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Notes on Supersymmetric Connes's Model

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Abstract

We introduce an example of noncommutative geometry-like supersymmetric standard model. We give a spectral triple based on Minkowskian superspace. Its Dirac operator fluctuated by its algebra gives supersymmetric matter kinematic term and mass term of the matter fields. The squared Dirac operator gives gauge kinematic terms and Higgs kinematic terms. Traceless condition of the gauge kinematic term gives every matter particle a hypercharge identical to that of standard model. Coupling constants of gauge symmetries coincide like as nonsupersymmetric noncommutative theory. We show mathematically unsettled problems of the model which we will overcome in a short span of time.

keyword: noncommutative geometry, supersymetric standard model, spectral triple

1 introduction

The standard model of elementary particles has some shortcomings[1]. Firstly, it has many free parameters, masses of particles, gauge coupling constants, mixing angles, coefficients of Higgs potential. Why on earth is the gauge symmetry $SU(3) \times SU(2) \times U(1)$? Secondly, there exists so-called hierarchy problem. The vacuum expectation value of Higgs field gives masses of particles, quarks,leptons, gauge particles and Higgs particle itself. One-loop corrections to the Higgs squared mass parameter m_H^2 include the square of very large energy scale Λ_{UV}, at which new physics enters to alter the high-energy behavior of the theory, so that masses of low energy scale directly or indirectly undergo fine tuning from the huge energy scale, for an example Plank scale[2].

Alain Connes and his co-workers proposed a method to describe the standard model in terms of noncommutative geometry(NCG)[3][4][5]. It is a full geometric description for interaction of elementary particles. The metric to determine the geometry is given by Dirac operator that is the denominator of Green function for matter fields and both gauge and Higgs fields are introduced by internal fluctuations of the operator. They have shown that if an involutive algebra $\mathcal{A}$ satisfies the conditions as follows:
1. A chirality operator γ and an antilinear isometry J exist,

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2. Relations between the above operators are given by

$$J^2 = \varepsilon, \tag{1}$$
$$JD = \varepsilon' DJ, \tag{2}$$
$$J\gamma = \varepsilon'' \gamma J,\ \gamma \mathcal{D} = -\mathcal{D}\gamma \tag{3}$$

whrere $\varepsilon, \varepsilon', \varepsilon'' = \{\pm 1\}^3$,

$$[a, b^0] = 0, \quad \forall a, b \in \mathcal{A},\ b^0 = Jb^* J^{-1} \text{ (order zero condition)}, \tag{4}$$
$$[[\mathcal{D}, a], b^0] = 0, \quad \forall a, b \in 1 \otimes \mathcal{A}_F \subset \mathcal{A} \text{ (order one condition)}, \tag{5}$$

the only possible gauge symmetry that $\mathcal{A}$ realizes is $U(3) \times SU(2) \times U(1)$[6]. While the three coupling constants of gauge symmetries in the usual standard model are independent of each other and even if they evolve according to renormalization group equations, they do not unify at any energy scale, they in the NCG model coincide like as SU(5) grand unified theory. On the top of that, the spectral action principle in curved space-time gives the Lagrangian of gravity as well as gauge fields coupled to matter[7].

In spite of these advantages, even in the NCG model, the hierarchy problem will not be solved unless it is extended to incorporate supersymmetry. In the supersymmetric standard model, there exist two Higgs supermultiplets and Higgs bosons accompany higsinos as their superpartners. Every loop correction of Higgs boson which contributes to Λ_{UV}^2 accompanies higgsino loop correction with the same size and with an opposite sign so that the leading term diverges like as logarithm of Λ_{UV} at most and we need not the fine tuning. This merit must inhabit supersymmetric NCG models, too.

In this paper, we use notations of supersymmetry given by Julius Wess and Jonathan Bagger[8]. We show the prescription to extend the Connes's style of construction to that of supersymmetric particle models, which do not have the hierarchy problem. We apply the NCG method of supersymmetric version to obtain an example model of minimum supersymmetric model(MSSM) in flat space. This model was given by S.Ishihara, H.Kataoka, A.Matsukawa, H.Sato and M.Shimojo. We describe some mathematical problems which make the model not full geometric but geometry-like. They will be soon overcome by an another model in [9].

2 The prescription for construction NC supersymmetic model

The local geometry of a manifold is determined by metric $g_{\mu\nu}$ or infinitesimal length element ds. In the NCG construction, the element ds is represented by the Dirac propagator $\mathcal{D}^{-1}$, and the noncommutative geometry for the standard model consists of "spectral triple", $(\mathcal{A}, \mathcal{H}, \mathcal{D})$, where $\mathcal{A}$ is an involutive algebra and $\mathcal{D}$ is a Dirac operator, both of which act on Hilbert space $\mathcal{H}$. We consider the product of the space-time manifold M by the finite geometry F and we have $\mathcal{A} = \mathcal{A}_M \otimes \mathcal{A}_F$, $\mathcal{H} = \mathcal{H}_M \otimes \mathcal{H}_F$. In order to extend the geometry to the supersymmetric version, we consider that M is the superspace$(x_\mu, \theta_\alpha, \overline{\theta}_{\dot{\alpha}})$ and that both the algebra $\mathcal{A}_M$ and the space $\mathcal{H}_M$ consist of chiral and antichiral superfields,

$$\mathcal{A} = (\mathcal{A}_+ \otimes \mathcal{A}_F) \oplus (\mathcal{A}_- \otimes \mathcal{A}_F), \tag{6}$$
$$\mathcal{H} = (\mathcal{H}_+ \otimes \mathcal{H}_F) \oplus (\mathcal{H}_- \otimes \mathcal{H}_F), \tag{7}$$

where

$$\mathcal{A}_+ = \{a_+ \in \mathcal{A}_M | \overline{D}_{\dot{\alpha}} a_+ = 0\}, \ \ \mathcal{H}_+ = \{x \in \mathcal{H} | \overline{D}_{\dot{\alpha}} x = 0\}, \tag{8}$$

$$\mathcal{A}_- = \{a_- \in \mathcal{A}_M | D_\alpha a_- = 0\}, \ \ \mathcal{H}_- = \{x \in \mathcal{H} | D_\alpha x = 0\}, \tag{9}$$

In order for a four component Dirac spinor to correspond to an element of the the space $\mathcal{H}$, we express a matter superfield as

$$\Psi = \begin{pmatrix} X \\ \Xi^* \end{pmatrix}, \tag{10}$$

where the chiral superfield X has χ_α as a fermionic component which describes the left-handed spinor and the antichiral superfield Ξ has $\overline{\xi}^{\dot{\alpha}}$ which is the right-handed spinor,

$$X = \varphi + i\theta\sigma^\mu\overline{\theta}\partial_\mu\varphi + \frac{1}{4}\theta\theta\overline{\theta\theta}\Box\varphi + \sqrt{2}\theta\chi - \frac{i}{\sqrt{2}}\theta\theta\partial_\mu\chi\sigma^\mu\overline{\theta} + \theta\theta F_\chi, \tag{11}$$

$$\Xi^* = \phi^* - i\theta\sigma^\mu\overline{\theta}\partial_\mu\phi^* + \frac{1}{4}\theta\theta\overline{\theta\theta}\Box\phi^* + \sqrt{2}\overline{\theta\xi} + \frac{i}{\sqrt{2}}\overline{\theta\theta}\sigma^\mu\partial_\mu\xi + \overline{\theta\theta}F^*_\xi. \tag{12}$$

The Dirac operator consists of $\mathcal{D}_M$ given by supersymmetric covariant derivatives D_α, $\overline{D}_{\dot{\alpha}}$ and $\mathcal{D}_F$ given by mass matrix m. The mass dimension of $\mathcal{D}_M$ and $\mathcal{D}_F$ shoud be one, since $ds = \mathcal{D}^{-1}$. Using operators $\mathcal{D}_0$, $\mathcal{D}_1$ defined by

$$\mathcal{D}_0 = -\frac{1}{4}(D^\alpha D_\alpha + \overline{D}_{\dot{\alpha}}\overline{D}^{\dot{\alpha}}) = -\frac{1}{4}(DD + \overline{DD}), \tag{13}$$

$$\mathcal{D}_1 = -\frac{1}{4}\overline{\eta}^{\dot{\alpha}}\eta^\alpha(\overline{D}_{\dot{\alpha}}D_\alpha + D_\alpha\overline{D}_{\dot{\alpha}}) = \overline{\eta}^{\dot{\alpha}}\eta^\alpha D_{\alpha\dot{\alpha}} = \frac{i}{2}(\eta\sigma^\mu\overline{\eta})\partial_\mu, \tag{14}$$

these operators of our example are given as follows:

$$\mathcal{D} = \mathcal{D}_M + \mathcal{D}_F, \tag{15}$$

$$\mathcal{D}_M = \mathcal{D}_0 \otimes \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + \mathcal{D}_1 \otimes \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \ \ \mathcal{D}_F = 1 \otimes \begin{pmatrix} 0 & m^* \\ m^T & 0 \end{pmatrix}, \tag{16}$$

where σ^μ is Pauli matrix, η^α,$\overline{\eta}^{\dot{\alpha}}$ are elements which obey Clifford algebra:

$$\eta^\alpha\eta^\beta - \eta^\beta\eta^\alpha = 2\varepsilon^{\alpha\beta}, \tag{17}$$

$$\overline{\eta}_{\dot{\alpha}}\overline{\eta}_{\dot{\beta}} - \overline{\eta}_{\dot{\beta}}\overline{\eta}_{\dot{\alpha}} = 2\varepsilon_{\dot{\alpha}\dot{\beta}}, \tag{18}$$

$$\eta^\alpha\overline{\eta}_{\dot{\beta}} - \overline{\eta}_{\dot{\beta}}\eta^\alpha = 0. \tag{19}$$

Here, we define a mapping C from chiral superfield X to antichiral superfield X^* and an operator $\mathcal{J}$ for Ψ as

$$\mathcal{J} = \begin{pmatrix} 0 & C \\ C & 0 \end{pmatrix}. \tag{20}$$

We introduce the real structure J for the basis $(\Psi, \Psi^c)^T$ as follows:

$$J = \begin{pmatrix} 0 & \mathcal{J} \\ \mathcal{J} & 0 \end{pmatrix}, \tag{21}$$

This real structure commutes with the Dirac operator $\mathcal{D}$.

For the basis $(\Psi, \Psi^c)^T$ of $\mathcal{H}$, the Dirac operator is expressed by

$$D = \begin{pmatrix} \mathcal{D}_M + \mathcal{D}_F & 0 \\ 0 & \mathcal{D}_M + \mathcal{D}_F^c \end{pmatrix}, \tag{22}$$

where

$$\begin{aligned} \mathcal{D}_F^c &= \mathcal{J}\mathcal{D}_F\mathcal{J}^{-1} \\ &= \begin{pmatrix} 0 & Cm^T C^{-1} \\ Cm^* C^{-1} & 0 \end{pmatrix} = \begin{pmatrix} 0 & m^\dagger \\ m & 0 \end{pmatrix}. \end{aligned} \tag{23}$$

In order to obtain the action with gauge symmetry, we consider the internal automorphism of the algebra $\mathcal{A}$. The left and right action of the automorphism on the superfield of (10) are elements of unitary group and expressed by

$$X' = e^{-i\Lambda_\chi^L} X, \quad X' = X e^{-i\Lambda_\chi^{RT}} = e^{-i\Lambda_\chi^R} X = C e^{i\Lambda_\chi^{R*}} C^{-1} X, \tag{24}$$

$$\Xi'^* = e^{i\Lambda_\xi^{L*}} \Xi^*, \quad \Xi'^* = \Xi^* e^{i\Lambda_\xi^{R\dagger}} = C e^{-i\Lambda_\xi^R} C^{-1} \Xi^* = e^{i\Lambda_\xi^{R*}} \Xi^*. \tag{25}$$

When we denote u as

$$u = \begin{pmatrix} e^{-i\Lambda_\chi^L} & 0 \\ 0 & e^{i\Lambda_\xi^{R*}} \end{pmatrix}, \tag{26}$$

the whole automorphism is expressed by

$$\begin{aligned} \Psi' &= u\mathcal{J}\overline{u}\mathcal{J}^{-1}\Psi \\ &= \begin{pmatrix} e^{-i\Lambda_\chi^L} e^{-\Lambda_\chi^R} & 0 \\ 0 & e^{i\Lambda_\xi^{R*}} e^{i\Lambda_\xi^{L*}} \end{pmatrix} \Psi \\ &= \begin{pmatrix} e^{-i\Lambda_\chi} & 0 \\ 0 & e^{i\Lambda_\xi^*} \end{pmatrix} \Psi \equiv U\Psi, \end{aligned} \tag{27}$$

where Λ_χ,Λ_ξ are algebras of the unitary groups, and we put $\Lambda_\chi = \Lambda_\chi^L + \Lambda_\chi^R$, $\Lambda_\xi = \Lambda_\xi^L + \Lambda_\xi^R$.

Under the automorphism, the replacement of the operator $\mathcal{D}$ is given by

$$\begin{aligned} \tilde{\mathcal{D}} &= U^\dagger \mathcal{D} U \\ &= U^\dagger \mathcal{D}_M U + U^\dagger \mathcal{D}_F U \\ &= \tilde{\mathcal{D}}_M + \tilde{\mathcal{D}}_F. \end{aligned} \tag{28}$$

With the fluctuated operator $\tilde{\mathcal{D}}_M$, the kinematic terms of matter superfields which interact with gauge field are given by

$$\begin{aligned} \mathcal{L}_{matter,gauge} &= \Psi^\dagger \tilde{\mathcal{D}}_M \Psi|_{\theta\theta,\overline{\theta\theta},\eta\overline{\eta}\to 0} \\ &= -\frac{1}{4} X^\dagger D D e^{V_\chi} X|_{\overline{\theta\theta}} - \frac{1}{4} \Xi^T \overline{DD} e^{V_\xi^T} \Xi^*|_{\overline{\theta\theta}}, \end{aligned} \tag{29}$$

where the both of V_χ and V_ξ are vector superfields defined by

$$e^{V_s} = e^{i\Lambda_s^\dagger} e^{-i\Lambda_s} \ (s = \chi, \xi). \tag{30}$$

In the Non-supersymmetric NCG, kinematic terms of gauge bosons and Higgs bosons are given by $Tr(f(\mathcal{D}^2/m_0^2))$, where f is a positive function. The calculation for the trace is carried out by heat kernel expansion[7]. In our example, the kinematic terms of superfields which include these bosons are simply given by the trace of squared Dirac operator. The gauge kinematic term is expressed as follows:

$$\mathcal{L}_{gauge} = \frac{1}{4} f_0 Tr\langle (K\tilde{\mathcal{D}}_M)^2 \rangle_{\theta\theta\overline{\theta}\overline{\theta},\eta\overline{\eta}\to 0}, \tag{31}$$

where

$$\begin{aligned} K &= (U^\dagger U)^{-1} \\ &= \begin{pmatrix} e^{-V_\chi} & 0 \\ 0 & e^{-V_\xi^T} \end{pmatrix}, \end{aligned} \tag{32}$$

and f_0 is an overall constant coefficient.

The fluctuation of $\mathcal{D}_F$ gives rise to superfield $\tilde{H}$ which includes Higgs scalar field as its bosonic component and is expressed as

$$\begin{aligned} \tilde{\mathcal{D}}_F &= U^\dagger \mathcal{D}_F U \\ &= \begin{pmatrix} 0 & \tilde{H}^* \\ \tilde{H}^T & 0 \end{pmatrix}, \end{aligned} \tag{33}$$

where

$$\tilde{H} = e^{-i\Lambda_\chi^T} m e^{-i\Lambda_\xi}. \tag{34}$$

The interaction term between this superfield and the matter superfields are given by

$$\mathcal{L}_{matter,Higgs} = \Psi^\dagger \tilde{\mathcal{D}}_F \Psi = X^\dagger \tilde{H}^* \Xi^* + \Xi^T \tilde{H}^T X, \tag{35}$$

which constitute superpotential of this theory.

While the squared $\tilde{\mathcal{D}}_M$ represents the interaction between gauge field and matter field, the squared $\tilde{\mathcal{D}}_F$ gives the Higgs kinematic term which includes the interaction with gauge field. It is expressed by

$$\mathcal{L}_{Higgs,gauge} = \frac{1}{2} Tr\langle (K\tilde{\mathcal{D}}_F)^2 \rangle_{\theta\theta\overline{\theta}\overline{\theta}} = Tr(e^{-V_\xi^T} \tilde{H}^T e^{-V_\chi} \tilde{H}^*). \tag{36}$$

While the fluctuation of Dirac operator(28) has been defined to keep it selfadjoint, $K\mathcal{D}$ in (31) and (36) is determined to respect gauge invariance. In fact, under the gauge transformation $U' = UV$, $Tr(K'\mathcal{D}'K'\mathcal{D}') = Tr(K\mathcal{D}K\mathcal{D})$, since the space-time derivatives in the trace are covariant derivatives or total derivatives.

3 NC geometry-like MSSM

3.1 Matter kinematic term and Hypercharge

Let us be engaged in the description of minimal supersymmetric standard model of noncommutative geometry-like version. We let the space $\mathcal{H}_F$ be the space of superfields each of which contains a matter fermion as spinor component. Each generation of fermions has dimension (15+15), where 15=12+3. The 12 corresponds to left- and right-handed quarks with gauge color degrees of freedom. The 3 corresponds to leptons and the other 15 to antiparticles.

In the quark sector, the chiral superfield X in (10) corresponds to one of left-handed quarks which is expressed as

$$\chi_{q\alpha} = \begin{pmatrix} u_{L\alpha} \\ d_{L\alpha} \end{pmatrix} \otimes 3, \tag{37}$$

where 3 represent the color gauge degrees of freedom. On the other hand, the antichiral superfield Ξ^* corresponds to right-handed quarks,

$$\bar{\xi}^{\dot{\alpha}} = \bar{\xi}^{\dot{\alpha}}_{u_R} \otimes 3,\ \bar{\xi}^{\dot{\alpha}}_{d_R} \otimes 3. \tag{38}$$

In the lepton sector, the chiral superfield X corresponds to a weak isospin multiplet $(\nu, e)_L$ and Ξ^* corresponds to right-handed electron-like fields, e_R,

$$\chi^\alpha = \chi^\alpha_l = \begin{pmatrix} \chi^\alpha_{\nu_L} \\ \chi^\alpha_{e_L} \end{pmatrix}, \bar{\xi} = \bar{\xi}^{\dot{\alpha}}_{e_R}. \tag{39}$$

In order to describe the internal automorphism of $\mathcal{A}$ which generates gauge symmetry, we adopt a notation as follows: for the fundamental and its complex conjugate representation of $U(3) \cong SU(3) \otimes U(1)$ gauge symmetry, we take

$$m_+ = e^{-i\frac{\lambda_i}{2}\beta_i - i\gamma'},\ m_- = e^{-i\frac{\lambda_i}{2}\beta^*_i - i\gamma'^*}, \tag{40}$$

as well as

$$m^*_- \equiv (m_+)^* = e^{i\frac{\lambda^*_i}{2}\beta^*_i + i\gamma'^*}, m^*_+ \equiv (m_-)^* = e^{i\frac{\lambda^*_i}{2}\beta_i + i\gamma'}, \tag{41}$$

$$m^T_+ = e^{-i\frac{\lambda^T_i}{2}\beta_i - i\gamma'} = (m^*_-)^\dagger,\ m^T_- = e^{-i\frac{\lambda^T_i}{2}\beta^*_i - i\gamma'^*} = (m^*_+)^\dagger, \tag{42}$$

where λ_i are Gell-Mann matrices and β_i and γ' are chiral scalar superfields. For the representation of $SU(2)$, we let

$$w_+ = e^{-i\frac{\tau_i}{2}\alpha_i},\ w_- = e^{-i\frac{\tau_i}{2}\alpha^*_i},\ w^\dagger_- \equiv (w_+)^\dagger = e^{i\frac{\tau_i}{2}\alpha^*_i}, \tag{43}$$

and For $U(1)$,

$$\lambda_+ = e^{-i\gamma},\ \lambda_- = e^{-i\gamma^*},\ \lambda^*_+ = (\lambda_-)^* = e^{i\gamma},\ \lambda^*_- = (\lambda_+)^* = e^{i\gamma^*} = \lambda^{-1}_-, \tag{44}$$

where τ_i are Pauli matrices, α_i and γ are also chiral scalar superfields.

In Table 1, We list up the superfields of the Hilbert space and the left and right actions of internal automorphisms for them. The actions in the Table enable us to calculate the vector superfields in (30). In the quark sector, they are given by

$$\begin{aligned} e^{V_\chi} = e^{i\Lambda^\dagger_\chi} e^{-i\Lambda_\chi} &= e^{i\frac{\lambda_i}{2}\beta^*_i} e^{-i\frac{\lambda_i}{2}\beta_i} e^{i\frac{\tau_i}{2}\alpha^*_i} e^{-i\frac{\tau_i}{2}\alpha_i} e^{i\gamma'^* - i\gamma'} \\ &\equiv e^{-2V(SU(3))} e^{-2V(SU(2))} e^{-2V(U(1)')} \equiv e^{-2V^{(q)}}, \end{aligned} \tag{45}$$

$$\begin{aligned} e^{V^T_\xi} &= e^{i\frac{\lambda_i}{2}\beta_i} e^{-i\frac{\lambda_i}{2}\beta^*_i} \otimes \begin{pmatrix} e^{-i\gamma^*} e^{i\gamma} \\ e^{i\gamma^*} e^{-i\gamma} \end{pmatrix} e^{-i\gamma'^*} e^{i\gamma'} \\ &= e^{2V(SU(3))} \otimes \begin{pmatrix} e^{2V(U(1))} \\ e^{-2V(U(1))} \end{pmatrix} e^{2V(U(1)')} \equiv \begin{pmatrix} e^{2V^{(u_R)}} \\ e^{2V^{(d_R)}} \end{pmatrix}, \end{aligned} \tag{46}$$

Table 1 :The list of superfields and internal automorphisms for them.

particle		
quark sector	left action	right action
$X:(q^\alpha)_L$	$e^{-i\Lambda_\chi^L}=w_+$	$m_+^T=e^{-i\frac{\lambda_i^T}{2}\beta_i-i\gamma'}=(e^{-i\Lambda_\chi^R})^T$
$\Xi^{\dagger T}:u_R^\alpha$	$e^{i\Lambda_\xi^{L*}}=\lambda_-$	$m_-^T=e^{-i\frac{\lambda_i^T}{2}\beta_i^*-i\gamma'^*}=(e^{i\Lambda_\xi^{R*}})^T$
$\Xi^{\dagger T}:d_R^\alpha$	$e^{i\Lambda_\xi^{L*}}=\lambda_-^*$	m_-^T
lepton sector		
$X:l_L$	$e^{-i\Lambda_\chi^L}=w_+$	λ_+^*
$\Xi^{\dagger T}:e_R$	$e^{i\Lambda_\xi^{L*}}=\lambda_-^*$	λ_-^*
antiparticle		
quark sector		
$X^{\dagger T}:(q_\alpha^c)_R$	m_-^*	$w_-^\dagger$
$\Xi:(u_\alpha^c)_L$	m_+^*	λ_+^*
$\Xi:(d_\alpha^c)_L$	m_+^*	λ_+
lepton sector		
$X^{\dagger T}:(l_R^c)$	λ_-	$w_-^\dagger$
$\Xi:(e^c)_L$	λ_+	λ_+

where we have defined vector superfield $V(G(n))$ which corresponds to gauge group $G(n)$ and $V^{(s)}$ which corresponds to matter field s. In the lepton sector, they are expressed by

$$e^{V_\chi}=e^{i\frac{\tau_i}{2}\alpha_i^*}e^{-i\frac{\tau_i}{2}\alpha_i}e^{-i\gamma^*+i\gamma}=e^{-2V(SU(2))}e^{2V(U(1))}\equiv e^{-2V^{(l)}}, \tag{47}$$

$$e^{V_\xi}=e^{2i\gamma^*}e^{-2i\gamma}=e^{-4V(U(1))}=e^{V_\xi^T}\equiv e^{2V^{(e_R)}}. \tag{48}$$

The vector superfield $V(G(n))$ is expanded in the Wess-Zumino gauge by

$$\begin{aligned}V(G(n))&=g_n(-\theta\sigma^\mu\overline{\theta}A_\mu^{(n)}+i\theta\theta\overline{\theta\lambda^{(n)}}-i\overline{\theta\theta}\theta\lambda^{(n)}+\theta\theta\overline{\theta\theta}D^{(n)})\\&=g_n(-\theta\sigma^\mu\overline{\theta}A_\mu^{(n)i}+i\theta\theta\overline{\theta\lambda^{(n)i}}-i\overline{\theta\theta}\theta\lambda^{(n)i}+\theta\theta\overline{\theta\theta}D^{(n)i})\Lambda_n^i,\end{aligned} \tag{49}$$

where $G(n)$ is expressed by

$$G(n)=\begin{cases}SU(n) & n=2,3\\ U(1) & n=1\\ U(1)' & n=1'\end{cases}, \tag{50}$$

and g_n is the coupling constant which corresponds to $G(n)$ and Λ_n^i is given by

$$\Lambda_n^i=\begin{cases}\frac{\lambda^i}{2} & n=3\\ \frac{\tau^i}{2} & n=2\\ \frac{1}{2} & n=1,1'\end{cases}. \tag{51}$$

The relation between $V^{(s)}$ and $V(G(n))$ are similar to that of their components. For example, $V^{(q)}=V(SU(3))+V(SU(2))+V(U(1)')$ (see (45)), so that $A_\mu^{(q)}=g_3A_\mu^{(3)}+g_2A_\mu^{(2)}+g_1'\frac{1}{2}A_\mu^{(1)\prime}$, $\lambda^{(q)}=g_3\lambda^{(3)}+g_2\lambda^{(2)}+g_1'\frac{1}{2}\lambda^{(1)\prime}$, and so on.

Now,according to (29), we can calculate the kinematic terms of matter superfields which include the interaction with vector superfields. They are expressed by

$$\mathcal{L}_{matter,gauge} = -\frac{1}{4}\sum_X X^\dagger DDe^{V_\chi}X|_{\overline{\theta\theta}} - \frac{1}{4}\sum_\Xi \Xi^T\overline{DD}e^{V_\xi^T}\Xi^*|_{\overline{\theta\theta}}$$
$$= \sum_{s=q,l}\varphi_s^\dagger\mathcal{D}_\mu^{(s)}\mathcal{D}^{(s)\mu}\varphi_s - i\overline{\chi}_s\overline{\sigma}^\mu\mathcal{D}_\mu^{(s)}\chi_s + F_s^\dagger F_s - i\sqrt{2}(\varphi_s^\dagger\lambda^{(s)}\chi_s - \overline{\chi}_s\overline{\lambda^{(s)}}\varphi_s) - \varphi_s^\dagger D^{(s)}\varphi_s$$
$$+ \sum_{s=u_R,d_R,e_R}\left(\phi_s^T\mathcal{D}_\mu^{(s)}\mathcal{D}^{(s)\mu}\phi_s^* - i\xi_s\sigma^\mu\mathcal{D}_\mu^{(s)}\overline{\xi}_s + F_s^T F_s^* - i\sqrt{2}(\phi_s^T\overline{\lambda^{(s)}}\overline{\xi}_s - \xi_s\lambda^{(s)}\phi_s^*) + \phi_s^T D^{(s)}\varphi_s^*\right), \tag{52}$$

where the covariant derivative $\mathcal{D}^\mu$ is given by

$$\mathcal{D}_\mu^{(s)} = \partial_\mu - iA_\mu^{(s)}. \tag{53}$$

In (52), the action of vector superfields V_χ,V_ξ^T for pairs of a component of $\Psi^\dagger$ and a component of Ψ form a matrix which represents the internal fluctuations of $\mathcal{D}_M$. Since a scalar multiple of identity does not affect the metric, i.e. the Dirac operator, we impose that these matrices are traceless. For example, when we take the part sandwiched between the spinor ($\overline{\chi}$,ξ) in $\Psi^\dagger$ and the spinor(χ,$\overline{\xi}$) in Ψ, it is expressed by

$$\begin{pmatrix} -(ig_2A_\mu^{(2)i}\frac{\tau^i}{2} + ig_1'\frac{A_\mu^{(1)\prime}}{2}\otimes 1_2)\otimes 1_3 & 0 & 0 \\ 0 & -i(g_1'\frac{A_\mu^{(1)\prime}}{2} + g_1\frac{A_\mu^{(1)}}{2})\otimes 1_3 & 0 \\ 0 & 0 & -i(g_1'\frac{A_\mu^{(1)\prime}}{2} - g_1\frac{A_\mu^{(1)}}{2})\otimes 1_3 \end{pmatrix}$$
$$+1_4\otimes(-ig_3)A_\mu^{(3)i}\frac{\lambda^i}{2}, \tag{54}$$

in the quark sector and

$$\begin{pmatrix} -(ig_2A_\mu^{(2)i}\frac{\tau_i}{2} - ig_1\frac{A_\mu^{(1)}}{2})\otimes 1_2 & 0 \\ 0 & i2g_1\frac{A_\mu^{(1)}}{2} \end{pmatrix}, \tag{55}$$

in the lepton sector, where 1_2 and 1_3 corresponds to the color and weak isospin gauge degrees of freedom. The traceless condition for this matrix is expressed by

$$g_1'A_\mu^{(1)\prime} = \frac{1}{3}g_1A_\mu^{(1)}. \tag{56}$$

Substituting (56), the covariant derivative for each field of (53) is given by

$$\begin{aligned} \mathcal{D}_\mu^{(q)} &= \partial_\mu - i(g_3A^{(3)} + g_2A^{(2)} + \frac{1}{3}g_1\frac{1}{2}A^{(1)})_\mu, \\ \mathcal{D}_\mu^{(u_R)} &= \partial_\mu - i(g_3A^{(3)} + \frac{4}{3}g_1\frac{1}{2}A^{(1)})_\mu, \\ \mathcal{D}_\mu^{(d_R)} &= \partial_\mu - i(g_3A^{(3)} - \frac{2}{3}g_1\frac{1}{2}A^{(1)})_\mu, \end{aligned} \tag{57}$$

$$\begin{aligned} \mathcal{D}_\mu^{(l)} &= \partial_\mu - i(g_2A^{(2)} - g_1\frac{1}{2}A^{(1)})_\mu, \\ \mathcal{D}_\mu^{(e_R)} &= \partial_\mu + i2g_1\frac{1}{2}A_\mu^{(1)}. \end{aligned} \tag{58}$$

In (57) and (58), the coefficient of $ig_1\frac{1}{2}A_\mu^{(1)}$ coincides with the weak hypercharge of each particle.

When we consider the part between the scalar $(\varphi^\dagger, \phi^T)$ in $\Psi^\dagger$ and the spinor $(\chi, \bar{\xi})$ in Ψ, we obtain the condition for gauginos as

$$g_1'\lambda^{(1)\prime} = \frac{1}{3}g_1\lambda^{(1)},\ g_1'\overline{\lambda^{(1)\prime}} = \frac{1}{3}g_1\overline{\lambda^{(1)}}, \tag{59}$$

which are similar to (56). In the same way, when we take the trace of the elements between scalar fields in $\Psi^\dagger$ and Ψ, we obtain

$$g_1'D^{(1)\prime} = \frac{1}{3}g_1 D^{(1)}. \tag{60}$$

Taking into account (56),(59) and (60), we obtain that

$$V(U(1)') = \frac{1}{3}V(U(1)). \tag{61}$$

3.2 Gauge kinetic term and coupling constants

The supersymmetric gauge kinematic term (31) is expressed in terms of vector superfields V_s by

$$\begin{aligned}\mathcal{L}_{gauge} =& f_0\sum_s Tr\left\{-\frac{1}{4}F_{\mu\nu}^{(s)}F^{(s)\mu\nu} + \frac{1}{2}D^{(s)2} - \frac{i}{4}\lambda^{(s)\alpha}\sigma^\mu_{\alpha\dot{\alpha}}D_\mu^{(s)}\overline{\lambda^{(s)}}^{\dot{\alpha}} + \frac{i}{4}D_\mu^{(s)}\overline{\lambda^{(s)}}^{\dot{\alpha}}\cdot\lambda^{(s)\alpha}\sigma^\mu_{\alpha\dot{\alpha}}\right.\\ &\left.+\frac{i}{4}D_\mu^{(s)}\lambda^{(s)\alpha}\sigma^\mu_{\alpha\dot{\alpha}}\overline{\lambda^{(s)}}^{\dot{\alpha}} - \frac{i}{4}\overline{\lambda^{(s)}}^{\dot{\alpha}}D_\mu^{(s)}\lambda^{(s)\alpha}\sigma^\mu_{\alpha\dot{\alpha}}\right\},\end{aligned} \tag{62}$$

where s runs over matter scalar superfields, but not over antimatter fields, because (62) already have taken into account the contribution from antimatter sector.

Since $A_\mu^{(q)} = g_3A_\mu^{(3)} + g_2A_\mu^{(2)} + g_1'\frac{1}{2}A_\mu^{(1)\prime}$, the contribution of $F_{\mu\nu}^{(q)}$ to $\mathcal{L}_{gauge}$ is given by

$$\begin{aligned}TrF_{\mu\nu}^{(q)}F^{(q)\mu\nu} =& \sum_{n=1',2,3}F_{\mu\nu}^{(n)i}F^{(n)\mu\nu j}Tr(\Lambda_i\Lambda_j)\\ =& 2\times\frac{1}{2}g_3^2F_{\mu\nu}^{(3)i}F^{(3)\mu\nu i} + 3\times\frac{1}{2}g_2^2F_{\mu\nu}^{(2)j}F^{(2)\mu\mu j} + 6\times\frac{1}{4}\left(\frac{1}{3}\right)^2 g_1^2F_{\mu\nu}^{(1)}F^{(1)\mu\nu},\end{aligned} \tag{63}$$

where $F_{\mu\nu}^{(n)}$ is the field strength of the gauge field $A_\mu^{(n)}$ expressed by

$$F_{\mu\nu}^{(n)} = \partial_\mu A_\nu^{(n)} - \partial_\nu A_\mu^{(n)} - ig_n[A_\mu^{(n)}, A_\nu^{(n)}] \tag{64}$$

In the r.h.s of (63), the factors $2, 3, 6$ are from the fact that the left-handed quark q transforms as (3,2) under the gauge group $SU(3)\times SU(2)$, $\frac{1}{2}, \frac{1}{4}$ are from $Tr(\Lambda_n^i\Lambda_n^j)$, $\frac{1}{3}$ is the weak hypercharge. In the same way, we obtain that

$$\sum_{s=u_R,d_R}Tr(F_{\mu\nu}^{(s)}F^{(s)\mu\nu}) = 2\frac{1}{2}g_3^2F_{\mu\nu}^{(3)i}F^{(3)\mu\nu i} + 3\times\frac{1}{4}\left(\left(\frac{4}{3}\right)^2 + \left(-\frac{2}{3}\right)^2\right)g_1^2F_{\mu\nu}^{(1)}F_{\mu\nu}^{(1)}, \tag{65}$$

$$\sum_{s=l,e_R}Tr(F_{\mu\nu}^{(s)}F^{(s)\mu\nu}) = \frac{1}{2}g_2^2F_{\mu\nu}^{(2)i}F^{(2)\mu\nu i} + \frac{1}{4}(2\times(-1)^2 + 1\times(-2)^2)g_1^2F_{\mu\nu}^{(1)}F^{(1)\mu\nu}. \tag{66}$$

The sum of (63),(65) and (66) amounts to

$$-\frac{1}{4}f_0\sum_s Tr(F^{(s)}_{\mu\nu}F^{(s)\mu\nu}) = -\frac{2f_0}{4}(g_3^2F^{(3)i}_{\mu\nu}F^{(3)\mu\nu i} + g_2^2F^{(2)i}_{\mu\nu}F^{(2)\mu\nu i} + \frac{5}{3}g_1^2F^{(1)}_{\mu\nu}F^{(1)\mu\nu}). \quad (67)$$

Normalizing the Yang-Mills terms to $-\frac{1}{4}F^{(n)i}_{\mu\nu}F^{(n)\mu\nu i}$ gives

$$g_3^2 = g_2^2 = \frac{5}{3}g_1^2, \quad 2f_0g_3^2 = 1 \quad (68)$$

We have also calculated the other terms of (62) and obtained the whole of supersymmetric gauge kinematic terms which is expressed by

$$\mathcal{L}_{gauge} = \sum_{n=1}^{3}\left\{-\frac{1}{4}F^{(n)i}_{\mu\nu}F^{(n)\mu\nu i} - \frac{i}{2}(\lambda^{(n)i}\sigma^\mu\mathcal{D}^{(n)}_\mu\overline{\lambda^{(n)i}} - \mathcal{D}^{(n)}_\mu\lambda^{(n)i}\sigma^\mu\overline{\lambda^{(n)i}}) + \frac{1}{2}D^{(n)i}D^{(n)i}\right\}, \quad (69)$$

where $\mathcal{D}^{(n)}_\mu$ is the covariant derivative given by

$$\mathcal{D}^{(n)}_\mu F = \partial_\mu F - i[A^{(n)}_\mu, F]. \quad (70)$$

3.3 Higgs-matter interaction

The Dirac operator $\mathcal{D}_F$ which acts on the finite space $\mathcal{H}_F$ is given by

$$\mathcal{D}_F = \begin{pmatrix} 0 & 0 & m_u^* & 0 \\ 0 & 0 & 0 & m_d^* \\ m_u^T & 0 & 0 & 0 \\ 0 & m_d^T & 0 & 0 \end{pmatrix}, \quad (71)$$

in the quark sector and

$$\mathcal{D}_F = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & m_e^* \\ 0 & m_e^T & 0 \end{pmatrix}, \quad (72)$$

in the lepton sector. Using left and right actions of the Table 1, the superfield which corresponds to (34) is expressed as follows: in the quark sector,

$$\begin{aligned}\tilde{H} &= e^{-i\Lambda_\chi^T}me^{-i\Lambda_\xi} \\ &= e^{-i\frac{\tau_i^T}{2}\alpha^i - i\frac{\lambda_i^T}{2}\beta_i}e^{-i\frac{\gamma}{3}}\begin{pmatrix} m_u & 0 \\ 0 & m_d\end{pmatrix}e^{i\frac{\lambda_i^T}{2}\beta_i}\begin{pmatrix} e^{i\frac{4}{3}\gamma} & \\ & e^{-i\frac{2}{3}\gamma}\end{pmatrix} \\ &= i\tau_2(H_u, H_d), \end{aligned} \quad (73)$$

where m_u/m_d is u/d quark mass and H_u,H_d are given by

$$H_u = e^{i\frac{\tau_i}{2}\alpha^i}\begin{pmatrix} 0 \\ m_u \end{pmatrix}e^{i\gamma}, \quad (74)$$

$$H_d = e^{i\frac{\tau_i}{2}\alpha^i}\begin{pmatrix} -m_d \\ 0 \end{pmatrix}e^{-i\gamma}. \quad (75)$$

In the lepton sector, in the same way, m_e is electron mass and

$$\tilde{H}_e = i\tau_2 H_e, \tag{76}$$

$$\begin{aligned} H_e &= e^{i\frac{\tau_i}{2}\alpha_i}\begin{pmatrix}-m_e\\0\end{pmatrix}e^{-i\gamma} \\ &= \mathbf{y_e}H_d, \end{aligned} \tag{77}$$

where $\mathbf{y_e}$ is a parameter which represents the ratio m_e to m_d.

The interaction (35) is expanded as follows:

$$\mathcal{L}_{matter,Higgs} = \mathbf{y_u}\Xi_u^T\tilde{H}_u^T X_u + \mathbf{y_d}\Xi_d^T\tilde{H}_d^T X_d + \mathbf{y_e}\Xi_e^T\tilde{H}_d^T X_e + h.c., \tag{78}$$

where $\mathbf{y_u}$ / $\mathbf{y_d}$ is the ratio of the mass of u-type / d-type quark of the generation to m_u/m_d, respectively.

Substituting (45),(46),(47),(48) along with (61),(77) to (36) , the whole of super-Higgs kinematic term is expressed by

$$\begin{aligned} &\mathcal{L}_{Higgs,gauge} = Tr_{quark}(e^{-V_\xi^T}\tilde{H}^T e^{-V_\chi}\tilde{H}^{\dagger T}) + Tr_{lepton}(e^{-V_\xi^T}\tilde{H}^T e^{-V_\chi}\tilde{H}^{\dagger T}) \\ =&Tr\left(\begin{pmatrix} e^{i\frac{4}{3}\gamma^*-i\frac{4}{3}\gamma}\mathbf{y_u}\tilde{H}_u^T \\ e^{-i\frac{2}{3}\gamma^*+i\frac{2}{3}\gamma}\mathbf{y_d}\tilde{H}_d^T\end{pmatrix}\tau_2 e^{i\frac{\tau_i}{2}\alpha_i}e^{-i\frac{\tau_i}{2}\alpha_i^*}e^{\frac{i}{3}(-\gamma^*+\gamma)}\tau_2(\mathbf{y_u}\tilde{H}_u^*, \mathbf{y_d}\tilde{H}_d^*)\right) \\ &+ e^{2i\gamma-2i\gamma^*}\mathbf{y_e}\tilde{H}_e^T\tau_2 e^{i\frac{\tau_i}{2}\alpha_i}e^{-i\frac{\tau_i}{2}\alpha^*}e^{i\gamma^*-i\gamma}\tau_2\mathbf{y_e}\tilde{H}_e^* \\ =&\mathbf{y_u}^2 H_u^\dagger e^{-2V(SU(2))}e^{-2V(U(1))}H_u + (\mathbf{y_d}^2+\mathbf{y_e}^2)H_d^\dagger e^{-2V(SU(2))}e^{2V(U(1))}H_d. \end{aligned} \tag{79}$$

When we expand the chiral Higgs superfield to

$$H_s = \varphi_{h_s} + i\theta\sigma^\mu\bar{\theta}\partial_\mu\varphi_{h_s} + \frac{1}{4}\theta\theta\bar{\theta}\bar{\theta}\Box\varphi_{h_s} + \sqrt{2}\theta\psi_{h_s} - \frac{i}{\sqrt{2}}\theta\theta\partial_\mu\psi_{h_s}\sigma^\mu\bar{\theta} + \theta\theta F_{h_s}, \tag{80}$$

and define vector superfield $V^{(h_u)}$, $V^{(h_d)}$ as

$$V^{(h_u)} = V(SU(2)) + V(U(1)), \quad V^{(h_d)} = V(SU(2)) - V(U(1)), \tag{81}$$

(79) is written down to

$$\begin{aligned} &\mathcal{L}_{Higgs,gauge} \\ =&\mathbf{y_u}^2\Big(-|\mathcal{D}_\mu^{(h_u)}\varphi_{h_u}|^2 - \varphi_{h_u}^\dagger D^{(h_u)}\varphi_{h_u} + F_{h_u}^\dagger F_{h_u} \\ &-i\psi_{h_u}\sigma^\mu\mathcal{D}_\mu^{(h_u)}\overline{\psi_{h_u}} + \sqrt{2}i(\overline{\psi_{h_u}\lambda^{(h_u)}}\varphi_{h_u} - \varphi_{h_u}^\dagger\lambda^{(h_u)}\psi_{h_u})\Big) \\ &+ (\mathbf{y_d}^2+\mathbf{y_e}^2)\Big(-|\mathcal{D}_\mu^{(h_d)}\varphi_{h_d}|^2 - \varphi_{h_d}^\dagger D^{(h_d)}\varphi_{h_d} + F_{h_d}^\dagger F_{h_d} \\ &-i\psi_{h_d}\sigma^\mu\mathcal{D}_\mu^{(h_d)}\overline{\psi_{h_d}} + \sqrt{2}i(\overline{\psi_{h_d}\lambda^{(h_d)}}\varphi_{h_d} - \varphi_{h_d}^\dagger\lambda^{(h_d)}\psi_{h_d})\Big), \end{aligned} \tag{82}$$

where the covariant derivative $\mathcal{D}^{(h_u)}$,$\mathcal{D}^{(h_d)}$ are given by gauge bosons of the vector field $V^{(h_u)}$, $V^{(h_d)}$, respectively,

$$\mathcal{D}_\mu^{(h_u)} = \partial_\mu - i(g_2 A_\mu^{(2)} + g_1\frac{1}{2}A_\mu^{(1)}), \tag{83}$$

$$\mathcal{D}_\mu^{(h_d)} = \partial_\mu - i(g_2 A_\mu^{(2)} - g_1\frac{1}{2}A_\mu^{(1)}). \tag{84}$$

At last, we have obtained the all terms of supersymmetric standard model in this section without soft-breaking terms.

4 conclusion and problems

We proposed a NC geometry-like construction of MSSM. We define the supersymmetric spectral triple $(\mathcal{H}, \mathcal{D}, \mathcal{A})$. The space $\mathcal{H}$ and the algebra $\mathcal{A}$ consist of chiral and antichiral subspaces, respectively. In addition we defined the real structure which commutes with the Dirac operator $\mathcal{D}$. The Dirac operator and fluctuations in Table 1 give the kinematic terms of matter superfield which interact with super-gauge field as well as super-Higgs field. The squared Dirac operator gives gauge kinematic terms and Higgs kinematic terms which interact with the super-gauge field. We impose traceless condition on the fluctuation of $\mathcal{D}_M$,i.e.,vector superfields and we obtained the correct weak hyperchares of matter fields. Normalizing the Yang Mills terms, we obtained the relations between coupling constants of the gauge symmetries identical to those of $SU(5)$ grand-unified theory. Altogether, we got the all exactly supersymmetric terms of MSSM.

So far, we seem to be satisfied physically with the situation. Regrettably, this model has some mathematically unsettled problems. At first, we must discover a Z_2 grading operator γ which obeys the condition (3). We need the condition to constrain the gauge group of the NCG model to $SU(3) \times SU(2) \times U(1)$. Secondly, this model is described by Grassmann variables and constructed on Minkowskian space so that the Dirac operator dose not have compact resolvent. When we denote the resolvent of $\mathcal{D}$ as T, $T = (\mathcal{D} - \lambda)^{-1}, \lambda \notin \mathrm{Spectrum}(\mathcal{D})$, T must be compact,i.e., the norm of T must be infinitesimal up to a finite dimensional subspace of $\mathcal{H}$. Already on a non-super Minkowskian space, the Dirac operator $(i\gamma^\mu \partial_\mu - m)$ is not elliptic and $ds = \mathcal{D}^{-1}$ is not always bounded. In addition to that, this model is based on superspace which includes Grassmann variables so that the Dirac operator is far from positive definite and the space $\mathcal{H}$ cannot be called "Hilbert" space, that is, inner product is not well-defined in $\mathcal{H}$. All calculations of non-supersymmetric NCG theories has always been developed on Euclidean space and the results are analytically-continued to Minkowskian form by Wick rotation. To overcome the above problems, we must rewrite our spectral triple from on the Minkowskian superspace to on the Euclidean space without Grassmann variables[9].

Finally, since there exist no squark, no slepton, no higgsino in our world, whether the theory is NC geometric or not, supersymmetry must be broken in low energy scale. We also have to introduce soft breaking terms in the NC geometric framework[2].

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