here, the term in { } is a SIF of an infinite length edge cracked beam subjected to pure bending moment $(-M_t)$, where $F_M(\xi=a/W)$ is the correction factor of finite width and $Z=W^2/6$ is the cross section modulus. M_t is the bending moment that produces an equivalent deformation due to thermal stress and defined by the following equation:

$$M_t=\frac{E\alpha}{1-\nu}\int_{-W/2}^{+W/2} T\eta d\eta \qquad (3)$$

where E, α and ν are Young's modulus, coefficient of thermal expansion and Poisson's ratio, respectively. Note that this moment $(-M_t)$ for thin-walled cylinder is determined by the temperature distribution and thus called as equivalent thermal moment in the text that follows. λ with subindexes are various compliances defined in thin-walled cylinder axisymmetric bending problems. ϕ_f and ψ_f are factors defined to give the resultant bending moment acting on the cracked plane for a thin-walled cylinder, subjected to external bending load pair. ϕ_f gives the ratio to the external bending load without crack and ψ_f gives the decrement ratio due the existence of the crack. Concrete expressions of these parameters are given in Appendix. Note that in case $T(\eta)$ can no more be regarded as linear, a term (δK_{cyl}) representing nonlinearity in temperature distribution will be necessary in Eq. (2). That case will be studied in an another paper.

We will now focus on the fact that the desired SIF K_{cyl} in Eq. (2) is expressed as a product of a term determined by structural parameters (such as ϕ_f and ψ_f) and that by negative equivalent thermal moment $(-M_t)$ determined by the temperature distribution. That is, by introducing a function Φ, that represents the effect of structural parameters on K_{cyl}, Eq. (2) is rewritten as follows.

$$K_{cyl}\equiv\Phi\cdot(-M_t) \qquad (4)$$

As the equivalent thermal moment determined by temperature M_t is independent of crack length, the fact that the tendency of the SIF to monotonously decrease as the crack becomes longer than a specific length[2] is due to the characteristics of the structure (function Φ) can be easily understood. Considering this point, we will study the characteristics of the function Φ, which represents the effect of structure, assuming temperature distribution is given (thus M_t is constant) in the following. Specifically, we will investigate the reason why the SIF decreases when crack becomes longer than a specific length, through studying the effect of structural parameters and material constants on Φ.

As a first step, we will define the following function F_{tcyl}.

$$F_{tcyl} \equiv \Phi \Big/ \left\{ \frac{1}{Z} \sqrt{\pi a} \cdot F_M(\xi) \right\} = \frac{\psi_f}{1+\phi_f(1-\psi_f)\lambda_{bf}^* / \lambda_{cylo}} \quad (5)$$

By substituting this function in Eq. (4), the desired SIF K_{cyl} can be rewritten as follows.

$$K_{cyl} = \Phi \cdot (-M_t) = F_{tcyl} \cdot \left\{ \frac{(-M_t)}{Z} \sqrt{\pi a} \cdot F_M(\xi) \right\} \quad (6)$$

Thus, F_{tcyl} can be understood as the ratio of the desired SIF K_{cyl} and the SIF of an edge cracked beam under pure bending moment $(-M_t)$ (i.e. { } in Eq. (6)). As this SIF of an edge cracked beam under pure bending monotonously increases as the crack becomes longer, characteristics of F_{tcyl} vs. crack length will be the key point for the function Φ to decrease when the crack becomes longer than a specific length. In the following study of the characteristics of Φ, we will focus our attention on the characteristics of F_{tcyl}.

3. Characteristics of Function Φ

In this section, we will study the effects of various structural parameters and material constants on the SIF.

3.1 Effect of cylinder length

As a first step, effect of cylinder length on $\Phi = K_{cyl}/(-M_t)$ was studied for constant mean radius to wall thickness ratio R_m/W. Wall thickness W of 10 mm and Poisson's ratio ν of 0.3 are used as standard values in numerical evaluation throughout the paper. Figure 2 shows a representative result for $R_m/W=10.5$.

From this figure, it can be said that $\Phi = K_{cyl}/(-M_t)$ shows a tendency to monotonously decrease after a specific crack length when crack becomes longer than a specific length, regardless of the cylinder length. Note that this characteristic of function Φ, which represents the effect of structural parameters on the SIF, is equivalent to the characteristic of the SIF itself under the condition M_t=constant. Thus, it can be said that the SIF of a circumferential crack in a cylinder under radial temperature distribution (which can be regarded as linear) decreases monotonously as the crack becomes longer than a specific length due to the characteristics of the structure, at least when M_t=constant.

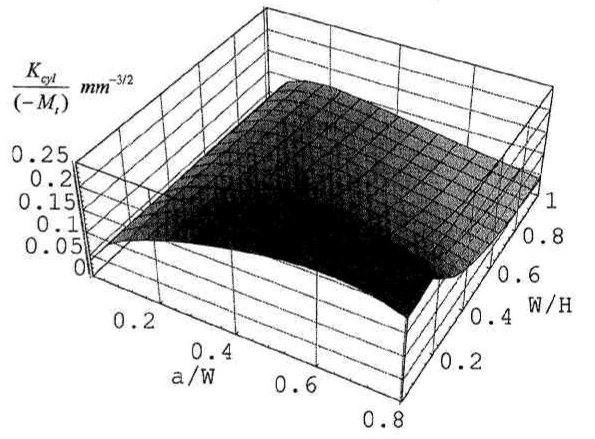

Fig. 2 The effect of cylinder length on Φ ($R_m/W=10.5$, $W=10$ mm, $\nu=0.3$)

In Fig. 2, the tendency for $\Phi = K_{cyl}/(-M_t)$ to generally increase as the cylinder length increases ($W/H \to 0$) and to saturate to a specific value for given crack length, can be read. Though it may be thought intuitively that Φ for infinite length cylinder is a safe evaluation from a practical standpoint, Φ seems to show a maximum value at a relatively small W/H.

To investigate this phenomenon in more detail, we will focus our attention on F_{tcyl}, as Φ in Eq. (5) is a product of F_{tcyl} (which is affected by cylinder length) and a term independent of cylinder length.

By substituting all the structural parameters defined in Appendix to Eq. (5), simplified equation of F_{tcyl} is obtained as follows.

$$F_{tcyl} = \frac{\cosh \beta H - \cos \beta H}{(\cosh \beta H - \cos \beta H) + \beta D \cdot \Delta\lambda (\sinh \beta H + \sin \beta H)} \quad (7)$$

Here, $\Delta\lambda$ is the increment of compliance due to the presence of crack for an infinitely long beam under pure bending, β and D are quantities used in replacing cylindrical shell by a beam on an elastic foundation given concretely in the Appendix.

We differentiated F_{tcyl} by (βH) to study the effect of cylinder length H on F_{tcyl}, as follows.

$$\frac{\partial F_{tcyl}}{\partial(\beta H)} = \frac{2\beta D \cdot \Delta\lambda \cdot \sinh \beta H \sin \beta H}{\{(\cosh \beta H - \cos \beta H) + \beta D \cdot \Delta\lambda (\sinh \beta H + \sin \beta H)\}^2} \quad (8)$$

From this equation, the fact that F_{tcyl} shows a peak value at $(\beta H) = n\pi$ (n : integer) can be read. As there are many peaks, the maximum of these peaks will be determined.

First, $(F_{tcyl})_\infty$, which is a limit value of F_{tcyl} for an infinitely long cylinder ($\beta H \to \infty$) was considered, as a reference value. Based on the knowledge of hyperbolic function,

$$\lim_{\beta H \to \infty} \sinh \beta H = \lim_{\beta H \to \infty} \cosh \beta H = \frac{\exp(\beta H)}{2} \quad (9)$$

and combining this with Eq. (7), $(F_{tcyl})_\infty$ is obtained as follows.

$$(F_{tcyl})_\infty=\lim_{\beta H\to\infty}F_{tcyl}=\frac{1}{1+\beta D\cdot\Delta\lambda} \quad (10)$$

Though $(F_{tcyl})_\infty$ is affected by the crack length through $\Delta\lambda$, which is the increment of compliance due to the presence of crack, it is no more affected by the cylinder length. So F_{tcyl} normalized by this $(F_{tcyl})_\infty$ is represented in Fig. 3 to intensify the characteristics of the peak values of F_{tcyl} for cylinder length. In this figure, it can be seen that F_{tcyl} (for respective crack lengths) increases for short H to reach the maximum at $H=\pi/\beta$, and gradually saturates to $(F_{tcyl})_\infty$ for long H.

From above-mentioned, F_{tcyl} can be said to show its maximum at a specific cylinder length $\beta H=\pi$ for a given crack length, mean radius to wall thickness ratio and material constants. That is, Φ becomes maximum at a cylinder length $\beta H=\pi$ and is larger than that for an infinite length cylinder, as shown in Fig. 2.

3.2 Effect of mean radius to wall thickness ratio R_m/W

Effect of mean radius to wall thickness ratio R_m/W on function Φ was studied. The fact that $\Delta\lambda$, the increment of compliance due to the presence of crack for an infinitely long beam under pure bending, can be expressed in the following form (detail in Appendix) is focused.

$$\Delta\lambda(\xi)=12f_M(\xi)/(EW^2) \quad (11)$$

By substituting Eq. (11) to Eq. (7), F_{tcyl} can be written as follows.

$$F_{tcyl}=\frac{\cosh\beta H-\cos\beta H}{[(\cosh\beta H-\cos\beta H)+\beta W\cdot(\sinh\beta H+\sin\beta H)\cdot f_M(\xi)/(1-\nu^2)]} \quad (12)$$

From this expression, it becomes clear that F_{tcyl} is a function of three non-dimensional parameters βH, βW and $\xi=a/W$. In addition, from the definition of β (in Appendix), βW can be rewritten as follows.

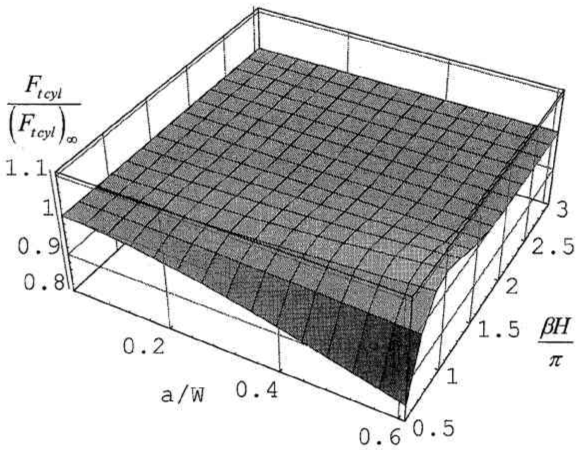

Fig. 3 Effect of cylinder length on function F_{tcyl} ($R_m/W=10.5$, $\nu=0.3$)

$$\beta W=\frac{\sqrt[4]{3(1-\nu^2)}}{\sqrt{R_m/W}} \quad (13)$$

From Eq. (13), the fact that defining βW is equivalent to defining mean radius to wall thickness ratio R_m/W can be read.

As F_{tcyl} is the only portion βW has its influence on function Φ, which represents the effect of whole structural parameters on the SIF in interest, it can be said that mean radius to wall thickness ratio R_m/W affects Φ as $(R_m/W)^{-1/2}$.

Quantitative effect of mean radius to wall thickness ratio R_m/W on Φ is studied next. In case $R_m/W\to\infty$, $F_{tcyl}\to1$ is easily obtained from Eqs. (12) and (13). This means that Φ shows the characteristics of the SIF for edge cracked beam under pure bending, which monotonously increases as the crack becomes long. So there should be a maximum R_m/W for Φ to show the tendency to decrease for a longer crack than a specific value.

Considering the complexity of Eqs. (12) and (13), it is difficult to derive this maximum R_m/W analytically. Thus non-dimensional Φ for cylinder length $\beta H=\pi$ and typical R_m/W were summarized in Fig. 4. The case of $R_m/W=1.5$, which exceeds the application limit of the closed form SIF Eq. (2), was plotted for reference to show the tendency due to further decrease in R_m/W. Note that Eq. (2) was derived based on the thin shell cylinder theory and its validity was shown for $R_m/W\geqq5.5$[2].

From this figure, the following can be read.

1. Φ for cylinder with its length $\beta H=\pi$ decreases as the crack becomes longer than a specific length for typical cylinders of $R_m/W\leqq20$.
2. This specific non-dimensional crack length a/W, for the Φ to show its maximum, and this maximum Φ increases as the cylinder becomes thinner.

3.3 Effects of material constants

From Eqs. (5) and (12), the fact that Young's modulus E does not affect Φ can be read. On the other hand, Poisson's ratio affects Φ, as it includes terms β and $(1-\nu^2)$. Quantitative study on the effect

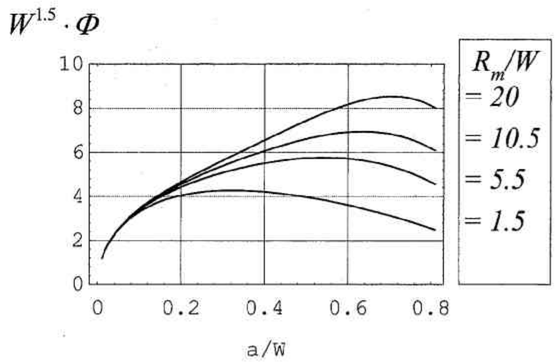

Fig. 4 Effect of R_m/W on Φ ($\beta H=\pi$)

of Poisson's ratio is summarized in Fig. 5.

As preceding numerical examples are based on Poisson's ratio $\nu=0.3$, results in Fig. 5 are normalized by Φ for $\nu=0.3$. From Fig. 5 and the fact that Poisson's ratios for all the materials satisfy $\nu\leqq 0.5$, the effect of Poisson's ratio on Φ can be neglected on the practical standpoint. Therefore, effect of Poisson's ratio in the following study is not considered.

3.4 Effect of crack length

Characteristic of Φ to monotonously decrease, as the crack becomes longer than a specific length, was studied. From Eq. (6), Φ can be understood as the SIF in interest K_{cyl} for $(-M_t)=1$. In other words, Φ is a product of F_{tcyl} and the term in { }, which is the SIF of a edge cracked beam under pure bending moment $(-M_t)=1$. This SIF of an edge cracked beam under pure bending monotonously increases as the crack becomes long. On the other hand, F_{tcyl} given by Eq. (7) monotonously decreases as the crack becomes long because the only term $\Delta\lambda$ affected by crack length in Eq. (7) is a monotonously increasing function of crack length. Thus the characteristic of the function Φ to monotonously decrease as the crack becomes longer than a specific length, is a result in the balance of characteristic of F_{tcyl} and that of the { } term.

Here K_{cyl} in Eq. (6) which is the SIF of a circumferential crack in a cylinder under radial temperature distribution, was derived through two steps[2]. First this cylinder problem was replaced with a problem of a rotary spring connected by two beams on an elastic foundation. Then the desired SIF was evaluated as the SIF of an edge cracked beam under pure bending, using the moment at the rotary spring (cracked section). Thus, $F_{tcyl}\cdot(-M_t)$ can be considered to represent the effective moment at the rotary spring.

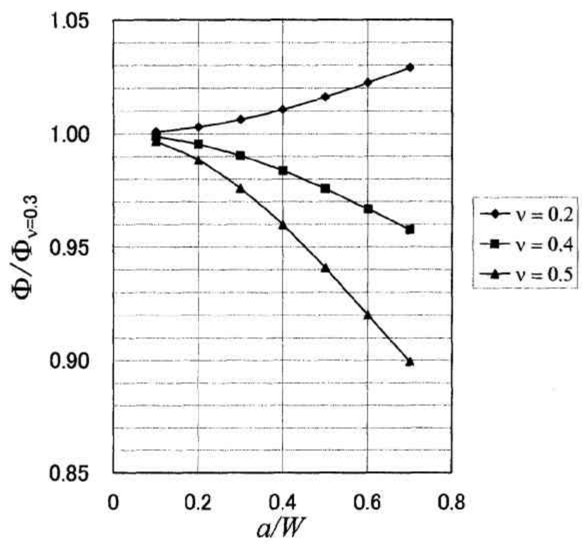

Fig. 2 The effect of cylinder length on Φ ($R_m/W=10.5$, $W=10$ mm, $\nu=0.3$)

Therefore, the fact that F_{tcyl} is a monotonously decreasing function of crack length indicates that the effective moment and the stress induced at the vicinity of the crack decreases though $(-M_t)$ is constant.

3.5 Dimensional analysis

F_{tcyl}, which is one of the factors of function Φ representing the effects of structural parameters on the circumferential crack in a cylinder under linear radial temperature distribution, is a non-dimensional function described by non-dimensional variables βH, βW and a/W as above-mentioned. Therefore, the portion with dimension in function Φ is the term { } in Eq. (5). By normalizing crack length a by wall thickness W and by substituting section modulus $Z=W^2/6$ into this term { }, Eq. (5) is deduced in the following form.

$$\Phi\equiv F_{tcyl}\cdot\left\{\frac{6}{W^{1.5}}\sqrt{\pi a/W}\cdot F_M(a/W)\right\} \tag{14}$$

From this equation, the fact that the scale factor of Φ is $W^{(-1.5)}$ can be read.

4. Conclusion

In this paper characteristics of the SIF of circumferential crack in a cylinder (with its edges rotation-restrained) under radial temperature distribution (which can be regarded as linear) were studied. The location of the crack was chosen as the midpoint of its length as the SIF shows its maximum for this crack location.

Considering the fact that the SIF in interest is a product of Φ (determined by structural parameters) and $-M_t$ (determined by temperature distribution), the effects of various structural parameters on Φ were studied under the condition M_t=constant. In conclusion, the following were obtained.

1. Φ is a function described by non-dimensional parameters βH, βW and a/W and wall thickness W. Scale factor of Φ is $W^{(-1.5)}$. Non-dimensional parameter βW has a direct relationship with the square root of the familiar parameter R_m/W.
2. Φ shows its maximum for a cylinder length satisfying $\beta H=\pi$, when R_m/W and a/W are given. This maximum value of Φ is larger than that for infinite cylinder length.
3. Φ for typical $R_m/W(\leqq 20)$ shows a tendency to monotonously decrease as the crack becomes longer (large a/W) than a specific value, when βH and R_m/W are given. This specific a/W for the Φ to show its maximum and this maximum Φ increases as the cylinder becomes thinner.
4. Characteristic of Φ to monotonously decrease for a longer crack length than a specific value can be understood as the decrease in the stress induced at the vicinity of the crack, due to moment redistribution

according to the increase in crack length.

Appendix

λ with subindexes in this paper are various compliances defined by replacing the problem of thin-walled cylinder under axisymmetric bending (Fig. A1 left) to a problem of a beam on an elastic foundation with bending loads on both ends (Fig. A1 right). Spring constant k in this case is given by the following equation, by formally writing the flexural rigidity of the beam as $D=EW^3/12(1-\nu^2)$[4].

$$k=4\beta^4 D\ ;\ \beta^4=\frac{EW}{4R_m^2 D} \tag{A1}$$

Here, R_m: mean radius, W: wall thickness, E: Young's modulus, ν: Poisson's ratio. Note that β has a dimension of inverse of length.

By using this β, compliances for a beam on an elastic foundation in Fig. A2 derived by Hetényi[5] was rewritten to be applied for thin-walled cylinder problem as follows[2].

$$\lambda_{bf}=\frac{1}{\beta D}\cdot\frac{\sinh\beta H+\sin\beta H}{\cosh\beta H+\cos\beta H-2} \tag{A2}$$

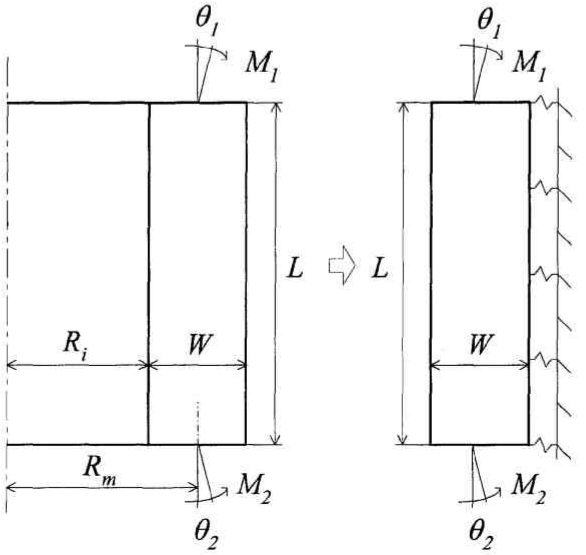

Fig. A1 Replacement of axisymmetric bending problem of a cylinder by a beam on an elastic foundation

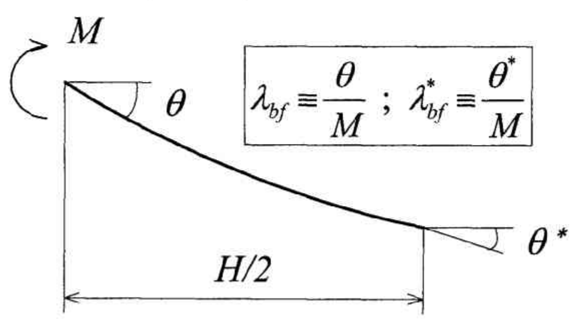

Fig. A2 Definition of compliance λ_{bf} and λ_{bf}^*

$$\lambda_{bf}^*=\frac{1}{\beta D}\cdot\frac{\sinh A\cos A+\sin A\cosh A}{\sinh^2 A-\sin^2 A}\ ;\ A=\frac{\beta H}{2} \tag{A3}$$

ϕ_f defined as the ratio of the resultant bending moment acting on the midpoint of cylinder length to the external bending load pair (case of $M_1=M_2=M$ in Fig. A1 left) can be written by using these compliances as follows.

$$\phi_f=\lambda_{bf}^*/\lambda_{bf} \tag{A4}$$

Note that the midpoint of cylinder length corresponds to the position circumferential crack is located for the study in this paper.

The resultant moment $\phi_f\cdot M$ at this location is redistributed by the existence of the crack. The redistribution factor ψ_f was written as follows[2].

$$\psi_f=\lambda_{bf}/(\lambda_{bf}+\Delta\lambda) \tag{A5}$$

The correction factor for finite width F_M and increment in compliance due to the existence of crack $\Delta\lambda$ used in numerical examples are Eq. (A6)[6] and (A7)[7], respectively.

$$F_M(\xi)=\sqrt{\frac{2}{\pi\xi}\tan\frac{\pi\xi}{2}}\times\frac{0.923+0.199[1-\sin(\pi\xi/2)]^4}{\cos(\pi\xi/2)} \tag{A6}$$

$$\Delta\lambda(\xi)=\frac{\pi(1.1215)^2}{2E}\cdot\frac{\xi^2}{(1-\xi)^2(1+2\xi)^2}\times[1+\xi(1-\xi)(0.44+0.25\xi)]\left(\frac{6}{W}\right)^2 \quad\cdots\cdots\cdots \text{(A7)}$$

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