Moving towards a control technique to help small firms monitor and control key marketing parameters: a survival aid

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Abstract
This article considers that one way to help the small- and medium-sized enterprise (SME) to survive is to offer it a robust but simple monitoring and control technique that would help it manage the business effectively and this, in turn, should help to increase its chances of survival. This technique should also be of interest to all people involved with monitoring or advising a large number of small enterprises or business units within a larger organization. For example, a bank manager or a small business consultant responsible for a portfolio of firms. The authors utilize process control techniques more often used in production and inventory control systems to demonstrate how one might monitor the marketing "health" of small firms.

Introduction

The authors are concerned with developing a quantitative method to help SMEs manage their operations more successfully, and thus hopefully increase their survival rate. It will also be of particular use to those who provide business advice and services to the small business. The behaviour of the SME is described by reference to a life cycle/stages framework. The proposal is that either a single, or several, key performance indicator(s) are monitored and that, when they fall outside an acceptable range, a warning message is generated. This requires an effective forecasting method, preferably one that utilises and "learns" from past data and a method by which to track unexpected deviations and generate a warning message. For the former it is argued that exponential smoothing models are suitable. For the tracking signal through which the data are monitored, a smoothed error tracking signal based on the work of Trigg *et al.* (Trigg, 1964; Trigg and Leach, 1967) is employed. The generation of the exception message must be related to a particular confidence level and for this cumulative probability tables for the tracking signal are needed.

Stage models

Most SMEs in the UK do not grow and thrive but fail within five years (Department of Trade and Industry, 1999). In trying to understand the dynamics behind such a performance then the adoption of a life-cycle model approach to the firm is appropriate. Such a framework encompasses the riskiest situation – the survival of the growing firm as it moves through its life-cycle. Equally this article has relevance for the SME in the "static" situation in which it remains at a particular stage but still has to address those problems peculiar to that stage. Many life-cycle models assume that the firm proceeds in a somewhat orderly fashion through defined stages: however, some variants of this approach introduce "crisis points" which delineate the stages. Such episodes have to be successfully addressed for the firm to move to the next stage of development (see, for example, Scott and Bruce, 1987). As shown by Hanks *et al.* (1993), whilst different authors subscribe to a different number of stages in their respective models, in general they can be reduced down to the generic stages of start-up, expansion, maturity, diversification, and decline. Critics of the life-cycle approach argue that these models are too restrictive due to their implicit assumptions, one of which is that the firm carries out functions concurrently rather than consecutively (Storey, 1994). This issue was addressed by Eggers *et al.* (1994), who, whilst still retaining the stages approach of the original Churchill and Lewis (1983) model, allowed for an organisation to develop in a more "organic" fashion – they can "hypergrow" (skip stages) or "backslide" to previous stages. This revisited and reworked model considered six stages: conception, survival, stabilisation, growth orientation, rapid growth, and resource maturity.

However, we suggest that if the models are treated as a general schema then they do serve to remind us of the practical problems that the firm needs to address and, roughly, when to expect such problems to manifest themselves. The attraction of the Scott and Bruce (1987) approach is that it is quite specific in suggesting what such managerial problems might be and their location in the life cycle. At worst such an approach serves to remind us that young, pioneering firms in respect of their behavior and, hence, their needs are very different from their maturer counterpart. Hanks *et al.* (1993) have demonstrated through statistical analysis of data patterns for their chosen industry sector

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(high technology ventures in Utah) that one can infer from actual data, rather than have to speculate upon, the existence of identifiable life cycle stages. They identified four such stages – start-up, expansion, maturity, and early diversification.

The factors that cause small business failure are depressingly familiar. They probably can be subsumed under the categories of any, or all, of the deficiencies in human capital, financial capital or external trading conditions (Cressy, 1996). Whilst the typical problems facing the SME need not manifest themselves in failure, the top three consistent problems faced by UK SMEs, as reported in the *NatWest SBRT Quarterly Survey of Small Business in Britain* (1999), are: low turnover/lack of business; governmental regulations and paper work, and cashflow/payments and debtors. The proposition in this article is that one might be able to increase its life expectancy if a simple but robust model could be found that would help the SME identify potential crisis points. The type of approach that we are proposing either could be used by the SME itself to monitor its key commercial variables or could be used by advisers and other interested parties who have responsibility for monitoring a large number of SMEs – for example, commercial bank small business advisers. Certainly the approach could help to track two of the aforementioned problem areas through the provision of more timely and accurate data or, perhaps, helping the SME to react more quickly, or to partly mitigate some of the consequences that would be caused by such problems.

The proposed method

The advantages of this proposed method are that it is easy to apply, can be calculated on a fairly simple Excel spreadsheet package (Greatorex) and the underlying statistical framework is well proven, as demonstrated by the age of our technical citations. What has made the model accessible to the SME is the calculation by one of the authors (Reynolds, 1986) of confidence limits for the smoothed error tracking signal, which then allows a tracking signal to be applied in practice. The disadvantages of the model are that it is data hungry in the sense that the more past data that are available, the more accurately the models predicts, and that to run the procedure some statistical knowledge and ability to "fine tune" the model are required. However, the former problem may not be insurmountable if the SME has, say, daily sales figures, and in the latter case some limited contact with an adviser would help with the fine tuning. The authors believe that the following criteria, which are necessary for the technique to be useful and valid, are met to a sufficient degree either in theory or in practice:

1. That the concept of identifiable stages and their concomitant and unique commercial problems and challenges within a life cycle schema is acceptable and realistic.
2. That exponential smoothing forecasting models are reliable and suitable for predicting future values of the appropriate data. At this stage the authors are concerned only with producing one period ahead forecasts. Indeed tracking the errors of the "n" period ahead forecasts produces a less sensitive tracking signal (Reynolds and Day, 1996).
3. That the monitoring scheme is accurate, simple to understand, economises on the storage of data and is robust to whatever data series is employed.
4. That Trigg's tracking signal is a reliable tracking device, capable of picking up both unexpected step (large) and ramp (gradual) changes in the underlying data pattern, and reporting such unexpected deviations as quickly as possible. Additionally cumulative probability values are available for the tracking signal which can be used to set control limits for a wide choice of sensitivity.
5. Suitable data are available to use as a leading indicator. For the purpose of illustration we have used sales turnover but realise that this may not be the best indicator, and that a combination of indicators may well be needed – particularly to capture more subtle effects such as entrepreneurial ability.

The main concern of this article is to establish the validity of items 2, 3 and 4 above.

The forecasting procedure

In monitoring the key marketing parameters of small firms the authors have made use of exponential smoothing to produce one period ahead forecasts of the parameter values. If the input data used in the forecasts are behaving as expected, then the forecasting errors will be normally distributed and will lie within certain bounds. These forecasting errors can be tracked with a tracking signal in order to identify as quickly as possible any unexpected patterns in the errors, which in turn could indicate possible unexpected changes in the underlying input data. If the tracking signal is computed as a derivative of the one period ahead forecast error, then it will be normally distributed. It should lie within certain cumulative probability boundaries, provided that the underlying input data are "well behaved" and within certain expected limits of variation, given the specific model used. The authors are not using this forecasting procedure with the intention of producing forecasts *per se* but as part of their process to enable the monitoring of selected performance variables. Exponential smoothing has been adopted to provide these forecasts, because: exponential smoothing has replaced moving averages as the predominant method used in short-term forecasting (Montgomery and Johnson, 1976; Makridakis and Hibon, 1979); second, evidence from the literature strongly suggests also that quantitative techniques are generally superior in accuracy to qualitative techniques (Hogarth, 1975; Sarbin, 1943; Slovic, 1972; Mabert, 1975).

Other studies by Bauman (1965), Geurts and Ibrahim (1975) and Newbould (1974) have concluded as well that simpler methods such as exponential smoothing in terms of accuracy do as well as or better than more sophisticated models. Both Geurts and Ibrahim (1975) and Makridakis and Hibon (1979) show that exponential smoothing outperforms the more sophisticated Box-Jenkins models. Interestingly, the comprehensive study by Witt and Witt (1992) on modelling and forecasting demand in tourism concluded, *inter alia*, that more complicated econometric models do not necessarily outperform more naïve models.

The tracking signal

The first tracking signal designed specifically for forecast control was used in inventory control and proposed by Brown (1962). This is defined as the sum of forecast errors divided by the mean absolute deviation (MAD) and is known as the Simple CUSUM technique. The smoothed error tracking signal, the procedure used in this article, was developed by Trigg (1964) and based on the earlier work of Brown (1962). The real difference in his method is that he uses a "smoothed error" in the numerator of the tracking signal instead of the sum of errors. In the simple CUSUM, Brown applied exponential smoothing to the modulus of the error to produce a smoothed MAD, and the sum of errors was calculated by summing the plus and minus values of successive errors. Trigg retains the smoothing of MAD but in addition applies simple exponential smoothing to the plus and minus errors to produce a smoothed error as the numerator of the tracking signal instead of the sum of errors. These variants are explained and discussed in the Appendix.

Tracking signal control limits

In this article the authors are using a forecasting procedure, not with the intention of producing forecasts for planning purposes, but as a means of monitoring performance and comparing it with an expected outcome. SMEs need to know as soon as possible when a forecast has gone '"out of control" in order to avoid the mistake of basing important decisions on poor information and, if necessary, to carry out corrective action. The tables of confidence limits have been produced for use with the smoothed error tracking signal for each of the main exponential smoothing models. That is Simple Exponential Smoothing, Brown's One-Parameter Linear Exponential Smoothing, Holt's Two-Parameter Linear Exponential Smoothing and Winter's (1960) Seasonal Method. Since Holt's method is more frequently used than Brown's method, the authors have only discussed the use of Holt's method. These cumulative probability tables contain many hundreds of individual confidence limit values that could be integrated into a computerised monitoring and control system. Their advantage is that they impart greater accuracy to the technique because they allow a greater permutation of smoothing coefficient values to be used. In essence the coefficient for the forecasting level equations can now be different from the coefficient used in the tracking signal equation. The first published report of a study using different values of smoothing coefficients in the forecasting equations from that used in the tracking signal ($\alpha > \alpha_1$ or $\alpha < \alpha_1$) was by McKenzie (1978). He showed that the performance of the smoothed error tracking signal (T) may be significantly improved by such a simple alteration in its application.

Methodology used to obtain control limits

Cumulative frequency tables were produced for the smoothed error tracking signal using "well behaved" data produced by simulation. The method involved setting the parameters for a particular model, e.g. Holt's (1957), and then generating random errors drawn from a normal distribution and adding these to the time series. Tables were produced using the forecasting models of Simple Exponential

Smoothing, Holt's Two-Parameter Linear Exponential Smoothing and Winter's Seasonal Method. Many possible permutations of smoothing coefficient values were used for level component, trend component, seasonal index (for Winter's (1960) Model) and tracking signal. These tables are able to serve as "tracking signal confident limit tables" when smoothed error tracking signal values from "real" one period ahead forecasting runs are compared to the theoretical, simulated values. If the value of the smoothed error tracking signal (T_t) in the forecast procedure exceeds the value given in the table (the modulus of T_t is shown in the tables) for a given permutation of smoothing coefficient values, i.e. trend component, seasonal components etc. – then the tracking signal T_t would be said to have "tripped" at a certain level of confidence, i.e. 95 per cent level, 99 per cent level etc. The tracking signal "critical value" produced using this simulation exercise agrees with the limited range of values reported by Batty (1969), Trigg (1964) and Gardner (1983) for simple exponential smoothing and agrees with the theoretical values produced from equations derived by McKenzie (1978) for Holt's Two-Parameter model of linear exponential smoothing. Critical values were produced for the Winter seasonal model, although there is nothing in the literature of either a theoretical or an empirical nature for this model to compare these critical values with. A detailed account of this methodology is reported by Reynolds (1986) and discussed by Reynolds and Greatorex (1988). Readers are referred to these for a more detailed coverage of the procedures used. It is not possible to show the comprehensive cumulative probability tables derived for T_t in this article, as, for example, the tables for Winter's method alone extend to some 20 pages. A full comprehensive set of cumulative probability tables for a wide range of smoothing coefficient value permutations and for use with all the exponential smoothing forecasting models discussed in this article is available on request from the authors.

Trialling the model

Figure 1 reports six runs of the model – five of these (a to e) are using hypothetical data and one (f) is using actual reported quarterly sales data from a local company for a ten-year period. The one set of actual data is reported from an exercise that applied this technique to four case study companies. Case (f) is a company in the authors' local economy with data being obtained from secondary published sources, the selection parameters being companies with a turnover of fewer than £2m per annum, who were employing fewer than 50 people, and were not a subsidiary of another company. The final criterion was that they had exhibited some variation in their financial performance over the last ten years, as indicated by their "ICC score" (a composite financial score of financial soundness). This case is presented also as Figure 2. This example demonstrates very well how difficult it is to identify unusual changes in the underlying data by merely looking at a plot of the data. The other three case study companies were located in the North East of England and had between 20 and 200 employees. The authors discuss them in the next section of this article. Standard forecasting models (those of Holt (1957) and Winter (1960) and simple exponential smoothing) were used to test the ability of the tracking signal to monitor step and ramp changes (shocks) in underlying input data for both the hypothetical simulated time series and real time series obtained from small firms. These forecasting and tracking signal programs, in conjunction with the tables of tracking signal "critical values" discussed earlier, were then used to test the reaction of T_t to a step or ramp shock for both the simulated and real time series data. In this sense the forecasting methodology is used as a marketing monitoring and control procedure.

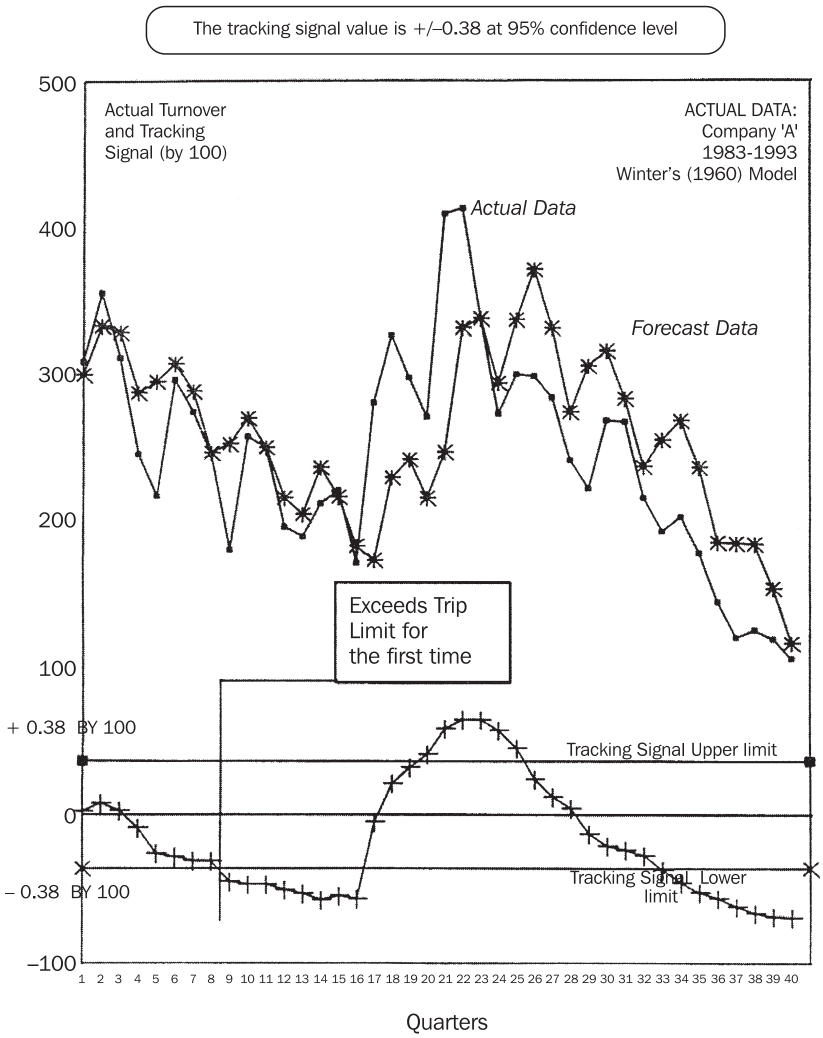
Discussion of results

As discussed above, the technique has been applied to four case study firms, only one of which is reported in Figure 1 (result f) and graphed as Figure 2. Results obtained from using the procedure look promising at present in cases where the input data exhibit some predictable pattern, and appear to be robust and suitable for a range of data patterns including data exhibiting seasonal or cyclical patterns. However, when the input data exhibit extreme variability, as in the case of the other three case study firms drawn from the North East of England, the method performs less well (Reynolds *et al.*, 1995). Because the data are virtually impossible to predict with any of the exponential smoothing forecasting models available, tracking the one period forecasting errors is a pointless exercise. The one period errors are so large that the resulting tracking signal exceeds its predetermined control limit virtually every time period. This is the one very serious drawback and limitation of

Figure 1
Six runs of the model using hypothetical (simulated) data and real data

Type		Data 'displays'	Model	Step change	Ramp change	Trips within (number of time periods)
(a)	Simulated	Linear trend	Holt's Linear Trend	yes, 5%		one
(b)	Simulated	Linear trend	Holt's Linear Trend		yes, 2%	two
(c)	Simulated	Stationarity	Simple Exponential Smoothing	yes, 5%		one
(d)	Simulated	Stationarity	Simple Exponential Smoothing		yes, 2%	two
(e)	Simulated	Seasonality	Winter's	yes, 5-8%		two
(f)	Real	Seasonality	Winter's		yes	two

Figure 2
Figure 1 in graph form



the proposed scheme. Of course the authors are simply using sales as input data; it may be possible to find, or even derive, a more stable leading indicator to use as input data and hence improve the general applicability of the procedure under a wider range of more volatile conditions.

In those cases where the underlying input data were predictable with an exponential smoothing forecasting model, the smoothed error tracking signal T_t was found to be extremely responsive. For example, when the simulated data reported in Figure 1 were subject to a step change of approximately 5 per cent of the mean level in the cases where the underlying mean level of the data was stationary or displaying a linear trend, the tracking signal "tripped" within one time period of the introduction of the step "shock." When, again, simulated data were subject to a "ramp" change of approximately 2 per cent in the underlying mean input data, the tracking signal "tripped" within two time periods, usually within one time period. When the real small firm data from the author's own area were used, the method proved to be highly robust and "tripped" within two time periods (Figure 2). These data were exhibiting a ramp change and were the most predictable of the four cases mentioned.

The idea of monitoring the commercial marketing health of small firms using some form of monitoring device or "tracking signal" seems to work well in principle for a wide range of situations where the underlying input data are time-dependent enough to be predicted by an established exponential forecasting model. In cases where predictability is impossible with such a model because of the chaotic nature of the underlying input data, the procedure fails and this is its main limitation. However, some derivative or composite of sales, or wider parameters, would probably provide more valuable and more responsive input data.

Given the importance of being able to monitor the commercial health of the SME so that crisis points can be identified before they occur, and thus remedial action can be considered and taken, we argue that our proposal merits further attention and development. Despite the limitations this method does have two strong points in its favour, namely:

1 An exponential smoothing forecasting system, coupled with a monitoring procedure in the form of the smoothed error tracking signal, is a suitable and robust procedure and one suited to the range of data characteristics found in commercial data.
2 The use of more sophisticated performance measures by potentially reducing the variability of the chosen leading indicator should reduce the main limitation of the procedure discussed above. The potential benefits in terms of improved survival and growth for the SME are sufficiently enticing to continue the search.

Conclusion

This article has suggested that a relatively simple but robust procedure is available to help the typical SME manage and interpret key performance data. Given that SMEs are vulnerable to failure, pressured by external market forces, and may well have to contend with turbulent and dangerous life cycles, we hope that such a procedure could help to contribute to a longer life span. The method we have chosen, and discussed, is not without problems but results so far are encouraging enough for further refinement to be undertaken.

References

Batty, M. (1969), "Monitoring an exponential smoothing forecasting system", *Operational Research Quarterly*, Vol. 20 No. 3, pp. 319-25.

Bauman, W.S. (1965), "The less popular stocks versus the most popular stocks", *Financial Analysis Journal*, Vol. 21, pp. 61-9.

Brown, R.G. (1962), reported in Brown. R.G. (1963), *Smoothing, Forecasting and Prediction of Discrete Time Series*, Prentice-Hall, New York, NY.

Churchill, N. and Lewis, V. (1983), "The five stages of small business growth", *Harvard Business Review*, Vol. 61 No. 3, pp. 30-50.

Cressy, D. (1996), "Small business failure: failure to fund or failure to learn by doing", *ICSB 41st World Conference*, Stockholm, pp. 385-410.

Department of Trade and Industry (1999), reported in *Quarterly Report on Small Business Statistics*, July, Domestic Finance Division, Bank of England, London, p. 35.

Eggers, J.H., Leahy, T. and Churchill, N. (1994), "Stages of small business growth revisited: insights into growth path and leadership/ management skills in low- and high-growth companies", *Frontiers of Entrepreneurship Research*, Babson College, Boston, MA, pp. 131-44.

Gardner, E.S. (1983), "Automatic monitoring of forecasting errors", *Journal of Forecasting*, Vol. 2, pp. 1-21.

Geurts, M.D. and Ibrahim, I.B. (1975), "Comparing the Box-Jenkins approach with the exponentially smoothed forecasting model – application to Hawaii tourists", *Journal of Marketing Research*, Vol. 12, pp. 182-8.

Hanks, S.H., Watson, C.J., Jansen, E. and Chandler, G. (1993), "Tightening the life-cycle construct: a taxonomic study of growth stage configurations in high technology organizations", *Entrepreneurship Theory and Practice*, Winter, pp. 5-29.

Hogarth, R.M. (1975), "Cognitive processes and assessment of subjective probability distributions", *Journal of the American Statistical Association*, Vol. 70, pp. 271-90.

Holt, C.C. (1957), "Forecasting seasonals by exponentially weighted moving average", *Office of Naval Research Memo*, No. 52.

Mabert, V.A. (1975), "Statistical versus salesforce executive opinion short-range forecasts: a time series analysis case study", working paper, Krannert Graduate School, Purdue.

McKenzie. E. (1978), "The monitoring of exponentially weighted forecasts", *The Journal of the Operations Research Society*, Vol. 29, May, pp. 449-58.

Makridakis, S. and Hibon, M. (1979), "Accuracy of forecasting: an empirical investigation", *Journal of the Royal Statistical Society* (A), Vol. 142 Part 2, pp. 97-145.

Makridakis, S. and Wheelwright, S.C. (1989), *Forecasting Methods for Management*, 5th ed., John Wiley & Sons, Chichester.

Montgomery, D.C. and Johnson, L.A. (1976), *Forecasting and Time Series Analysis*, McGraw-Hill, New York, NY.

NatWest SBRT Quarterly Survey of Small Business (1999), reported in *Quarterly Report on Small Business Statistics*, July, Domestic Finance Division, Bank of England, London, p. 18.

Newbould, P. (1974), "The exact likelihood function for a mixed autoregressive moving average process", *Biometrika*, Vol. 61, pp. 423-6.

Reid, D.J. (1969), "A comparative study of time series prediction techniques on economic data", PhD thesis, University of Nottingham, October, p. 80.

Reynolds. P.L. (1986), "The monitoring of short term sales forecasts produced by exponential smoothing", MSc dissertation, Manchester School of Management, UMIST, pp. 130-42.

Reynolds, P.L. and Day, J. (1996), "A semi-automatic scheme for monitoring the adequacy of short-term sales Forecasts", Marketing Science Conference, University of Florida, Gainesville, FL.

Reynolds, P.L. and Greatorex, M. (1988), "The monitoring of short-term forecasts using exponential smoothing", *Proceedings of the 21st Annual Conference Marketing Education Group*, pp. 558-86.

Reynolds, P.L., Richards, G. and Day, J. (1995), "Marketing management for the SME – a novel approach to monitoring and control", 1995 MEG Conference, University of Bradford, Bradford.

Sarbin, T.R. (1943), "Contributions to the study of actuarial and individual methods of prediction", *American Journal of Sociology*, Vol. 48, pp. 593-602.

Scott, M. and Bruce, R. (1987), "Five stages of growth in small business," *Long Range Planning*, Vol. 20 No. 10, pp. 45-52.

Slovic, P. (1972), "Psychological study of human judgement: implications for investment decision making", *Journal of Finance*, Vol. 27, pp. 779-99.

Storey, D. (1994), *Understanding the Small Business Sector*, International Thompson Business Press, London.

Trigg, D.W. (1964), "Monitoring a forecasting system", *Operational Research Quarterly*, Vol. 15 No. 3, pp. 271-4.

Trigg, D.W. and Leach, A.G. (1967), "Exponential smoothing with an adaptive response rate", *Operational Review Quarterly*, Vol. 18 No. 1, pp. 53-9.

Winter, P.R. (1960), "Forecasting sales by exponentially weighted moving averages", *Management Science*, Vol. 6, pp. 271-4.

Witt, S.F. and Witt, C.A. (1992), *Modelling and Forecasting Demand in Tourism*, Academic Press, London.

Appendix. Technical details

For *Simple Exponential Smoothing* the one step ahead forecast produced in current time is denoted by F_{t+1} and the actual current demand value X_t. Using this we get:

$$F_{t+1} = \alpha X_t + \alpha(1-\alpha)X_{t-1} + \alpha(1-\alpha)^2 X_{t-2} + \alpha(1-\alpha)^3 X_{t-3} \text{ etc.} \tag{1}$$

Transcribing the equation for F_{t+1} into F_t by subtracting 1 from all the subscripts, we obtain:

$$F_{t+1} = \alpha X_{t-1} + \alpha(1-\alpha)X_{t-2} + \alpha(1-\alpha)^2 X_{t-3} + \alpha(1-\alpha)^3 X_{t-4} \text{ etc.} \tag{2}$$

If the equation is rewritten as:

$$F_{t+1} = \alpha X_t + (1-\alpha)[\alpha X_{t-1} + \alpha(1-\alpha)X_{t-2} + \alpha(1-\alpha)^2 X_{t-3} \text{ etc.} \tag{3}$$

It can be seen then that the equation for F_t is exactly the same as that which appears in brackets in the equation for F_{t+1}.

Substituting F_t for this we obtain:

$$F_{t+1} = \alpha X_t + (1-\alpha)f_T \tag{4}$$

This is the basic equation defining a simple exponentially weighted moving average given by Holt (1957), and from which all other models of exponential smoothing derive. More correctly, the process is a geometrically weighted moving average, the exponentially weighted moving average being its analogue in continuous time (see Reid, 1969).

For *Holt's Two-Parameter Linear Exponential Smoothing* the forecast is formed by using two smoothing coefficients, $0 \leq \alpha \leq 1$ for the original series, and $0 \leq \beta \leq 1$ for the trend. The updating equations for Holt's (1957) method are:

$$\text{LEVEL } S_t = \alpha X + (1-\alpha)(S_{t-1} + Z_{t-1}) \tag{1}$$

$$\text{TREND } Z_t = \beta(S_t - S_{t-1}) + (1-\beta)Z_{t-1} \tag{2}$$

$$\text{FORECAST } F_{t+m} = S_t + mZ_t \tag{3}$$

where S_t is the level in time "t", Z_t is the trend component and F_{t+m} is the forecast

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produced in time "t" for "m" periods ahead of current time. Basically the trend Z_t is multiplied by the number of periods ahead to be forecast m, and added to the value of the level S_t.

Winter's (1960) Three-Parameter Linear and Seasonal Exponential Smoothing model is an extension of Holt's (1957) linear model, in that it includes an extra equation that is used to estimate seasonality. The updating equations for the Winter model are given by Makridakis and Wheelwright (1989, p. 98) as follows:

$$\text{OVERALL SMOOTHING } S_t = \alpha(X_t/I_{t-L}) + (1-\alpha)(S_{t-1}+Z_{t-1}) \quad (1)$$

$$\text{TREND } Z_t = \varepsilon(S_t - S_{t-1}) + (1-\varepsilon)Z_{t-1} \quad (2)$$

$$\text{SEASONALITY } I_t = \beta(X_t/S_t) + (1-\beta)I_{t-L} \quad (3)$$

$$\text{FORECAST } F_{t+m} = (S_t + mZ_t)I_{t-L+m} \quad (4)$$

where L is the length of seasonality (e.g. number of months or quarters in the year etc.), Z_t is the trend component, I_t is the seasonal adjustment factor, and F_{t+m} is the forecast for m periods ahead: α, ε, and β are the smoothing coefficients for overall smoothing, trend and seasonal components respectively.

The overall smoothing equation (1) differs slightly from Holt's equation (1) in that the first term is divided by the seasonal number I_{t-L} which adjusts X_t for seasonality by reviewing the seasonal effects which may exist in X_t. The estimate of seasonality, calculated with equation (3), is given as an index fluctuating around 1. The seasonal index is a ratio of the current value of the series X_t divided by the current single smoothed value for the series S_t. If X_t is greater than S_t, the ratio will be greater than 1, while if X_t is less than S_t the ratio will be less than 1. S_t is a smoothed average value of the series that does not include any seasonality, the values of X_t contain both seasonality and any randomness in the series. To smooth out this randomness, equation (3) weighs the newly computed seasonal factor X_t/S_t, with β and the most recent seasonal number corresponding to the same season with (1–β). This prior seasonal factor was computed in period $_{t-L}$, where L is the length of seasonality.

The form of equation (3), used to calculate the seasonal component, is similar to that of other smoothing equations; there is a value, in this case the ratio X_t/S_t, which is multiplied by a smoothing coefficient β and is then added to its previous smoothed estimate which has been multiplied by (1–β). Equation (2) used for the smoothing the trend is exactly the same as Holt's trend equation (2) discussed earlier. Equation (4), used to produce the forecast in Winter's model, is the same as the corresponding formula used to produce a forecast in Holt's (1957) model (equation (3) for Holt), except that the estimate for the future period $_{t+m}$ is multiplied by I_{t-L+m}. In the equation for overall smoothing (1) X_t was divided by I_{t-L} to remove any seasonal effects that may exist in X_t. Multiplying the value of $(S_t + mZ_t)$ by I_{t-L+m} in equation (4) readjusts the forecast for seasonality by reintroducing seasonal effects into the forecast. The updating equations for the smoothed error tracking signal are given by Trigg (1964, p. 272) as follows:

Smoothed Error = (1–α) previous smoothed error + α latest error.
Mean Absolute Deviation (MAD) = (1–α) previous MAD + αlatest absolute error.
Tracking Signal = Smoothed Error/MAD.

These updating equations are expressed in a more concise form by Gardner (1983, p. 10) as:

$$E_t = (\alpha e_t + (1-\alpha)E_{t-1} \quad (1)$$

$$\mathrm{MAD}_t = \alpha|e_t| + (1-\alpha)\mathrm{MAD}_{t-1} \quad (2)$$

$$T_t = E_t/\mathrm{MAD}_t \quad (3)$$

where t represents current time, E_t is the smoothed error, e_t is the present error, MAD_t is the mean absolute deviation, $|e_t|$ is the modulus or absolute value of the forecast error, T_t is the smoothed error tracking signal, and α is the smoothing coefficient $0 \le \alpha \le 1$.

Application questions

1 What are the particular crises, at different stages of its life, that a small firm must most critically deal with?

2 How effective would this procedure be for autonomous business units in a larger organization?