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Application of the Mutual Information to the Zn Gauge Model

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We introduce the mutual information to the Zn lattice gauge theory for testing the degree of correlations. It is shown that our simulation result for the plaquettes gives a clear peak on the critical point.

1. Introduction

When we know the Hamiltonian of a system, we can calculate a physical quantity of the system using a Monte Calro method. We will apply the method to a system of elementary particles.

The method proposed by Wilson [1] is as follows. With a path integral theory an expectation value of operator F is written as

$$< F >= \frac{\int \mathcal{D}A F[A] \exp\{i\mathcal{S}[A]\}}{\int \mathcal{D}A \exp\{i\mathcal{S}[A]\}}, \tag{1.1}$$

where $\mathcal{S}$ is an action. Regarding the time variable as an imaginary one, and to look for a Lagrangian on a time-space lattice, Eq. (1.1) translates

$$< F >= \frac{\sum_A F[A] \exp\{-\mathcal{S}[A]\}}{\sum_A \exp\{-\mathcal{S}[A]\}}. \tag{1.2}$$

It means that the local field follows a canonical distribution. It is known that there is a phase transition in the system of gluon or photon. A new signal of the transition has been investigated by many physicists.

On the other hand, Matuda verified that a mutual information [3] is a good signal [2] to find a critical point of the Ising system. So, we investigate the validity of the mutual information to the lattice gauge theory.

This report is organized as follows. We introduce Wilson's formulation of the gauge fields on a space-time lattice in section 2. The mutual information

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is explained in section 3. And our result on the mutual information for lattice gauge theory by the Monte Calro simulation is shown in section 4. Section 5 is devoted to discussion.

2. Z_n Lattice Gauge Model

We work on the space-time lattice with lattice spacing a. For each pair of nearest-neighbor sites i and j we have a gauge or link variable U_{ij} in the group $\mathrm{Z_n}$,

$$U_{ij} \in \left\{ \exp\left(\frac{2\pi i}{a g_0 N} n\right) \mid n = 1, \ldots, N \right\}, \tag{2.1}$$

where g_0 stands for the coupling constant. The dynamics of this gauge system follows from the action,

$$\mathcal{S} = \sum_{\square} \mathcal{S}_{\square}. \tag{2.2}$$

The sum in Eq. (2.2) is over all elementary squares or so called plaquettes which contribute

$$\beta\{1 - \mathrm{Re}(U_{ij}U_{jk}U_{kl}U_{li})\} \tag{2.3}$$

where the site i, j, k, and l circulate around the square. When a lattice spacing a tends to 0, the action in Eq. (2.2) becomes

$$\mathcal{S}_{\mathrm{gauge}} = \int d^4x \frac{1}{4} F_{\mu\nu} F^{\mu\nu} \tag{2.4}$$

for a U(1) gauge system, where $F_{\mu\nu}$ is field strength,

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu, \tag{2.5}$$

under the condition,

$$\beta = \frac{2}{g_0^2}. \tag{2.6}$$

3. Mutual Information

Information is defined as

$$S = -\sum_i P_i \ln P_i, \tag{3.1}$$

where P_i stands for a probability of state i. When the system is not deterministic, that is to say all the probability fill with

$$P_1 =, \ldots, P_n = \frac{1}{n}, \tag{3.2}$$

the information becomes maximum. When the system is deterministic, i.e., the probability of the system fill with

$$P_1 = 1, P_2, \ldots, P_n = 0, \tag{3.3}$$

the information becomes 0.

To consider the amount of the information, the mutual information [3] is defined as

$$I_M = \sum_{ij} P_{ij} \ln P_{ij} - \sum_i P_i \ln P_i - \sum_j P_j \ln P_j, \tag{3.4}$$

where P_{ij} stands for a probability of state i on a system 1, and of state j on a system 2.

The mutual information behave as follows. If there is no correlation between the system 1 and 2, I_M becomes 0. If two system are correlated, I_M becomes finite value.

4. Result

For simplicity, we choose the Z_2 gauge model. But it is a non-trivial model for the reasons it has a respectable phase transition like U(1) gauge theory. We show the average plaquette P versus β in fig. 1 for the Z_2 group on a four dimensional lattice with lattice size $L = 4$. This figure tells us that a phase transition is occurred at $\beta \simeq 0.47$ in the system by the discontinuity in P.

We calculate the temporal mutual information I_M^t for the plaquette, which is gauge invariant. We choose two system which is far separated from each other in MC time τ on each. I_M^t versus β is drawn in fig. 2 for $\tau = 1$. It shows that I_M^t has a maximum value at a critical point β_c. We checked that I_M^t has a maximum at β_c for any values of n. There is the reason why I_M^t has a maximum value on the critical point next. If β is smaller than β_c, the local state has almost random. So, even if the information of each system is large, the mutual information is small. If β is lager than β_c, the local state has almost the same value, and freezes. In such a case the mutual information is small. On the critical point the local state shows complex behavior.

5. Conclusion

Using a Z_n lattice gauge model we checked that I_M^t is an effective method for finding a critical point in lattice gauge theory. I_M^t applied to the plaquette, which is gauge invariant, is a good signal to find the critical point. But I_M^t applied to a link variable, which is not a gauge invariant quantity, there is no guarantee I_M^t has a maximum at β_c.

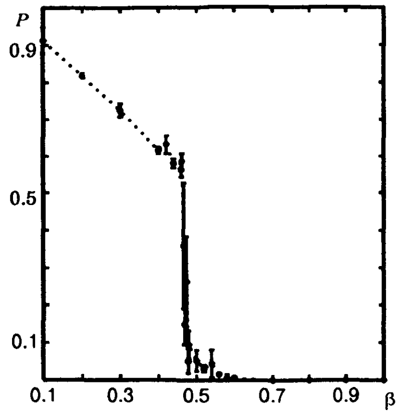

fig. 1. The average of plaquette versus β.

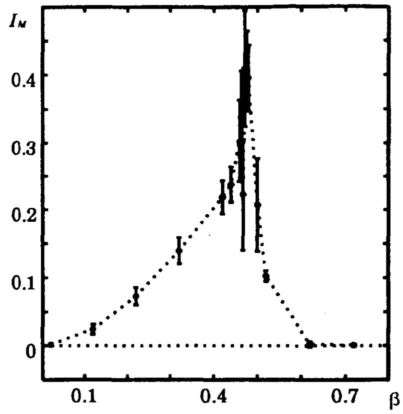

fig. 2. I_M^t versus β.

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